Estimating spatial processes in a simulation context with TMB.

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Introduction

Template Model Builder (TMB) offers a framework to estimate random effects. Here, we are concerned with the ability of TMB to estimate spatially explicit random effects through the use of sparse covariance matrices, or Gaussian Markov Random Fields (GMRFs).

We perform a simulation experiment where spatially referenced count data were sampled from a true population generated using the `RandomFields` package in **R** and fit to a spatially-explicit Gompertz population dynamics model in TMB. Specific parameterizations of the operating model (OM; generator of the truth) and estimation method (EM) may or may not affect the the ability of TMB to estimate parameters of importance without bias.

Methods

The true population was governed by a spatially-explicit Gompertz population dynamics model,

$$log(\boldsymbol{n}_t) = \boldsymbol{\omega} + \rho * log(\boldsymbol{n}_{t-1}) + \boldsymbol{\epsilon}_t,$$

where $\boldsymbol{n}_t$ a vector representing the density of the population at each sampled location in time t, $\boldsymbol{\omega}$ is a vector of spatially-explicit deviates in productivity from the mean productivity, α, ρ represents the degree of density dependence, and $\boldsymbol{\epsilon}_t$ is a vector of process error in year t. The initial conditions (i.e., $t = 1$) were specified as,

$$log(\boldsymbol{n}_1) = \frac{\boldsymbol{\omega}}{1-\rho} + \phi\mathbf{1} + \boldsymbol{\epsilon}_1,$$

where $\phi\mathbf{1}$ is the log-ratio of expected abundance in the initial year and the median of the stationary distribution at equilibrium. Stochastic processes were assumed to follow multivariate normal distributions, $\boldsymbol{\omega} \sim MVN(\alpha\mathbf{1}, \sigma^2_\omega\boldsymbol{R})$ and $\boldsymbol{\epsilon_t} \sim MVN(0, \sigma^2_\epsilon\boldsymbol{R})$. Marginal standard deviations were assumed to be independent, but the Matérn spatial correlation function, $\boldsymbol{R}$, that approximates the stochastic process as a GRF was assumed to be the same for productivity and process error.

Each simulation included 10 years of data from 100 unique sampling locations, where all sampling locations were sampled on a yearly basis, repeated 100 times. The capabilities of the EM was assessed using relative error (RE) and median absolute relative error (MARE). RE was calculated using $\frac{(true-estimate)}{true}$ and $\frac{(e^{true}-e^{estimate})}{e^{true}}$ for parameters where the true value was non-zero and zero, respectively. The latter allows for non-infinite RE values.

Sensitivity analyses

Two sensitivity analyses were run, one where α was fixed at its true value and the one where ρ was fixed at its true value. These parameters were chosen because initial exploration indicated that they exhibited the highest correlation with the remaining model parameters, mainly with the marginal standard deviation of $\boldsymbol{\omega}$, σ_ω.

Results

In general, the EM can estimate the marginal variation in process error, the scale of the spatial effects, and ϕ, but not the remaining parameters (Figures 1 and 2). The Poisson and Poisson lognormal ('Poissonln') objective functions performed similarly (Figure 1), namely because the model was able to estimate both of the additional parameters governing the Poissonln function (Figure 3). Additionally, there was no appreciable difference between the two available functions in the `RandomFields` package used to simulate MVN processes (i.e., `RMgauss` and `RMgneiting`; where the latter is supposed to be less biased).

Parameter estimates improved when either α (Figure 4) or ρ (Figure 5) were fixed at their true values.

