Random effect estimation of time-varying factors in Stock Synthesis

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Manuscripts

1 **Random effect estimation of time-varying factors in Stock Synthesis**

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Abstract

There is increasing recognition that biological processes such as fishery selectivity, natural

mortality, and somatic growth vary over time, and that time-varying processes are important to

include in marine population dynamics models. However, previous penalized-likelihood

estimation approaches provide little guidance regarding the variance of time-varying processes,

and random-effect approaches are computationally infeasible or not implemented for many

models and software packages. We therefore show that existing stock assessment models and

software can be used to calculate the Laplace approximation to the marginal likelihood of

parameters representing variability over time, and use Stock Synthesis to show how this can be

used to select an appropriate value for variation over time in stock assessment models. Using

North Sea cod and Pacific hake models as case studies, we show that this method has little bias

in estimating variances for simulated data. It also provides a similar estimate of hake recruitment

(log-SD = 1.43) to Markov chain Monte Carlo (MCMC) methods (log-SD = 1.68), and estimates

a non-trivial magnitude (log-SD = 0.07) of variation in growth for North Sea cod. We conclude

by discussing the generality of the proposed method and by recommending future research

regarding its performance relative to MCMC, particularly when estimating multiple variances

simultaneously.

Keywords: random effect; Bayesian; maximum likelihood; hierarchical model; time-varying

parameter; mixed effect; Laplace approximation; stock assessment; penalized likelihood;

recruitment

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Introduction

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37   Population dynamics models that include the effects of human-caused mortality (called stock

38   assessment models) are an important tool in the management of aquatic and terrestrial species.

39   These models are routinely used to assess whether population abundance and harvest rates are

40   respectively above and below sustainable targets, as well as to forecast population responses

41   given different proposed management actions (Hilborn and Walters 1992). They are also

42   increasingly used for ecological purposes, e.g., to estimate life history relationships (Thorson et

43   al. In press), evaluate environmental impacts on populations (Schirripa et al. 2009, Thorson et al.

44   2013), and to explore trophic interactions among species (Plagyani et al. In press.).

45   Integrated stock assessment models account for measurement processes (i.e. the process by

46   which data are generated) in increasingly realistic ways (Maunder and Punt 2013). Most of these

47   models account for stochastic processes in the dynamics (e.g., growth, productivity, or

48   availability to harvest) through a time series of fixed effects that are penalized towards zero, and

49   where the strength of this penalty is selected by the user *a priori*. State-space population models

50   (Gudmundsson and Gunnlaugsson 2012, Mäntyniemi et al. 2013) expand on this capability by

51   allowing estimation of the variance of stochastic processes such as recruitment, selectivity,

52   natural mortality, immigration/emigration, and other demographic processes. State-space models

53   accomplish this by using random effects, where stochasticity in one or more dynamical processes

54   is 'integrated out' of the marginal likelihood (de Valpine 2002), and hence avoids statistical

55   limitations of parameter confounding and asymptotic biases in standard 'fixed effect' models.

56   Recent research has demonstrated that random effects can be used to approximate many

57   population processes, e.g., variability in fishing gear efficiency (Wilberg et al. 2010), and to

58   approximate unknown functional forms in population dynamics (Thorson and Taylor In review,

59   Thorson et al. in press). <u>Penalized likelihood methods are often used to estimate time-varying</u>

60   <u>parameters such as juvenile survival (recruitment, Maunder and Deriso 2003), but this will often</u>

61   <u>result in biased parameter estimates (De Valpine and Hilborn 2005).</u> Methot and Taylor (2011)

62   developed a bias correction procedure to estimate recruitment variability, but it is unclear how to

63   generalize their method to other time-varying parameters. *Need to read Methot and Taylor to see how this works, also I wasn't aware the bias correction procedure was estimating sigmaR*

64   Random effect and state-space models require integration (or its approximation, e.g., the

65   Kalman filter comparison in Gudmundsson and Gunnlaugsson 2012) when fitting a model to a

66   data set. Existing computation approaches to random effect estimation include Markov chain

67   Monte Carlo (MCMC) in either a maximum likelihood (De Valpine 2004) or Bayesian

68   framework (Meyer and Millar 1999) or the Laplace approximation as implemented in ADMB-

69   RE (Skaug and Fournier 2006). This integral may be high-dimensional, rendering simple

70   integration methods computationally infeasible. For example, MCMC sampling may be

71   computationally impractical for many population dynamics models due to irregularly shaped

72   likelihood or posterior distributions (Polansky et al. 2009, Stewart et al. 2013). Similarly,

73   implementing a model in ADMB-RE requires a large overhead of time and expertise due to the

74   practical necessity of finding a 'separable' formulation of the population model (Bolker et al.

75   2013) and still may take a considerable amount of time during optimization.

