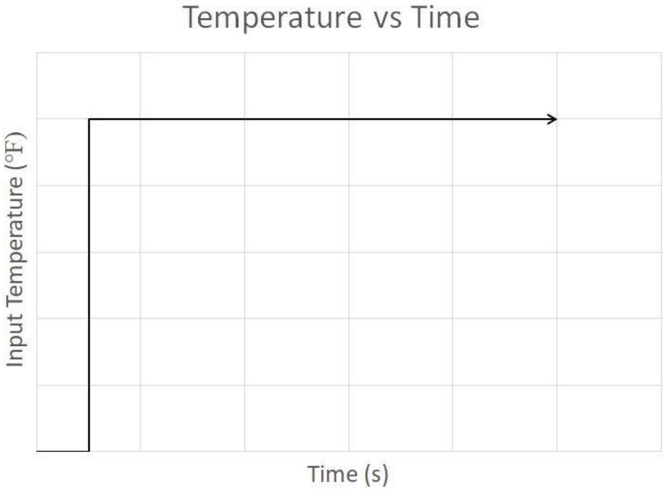
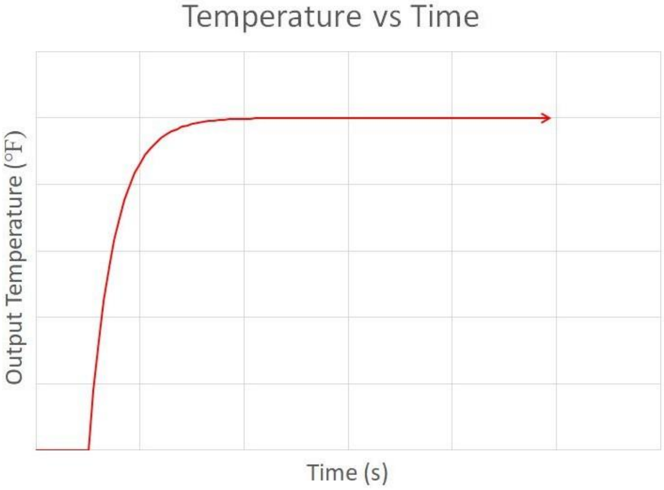
Dynamic Measurements

Dynamic Measurements

It often takes a measurement system time to reach equilibrium, even for static measurands (like when taking your temperature).





The situation is further complicated when the input temperature changes with time.

Dynamic Measurements

Heat transfer and energy absorption by thermometer bulb

$$q = hA(T_{mouth} - T_{bulb}) = mc\frac{dT}{dt}$$

where

- A = bulb surface area
- c = heat capacity of bulb
- h = heat transfer coefficient
- m = mass of bulb

Rearranging,

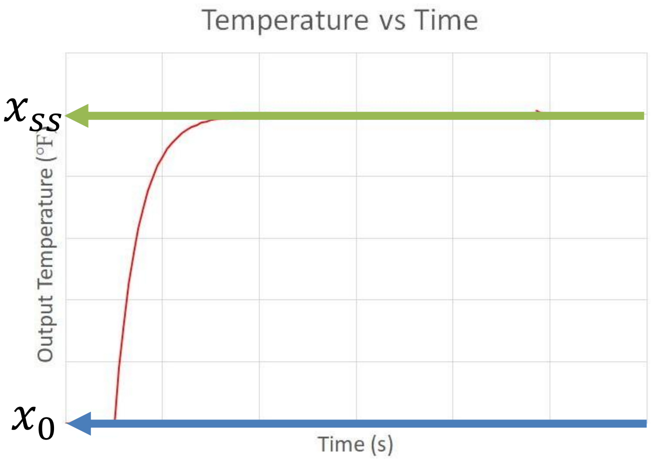
$$T_{mouth} - T_{bulb} = \frac{mc}{hA}\frac{dT}{dt}$$

Time constant - As the mass of the bulb decreases, the response time decreases

Dynamic Measurements

$$\tau = time\ constant = \frac{mc}{hA}$$

The thermometer is an example of a 1st order system. These systems have energy storage effects.



