Problem 1: Suppose $(x_i, y_i)_{1 \leq i \leq n}$ are a collection of data points.

Write a code in R for finding the equation of the line $y = ax + b$ such that "*the maximum error is minimized*" i.e. minimize the maximum error. More specifically if $\varepsilon_i = |ax_i + b - y_i|$ then we wish to minimize the quantity,

$$\max(\varepsilon_1, ..., \varepsilon_n)$$

by choosing a and b.

The objective function,

$$f(a, b) = \max_{1 \leq i \leq n} |ax_i + b - y_i|$$

is not a linear function. Therefore, extra tricks will be required to convert this problem into a linear optimization problem.

Instead consider the linear function of three variables,

$$g(a, b, \theta) = \theta$$

Minimize this linear function subject to the constraints that,

$$|ax_i + b - y_i| \leq \theta$$

(Note, in order for θ to be small as possible but to still $\geq \varepsilon_i$, it means that θ must be the maximum of all the ε's. Therefore, this new linear version of the property will achieve the minimum that we seek.)

Call your code `best.line.minmax(x,y)`. In constrast, in class we developed a code called `best.line(x,y)`. The line-of-best-fit was optimal from all lines that minimized the total error.

Problem 2: The package `datasets` is probably automatically loaded when you start R. If you are starting R Studio Cloud then it will automatically be loaded. This package is just a collection of various data that was used in research. The only purpose to these data sets is to practice.

One of the data sets present in this package is called `women`. If you run `women` in R you will get a table of 15 women listed according to their height (inches) and weight (lbs) that was used in someone's research. Type,

```
x=women$height
y=women$weight
```

You can check now that `x` and `y` are vectors consisting 15 numbers each. If you type `plot(x,y)` will get a scatter plot of women's height vs women's weight. Notice how close to a straight line these lines are positioned.

Find the line-of-best-fit which minimizes the maximum error (previous problem) and find the line-of-best-fit which minimizes the total error (from class). Compare your two answers. Plot both of these lines visually together in the same plot. You should notice that even though you used two different methods the very close to each other.