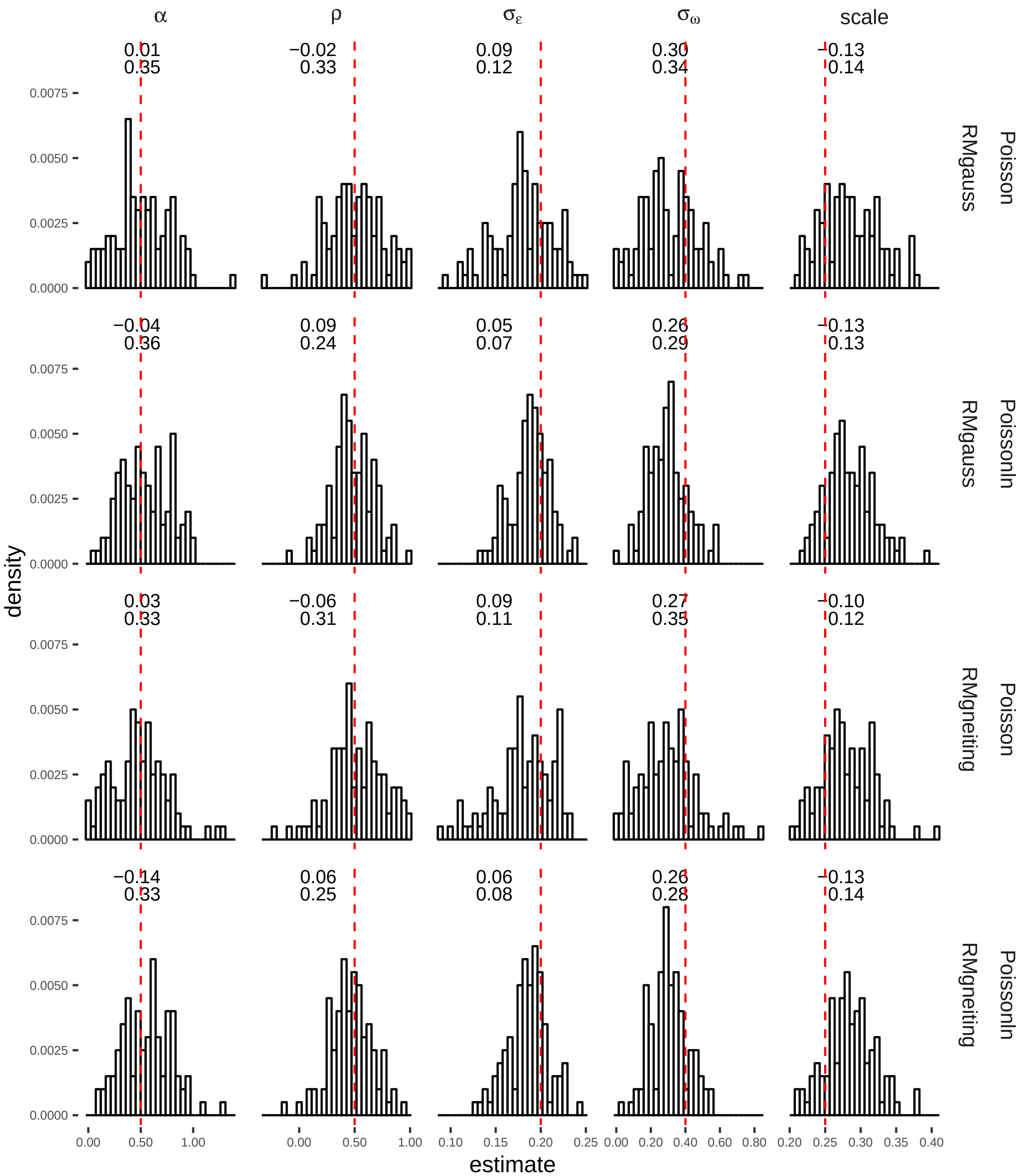
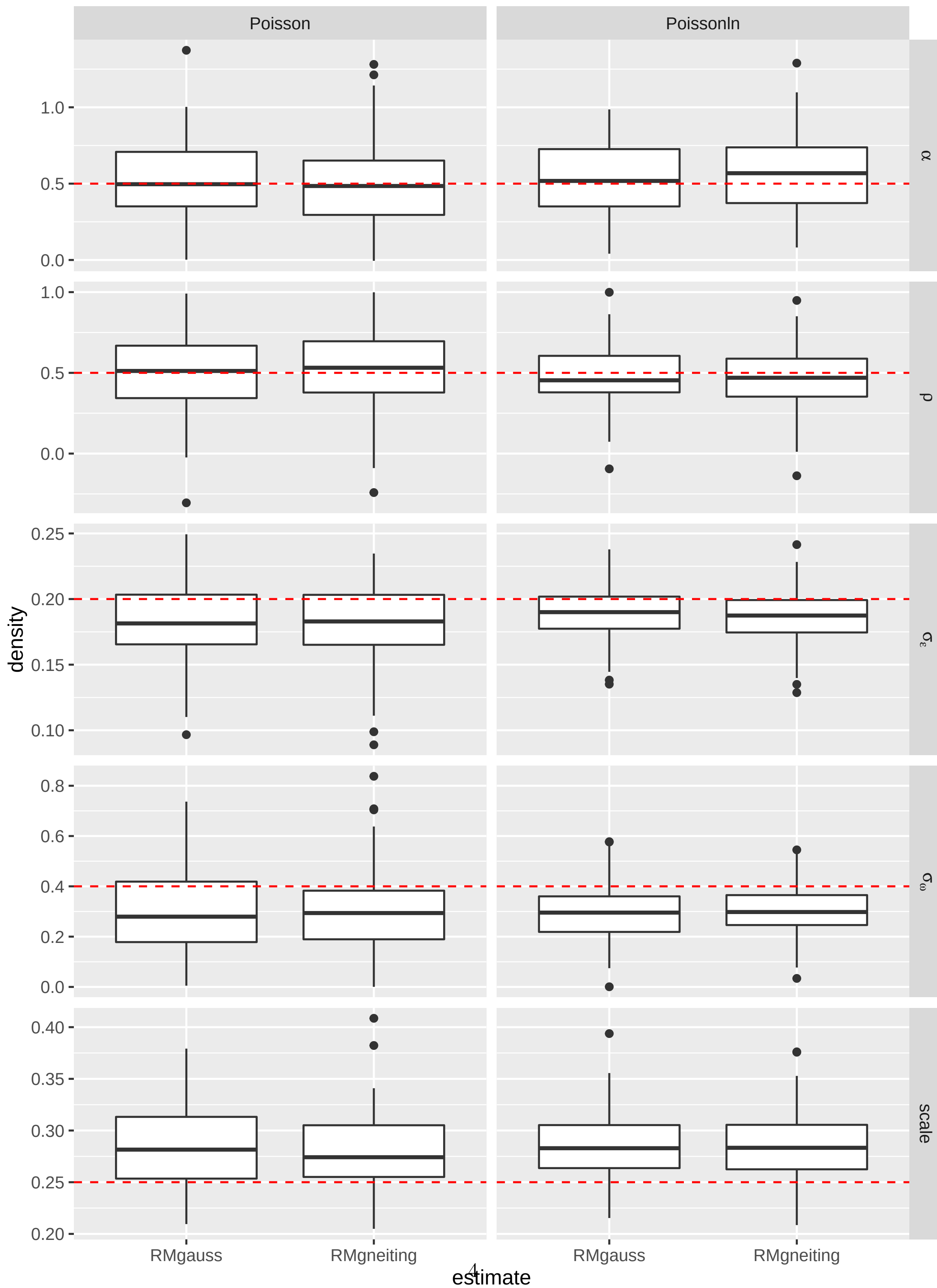


Figure 1: Parameter estimates and their median relative error and median absolute relative error. Red dashed line indicates the true parameter value.

