Regional Optimizations

Martin Kellogg

Course Announcements

- Graded midterms are at the front of the room
  - If you don't have it yet, pick it up after class
  - If you take it with you, I won't accept regrade requests
- A problem with the PA3c3 autograder was found over the weekend
  - I've therefore granted an extension to today (AoE)
    - Same extension for PA3
- We recently fixed a bug in the reference compiler's x86-64 module. Only use Cool **version 1.39** for compiling to x86.

Agenda

- Finish survey of local optimizations
- Deep dive on how to implement local value numbering
- Intro to regional optimizations
- Lifting local value numbering to a regional optimization
- Other regional optimizations

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- Common Subexpression Elimination replaces duplicated right-hand sides of expressions
  - e.g., if `x := y + z` and `w := y + z`, then `w := x`

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- **Single Static Assignment** (**SSA**) form is a useful IR for optimizations, because all variables are referentially transparent
- **Common Subexpression Elimination** replaces duplicated right-hand sides of expressions
  - e.g., if `x := y + z` and `w := y + z`, then `w := x`
- **Copy Propagation** replaces the LHS of assignments with the RHS
  - e.g., if `x := y`, replace subsequent uses of `x` with `y`

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- Example (assume that a is not used anywhere else)

```
x := z + y
a := x          ⟶
x := 2 * a
```

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x := z + y          b := z + y
a := x      →       a := b      →
x := 2 * a          x := 2 * b
```

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- Typically optimizations **interact**
  - Performing one optimization enables (or disables!) other optimizations
- Typical optimizing compilers repeatedly perform optimizations until no improvement is possible
  - **Phase ordering problem** again: must beware of local minima
- Interpreters and JITs must be fast!
  - The optimizer can also be stopped at any time to limit the compilation time

An Example

- Initial code:

```
a := x ** 2
b := 3
c := x
d := c * c
e := b * 2
f := a + d
g := e * f
```

An Example

- Algebraic simplification:

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- Algebraic simplification:

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An Example

- Copy propagation:

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a := x * x
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- Copy propagation:

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a := x * x
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e := 3 + 3
f := a + d
g := e * f
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An Example

- Constant folding:

```
a := x * x
b := 3
c := x
d := x * x
e := 3 + 3
f := a + d
g := e * f
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An Example

- Constant folding:

```
a := x * x
b := 3
c := x
d := x * x
e := 6
f := a + d
g := e * f
```

An Example

- Common subexpression elimination:

```
a := x * x
b := 3
c := x
d := x * x
e := 6
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f := a + a
g := 6 * f
```

Could we get to `g = 12 * a`?

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  - e.g., can we safely constant fold in a block that may divide by zero?
- Programmers might protest that they don't write code that can be easily improved by such "simple" optimizations
  - But keep in mind that **the compiler** is actually generating most of the code you're optimizing (e.g., array accesses)

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- The key idea of LVN is to assign a **distinct number** (called the “***value number***”) to **each value** computed by the basic block
  - LVN’s goal: assign the same number to two different expressions iff they are **provably equal**

LVN: Core Algorithm

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    - if found, use already-assigned value number
    - if not found, assign a new value number
  - creates a new string key `k` = VN_L `Op` VN_R
  - looks up `k` in the table, assigning a new value number if not found

LVN: Example

```
a := b + c
b := a - d
c := b + c
d := a - d
```

Value table:

key	value num

LVN: Example

```
a := b + c
b := a - d
c := b + c
d := a - d
```

Value table:

key	value num
b	0

LVN: Example

```
a := b + c
b := a - d
c := b + c
d := a - d
```

Value table:

key	value num
b	0
c	1

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a := b + c
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LVN: Example

```
a := b + c
b := a - d
c := b + c
d := a - d
```

Value table:

key	value num
`b`	0
`c`	1
`0 + 1`	2

LVN: Example

```
a := b + c
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```

Value table:

key	value num
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c	1
0 + 1	2
a	2

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a := b + c
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```

Value table:

key	value num
`b`	0
`c`	1
`0 + 1`	2
`a`	2

Note how `a` and `0 + 1` get the **same** value number!

LVN: Example

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a := b + c
b := a - d
c := b + c
d := a - d
```