The solution to a 1st order system is

$$\frac{x(t) - x_{ss}}{x_0 - x_{ss}} = e^{\frac{-t}{\tau}}$$

where

- x is the response (in the thermometer example it is temperature)
- x_o and x_{ss} are the initial and steady state value of the system
- t is time
- τ is the time constant

Example

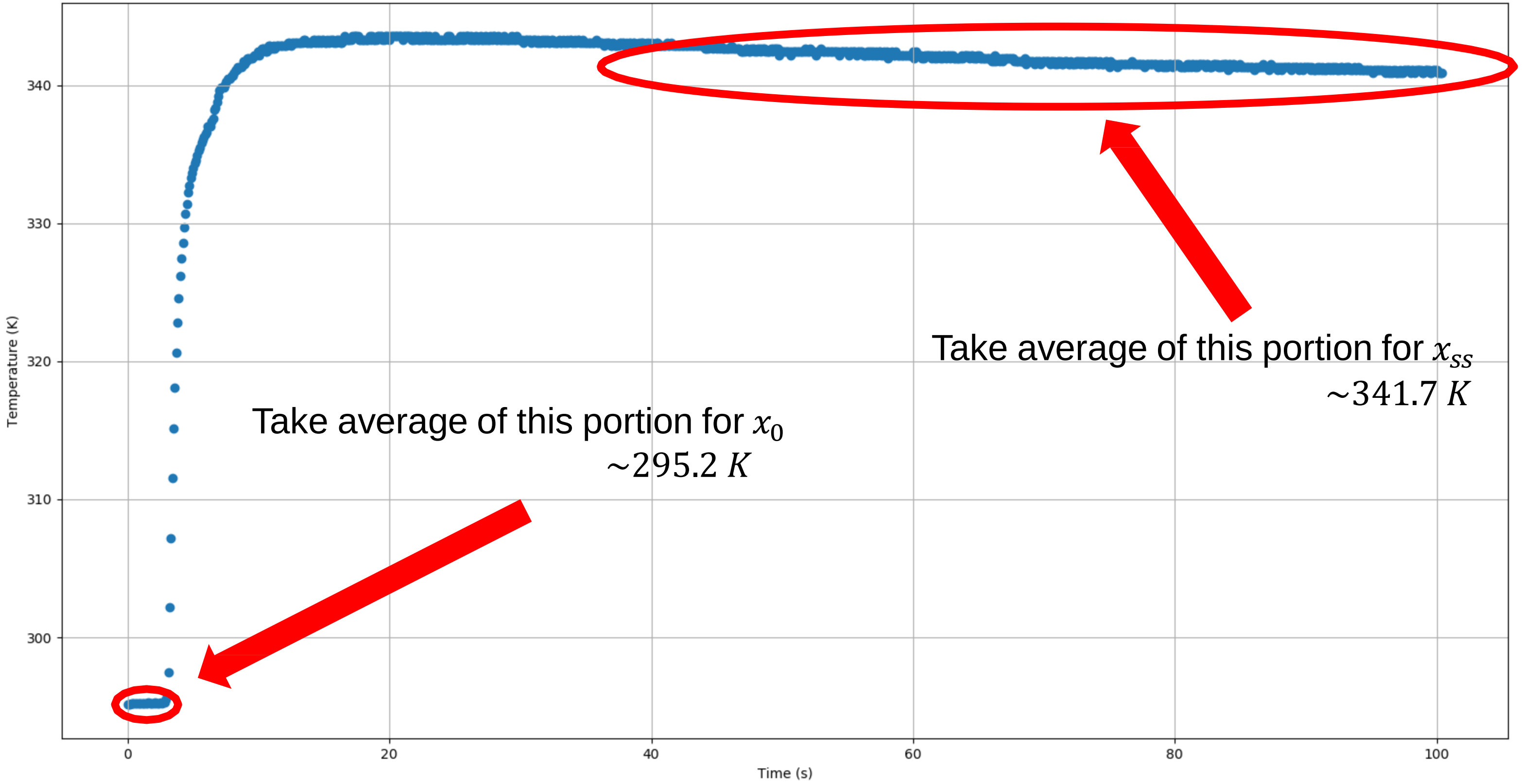
Thermistor data here

ADC equation: $V_2 = aR\frac{5V}{1023}$

Voltage divider equation: $R_t = R_2\left(\frac{V_s}{V_2} - 1\right)$

Manufacturer's Temperature equation: $T = \frac{B}{ln\left(\frac{R_t}{R_1}e^{\frac{B}{T_1}}\right)}$

Example (contd.)



Example (contd.)

Special point in 1st order response equation. $$\frac{x(t) - x_{ss}}{x_0 - x_{ss}} = e^{\frac{-t}{\tau}}$$

when $t = \tau$

then $$\frac{x(\tau) - x_{ss}}{x_0 - x_{ss}} = e^{-1} \approx 0.3679$$

This is useful!