Figures

Sample size of 100 per year



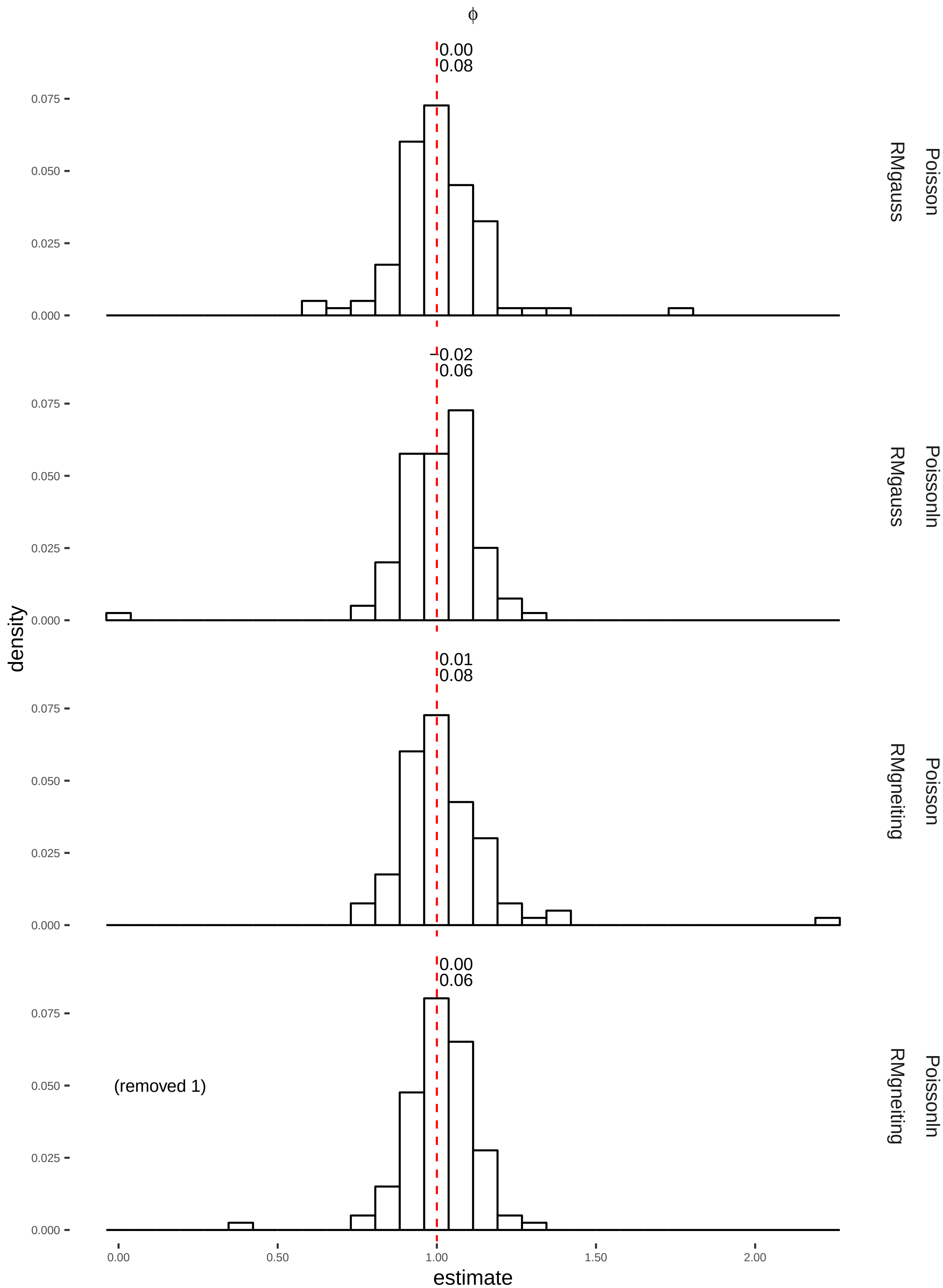
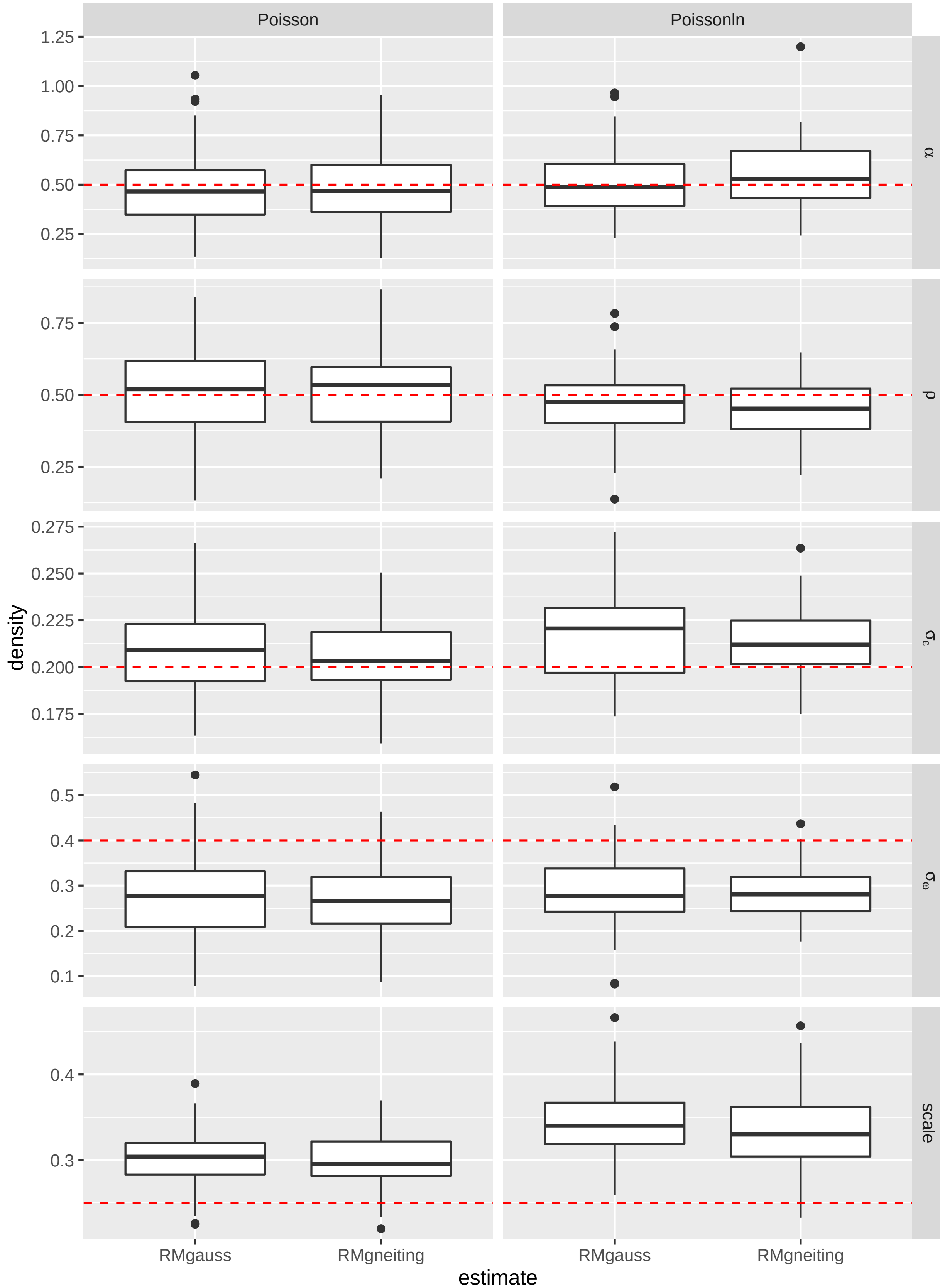


