Estimating PATT with noncompliance

April 1, 2015

Let Y_{ist} be the potential outcome for individual i in group s, where $s = 0$ for the population and $s = 1$ for the randomized control trial, and t be the treatment assigned. Let W_i^T and W_i^C denote individual i's observable covariates related to the sample selection mechanism for membership in the RCT under treatment and control assignment, respectively. For a generic value, we drop the subscript i.

We make the following assumptions:

- Consistency under parallel studies: for all i and for $t = 0, 1$,

$$Y_{i0t} = Y_{i1t} \tag{1}$$

- Strong ignorability of sample assignment for treated:

$$(Y_{01}, Y_{11}) \perp\!\!\!\perp S \mid (W^T, T = 1), 0 < P(S = 1 \mid W^T, T = 1) < 1 \tag{2}$$

- Strong ignorability of sample assignment for controls:

$$(Y_{00}, Y_{10}) \perp\!\!\!\perp S \mid (W^C, T = 1), 0 < P(S = 1 \mid W^C, T = 1) < 1 \tag{3}$$

- Stable unit treatment value assumption (SUTVA):

$$Y_{ist}^{L_i} = Y_{ist}^{L_j}, \forall i \neq j \tag{4}$$

  where L_j is the treatment and sample assignment vector for unit j. This means that the treatment assignment for all other individuals j does not affect the potential outcomes of individual i.

The estimand of interest is

$$\tau_{\text{PATT}} = E\left(Y_{01} - Y_{00} \mid S = 0, T = 1\right) \tag{5}$$

Let p be the probability that an individual is a complier and $1 - p$ be the probability of a never-treat subject. Assume that there are no defiers, and crossover is only possible from treatment to control. For non-compliers, $Y_{11} = Y_{10}$. In the population $S = 0$, it doesn't make sense to talk about compliance, as treatment isn't assigned at random.

$$\begin{aligned}
E\left(Y_{01} \mid S = 0, T = 1\right) &= E\left(Y_{11} \mid S = 0, T = 1\right) && \text{by (1)} \\
&= E_{01}\left[E\left(Y_{11} \mid S = 1, T = 1, W^T\right)\right] && \text{by (2)} \\
&= E_{01}\left[pE\left(Y_{11} \mid S = 1, T = 1, W^T, \text{complier}\right)\right. \\
&\qquad \left. + (1-p)E\left(Y_{11} \mid S = 1, T = 1, W^T, \text{non-complier}\right)\right] \\
&= E_{01}\left[pE\left(Y_{11} \mid S = 1, T = 1, W^T, \text{complier}\right)\right. \\
&\qquad \left. + (1-p)E\left(Y_{10} \mid S = 1, T = 1, W^T, \text{non-complier}\right)\right] && \text{by non-compliance}
\end{aligned}$$

$$
\begin{aligned}
E\left(Y_{00} \mid S=0, T=1\right) &= E\left(Y_{10} \mid S=0, T=1\right) && \text{by (1)} \\
&= E_{01}\left[E\left(Y_{10} \mid S=1, T=1, W^C\right)\right] && \text{by (3)} \\
&= E_{01}\left[E\left(Y_{10} \mid S=1, T=0, W^C\right)\right] && \text{by randomization, i.e. } Y_{10} \perp\!\!\!\perp T \mid (W^C, S=1) \\
&= E_{01}\left[pE\left(Y_{10} \mid S=1, T=0, W^C, \text{complier}\right) + (1-p)E\left(Y_{10} \mid S=1, T=0, W^C, \text{non-complier}\right)\right] \\
&= E_{01}\left[pE\left(Y_{10} \mid S=1, T=0, W^C, \text{complier}\right) + (1-p)E\left(Y_{10} \mid S=1, T=0, W^C, \text{non-complier}\right)\right]
\end{aligned}
$$

If we additionally assume that

$$
E\left(Y_{10} \mid S=1, T=1, W^T, \text{non-complier}\right) = E\left(Y_{10} \mid S=1, T=0, W^C, \text{non-complier}\right)
$$

then the terms corresponding to the non-compliers cancel and

$$
\tau_{\text{PATT}} = pE_{01}\left[E\left(Y_{11} \mid S=1, T=1, W^T, \text{complier}\right)\right] - pE_{01}\left[E\left(Y_{10} \mid S=1, T=0, W^C, \text{complier}\right)\right] \tag{6}
$$