Estimating PATT with noncompliance

April 2, 2015

Let Y_{ist} be the potential outcome for individual i in group s, where $s = 0$ for the population and $s = 1$ for the randomized control trial, and t be the treatment assigned. Let W_i^T and W_i^C denote individual i's observable covariates related to the sample selection mechanism for membership in the RCT under treatment and control assignment, respectively. For a generic value, we drop the subscript i.

We make the following assumptions:

- Consistency under parallel studies: for all i and for $t = 0, 1$,

$$Y_{i0t} = Y_{i1t} \tag{1}$$

- Strong ignorability of sample assignment for treated:

$$(Y_{01}, Y_{11}) \perp\!\!\!\perp S \mid (W^T, T = 1), 0 < P(S = 1 \mid W^T, T = 1) < 1 \tag{2}$$

- Strong ignorability of sample assignment for controls:

$$(Y_{00}, Y_{10}) \perp\!\!\!\perp S \mid (W^C, T = 1), 0 < P(S = 1 \mid W^C, T = 1) < 1 \tag{3}$$

- Stable unit treatment value assumption (SUTVA):

$$Y_{ist}^{L_i} = Y_{ist}^{L_j}, \forall i \neq j \tag{4}$$

  where L_j is the treatment and sample assignment vector for unit j. This means that the treatment assignment for all other individuals j does not affect the potential outcomes of individual i.

- Conditional independence of compliance and assignment:

$$C \perp\!\!\!\perp (S = 1, T = 1) \mid (W^T, W^C), 0 < P(C = 1 \mid (W^T, W^C)) < 1 \tag{5}$$

The estimand of interest is

$$\tau_{\text{PATT}} = E\left(Y_{01} - Y_{00} \mid S = 0, T = 1\right) \tag{6}$$

Let C be an indicator for compliers. Assume that there are no defiers, and crossover is only possible from treatment to control. For non-compliers, $Y_{11} = Y_{10}$. In the population $S = 0$, it doesn't make sense to talk about compliance, as treatment isn't assigned at random.

$$\begin{aligned}
E\left(Y_{01} \mid S = 0, T = 1\right) &= E\left(Y_{11} \mid S = 0, T = 1\right) && \text{by (1)} \\
&= E_{01}\left[E\left(Y_{11} \mid S = 1, T = 1, W^T\right)\right] && \text{by (2)} \\
&= E_{01}\left[E_C\left[E\left(Y_{11} \mid S = 1, T = 1, W^T, C\right)\right]\right] \\
&= E_{01}\left[P(C = 1 \mid W^T)E\left(Y_{11} \mid S = 1, T = 1, W^T, C = 1\right)\right. \\
&\quad \left. + (1 - P(C = 1 \mid W^T))E\left(Y_{10} \mid S = 1, T = 1, W^T, C = 0\right)\right] && \text{by (5)} \\
&= E_{01}\left[P(C = 1 \mid W^T)E\left(Y_{11} \mid S = 1, T = 1, W^T, C = 1\right)\right. \\
&\quad \left. + (1 - P(C = 1 \mid W^T))E\left(Y_{11} \mid S = 1, T = 1, W^T, C = 0\right)\right] && \text{by non-compliance}
\end{aligned}$$

$$
\begin{aligned}
E\left(Y_{00} \mid S=0, T=1\right) &= E\left(Y_{10} \mid S=0, T=1\right) && \text{by (1)} \\
&= E_{01}\left[E\left(Y_{10} \mid S=1, T=1, W^C\right)\right] && \text{by (3)} \\
&= E_{01}\left[E\left(Y_{10} \mid S=1, T=0, W^C\right)\right] && \text{by randomization, i.e. } Y_{10} \perp\!\!\!\perp T \mid (W^C, S=1) \\
&= E_{01}\left[E_C\left[E\left(Y_{10} \mid S=1, T=0, W^C, C\right)\right]\right] \\
&= E_{01}\left[P(C=1 \mid W^C) E\left(Y_{10} \mid S=1, T=0, W^C, C=1\right)\right. \\
&\qquad \left.+(1-P(C=1 \mid W^C)) E\left(Y_{10} \mid S=1, T=0, W^C, C=0\right)\right] && \text{by (5)}
\end{aligned}
$$

If we additionally assume that

$$
E\left(Y_{10} \mid S=1, T=1, W^T, C=0\right) = E\left(Y_{10} \mid S=1, T=0, W^C, C=0\right)
$$

then the terms corresponding to the non-compliers cancel and

$$
\tau_{\text{PATT}} = E_{01}\left[P(C=1|W^T) E\left(Y_{11} \mid S=1, T=1, W^T, C=1\right)\right] - E_{01}\left[P(C=1|W^C) E\left(Y_{10} \mid S=1, T=0, W^C, C=1\right)\right] \tag{7}
$$

There are two issues:

- We must estimate the conditional probability of being a complier given a set of covariates. This can be done using a standard logistic or probit regression, or can be done using a nonparametric method.
- We cannot observe who among the RCT controls is a complier or a "never-treat". However, the second term in (7) involves an expectation over the compliers assigned to control in the RCT. We will use the model for compliers that we fit previously to predict who among these controls is a complier, given their observed characteristics.