

# Depth From Defocus: A Real Aperture Imaging Approach



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# Depth From Defocus: A Real Aperture Imaging Approach

With a Foreword by Alex (Sandy) Pentland

With 58 Illustrations



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*To*

*Dearest Sucharita and Ushasi.*

– *SC*

*Dearest Dad, Mom, Lakshmi, and (late) Shridhar.*

– *ANR*

# Foreword

It is now more than fifteen years since I first noticed that defocus in an image could be an important source of depth information. Although this period is brief when compared to the history of stereo or structured light range methods, already depth from defocus has taken its place as an important sensing method. This rapid progress has been because of the intrinsic simplicity of the method: there is no expensive searching for corresponding points in two images, and because there is no correspondence problem there are none of the seemingly-inevitable large errors that come from bad matches.

The main limiting factors for depth from defocus are camera quality, and the sophistication of the signal processing. Progress in cameras has been extremely rapid over the last few years: you can now buy megapixel cameras with an honest 14 bits of intensity resolution. The great accuracy of modern cameras makes possible great accuracy in depth from defocus.

However good cameras alone are not enough. The signal processing that extracts depth must also be extremely sophisticated, and that is the subject of this book. The authors have done a superb job of describing and comparing the various depth-from-defocus methods that have been proposed, an important topic for anyone who has been following this topic. They then move substantially beyond the previous state-of-the-art to develop (and experimentally compare) sophisticated Maximum Likelihood (ML) and Markov Random Field (MRF) methods for depth extraction. In doing so they have made a very significant contribution to the mathematical underpinning of depth-from-defocus, and have in addition made it a much more accurate and practical method.

In summary, I would like to congratulate the authors on their very careful and complete work, and strongly recommend this book to you the reader. I think that you will find reading this book an excellent use of your time.

September 1998

Alex (Sandy) Pentland  
Academic Head, The Media Laboratory  
Toshiba Professor of Media Arts and Sciences  
Massachusetts Institute of Technology

# Preface

The recovery of depth from defocused images involves calculating the depth of various points in a scene by modeling the effect that the focal parameters of the camera have on images acquired with a small depth of field. Given two images of a scene recorded with different but known camera parameter settings, the relative blur between the defocused images is measured to obtain an estimate of the depth of the scene. Research on depth from defocus (DFD) was first initiated by Alex Pentland in 1982. Since then, new algorithms have been continuously emerging in the literature for tackling the DFD problem.

The literature on vision abounds with methodologies for extracting depth (or structure) information from various cues in the scene. Among these, motion and stereo disparity are most common and extensively used. Other popular cues include shading and defocus blur. Methods based on these different cues have their own merits and limitations in terms of accuracy, computational complexity, calibration procedures, and associated assumptions. However, one aspect that is very clear is that except for the methods based on measuring the defocus blur, all methods uniformly assume a pin-hole model for the camera, or in other words, the images are not captured with a real aperture camera. This necessitates a longer shutter time for frame grabbing and the captured image may suffer from additional motion blur. The DFD methods use the properties of a real aperture camera to their advantage and provide us with yet another cue to estimate the structure of a scene. This book looks at various ways in which the above cue can be used to recover the depth accurately but refrains from comparing the performance of the DFD technique with methods based on other visual cues.

The area of DFD has witnessed a steady growth of researchers over time. Unfortunately, no single compendium of literature currently exists to facilitate



a comprehensive discussion on DFD techniques. This book is an outgrowth of the Ph.D. dissertation work of the author A.N. Rajagopalan under the guidance of the author Subhasis Chaudhuri at the Indian Institute of Technology, Bombay. This book is a revised and extended version of the thesis. A substantial part of this book has appeared (or is due to appear) in many conference proceedings and international journals. Additional theorems and more results have been added in this book for the completeness of the topics discussed. What we emphasize in this book are the concepts that underlie the DFD principle and the recent trends in this area. The book presents several new approaches for computing the depth of a scene when multiple, defocused images of the scene are given. The aim of this book is to show that a rich and interesting theory exists with results that are practically applicable. Since the DFD method is already in use in several industrial applications, we believe that the material herein is both timely and relevant.

Even though the principle of DFD has its origin in physical optics, our analysis is largely from a computer vision/image-processing perspective. The manner of presentation has been designed to make the material suitable to a wide audience ranging from engineering and computer science to mathematics and physics. The text is essentially self-contained and is, for the most part, theoretically complete. The mathematical complexity of the book remains at a level well within the grasp of graduate students and requires only introductory preparation in mathematical analysis, matrix algebra, and probability theory.