Figure 2: Estimates of ϕ and its median relative error (RE) and median absolute RE. Red dashed line indicates the true parameter value. RE was calculated using exponentiated values to circumvent RE values of infinity.

Sample size of 500 per year



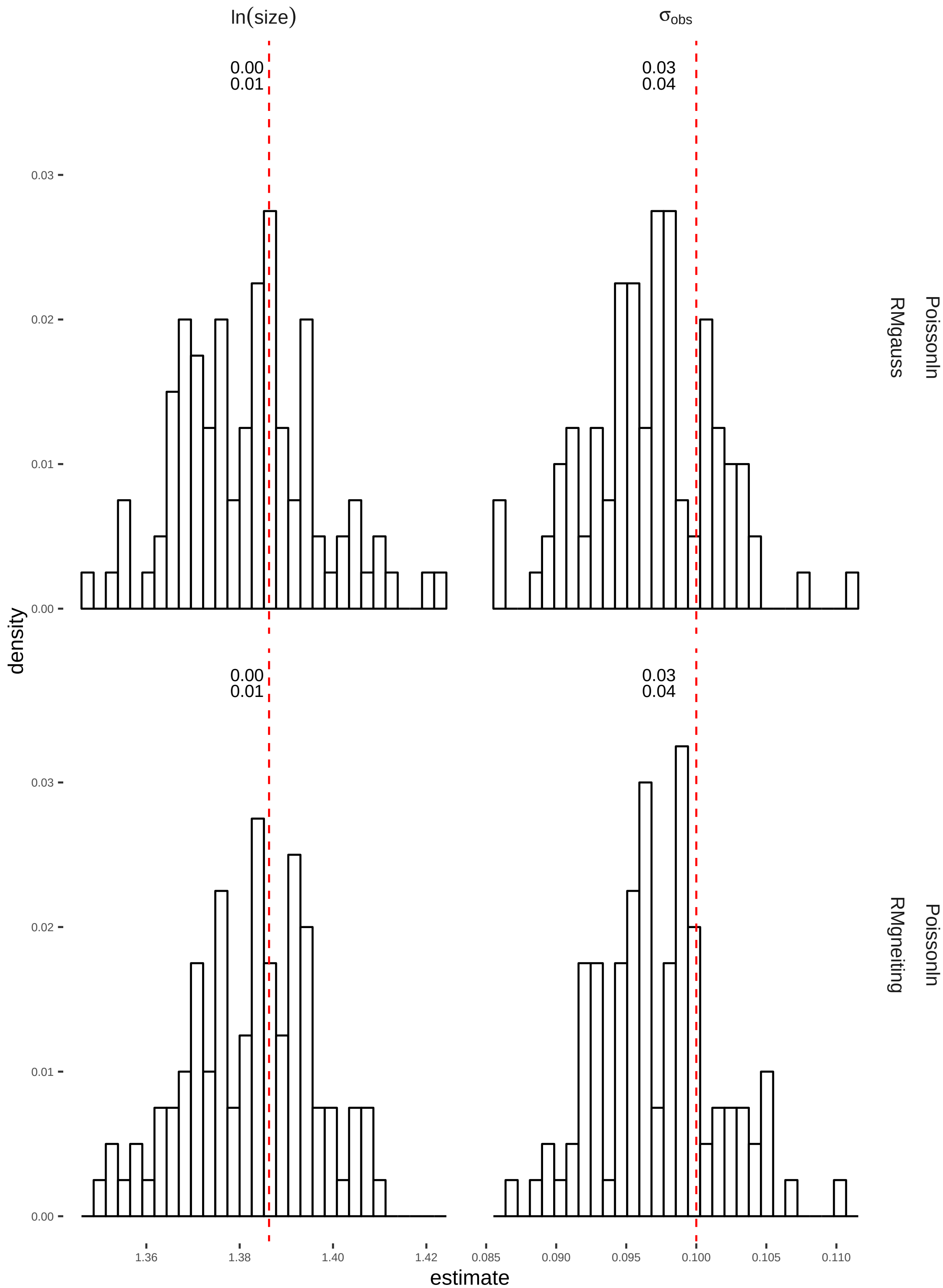


Figure 3: Parameter estimates and their median relative error and