Register Allocation

Martin Kellogg

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  - Unfortunately the technical problems here were tougher than I expected, so I am changing how it works
    - More details to come in the next day or two…
- Due to popular demand, I have **changed** PA4c1's specification to also allow the input to be a `.cl-type` file (originally was TAC)
  - PA4c1 is due April 28, and is mostly optional

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    - this happens when there aren't enough physical registers
  - Insert code to move values between registers and memory if needed ("***spill code***")
    - Typically we will re-run the instruction scheduler if we ever have to spill a register

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- However, one of the **first optimizations** that I suggest you implement for PA4 is a simple register allocator
  - avoiding memory operations is **extremely profitable** 🤑🤑🤑
  - I recommend waiting until later in PA4 to try more complex schemes

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- Regional register allocation + complications
  - **liveness analysis** strikes again!
- Global register allocation via **reduction to graph coloring**
  - including a **union-find** algorithm for fun

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  - Hopefully, this is where you are after PA3. I will assume this memory model for the rest of this lecture.

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In this discussion of local allocation, I'm going to assume that all values must be stored in memory **between** basic blocks

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  - any control flow (i.e., more than one basic block)

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Recall that a **reduction** is a proof that we could use an oracle for one problem to solve another, so for a problem to be NP-complete we need to prove both that if we can solve it, we could solve SAT and vice-versa

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In practice, for both NP-complete and undecidable problems we typically use **approximate** algorithms ("heuristics")

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- If ***k*** < ***v***, there is another complication: spilling a register to memory may require **more registers**!
  - We have to account for this when we choose ***k***

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- Advantage of this heuristic: **easy to understand + implement**
- Big disadvantage: it dedicates a physical register to one virtual register for the **entire basic block**
  - even if the virtual register is only used in the first half!

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"whose next use is farthest in the future" is easy to pre-compute in a single pass (because it's a basic block)

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  - this is part of why the register allocation problem is so hard!
- Let's see an example…

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- Consider a two-register machine with this state:

phys. reg.	value/VR	clean?
r1	x1	clean
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but, if we spill **x1** instead, we get:

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This example suggests that it might be better to **preferentially spill clean** values. Unfortunately, it's not that simple…

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spilling the dirty value results in less spill code!

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- No. In practice, many virtual registers contain values that are **dead** after the block (e.g., results of intermediate computations)
- As an optimization, we can **not store to memory** any value that is dead after the block.
  - A **global liveness analysis** computes exactly this information
    - i.e., we can easily inspect its “live out” sets to determine which virtual registers (= temporaries/IR names) we don’t need to store

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  - though it is **NP-hard** to produce **optimal** local allocations for single basic blocks, because of clean vs dirty values
  - frequency count allocator is **simpler**, but its allocations are usually worse
- If you are going to implement a local allocator in PA4, this greedy strategy is a good option
  - the textbook has a more-detailed version of this algorithm
  - if you want to do register allocation, I recommend building a local allocator first, regardless

Regional Register Allocation

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- Unfortunately, this is a bit of a **false hope**.
  - There are a lot of complications that arise at basic block boundaries! Let's take a look at some examples to see why.

Regional Register Allocation: Strawman

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Why won't this work?

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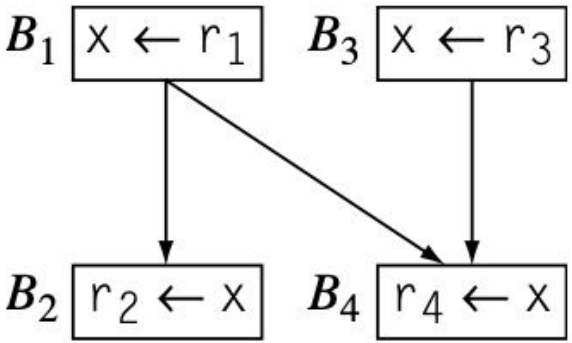