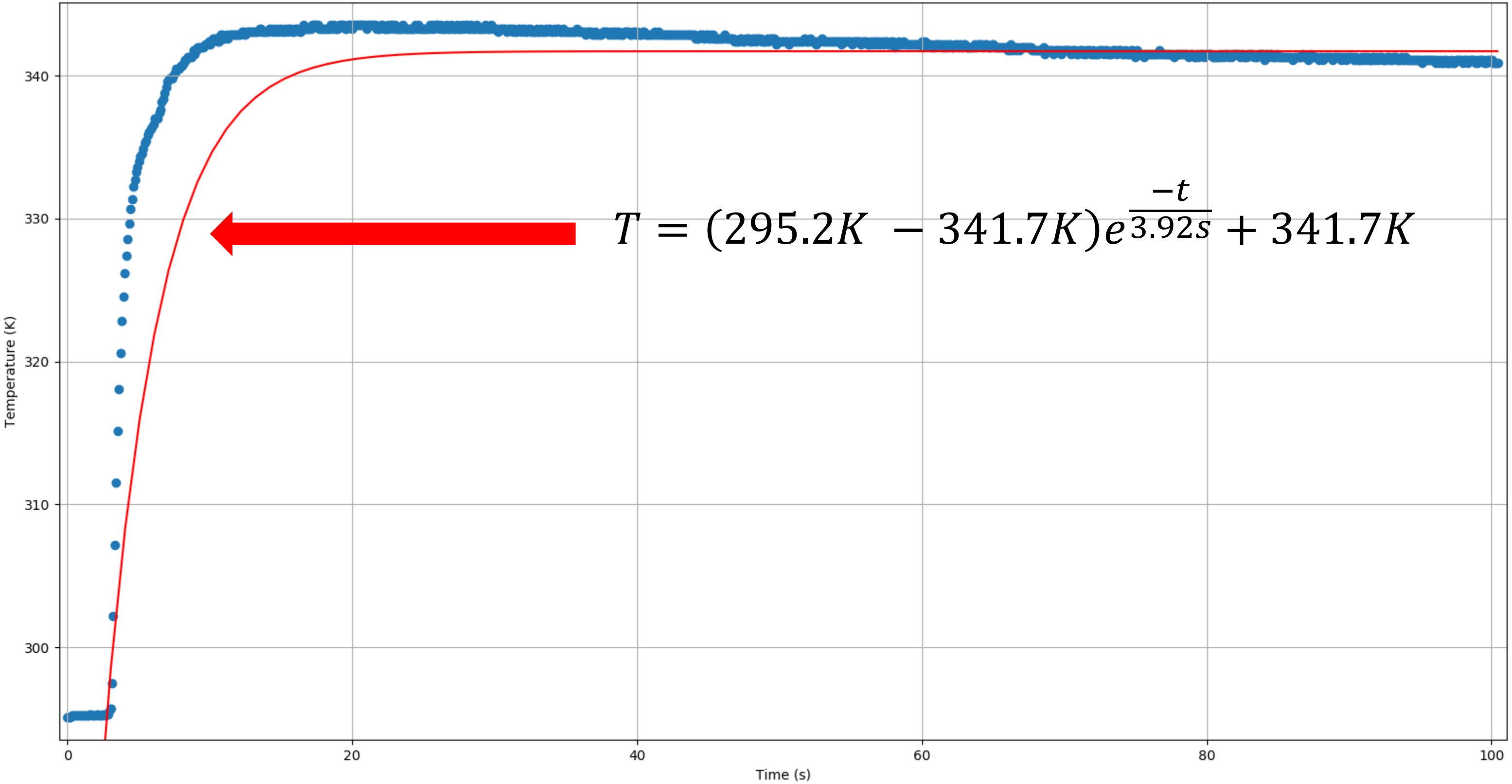
$$x(\tau) \approx 0.3679(x_0 - x_{ss}) + x_{ss}$$

$$x(\tau) \approx 0.3679(295.2K - 341.7K) + 341.7K$$

$$x(\tau) \approx 324.6K$$

Lookup $T = 324.6K$ and find t. This give you τ. $\tau \approx 3.92s$

Example (contd.)



$$T = (295.2K - 341.7K)e^{\frac{-t}{3.92s}} + 341.7K$$

Dynamic Measurements

General differential equation form of 1st order system subject to a step input

$$\tau\dot{x}(t) + x(t) = x_{ss}\Phi(t)$$

where $\Phi(t)$ is known as the Heaviside function. It is a unit step function.