Value table:

key	value num
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LVN: Example

```
a  := b + c
b  := a - d
c  := b + c
d  := a - d
```

already have an entry for **a**

Value table:

key	value num
b	0
c	1
0 + 1	2
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LVN: Example

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a := b + c
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Value table:

key	value num
`b`	0
`c`	1
`0 + 1`	2
`a`	2
`d`	3

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a := b + c
b := a - d
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d := a - d
```

Value table:

key	value num
b	0
c	1
0 + 1	2
a	2
d	3
2 - 3	4

LVN: Example

```
a := b + c
b := a - d
c := b + c
d := a - d
```

Value table:

key	value num
b	~~0~~ 4
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0 + 1	2
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LVN: Example

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a  := b + c
b  := a - d
c  := b + c
d  := a - d
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already have entries for `b, c`

Value table:

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Value table:

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a := b + c
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already have an entry for **2 − 3**!

Value table:

key	value num
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0 + 1	2
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d	3
2 - 3	4
1 + 4	5

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How to use this information?

We can replace a - d with anything with value number 4

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- LVN can incorporate **other optimizations**
  - e.g., if operands are constants, storing that info directly in the table enables constant folding during LVN
- LVN is highly **order-dependent**: rewriting the code to change the order of operations may change the results

Trivia Break: Literature