76   In this study, we demonstrate a novel approach to estimating random effects in existing stock

77   assessment software that implement stochastic processes as a sequence of fixed effects.

78   Specifically, we show that the Laplace approximation can be applied to the penalized likelihood

79   of estimated parameters in combination with the matrix of second derivatives (i.e., the model

80   Hessian) to estimate the variance of random effects. This approach is demonstrated specifically

81   for the widely used Stock Synthesis software (Methot and Wetzel 2013) written in the AD Model

Hessian = matrix of second derivatives

82 Builder optimization software (ADMB, Fournier et al. 2012) and is distributed using the *r4ss*

83 package (Taylor et al. 2012) in R (R Core Development Team 2012).

Is the approximation in r4ss? How difficult would it be to use the approximation code with another framework?

Methods

85 We first review the estimation of random-effect models. Parameters in maximum-likelihood

86 models that contain both random and fixed-effects ("mixed-effects models") are estimated by

87 maximizing the marginal likelihood that is obtained when integrating across random effects.

88
$$\hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\tau}} = \max_{\boldsymbol{\theta}, \boldsymbol{\tau}} \left[L(\boldsymbol{\theta}, \boldsymbol{\tau} \mid \boldsymbol{D}) \right] = \max_{\boldsymbol{\theta}, \boldsymbol{\tau}} \left[\int_{\varepsilon} L(\boldsymbol{\theta}, \boldsymbol{\varepsilon} \mid \boldsymbol{D}) \cdot \pi(\boldsymbol{\varepsilon} \mid \boldsymbol{\tau}) \cdot d\varepsilon \right] \tag{1}$$

89 where $L(\boldsymbol{\theta}, \boldsymbol{\tau} | \boldsymbol{D})$ is the marginal likelihood that is maximized to identify values of the fixed-effect

90 parameters $\boldsymbol{\theta}$ and hyper-parameters $\boldsymbol{\tau}$ given data $\boldsymbol{D}$, $L(\boldsymbol{\theta}, \boldsymbol{\varepsilon} | \boldsymbol{D})$ is the joint likelihood of fixed-effect

91 $\boldsymbol{\theta}$ and random-effect parameters $\boldsymbol{\varepsilon}$, and $\pi(\boldsymbol{\varepsilon} | \boldsymbol{\tau})$ is the probability of random effects given

92 hyperparmeters. We will call the product of the joint likelihood and the probability of random

93 effects ($L(\boldsymbol{\theta}, \boldsymbol{\varepsilon} | \boldsymbol{D}) \pi(\boldsymbol{\varepsilon} | \boldsymbol{\tau})$) the 'penalized likelihood'. If the penalized likelihood is maximized

94 directly (rather than integrating across random effects), this generally results in biased estimates

95 of model parameters (De Valpine and Hilborn 2005).

96   Notably, Eq. 1 includes a multivariate integral of the 'penalized likelihood' across random

97 effects, and this integral is the main computational impediment to the wide adoption of mixed-

98 effect models in ecology (de Valpine 2009). By contrast, Bayesian estimation of mixed-effect

99 models are estimated by approximating the integral across all coefficients:

100
$$\boldsymbol{\theta}_i, \boldsymbol{\tau}_i, \boldsymbol{\varepsilon}_i \approx \int_{\boldsymbol{\theta}, \boldsymbol{\tau}, \boldsymbol{\varepsilon}} L(\boldsymbol{\theta}, \boldsymbol{\varepsilon} \mid \boldsymbol{D}) \cdot \pi(\boldsymbol{\varepsilon} \mid \boldsymbol{\tau}) \cdot p(\boldsymbol{\theta}, \boldsymbol{\tau}) \cdot d\varepsilon \cdot d\theta \cdot d\tau \tag{2}$$

101 where samples $\boldsymbol{\theta}_i, \boldsymbol{\tau}_i, \boldsymbol{\varepsilon}_i$ from the integral are obtained using MCMC, and $p(\boldsymbol{\theta}, \boldsymbol{\tau})$ is the prior

102 probability of parameters $\boldsymbol{\theta}$ and $\boldsymbol{\tau}$. Thus, maximum likelihood and Bayesian estimates differ in

103 what is 'integrated across': Maximum likelihood integrates across random effects only, while

104   Bayesian integrates across all model coefficients. Bayesian methods also require a prior $p(\boldsymbol{\theta}, \boldsymbol{\tau})$

105   on fixed effects $\boldsymbol{\theta}$ and hyper-parameters $\boldsymbol{\tau}$, while this prior is not necessary in maximum

106   likelihood models. However, many 'maximum likelihood' assessment models already include a

107   penalty on these parameters, so the distinction is somewhat blurred in this regard.