median absolute relative error for the estimation method that assumed a Poisson lognormal response variable. Red dashed line indicates the true parameter value.

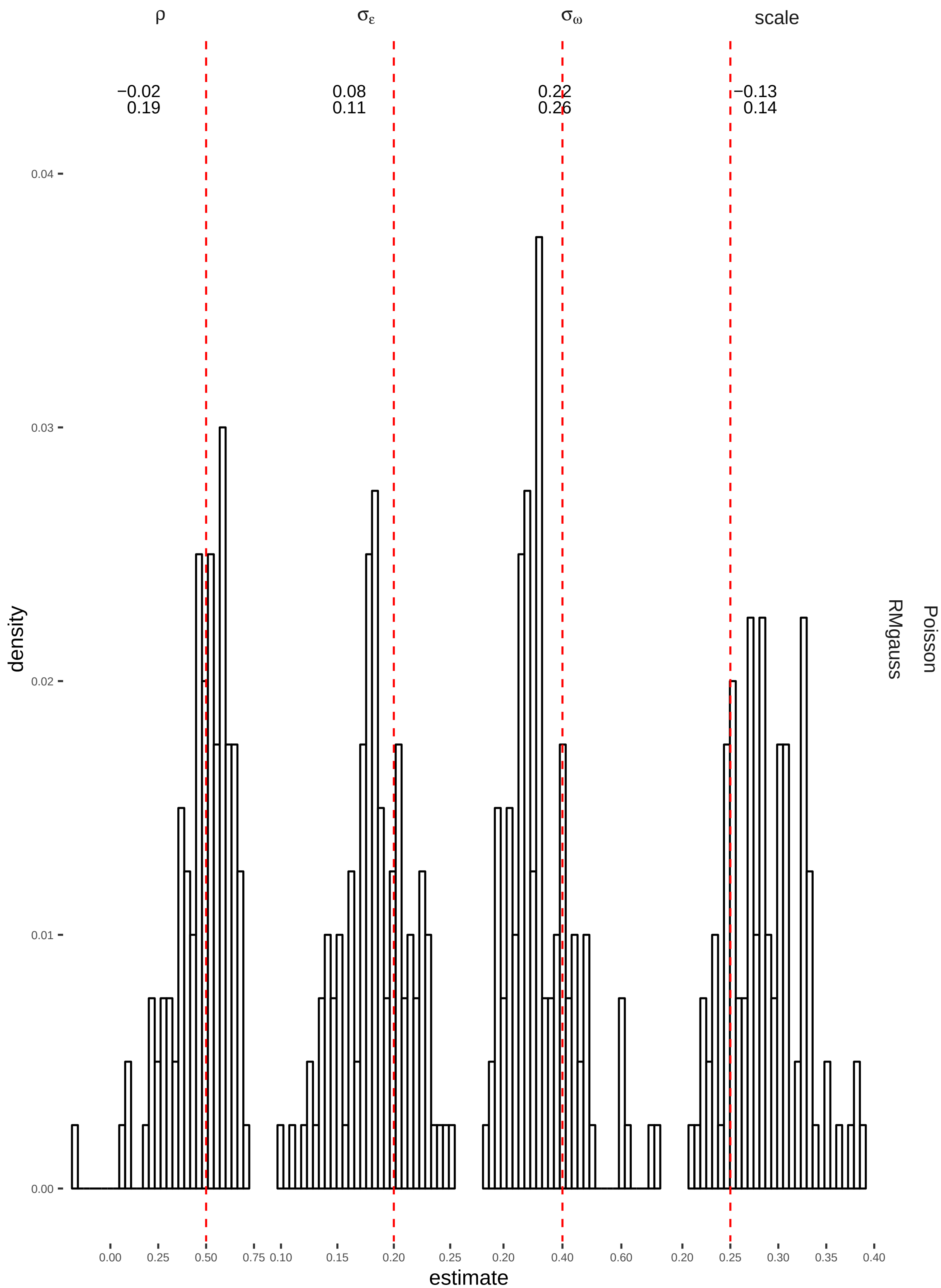


Figure 4: Parameter estimates and their median absolute relative error when α was fixed at its true value. Red dashed line indicates the true parameter value.