In particular, this book will be of great value to computer vision/image processing researchers and practitioners who are interested in the use of the depth data and the restored image of a scene for 3D structure recovery and image interpretation, object recognition, and navigation. The intended audience includes both the novice and the expert: to the novice we provide a lucid introduction, and to the expert a fresh and stimulating perspective to breed new ideas. It is our hope that this book will arouse more curiosity and interest in this exciting and vibrant area, and that it will serve as a stepping stone towards an improvement in our collective understanding of this subject.

A very brief synopsis of the contents of the book is as follows. In Chapter 1, we cover some of the well known passive range finders in brief. The basic theory of real aperture imaging is introduced in Chapter 2. The depth from defocus technique is described, and related available literature on DFD is reviewed. Chapter 3 provides a necessary mathematical background on the time-frequency representations, the calculus of variations, and the Markov random fields (MRF) for use in subsequent chapters. A block shift-variant blur model is described in Chapter 4 to incorporate the interaction of blur among neighboring subimages to obtain improved estimates of the blur, and hence, the depth. In Chapter 5, space-variant filtering models derived from space-frequency representations are shown to successfully recover the depth. In Chapter 6, a maximum likelihood estimator is discussed for recovering the depth from blurred images. Based on the Cramér-Rao bound of the variance of the error in the estimate of blur, an optimality criterion is suggested for selecting the camera parameters. Depth recovery using the maximum likelihood principle and multiple blurred versions of the original image is studied in Chapter 7. A

computationally efficient, recursive algorithm is given for computing the likelihood function as more observations are added. The problem of recovering depth given both the focused and the defocused images of a scene is addressed in Chapter 8. The space-variant blur parameter is modeled as an MRF and its maximum *a posteriori* (MAP) estimates are obtained. The problem of simultaneous depth recovery and image restoration is solved in Chapter 9. The space-variant blur parameter and the scene intensity are both modeled as separate MRFs. The book concludes with Chapter 10, wherein the discussions are summarized and suggestions for future work are indicated. A large number of experimental results have been provided in each chapter for a proper appreciation of the methods discussed.

Needless to say, we welcome comments and suggestions from the readers.

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A.N. Rajagopalan

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# Contents

<b>Foreword</b>	<b>vii</b>
<b>Preface</b>	<b>ix</b>
<b>Acknowledgments</b>	<b>xiii</b>
<b>List of Symbols</b>	<b>xix</b>
<b>1 Passive Methods for Depth Recovery</b>	<b>1</b>
1.1 Introduction . . . . .	1
1.2 Different Methods of Depth Recovery . . . . .	2
1.2.1 Depth from Stereo . . . . .	2
1.2.2 Structure from Motion . . . . .	4
1.2.3 Shape from Shading . . . . .	6
1.2.4 Range from Focus . . . . .	8
1.2.5 Depth from Defocus . . . . .	10
1.3 Difficulties in Passive Ranging . . . . .	11
1.4 Organization of the Book . . . . .	12
<b>2 Depth Recovery from Defocused Images</b>	<b>14</b>
2.1 Introduction . . . . .	14
2.2 Theory of Depth from Defocus . . . . .	16
2.2.1 Real Aperture Imaging . . . . .	16
2.2.2 Modeling the Camera Defocus . . . . .	18
2.2.3 Depth Recovery . . . . .	19

2.2.4	Sources of Errors . . . . .	21
2.3	Related Work . . . . .	22
2.4	Summary of the Book . . . . .	25
<b>3</b>	<b>Mathematical Background</b>	<b>28</b>
3.1	Introduction . . . . .	28
3.2	Time-Frequency Representation . . . . .	29
3.2.1	The Complex Spectrogram . . . . .	30
3.2.2	The Wigner Distribution . . . . .	31
3.3	Calculus of Variations . . . . .	32
3.4	Markov Random Fields and Gibbs Distributions . . . . .	34
3.4.1	Theory of MRF . . . . .	35
3.4.2	Gibbs Distribution . . . . .	36
3.4.3	Incorporating Discontinuities . . . . .	38
<b>4</b>	<b>Depth Recovery with a Block Shift-Variant Blur Model</b>	<b>40</b>
4.1	Introduction . . . . .	40
4.2	The Block Shift-Variant Blur Model . . . . .	41
4.2.1	Estimation of Blur . . . . .	43
4.2.2	Special Cases . . . . .	43
4.3	Experimental Results . . . . .	44
4.4	Discussion . . . . .	52
<b>5</b>	<b>Space-Variant Filtering Models for Recovering Depth</b>	<b>55</b>
5.1	Introduction . . . . .	55
5.2	Space-Variant Filtering . . . . .	56
5.3	Depth Recovery Using the Complex Spectrogram . . . . .	57
5.4	The Pseudo-Wigner Distribution for Recovery of Depth . . . . .	58
5.5	Imposing Smoothness Constraint . . . . .	59
5.5.1	Regularized Solution Using the Complex Spectrogram . . . . .	59
5.5.2	The Pseudo-Wigner Distribution and Regularized Solution . . . . .	60
5.6	Experimental Results . . . . .	61
5.7	Discussion . . . . .	66
<b>6</b>	<b>ML Estimation of Depth and Optimal Camera Settings</b>	<b>70</b>
6.1	Introduction . . . . .	70
6.2	Image and Observation Models . . . . .	72
6.3	ML-Based Recovery of Depth . . . . .	73
6.4	Computation of the Likelihood Function . . . . .	75
6.5	Optimality of Camera Settings . . . . .	77
6.5.1	The Cramér-Rao Bound . . . . .	77
6.5.2	Optimality Criterion . . . . .	79
6.6	Experimental Results . . . . .	81
6.7	Discussion . . . . .	89