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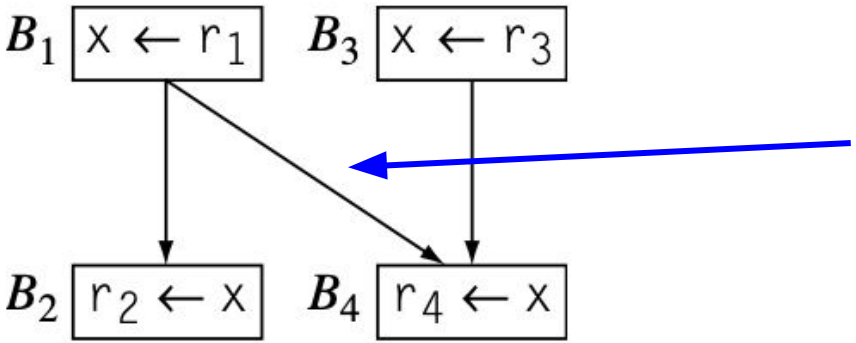

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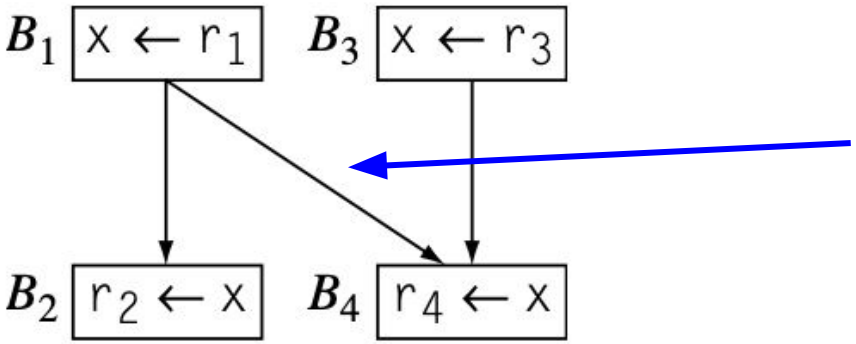

this edge B_1 -> B_4 is a ***critical*** edge: there is no safe choice for the register to store x, because of B_2 and B_3

- e.g., consider adding copy operations to keep x in a register. Where would they go?

Regional Register Allocation: Strawman

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B_1 $\boxed{x \leftarrow r_1}$ B_3 $\boxed{x \leftarrow r_3}$

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store x,

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Critical edges are just one reason that extending a local allocator beyond a single block is a bad idea. Instead, a global allocator should **coordinate** allocation decisions.

- it turns out that global allocators are fundamentally quite different than local allocators

Trivia Break: Popular Culture

This American science fiction media franchise began with a television show in the mid-1960s. The franchise is noted for its cultural influence beyond works of science fiction and for its progressive stances on civil rights; for example, in the 1960s, it was one of the first shows with a multiracial cast on American television. The franchise contains 11 spin-off television series and a film franchise; further adaptations also exist in several media. The series greatly influenced public interest in the U.S. Space Program and in education on the topic of space exploration. For example, NASA named one of its space shuttles (now on display at the Intrepid Museum in NYC) after something from this series.

Trivia Break: Physics

The creator of this speculative theory stated in an email to William Shatner (who played Captain Kirk in the *Star Trek* original series) that his theory was directly inspired by the term used in the show. The theory posits a speculative warp drive idea according to which a spacecraft could achieve apparent faster-than-light travel by contracting space in front of it and expanding space behind it, under the assumption that a configurable energy-density field lower than that of vacuum (that is, negative mass) could be created. The idea remains a hypothetical concept with seemingly difficult problems, although the amount of energy required is no longer thought to be unobtainably large.

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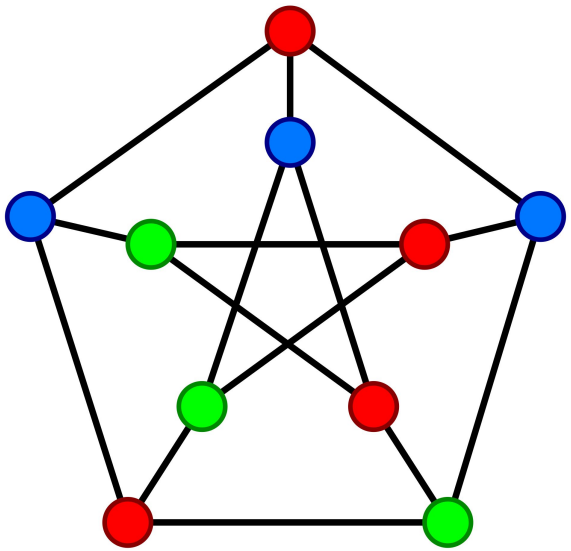