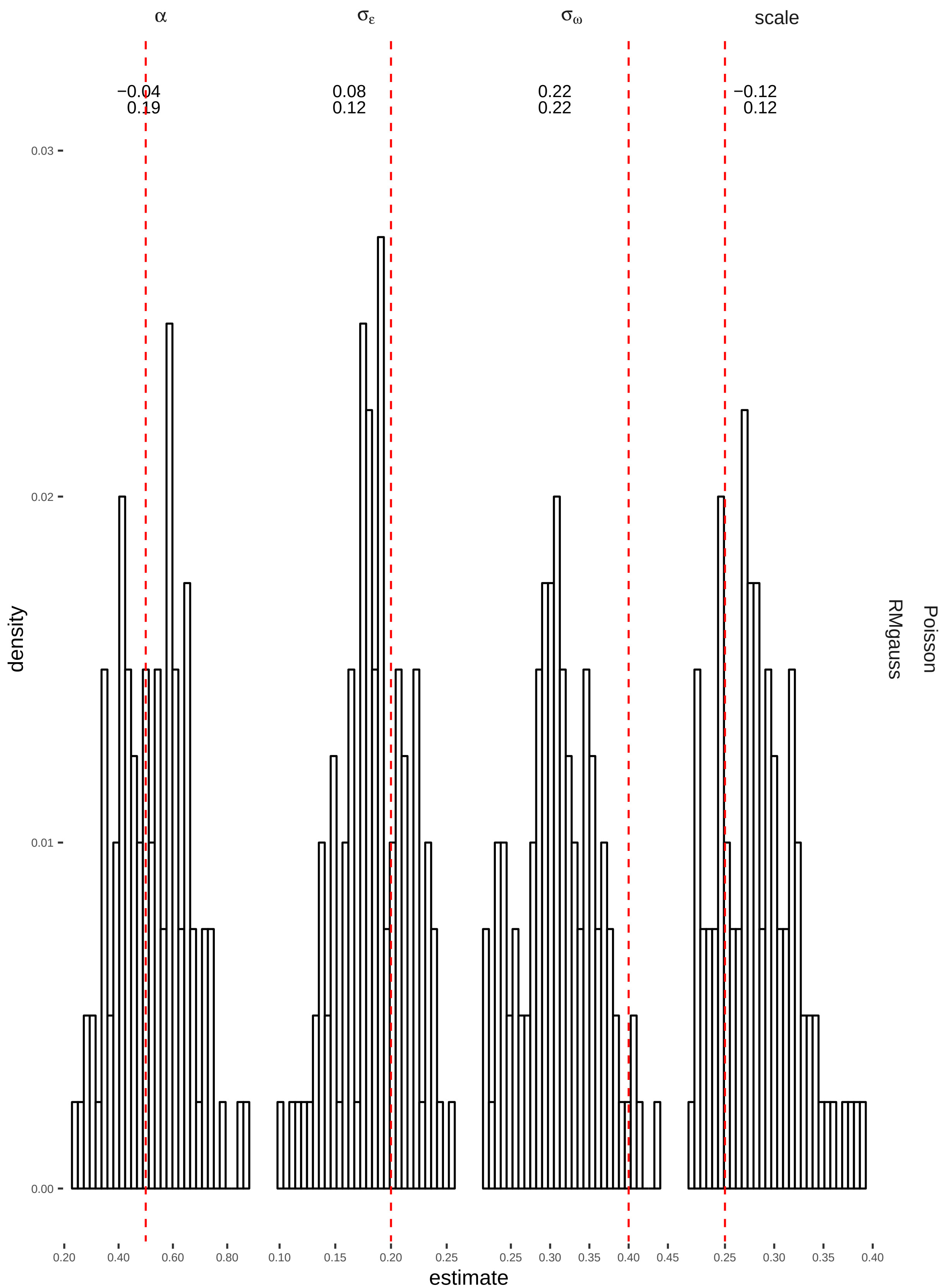


Figure 5: Parameter estimates and their median absolute relative error when ρ was fixed at its true value. Red dashed line indicates the true parameter value.