This epic 1862 novel follows the lives and interactions of several characters from 1815 until the 1832 June Rebellion in Paris. It is one of the longest novels ever written in French, at 655,478 words. The novel contains many digressions - comprising more than a quarter of its pages - that do not advance the plot in any way. Despite this, Upton Sinclair described it as "one of the half-dozen greatest novels of the world." It has been has been popularized through numerous adaptations for film, television, and the stage, including a musical. Its author is Victor Hugo, whose other works include *The Hunchback of Notre-Dame.*

Trivia Break: Mathematics

This French republican political activist was repeatedly arrested as a teenager in the late 1820s and early 1830s in the lead up to the June Rebellion (which is famously the setting for *Les Misérables*), before dying in a duel just a few days before the uprising began, aged just 20. Despite his involvement in politics, he was an active research mathematician. His work in mathematics, though not appreciated during his lifetime, laid the foundations for two major branches of abstract algebra, one of which is named for him. He also solved a problem open for over 350 years: determining a necessary and sufficient condition for a polynomial to be solvable by radicals.

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    - Peanut gallery commentary is **encouraged**: I am by no means the world's best systems programmer
- The problem: we're generating assembly code for Cool programs that call into libc, and printf is segfaulting
    - You will **not** be able to reproduce this behavior on v1.36 of the reference compiler :)

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- Primary difference between local and regional optimizations is the need to handle **control flow**
  - e.g., if/else, jumps, etc.
- We will look at two examples:
  - extending local value numbering to regions
  - loop unrolling

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  - all basic blocks besides the entry must only have predecessor blocks that are members of the EBB

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Extended Basic Blocks

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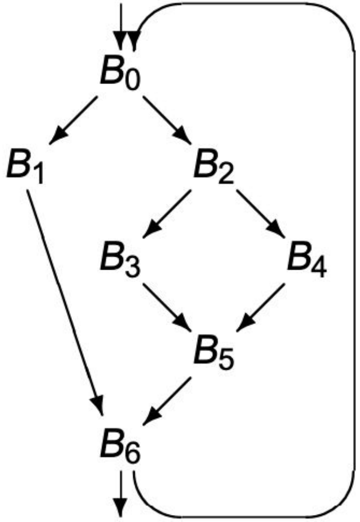
- I will show how we extend **local value numbering** to a regional optimization; others are left as an exercise

Extended Basic Blocks: Example