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  - Once we have a graph coloring problem, how do we actually **solve** it? It's NP-complete, after all…
    - We will see an algorithm that takes advantage of a special property of our graph: we can spill registers to **simplify** it

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- To make this concrete, let's start with **local live ranges**…

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1  loadI   ...        ⇒ r_arp
2  loadAI  r_arp,@a   ⇒ r_a
3  loadI   2          ⇒ r_2
4  loadAI  r_arp,@b   ⇒ r_b
5  loadAI  r_arp,@c   ⇒ r_c
6  loadAI  r_arp,@d   ⇒ r_x
7  mult    r_a,r_2    ⇒ r_a
8  mult    r_a,r_b    ⇒ r_a
9  mult    r_a,r_c    ⇒ r_a
10 mult    r_a,r_d    ⇒ r_a
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Global Reg. Alloc.: Loca

What are the live ranges for some of these (virtual) registers?

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	Register	Interval
1	r_{arp}	[1,11]
2	r_a	[2,7]
3	r_a	[7,8]
4	r_a	[8,9]
5	r_a	[9,10]
6	r_a	[10,11]
7	r_2	[3,7]
8	r_b	[4,8]
9	r_c	[5,9]
10	r_d	[6,10]

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  - Logically, a global live range is a **closure** of definitions and uses

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  - **insight**: we can simplify live range construction by converting the input to **SSA form** to guarantee one live range per variable
    - I will assume this from now on

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  - this data structure is a key component in other algorithms, such as Kruskal's (graph minimum spanning trees) and unification (for equation solving)

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  - after all phi functions have been processed, the resulting sets exactly correspond to live ranges

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      - We can easily derive this information from the live ranges of the values for n1 and n2

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z <- x + y
t <- y
u <- x + t
print z
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Global Reg. Alloc.: Interference Graph Example

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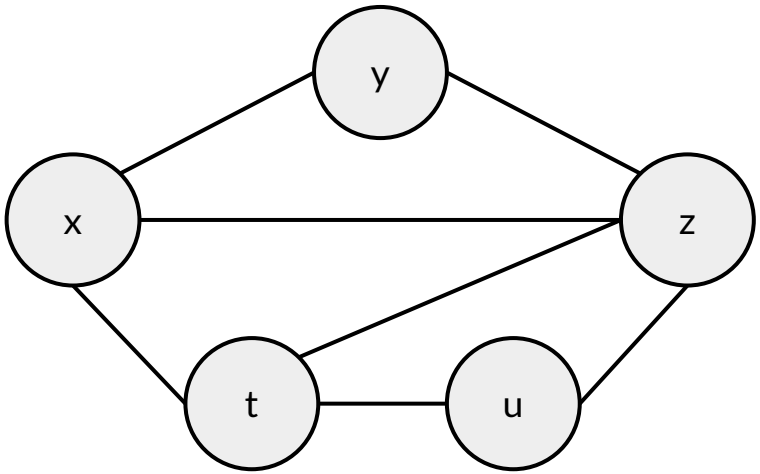

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  - We can map "spilling a value" back into the graph to **simplify** the interference graph (and therefore the graph coloring problem)
    - To be clear: this approach is **heuristic** (graph coloring is NP-complete). You could solve the graph coloring problem here using any graph coloring algorithm.

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- More formally, if the simplified graph is ***K-colorable***, then so is G: since n has less than K neighbors, those use at most K-1 colors, and there is therefore at least one color available for n.

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When a node is really spilled, the program has to be rewritten, which changes the interference graph. When this happens, the allocation process must be **restarted completely**. In practice, it converges in one or two iterations in most cases.