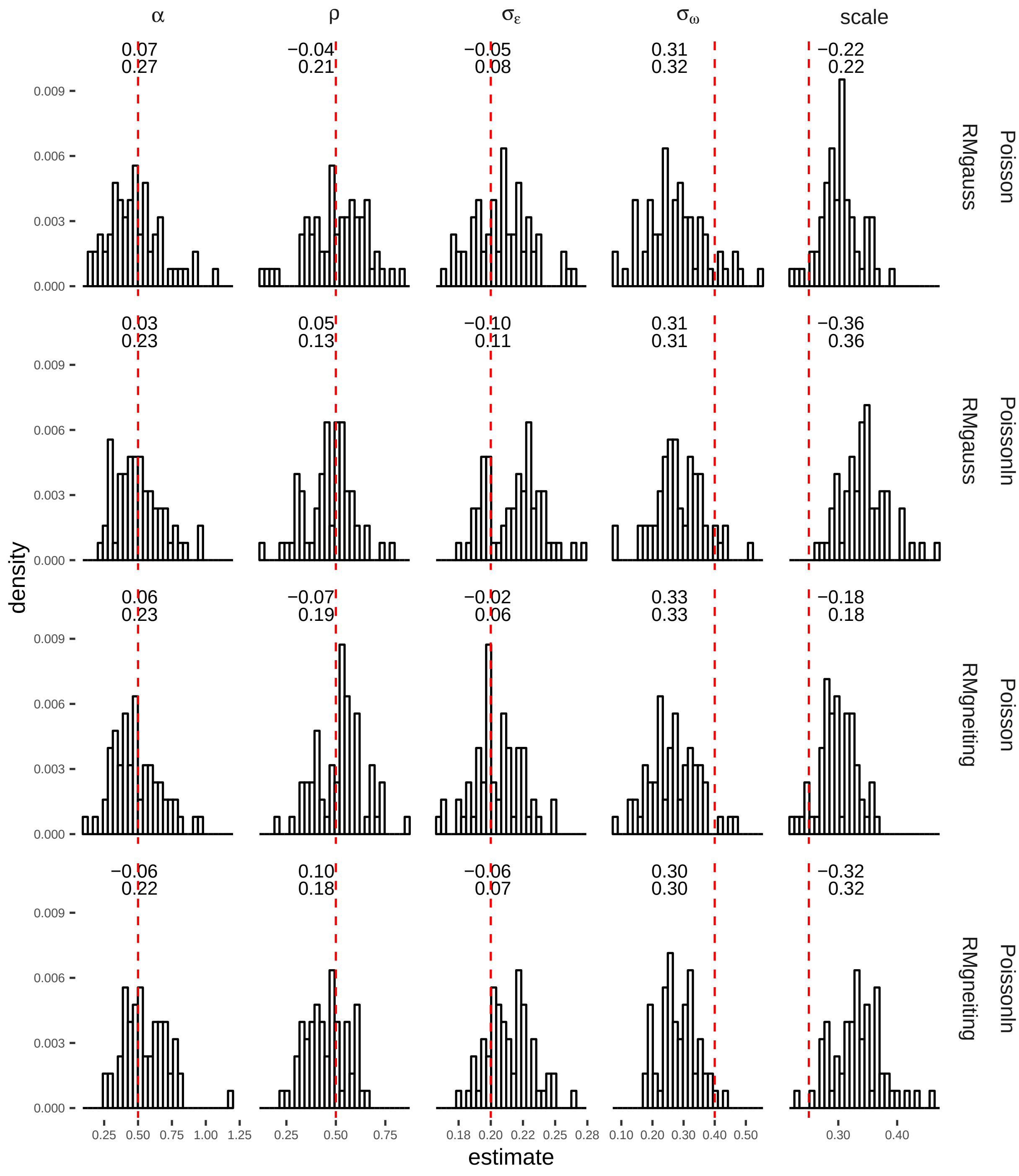


Figure 6: Parameter estimates and their median relative error and median absolute relative error. Red dashed line indicates the true parameter value.