108   We instead propose a third option for estimating the variance of random effects.

109   Specifically, we propose to integrate across all coefficients except the hyperparameters $\boldsymbol{\tau}$. This

110   results in the following optimization problem:

111    $$\hat{\boldsymbol{\tau}} = \max_{\boldsymbol{\tau}} \left[L(\boldsymbol{\tau} \mid \boldsymbol{D}) \right] = \max_{\boldsymbol{\tau}} \left[\int_{\boldsymbol{\theta}, \boldsymbol{\varepsilon}} L(\boldsymbol{\theta}, \boldsymbol{\varepsilon} \mid \boldsymbol{D}) \cdot \pi(\boldsymbol{\varepsilon} \mid \boldsymbol{\tau}) \cdot p(\boldsymbol{\theta}) \cdot d\varepsilon \cdot d\theta \right] \tag{3}$$

112   where $p(\boldsymbol{\theta})$ is the prior probability of parameters $\boldsymbol{\theta}$. This third option essentially sets

113   hyperparameters $\boldsymbol{\tau}$ to the value that maximizes the marginal likelihood of all other model

114   parameters. We subsequently show that this optimization problem can be solved using most

115   stock assessment models and their existing software implementations, by means of the Laplace

116   approximation.

117   The Laplace approximation is derived from a Taylor's series expansion of a distribution:

118    $$f(z) \approx f(\hat{z}) + f'(\hat{z})(z - \hat{z}) + \frac{1}{2} f''(\hat{z})(z - \hat{z})^2 \tag{4}$$

119   where $f(z)$ is a distribution, i.e. the logarithm of the penalized likelihood, $\hat{z}$ is a point, i.e., the

120   vector of parameters z that maximizes the log-penalized likelihood, and f' and f'' are the first and

121   second derivatives of f. At the maximum penalized likelihood estimate $\hat{z}$, the first derivatives

122   of the log-penalized likelihood are 0 so the second term of the right-hand side drops out, and the

123   second derivatives are all negative. Therefore:

124    $$e^{f(z)} \approx e^{f(\hat{z}) - \frac{1}{2} |f''(\hat{z})|(z - \hat{z})^2} \tag{5}$$

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125 This approximation is useful while taking the integral of the penalized likelihood $e^{f(z)}$:

126 $$\int e^{f(z)}dz \approx e^{f(\hat{z})} \int e^{-\frac{1}{2}|f''(\hat{z})|(z-\hat{z})^2} dz \tag{6}$$

127 where the second-derivatives of the log-penalized likelihood $f''(\hat{z})$ is also known as the

128 Hessian, and is routinely reported by most maximum likelihood optimization software. The term

129 $e^{-\frac{1}{2}|f''(\hat{z})|(z-\hat{z})^2}$ resembles a multivariate normal likelihood (although without its normalization

130 constant). Therefore, the Laplace approximation can be interpreted as approximating the

131 penalized likelihood with a multivariate normal distribution, and integrating across this

132 approximation (by calculating the area under this multivariate normal distribution; Skaug and

133 Fournier 2006).

134   We now apply the Laplace approximation to our specific problem. Given the vector of

135 coefficients $\tilde{\boldsymbol{\theta}}, \tilde{\boldsymbol{\varepsilon}}$ that maximizes the penalized likelihood (given data $\boldsymbol{D}$ and proposed values of

136 the hyperparameters $\boldsymbol{\tau}$) and its Hessian matrix (the matrix of second derivatives of the log-

137 penalized likelihood):

138 $$H(\tilde{\boldsymbol{\theta}}, \tilde{\boldsymbol{\varepsilon}} \mid \boldsymbol{\tau}, \boldsymbol{D}) = \begin{bmatrix} \dfrac{\delta^2 \left(\ln(L(\tilde{\boldsymbol{\theta}}, \tilde{\boldsymbol{\varepsilon}} \mid \boldsymbol{D}) \cdot \pi(\tilde{\boldsymbol{\varepsilon}} \mid \boldsymbol{\tau}))\right)}{\delta \tilde{\theta_1}^2} & \cdots & \dfrac{\delta^2 \left(\ln(L(\tilde{\boldsymbol{\theta}}, \tilde{\boldsymbol{\varepsilon}} \mid \boldsymbol{D}) \cdot \pi(\tilde{\boldsymbol{\varepsilon}} \mid \boldsymbol{\tau}))\right)}{\delta \tilde{\theta_1} \delta \tilde{\varepsilon}_n} \\ \cdots & \cdots & \cdots \\ \dfrac{\delta^2 \left(\ln(L(\tilde{\boldsymbol{\theta}}, \tilde{\boldsymbol{\varepsilon}} \mid \boldsymbol{D}) \cdot \pi(\tilde{\boldsymbol{\varepsilon}} \mid \boldsymbol{\tau}))\right)}{\delta \tilde{\theta_1} \delta \tilde{\varepsilon}_n} & \cdots & \dfrac{\delta^2 \left(\ln(L(\tilde{\boldsymbol{\theta}}, \tilde{\boldsymbol{\varepsilon}} \mid \boldsymbol{D}) \cdot \pi(\tilde{\boldsymbol{\varepsilon}} \mid \boldsymbol{\tau}))\right)}{\delta \tilde{\varepsilon}_n^2} \end{bmatrix} \tag{7}$$