Coloring by Simplification Example

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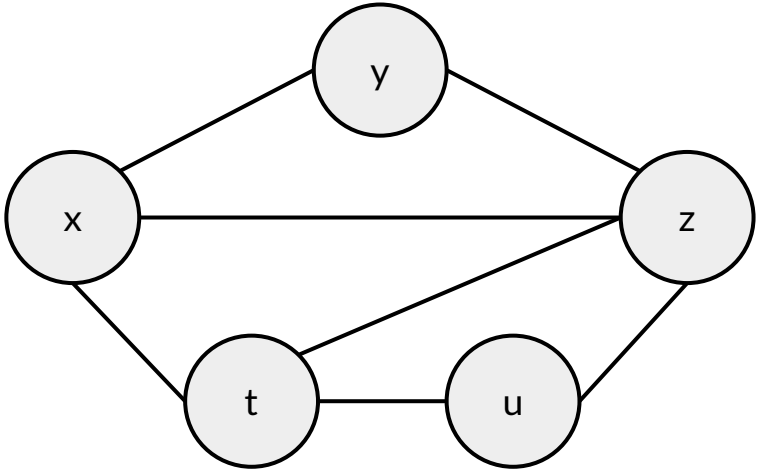

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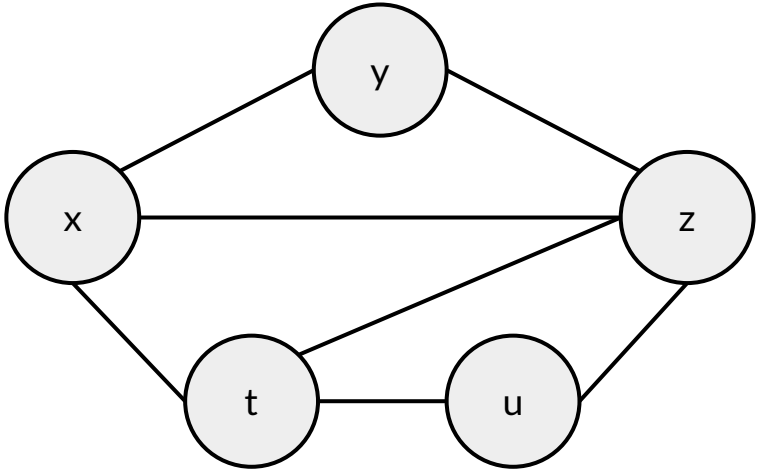

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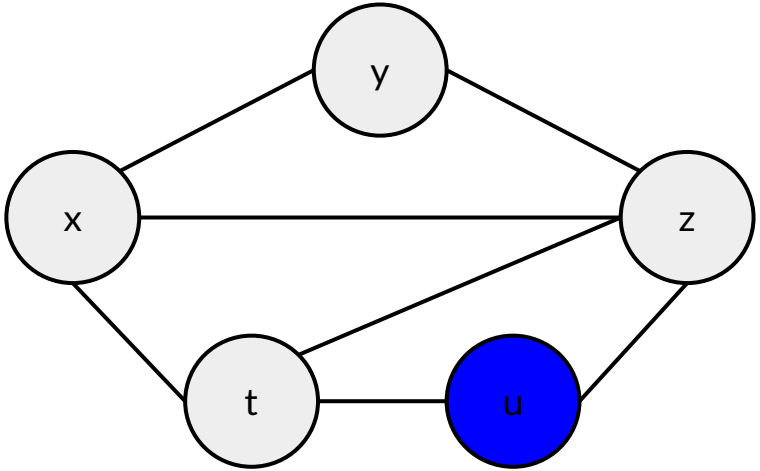

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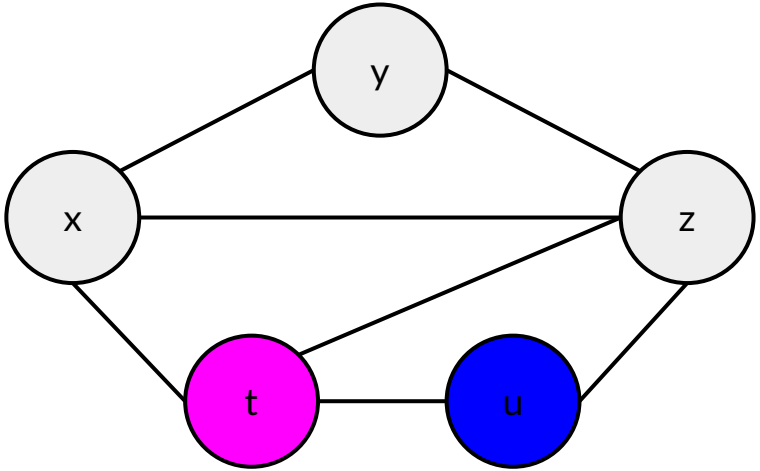