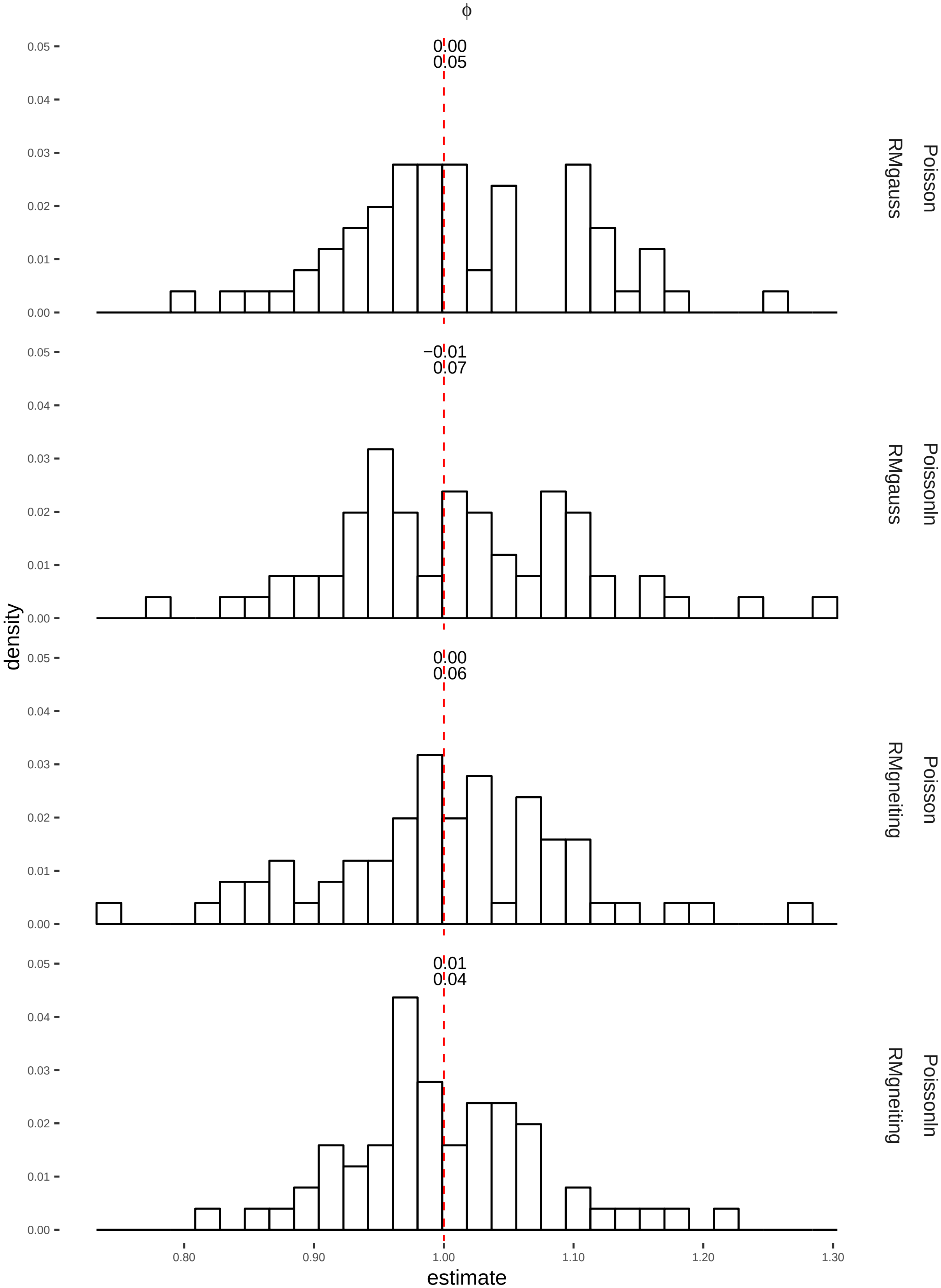


Figure 7: Estimates of ϕ and its median relative error (RE) and median absolute RE. Red dashed line indicates the true parameter value. RE was calculated using exponentiated values to circumvent RE values of infinity.

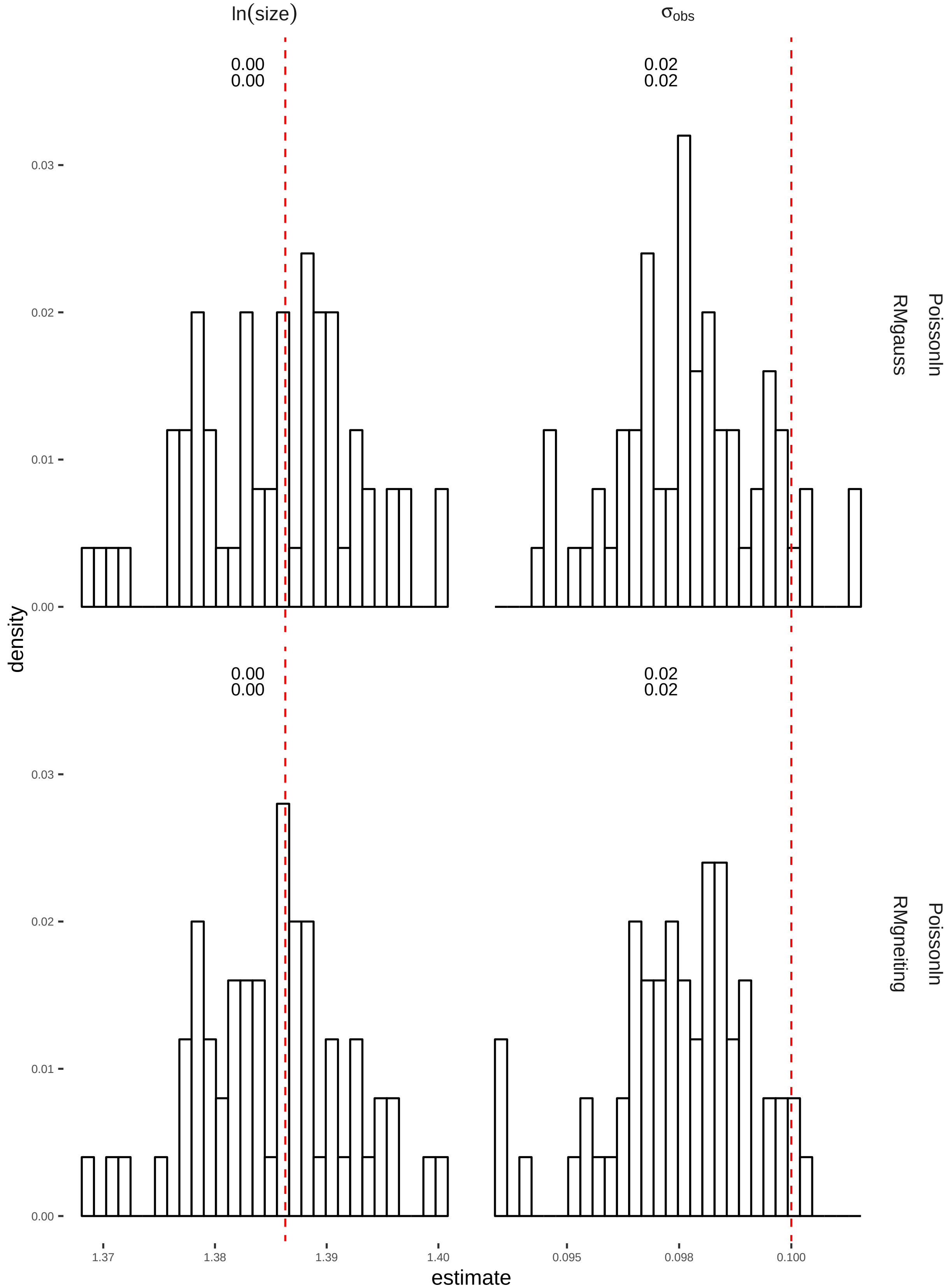


Figure 8: Parameter estimates and their median relative error and median absolute relative error for the estimation method that assumed a Poisson lognormal response variable. Red dashed line indicates the true parameter value.