139 the Laplace approximation is equal to the area under a multivariate normal distribution with

140 maximum equal to the 'maximum penalized likelihood' and covariance equal to the inverse of

141 this Hessian matrix:

142 $$L(\boldsymbol{\tau} \mid \boldsymbol{D}) = \int_{\boldsymbol{\theta}, \boldsymbol{\varepsilon}} L(\boldsymbol{\theta}, \boldsymbol{\varepsilon} \mid \boldsymbol{D}) \cdot \pi(\boldsymbol{\varepsilon} \mid \boldsymbol{\tau}) \cdot p(\boldsymbol{\theta}) \cdot d\varepsilon \cdot d\theta \quad \approx \quad \frac{2 \cdot L(\tilde{\boldsymbol{\theta}}, \tilde{\boldsymbol{\varepsilon}} \mid \boldsymbol{D}) \cdot \pi(\tilde{\boldsymbol{\varepsilon}} \mid \boldsymbol{\tau})}{\sqrt{\left| H(\tilde{\boldsymbol{\theta}}, \tilde{\boldsymbol{\varepsilon}} \mid \boldsymbol{\tau}, \boldsymbol{D}) \right|}} \tag{8}$$

143   where $\tilde{\boldsymbol{\theta}}$ and $\tilde{\boldsymbol{\varepsilon}}$ are again the coefficients that maximize the penalized likelihood given proposed

144   values of hyperparameters $\boldsymbol{\tau}$, and $\left|H(\tilde{\boldsymbol{\theta}},\tilde{\boldsymbol{\varepsilon}}\,|\,\boldsymbol{\tau},\boldsymbol{D})\right|$ is the determinant of the Hessian matrix. This

145   approximation is exact whenever the penalized likelihood has a multivariate-normal distribution,

146   and hence performs well given large-sample asymptotic theory, where likelihood and posterior

147   distributions will converge on a multivariate normal distribution (Gelman et al. 2003).

148   **Application to Stock Synthesis**

149   We briefly summarize our implementation of this proposed approximation in Stock Synthesis.

150   The algorithm starts with an initial value for all variance parameters, $\boldsymbol{\tau}_0$. Given these values,

151   Stock Synthesis is run once to obtain estimates of parameters and their standard deviations, and

152   may optionally be run again after adjusting the recruitment bias-correction ramp following

153   methods in Methot and Taylor (2011). The Hessian is then extracted from ADMB using

154   auxiliary code (C. Monnahan, pers. comm.), integration constants are added where necessary to

155   the log-penalized likelihood derived from Stock Synthesis, and these two components are used to

156   calculate the marginal likelihood (Eq. 8). A new set of parameters is then proposed $\boldsymbol{\tau}_1$, and this

157   process is iterated until a local maximum is identified. Any application of this algorithm where

158   the first *5* runs of Stock Synthesis fail to converge to a satisfactory degree (i.e. the final gradient

159   of any Stock Synthesis optimization is greater than *1*) can be identified, and may not have

160   adequately converged to the value of $\boldsymbol{\tau}$ that maximizes the marginal likelihood. This algorithm is

161   distributed as the function *NegLogInt_Fn* in the *r4ss* package.

162   **Case study 1: Variable recruitment in Pacific hake**

163   Pacific hake (*Merluccius productus*) is a commercially important species whose harvest is split

164   between fisheries originating in the U.S. West Coast and Western Canada. It has recently been

165   managed under an international treaty between these two countries, and has been assessed using

166 Stock Synthesis. The assessment includes fishery catches from 1966-2012, fishery age

167 composition samples from 1975-2012, an intermittent acoustic survey starting in 1995 that

168 provides an index of abundance and age-composition samples, and 'empirical' weight-at-age

169 values that are assumed to be known without error. Assessment results indicate that population

170 dynamics are driven by very high variability in early juvenile survival ("recruitment").

171 Recruitment in the hake assessment model is treated as a penalty towards a parametric stock-

172 recruit curve. For our investigations, we therefore treat the magnitude of recruitment variability

173 as a hyperparameter in the proposed Laplace approximation.

174 **Case study 2: Variable growth in North Sea cod**

175 Cod (*Gadus morhua*) supports a commercially important and iconic fishery in the North Sea.

176 The Skagerrak (northern section of Division IIIa) and eastern Channel (Division VIId) division

177 has been assessed previously using catch, age-composition, and weight-at-age data from 1963-

178 2011, an index of adult abundance 1963-2011, and an index of juvenile abundance 1983-2011.