$$
\begin{aligned}
B_0:\ & m_0 \leftarrow a_0 + b_0 \\
& n_0 \leftarrow a_0 + b_0 \\
& (a_0 > b_0) \rightarrow B_1, B_2
\end{aligned}
$$

$$
\begin{aligned}
B_1:\ & p_0 \leftarrow c_0 + d_0 \\
& r_0 \leftarrow c_0 + d_0 \\
& \rightarrow B_6
\end{aligned}
$$

$$
\begin{aligned}
B_2:\ & q_0 \leftarrow a_0 + b_0 \\
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$$
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B_3:\ & e_0 \leftarrow b_0 + 18 \\
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& \rightarrow B_5
\end{aligned}
$$

$$
\begin{aligned}
B_4:\ & e_1 \leftarrow a_0 + 17 \\
& t_0 \leftarrow c_0 + d_0 \\
& u_1 \leftarrow e_1 + f_0 \\
& \rightarrow B_5
\end{aligned}
$$

$$
\begin{aligned}
B_5:\ & e_2 \leftarrow \phi(e_0, e_1) \\
& u_2 \leftarrow \phi(u_0, u_1) \\
& v_0 \leftarrow a_0 + b_0 \\
& w_0 \leftarrow c_0 + d_0 \\
& x_0 \leftarrow e_2 + f_0 \\
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$$
\begin{aligned}
B_6:\ & r_2 \leftarrow \phi(r_0, r_1) \\
& y_0 \leftarrow a_0 + b_0 \\
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(don't worry about the details)

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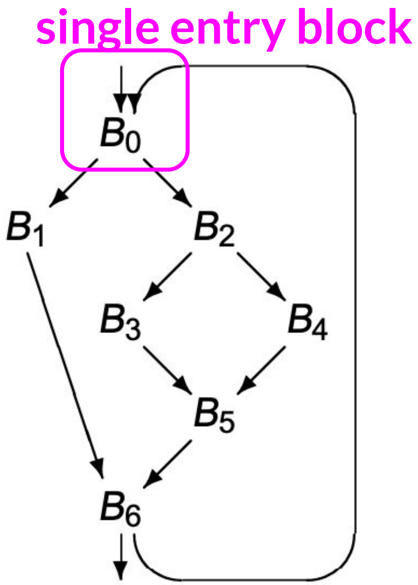

B_0: $m_0 \leftarrow a_0 + b_0$
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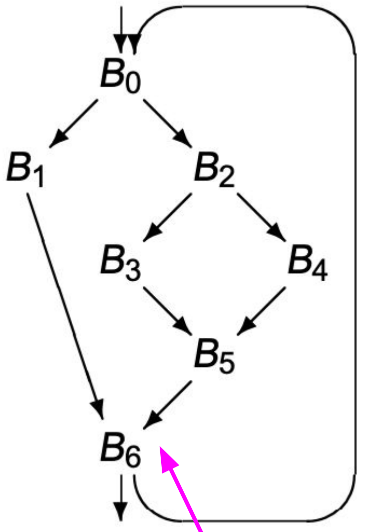

control-flow joins

B_0: $m_0 \leftarrow a_0 + b_0$
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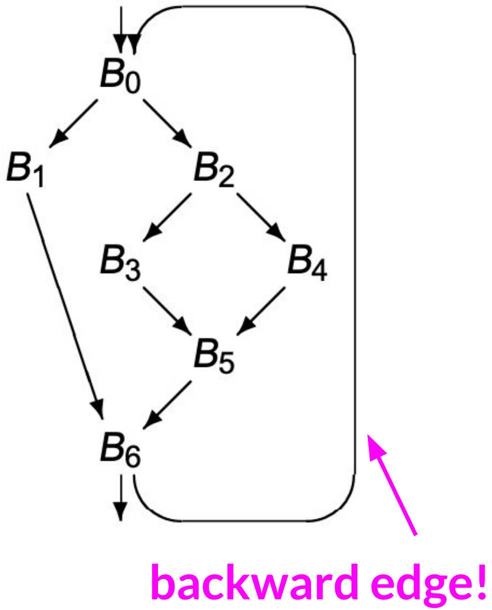

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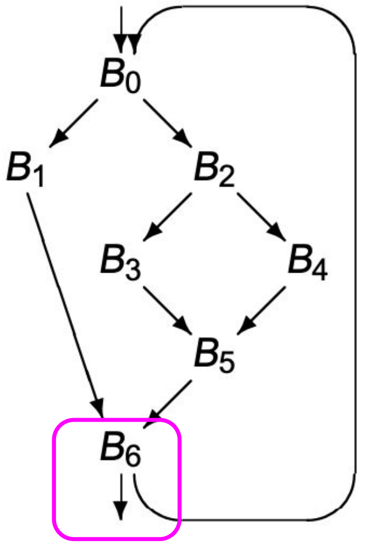

single exit block

$$
\begin{aligned}
B_0:\;& m_0 \leftarrow a_0 + b_0\\
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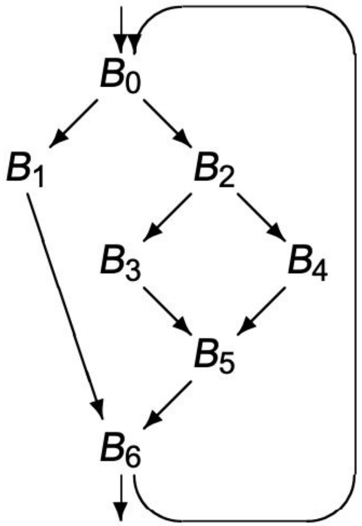
Super-local Value Numbering



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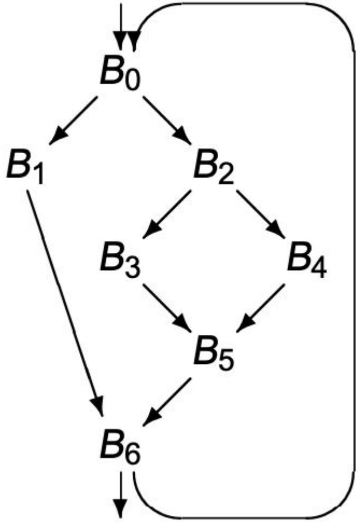

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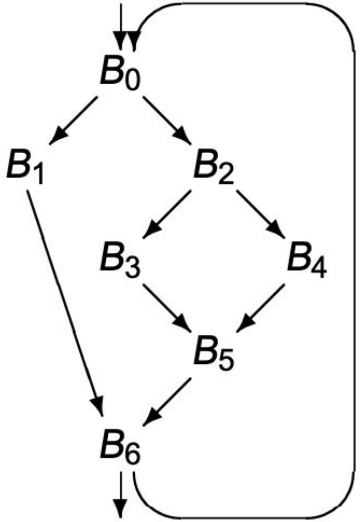

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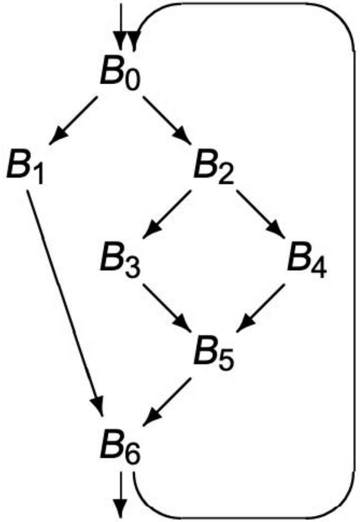

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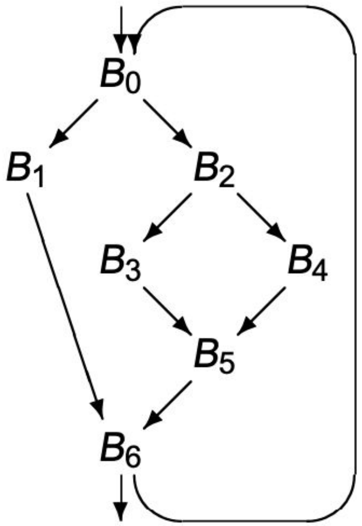

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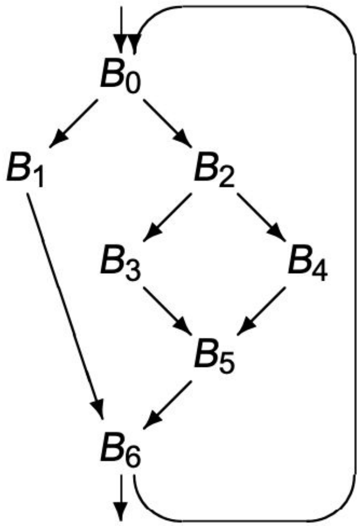