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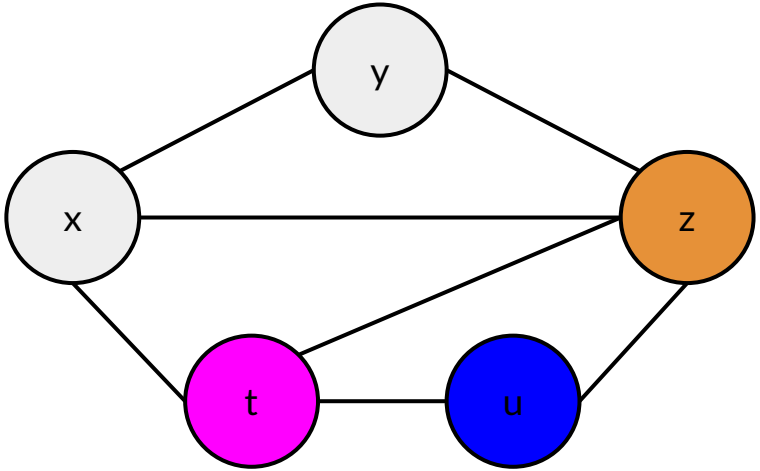

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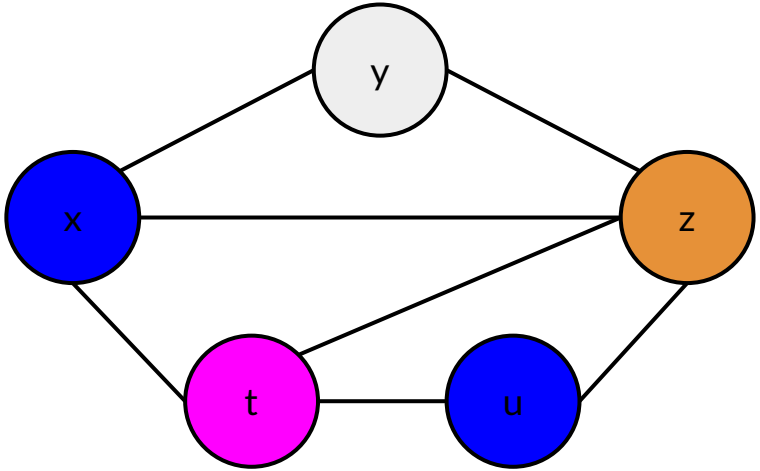

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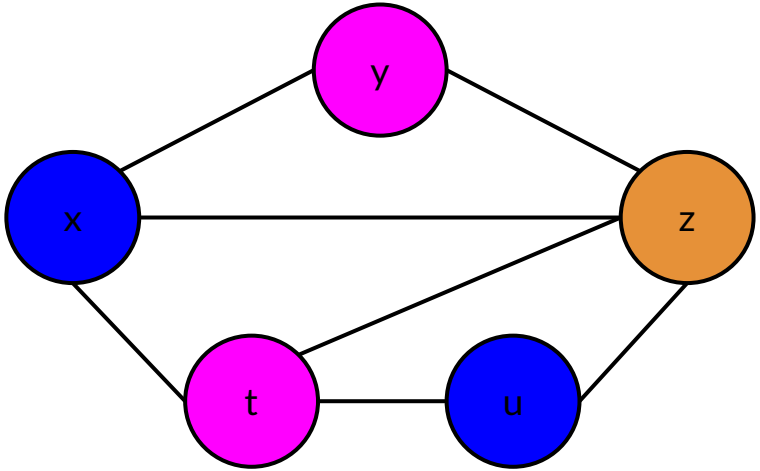

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  - this is easy to handle: just **pre-color** those nodes in the graph

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- Implementing a global register allocator correctly is a **challenge**
  - I don't expect all (or even most) of you to succeed at this, and it is not required for PA4

Course Announcements

- The PA4 leaderboard is still Coming Soon™
  - Unfortunately the technical problems here were tougher than I expected, so I am changing how it works
    - More details to come in the next day or two…
- Due to popular demand, I have **changed** PA4c1's specification to also allow the input to be a `.cl-type` file (originally was TAC)
  - PA4c1 is due April 28, and is mostly optional