179 These data have been assessed using a state-space assessment model (SAM; Gudmundsson and

180 Gunnlaugsson 2012), and has been transcribed to an integrated catch-at-age model (Maunder and

181 Punt 2013) using the Stock Synthesis model software (Methot and Wetzel 2013) in preparation

182 for the World Congress on Stock Assessment Methods in 2013. Although SAM assumes that

183 weight at age are known without error, the transcribed Stock Synthesis model includes weight at

184 age as data and can estimate cohort-specific deviations in growth. These deviations are

185 penalized towards zero, so that cohort-specific growth patterns are penalized towards an average

186 growth schedule. The magnitude of this penalty cannot be estimated easily using penalized

187 likelihood, and is treated as a hyperparameter τ in the proposed Laplace approximation. For

188 simplicity, we assume that recruitment deviations have a standard deviation of *0.7* (as identified

189   using methods in Methot and Taylor 2011). We do not explore estimating the variance of both

190   processes simultaneously, although there is no reason this could not be explored in the future.

191   **Model testing and application**

192   For each case study, we apply our proposed Laplace approximation using existing Stock

193   Synthesis assessment files and record the estimated variance for the random effect. We also

194   perform a simulation study for each species, where the data sample sizes and estimated

195   parameters are used to simulate a new data set, which is then fitted again using the same Stock

196   Synthesis model where the variance of random effects is estimated using the Laplace

197   approximation. For the hake example, we additionally re-simulate recruitment deviations for all

198   years from a normal distribution with mean *0* and standard deviation of either *1.5* or *0.5*

199   (depending upon simulation scenario) and compare Laplace approximation estimates of the

200   standard deviation with this known, true value. For the cod example, we re-simulate recruitment

201   deviations for all years with mean *0* and standard deviation *0.7* (i.e. the value assumed in the

202   estimation model), re-simulate growth deviations with a mean of *0* and standard deviation of

203   either *0.1* or *0.02* (depending upon simulation scenario), and re-simulate fishing mortality to

204   follow an ascending and then descending pattern.

205   Finally, we estimate the original hake case-study model using Markov chain Monte Carlo to

206   compare this estimate of recruitment variability with our Laplace approximation estimate. We

207   do not do this for the cod case study because growth variability is not programmed as a

208   'parameter' in that model and hence cannot be sampled using MCMC using existing model

209   software.

210   **Results**

211   The Laplace approximation to the log-marginal likelihood is the sum of the log-penalized

212   likelihood and the log-determinant of the Hessian (Eq. 8). We therefore first show these three

213   curves as a function of random effect variability for hake (Fig. 1) and cod (Fig. 2). In the hake

214   model, the penalized likelihood is maximized by a standard deviation of approximately *0.9* for

215   recruitment variability. By contrast, the determinant of the Hessian is monotonically increasing

216   for all values from *0.5* to *2.5*. Their sum has a maximum at *1.43*, and represents a compromise

217   between both components. Similarly, the penalized likelihood for the cod model has maximum

218   at approximately *0.046*, while the determinant of the Hessian is increasing monotically for all

219   values *0.02* to *0.15*. The Laplace approximation again indicates an intermediate value of *0.068*.

220   Fig. 1-2 illustrate a general principle of this method, i.e., that the Laplace approximation estimate

221   will always be higher than maximizing the penalized likelihood by itself.

222   The simulation testing of this approximation for hake (Fig. 3) and cod (Fig. 4) shows that it

223   has little bias, and that the direction of this small bias depends upon the simulation scenario.

224   When estimating recruitment variability for Pacific hake (Fig. 3), for example, simulation testing

225   shows a negative bias when the true value is *1.5* (median converged estimate = *1.34*), and a

226   positive bias when the true value is *0.5* (median estimate = *0.60*). Estimating growth variability

227   in Atlantic cod shows trivial bias either when the true value is 0.10 (median converged estimate

228   = *0.099*) or when the true value is 0.02 (median converged estimate = *0.022*). However, the

229   algorithm has evidence of non-convergence in as many as 40% of replicates (i.e., for the case of

230   cod with 0.02 variability in growth), and generally has lower convergence rates when simulating

231   data given a low process variability. These cases of non-convergence could often be corrected

232   by applying additional user guidance (i.e. better starting values for parameters) rather than an

233   automated simulation-testing procedure.