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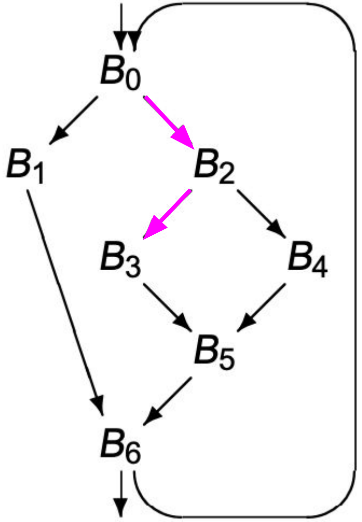
- Any block with multiple predecessors, such as B_5, can use a fresh hashtable

S-LVN: Example



consider the path B_0, B_2, B_3

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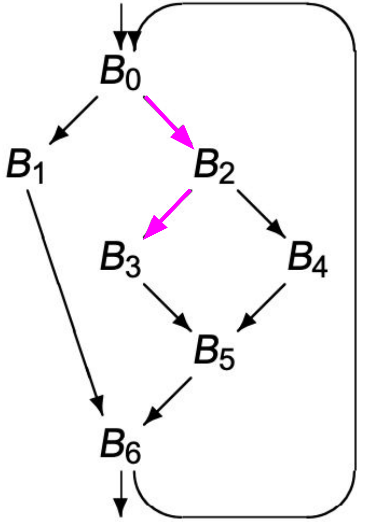

consider the path B_0, B_2, B_3

combine into a single logical block

$$
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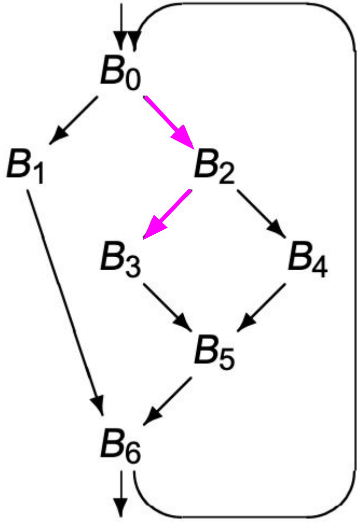

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LVN can discover this redundancy on its own…

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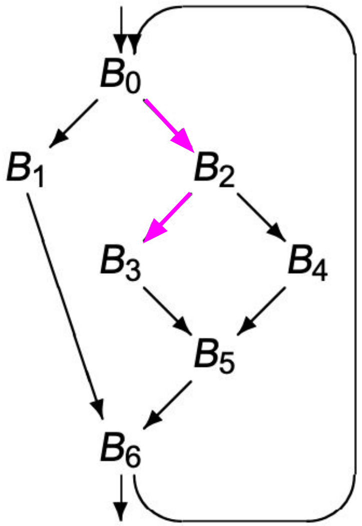

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...but we need S-LVN to discover these two!

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For more details on this algorithm, see the book.

Other Regional Optimizations

- Loop unrolling
- Code motion
- Loop induction variable elimination

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- Loop unrolling changes the **ratio** of arithmetic to memory operations in the loop

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Whether or not to unroll a loop often depends on these factors, so there is **no one-size-fits-all algorithm** for deciding whether to unroll

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- Benefit: avoids **redundant computation** each time around the loop

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```
for (p = &a[0]; p < &a[10]; p = p+4){
    *p = *p + x;
}
```

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- Nearly all local optimizations can be extended to work at the regional level
  - Which you want to use is up to you!

Course Announcements

- Graded **midterms** are at the front of the room
  - If you don't have it yet, pick it up after class
  - If you take it with you, I won't accept regrade requests
- A problem with the PA3c3 autograder was found over the weekend
  - I've therefore granted an extension to **today** (AoE)
    - Same extension for PA3
- We recently fixed a bug in the reference compiler's x86-64 module. Only use Cool **version 1.39** for compiling to x86.