Example 1 : Recursive Bayes Learning (pp. 98)

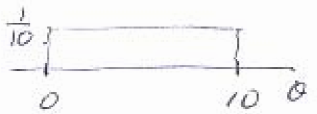
$$p(x|\theta) \sim U(0,\theta) = \begin{cases} \frac{1}{\theta} & 0 \le x \le \theta \\ 0 & \text{otherwise} \end{cases}$$

· assume : θ is bounded and $0 < \theta \le 10$. (flat prior) noninformative.

$D = \{4, 7, 2, 8\}$. : randomly selected.

· Recursive Bayes to estimate θ & the underlying density.

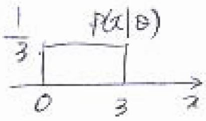
$$p(\theta|D^0) = p(\theta) = U(0,10)$$



$$p(\theta|D^1) = \frac{p(x_1|\theta)\, p(\theta|D^0)}{\int \quad '' \quad d\theta} \propto p(x_1|\theta)\, p(\theta|D^0)$$

$$= \begin{cases} \frac{1}{\theta} & 4 \le \theta \le 10 \\ 0 & \text{otherwise} \end{cases} \leftarrow \quad p(x=4|\theta) = \begin{cases} \frac{1}{\theta} & \text{if } 4 \le \theta \le 10 \\ 0 & \text{otherwise} \end{cases}$$

e.g.) if $\theta = 3$, then



i.e. $p(x=4|\theta) = 0$.

$$p(\theta|D^2) \propto p(x_2|\theta)\, p(\theta|D^1)$$

$$= \begin{cases} \frac{1}{\theta^2} & 7 \le \theta \le 10 \\ 0 & \text{otherwise} \end{cases}$$

$$p(\theta|D^3) = \begin{cases} \frac{1}{\theta^3} & 7 \le \theta \le 10 \\ 0 & \text{otherwise} \end{cases} \qquad p(\theta|D^4) = \begin{cases} \frac{1}{\theta^4} & 8 \le \theta \le 10 \\ 0 & \text{otherwise} \end{cases}$$

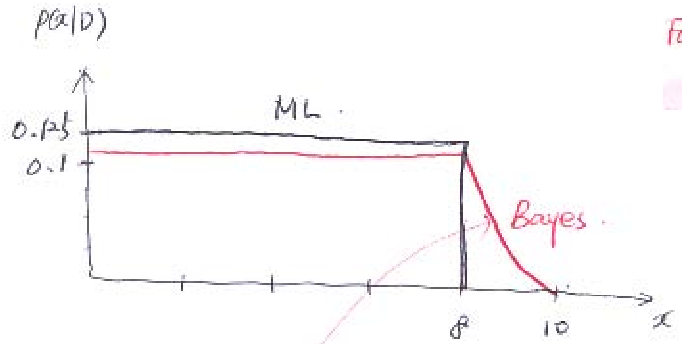
$$p(\theta|D^n) = \begin{cases} \frac{1}{\theta^n} & \max_x(D_n) \le \theta \le 10 \\ 0 & \text{otherwise} \end{cases}$$

$$p(D|\theta) = p(x_1|\theta)\, p(x_2|\theta)\, p(x_3|\theta)\, p(x_4|\theta) = \begin{cases} \frac{1}{\theta^4} & 0 \le x \le \overset{8}{\overset{\shortparallel}{\theta}} \\ 0 & \text{otherwise} \end{cases}$$

MLE : $\theta_{ML} = 8$.

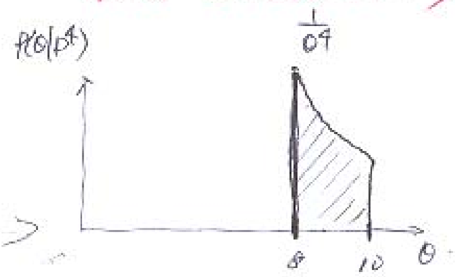
$\because$ 1) if $\theta < 8$ then at least one term (e.g. $p(x=8|\theta)$) equals 0.
Therefore $p(D^4|\theta) = 0$.

2) if $\theta > 8$ the value of each term $(= \frac{1}{\theta})$ decreases $\Rightarrow p(D^4|\theta)$ decreases.



For ML, values of x larger than 8 is impossible while they are possible for Bayes.

in prior information too. ($p(x|\theta) \sim U(0,\theta)$, $0 \le \theta \le 10$)



Bayes Est.

$$p(x|D) = \int_8^{10} p(x|\theta)\,p(\theta|D)\,d\theta$$

$$= \begin{cases} \int_8^{10} \frac{1}{\theta} \cdot \frac{1}{\theta^4}\,d\theta & \text{if } x \le 8 \\ \int_x^{10} \frac{1}{\theta} \cdot \frac{1}{\theta^4}\,d\theta & \text{if } 8 < x \le 10 \end{cases}$$

$\because$ $p(x|\theta) = 0$ for $8 \le \theta < x$