234   Finally, the comparison of MCMC and Laplace approximation estimate of recruitment

235   variability for Pacific hake (Fig. 5) illustrates a small difference in estimates between these

236   approaches. Specifically, the mean Bayesian estimate is *1.68*, compared with the *1.43* estimated

237   by the Laplace approximation. However, the Laplace approximation acknowledges relatively

238   high support for the value of *1.68*; the difference in the log-likelihood for $\sigma_R=1.43$ and $\sigma_R=1.68$

239   is approximately *0.90*, and *1.05 < σ_R < 1.95* all have a log-likelihood within *3.0* of the maximum

240   log-likelihood. Therefore, the Laplace approximation signals that the strength of evidence to

241   distinguish between $\sigma_R = 1.05\text{-}1.95$ is relatively low. Similarly, the MCMC sample has a 95%

242   credible interval ranging from *1.33-2.06*, and similarly shows little strength-of-evidence for

243   distinguishing between MCMC and Laplace approximation estimates.

Discussion

245   Random-effects models are increasingly used in ecology to account for spatial and temporal

246   variability in biological dynamics and measurement processes (Royle and Dorazio 2008, de

247   Valpine 2009). As one example, a recent state-space model used count data in isolation to

248   estimate site- and age-structured abundance for northern dusky salamanders (*Desomgnathus*

249   *fuscus*) given variability in recruitment, immigration, and natural mortality (Zipkin et al. In

250   press). Models such as this are possible because states (i.e. local densities) and variability over

251   time are treated as random effects, and hence avoid identifiability problems that arise when only

252   using fixed effects or penalized likelihood (De Valpine and Hilborn 2005).

253   Random-effect models have also been advocated within fisheries science for many years

254   (Maunder and Deriso 2003). These models can update expectations about the shape of species'

255   dynamics (Thorson et al. in press) while simultaneously accounting for the many types of

256   variability that are likely to occur for marine species (Mäntyniemi et al. 2013). However,

257 existing computational approaches to random-effect models are either slow or difficult to

258 implement (Stewart et al. 2013), and this has prevented wide-spread application of random

259 effects in stock assessment models (Maunder and Punt 2013).

260   To rectify this problem, we have shown that the Laplace approximation can be applied to

261 existing stock assessment software to estimate the likelihood when integrating across all fixed

262 and random effects, and that this can be used to identify an appropriate value for the variance of

263 time-varying processes. We have also developed software to do this when using Stock Synthesis

264 (Methot and Wetzel 2013), which is freely available using the *r4ss* package (Taylor et al. 2012).

265 This approach performs well during simulation testing for both variability in recruitment somatic

266 growth. This approach also performs similarly to Markov chain Monte Carlo when estimating

267 recruitment variability, although remains a small difference in estimates between these two

268 approaches (Laplace approximation: $\sigma_R = 1.68$; MCMC: $\sigma_R = 1.43$). This discrepancy likely

269 arises because the Laplace approximation uses a multivariate normal distribution to approximate

270 the posterior distribution, while MCMC samples from the distribution directly. Therefore, any

271 divergence between the posterior and a multivariate normal distribution will likely cause the

272 Laplace approximation and MCMC to result in different estimates in general.

273   We therefore conclude that our proposed approximation represents a viable approach to

274 identifying the variance of random effects in stock assessment models. However, we

275 recommend further case-study and simulation-modeling comparisons between MCMC,

276 maximum marginal likelihood, and our proposed approach. These comparisons can hopefully

277 identify when each method is fastest and can most appropriately identify the magnitude of one or

278 more time-varying processes. We also recommend future research regarding the simultaneous

279 estimation of multiple time-varying processes. Some assessment models can include multiple

280   time-varying processes (Gudmundsson and Gunnlaugsson 2012, Mäntyniemi et al. 2013), but

281   there are few existing diagnostics for determining which are supported by the data or

282   substantially affect model results. The proposed Laplace approximation method will in some

283   cases be faster than MCMC estimation approaches, and also permits the use of pre-existing

284   maximum likelihood model diagnostics (Taylor et al. 2012), so it may ease model building and

285   evaluation given multiple time-varying processes. In other cases, however, Bayesian approaches

286   will allow for better estimates of variance in time-varying processes. In particular, Bayesian

287   methods can identify instances when multiple processes (e.g., time-varying selectivity or growth)

288   can equally explain the data, i.e., by showing a ridge in the joint posterior of these variances. In

289   these cases, maximum likelihood approaches often estimate one or the other process as having

290   zero variance, or fail to propagate uncertainty about variances (see Gelman 2005 for details).

291   For this reason, we advocate continued testing of models with multiple time-varying processes

292   and comparison of computational approaches in this circumstance. Such a comparisons will be

293   vital for stock assessment practitioners that seek to provide a biologically realistic portrayal of

294   marine populations and processes (Kuparinen et al. 2012), where many processes will vary

295   simultaneously over time, and may improve estimates of forecast imprecision, which is poorly

296   estimated by existing models with parameters that are constant over time (Ralston et al. 2011).