<b>7</b>	<b>Recursive Computation of Depth from Multiple Images</b>	<b>91</b>
7.1	Introduction . . . . .	91
7.2	Blur Identification from Multiple Images . . . . .	92
7.3	Minimization by Steepest Descent . . . . .	94
7.4	Recursive Algorithm for Computing the Likelihood Function . . . . .	95
7.4.1	Single Observation . . . . .	97
7.4.2	Two Observations . . . . .	97
7.4.3	General Case of $M$ Observations . . . . .	98
7.5	Experimental Results . . . . .	99
7.6	Discussion . . . . .	102
<b>8</b>	<b>MRF Model-Based Identification of Shift-Variant PSF</b>	<b>105</b>
8.1	Introduction . . . . .	105
8.2	A MAP-MRF Approach . . . . .	107
8.3	The Posterior Distribution and Its Neighborhood . . . . .	109
8.4	MAP Estimation by Simulated Annealing . . . . .	112
8.5	Experimental Results . . . . .	113
8.6	Discussion . . . . .	120
<b>9</b>	<b>Simultaneous Depth Recovery and Image Restoration</b>	<b>122</b>
9.1	Introduction . . . . .	122
9.2	Depth Recovery and Restoration using MRF Models . . . . .	124
9.3	Locality of the Posterior Distribution . . . . .	126
9.4	Parameter Estimation . . . . .	129
9.5	Experimental Results . . . . .	131
9.6	Discussion . . . . .	139
<b>10</b>	<b>Conclusions</b>	<b>141</b>
<b>A</b>	<b>Partial Derivatives of Various Quantities in CRB</b>	<b>150</b>
<b>B</b>	<b>CRB of <math>Var(\tilde{s})</math> for a Single Image</b>	<b>154</b>
	<b>References</b>	<b>157</b>
	<b>Index</b>	<b>169</b>

# List of Symbols

$D$	Depth of the scene
$F_l, F_{l_1}, F_{l_2}$	Focal length of camera
$v_0, v_1, v_2$	Image plane to lens distance
$r_b$	Blur radius
$r_0, r_1, r_2$	Lens aperture
$\sigma, \sigma_1, \sigma_2$	Blur parameter
$\rho, \rho_1, \rho_2$	Constants of camera
$f(\cdot)$	Focused image of the scene
$h(\cdot)$	Point spread function of camera
$g(\cdot), g_1(\cdot), g_2(\cdot)$	Defocused images
$C_x(\cdot, \cdot)$	Complex spectrogram of $x(\cdot)$
$\tilde{W}_x(\cdot, \cdot)$	Pseudo-Wigner distribution of $x(\cdot)$
$\check{s}(i, j)$	$\sigma_2^2(i, j) - \sigma_1^2(i, j)$
$\lambda$	Regularization parameter
$a(\cdot)$	AR coefficients
$\sigma_w^2$	Variance of observation noise
$\sigma_v^2$	Variance of noise in AR process
$\hat{s}$	$\sigma_1^2$
$\eta$	MRF neighborhood without line fields
$\bar{\eta}$	MRF neighborhood with line fields
$\bar{U}(\cdot)$	Energy function in MRF model
$V_\epsilon(\cdot)$	Clique potential
$l^{\epsilon}$	Horizontal line field
$v^s$	Vertical line field
$\mu$	Weightage for smoothness in MRF model
$\gamma$	Penalty associated with line fields
$s_{i,j}$	$\sigma_{i,j}$