"Normal" proofs: want $\underline{A \Rightarrow E}$

$A \Rightarrow B$ (Axiom 1)

$B \Rightarrow C$ (algebra)

$C \Rightarrow D$ (other theorem)

$D \Rightarrow E$ (other other theorem)

New technique: <u>induction</u>.

Usual setting: have some <u>parameterized</u> statement, with parameter $n \in \mathbb{N}$.

Call this $T(n)$.

E.g., $T(n)$ might state that

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Blueprint for induction:

① Prove a "base case": prove $T(n)$ explicitly for small value of n, e.g. $n=1$.

In the above: $\sum_{i=1}^{1} i = \frac{1(1+1)}{2} = 1$ ✓

② Show $T(n) \Rightarrow T(n+1)$:

Assuming $T(n)$, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

Then $\sum_{i=1}^{n+1} i = \sum_{i=1}^{n} i + (n+1)$

$$= \frac{n(n+1)}{2} + (n+1)$$ (using assumption $T(n)$ true)

$$= \frac{n(n+1) + 2(n+1)}{2}$$

$$= \frac{(n+1)(n+2)}{2} \quad \checkmark \quad \equiv T(n+1).$$

Note: we've proved that a "normal" proof <u>exists</u> for any value of $n \geq 1$:

$$T(1) \underset{②}{\Longrightarrow} T(2) \underset{②}{\Longrightarrow} T(3) \underset{②}{\Longrightarrow} \cdots \underset{②}{\Longrightarrow} T(n)$$

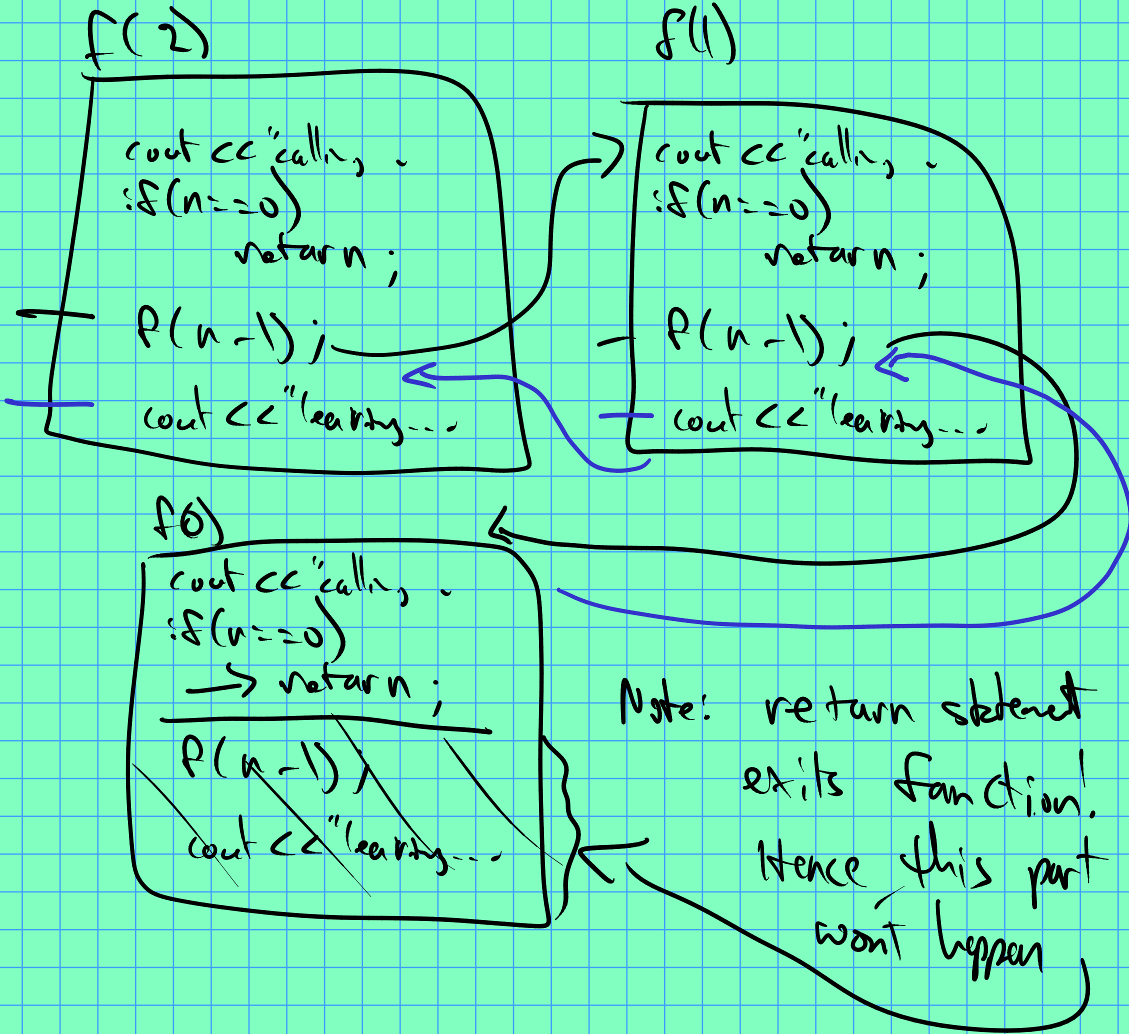
explicit in ① (base case)

Recursion: example: compute $n!$

```
int fac (int n)
{   //base case:
  → if (n ==0) return 1;
    // note n! = n·(n-1)!
    return n· fac(n-1);
}
```

$$fac(3) \longrightarrow fac(2) \longrightarrow fac(1) \longrightarrow fac(0)$$

$3 \cdot 2$ $\quad$ $2 \cdot 1$ $\quad$ $1 = 1 \cdot 1$ $\quad$ 1




```
int fib(int n)
{
    if (n < 2) return 0;
    return fib(n-1) + fib(n-2);
}
```