Acknowledgements

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392   Fig. 1 – Profile showing the log-penalized likelihood (dotted line), 0.5 times the log-determinant

393   of the Hessian (dashed line), and the resulting log-marginal likelihood (solid line) arising from

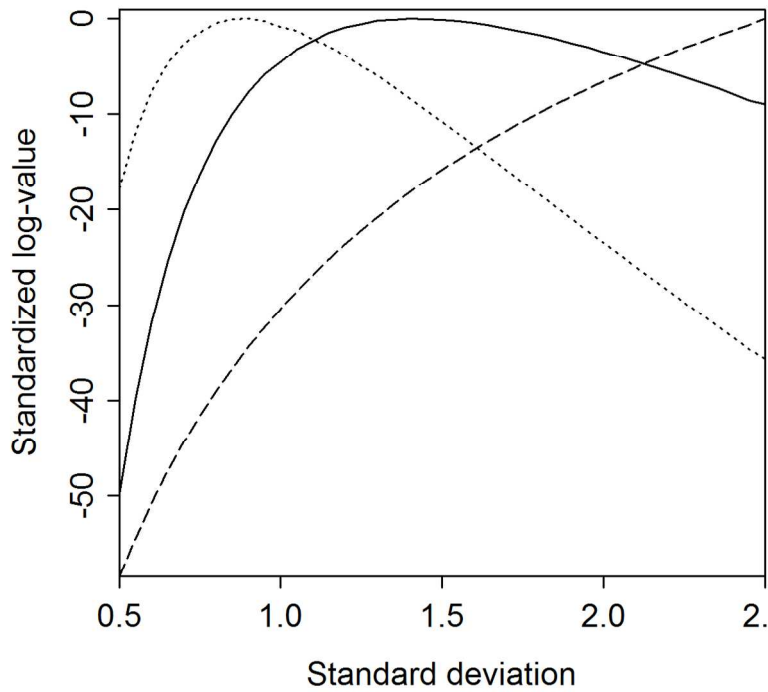394   the proposed Laplace approximation for recruitment variability in Pacific hake

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396   Fig. 2 – Profile showing the log-penalized likelihood (dotted line), *0.5* times the log-determinant
397   of the Hessian (dashed line), and the resulting log-marginal likelihood (solid line) arising from
398   the proposed Laplace approximation for variability in growth for North Sea cod
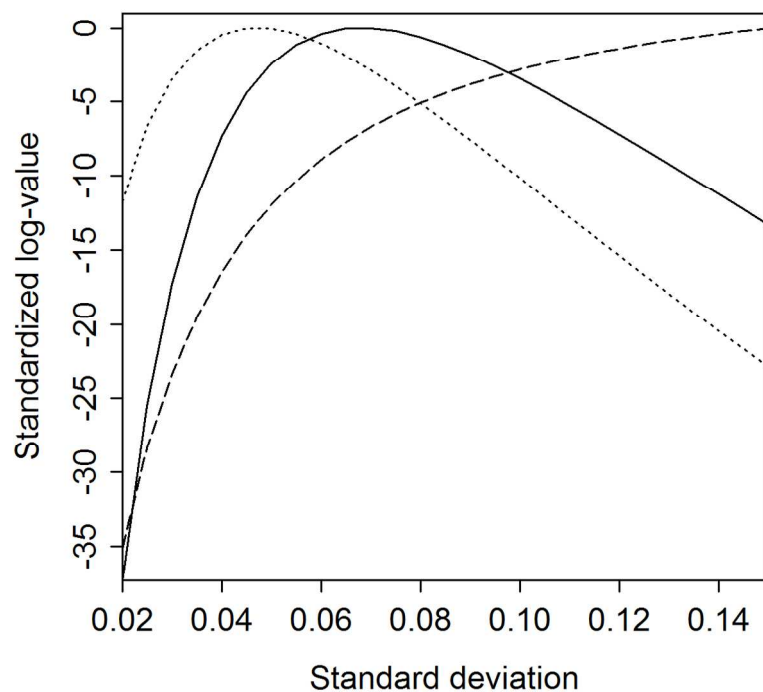

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400   Fig. 3 – Histogram of estimated recruitment variability for Pacific hake for *200* simulated data

401   sets, given a true value (solid vertical line) of *1.5* (panel A) or *0.5* (panel B), where the number

402   of algorithm runs that converged (i.e. where the first *5* iterations had a final gradient less than *1*)

403   and did not converge (i.e. one of the first *5* iterations had a final gradient greater than *1*) are

404   shown in the upper right (the dark-grey area is the histogram for 'converged' runs, while light-