At $t = 0$,

$$\tau\dot{x}(0) + x(0) = x_{ss}\Phi(0)$$

$$\tau\dot{x}(0) + x_0 = x_{ss}$$

Solving for the time constant,

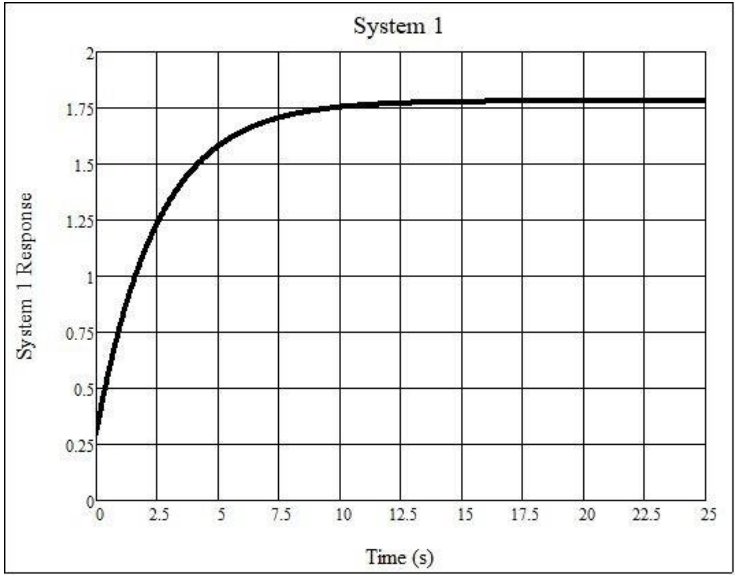
$$\tau = \frac{x_{ss} - x_0}{\dot{x}(0)}$$

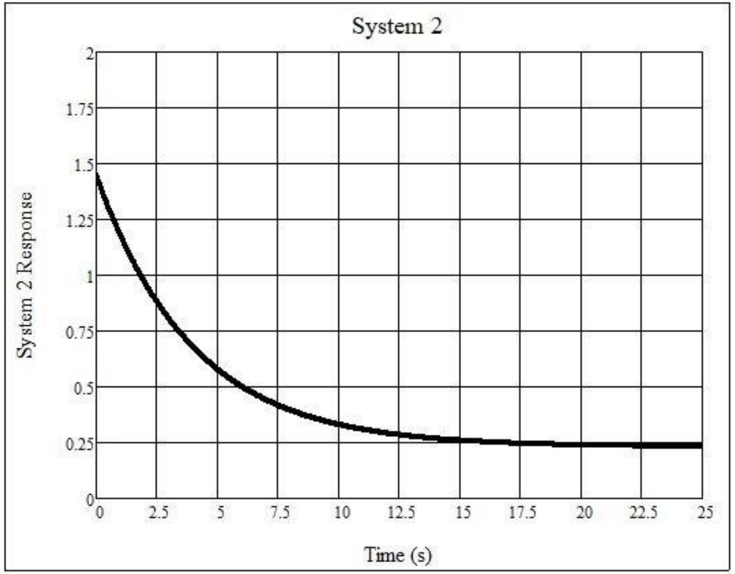
This is useful!

- x_{ss} - steady state value
- x_0 - initial value
- $\dot{x}(0)$ - slope at $t = 0$

Example

The 1st order responses of two different systems subject to a step input are shown below. Which has the higher time constant? What are the response equations for each system?





System 1

From the plot

$$x_0 \approx 0.3$$
$$x_{ss} \approx 1.8$$

For 1st Order system

$$\frac{x(t) - x_{ss}}{x_0 - x_{ss}} = e^{\frac{-t}{\tau}}$$

When $t = \tau$

$$x(\tau) = (x_0 - x_{ss})e^{-1} + x_{ss}$$

$$x(\tau) \approx (0.3 - 1.8)0.3679 + 1.8 = 1.25$$

On the plot where $x(\tau) \approx 1.25$ then $t \approx 2.5s$

So $\tau_1 \approx 2.5s$ and $\boxed{x_1(t) \approx (0.3 - 1.8)e^{\frac{-t}{2.5s}} + 1.8}$

System 2

From the plot

$$x_0 \approx 1.4$$

$$x_{ss} \approx 0.25$$

$$\dot{x}(0) \approx \frac{x_2 - x_1}{t_2 - t_1} \approx \frac{0.5 - 1.4}{2.5 - 0} \approx -0.36$$

For 1st Order system

$$\tau_2 = \frac{x_{ss} - x_0}{\dot{x}(0)} \approx \frac{0.25 - 1.4}{-0.36} \approx 3.19s$$

$$\boxed{x_2(t) \approx (1.4 - 0.25)e^{\frac{-t}{3.19s}} + 0.25}$$

System 2 has a larger time constant.