405   grey is the additional distribution of 'non-converged' runs).
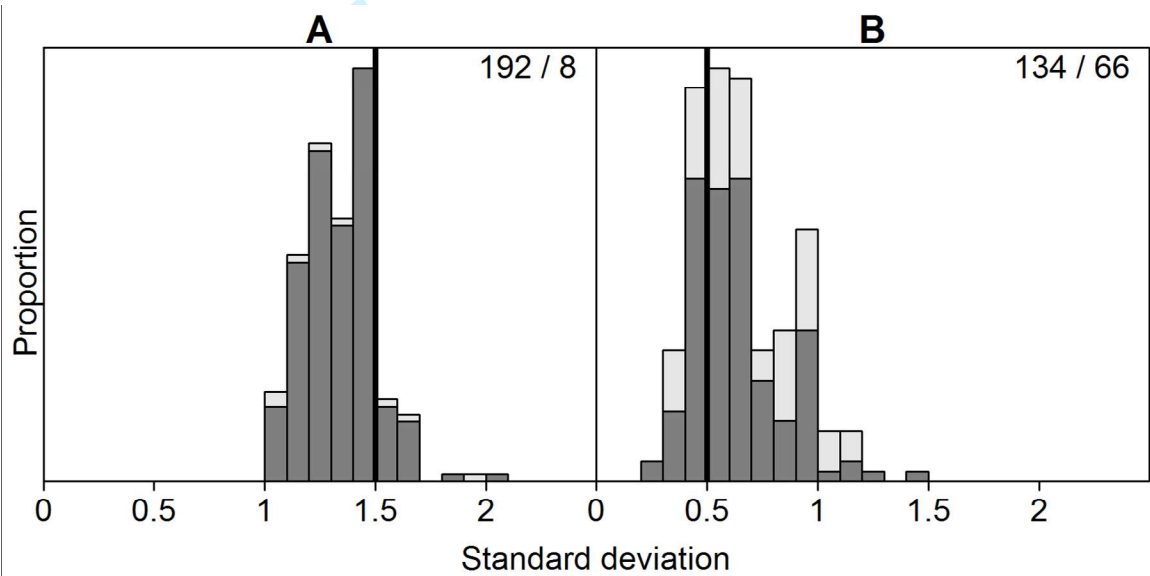

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408   Fig. 4 – Histogram of estimated growth variability for North Sea cod for *200* simulated data sets,

409   given a true value (solid vertical line) of *0.1* (Panel A) or *0.02* (Panel B), where the number of

410   algorithm runs that converged (i.e. where the first *5* iterations had a final gradient less than *1*)

411   and did not converge (i.e. one of the first *5* iterations had a final gradient greater than *1*) are

412   shown in the upper right (the dark-grey area is the histogram for 'converged' runs, while light-

413   grey is the additional distribution of 'non-converged' runs).

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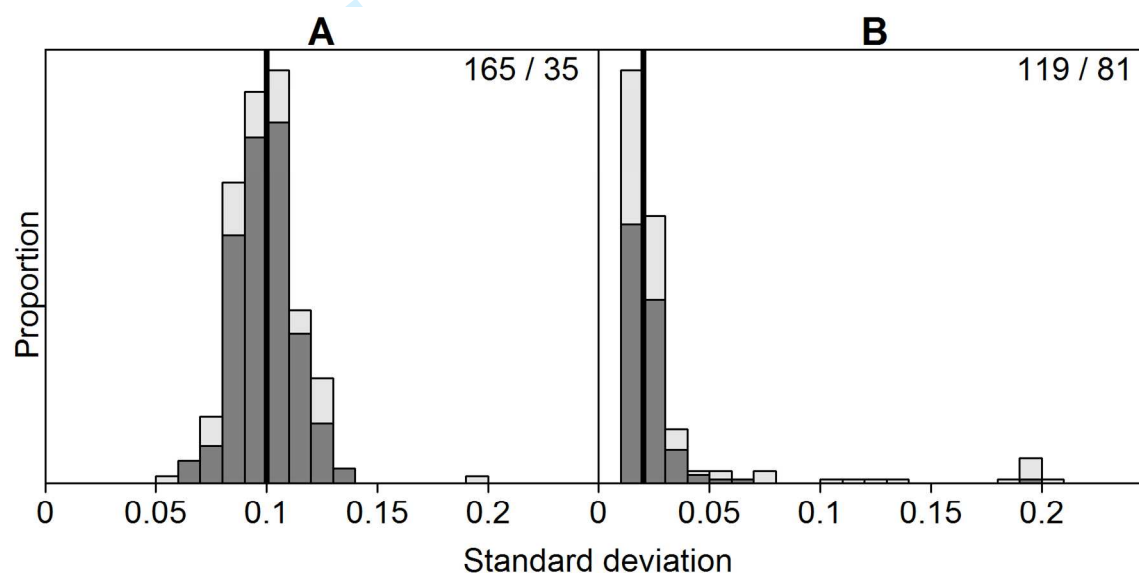51
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415   Fig. 5 – Markov chain Monte Carlo approximation of the posterior distribution for recruitment

416   variability for Pacific hake, showing the posterior mean (solid line) and 5% and 95% quantiles

417   (dotted lines)

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