$$= x \begin{vmatrix} x & y \\ x & y \\ \vdots \\ x & y \end{vmatrix} + (-1)^{n+1} y \begin{vmatrix} y \\ x & y \\ \vdots \\ x & y \end{vmatrix}$$
$$= x^{n} + (-1)^{n+1} y^{n}$$

$$1(5)\begin{vmatrix}
1 & 2 & 2 & \cdots & 2 & 2 \\
2 & 2 & 2 & \cdots & 2 & 2 \\
2 & 2 & 3 & \cdots & 2 & 2 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
2 & 2 & 2 & \cdots & n-1 & 2 \\
2 & 2 & 2 & \cdots & 2 & n
\end{vmatrix}$$

$$= r_1 + r_i \begin{vmatrix} 1 & 2 & 2 & \cdots & 2 & 2 \\ 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 0 & 0 & \cdots & n-3 & 0 \\ 2 & 0 & 0 & \cdots & 0 & n-2 \end{vmatrix} = (-1)^{2+1} \begin{vmatrix} 2 & 2 & 2 & \cdots & 2 \\ 0 & 1 & 0 & & 0 \\ 0 & 0 & 2 & & 0 \\ & & & \ddots & \\ 0 & 0 & 0 & & n-2 \end{vmatrix}$$

$$=2(n-2)!$$

$$\frac{c_n + c_{n-1}}{2} = \begin{vmatrix}
1 & 2 & 3 & (n-1) + n & n \\
1 & -1 & 0 & 0 & 0 \\
2 & -2 & 0 & 0 \\
& & \ddots & & \\
0 & 0 & 0 & 0 & -(n-2) & 0 \\
0 & 0 & 0 & 0 & 0 & -(n-1)
\end{vmatrix}$$

$$\frac{c_{n}+c_{n-1}}{c_{n-1}+c_{n-2}} = \begin{vmatrix}
1 & 2 & 3 & (n-1)+n & n \\
1 & -1 & 0 & 0 & 0 \\
2 & -2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -(n-2) & 0 \\
0 & 0 & 0 & 0 & -(n-2) & 0 \\
0 & 0 & 0 & 0 & -(n-1)+n & n
\end{vmatrix}$$

$$\frac{c_{n-1}+c_{n-2}}{c_{n-1}+c_{n-2}} = \frac{\sum_{k=1}^{n} k \sum_{k=2}^{n} k \sum_{k=3}^{n} k (n-1)+n & n}{0 & -1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & -2 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & -(n-2) & 0 \\
0 & 0 & 0 & 0 & -(n-2) & 0 \\
0 & 0 & 0 & 0 & -(n-1)! = (-1)^{n-1}(n-1)! \frac{n(n-1)}{2} = (-1)^{n-1}\frac{(n+1)!}{2}$$

$$2(\beta_n) = \begin{bmatrix} \cos\theta & 1 & \cdots & & & \\ 1 & 2\cos\theta & 1 & \cdots & & \\ & 1 & 2\cos\theta & \cdots & & \\ & & \vdots & \ddots & \vdots & & \\ & & & \cdots & 2\cos\theta & \\ & & & \cdots & 1 & 2\cos\theta & 1 \end{bmatrix} = \cos n\theta$$

证明: 
$$(1)$$
 当 $n=1$ 时,  $D_1=\cos\theta$ 

(2) 假设当
$$n \leq k$$
时, $D_k = \cos k\theta$ 

$$D_{k+1} = 2\cos\theta D_k + (-1)^{k+1+k}$$

$$\cos\theta \qquad 1$$

$$1 \qquad 2\cos\theta \qquad 1$$

$$\vdots \qquad 1$$

$$2\cos\theta \qquad 0$$

$$1 \qquad 1$$

$$D_{k+1} = 2\cos\theta D_k + (-1)^{k+1+k}$$

$$Cos\theta 1$$

$$1 \quad 2\cos\theta \quad 1$$

$$1 \quad 2\cos\theta \quad 1$$

$$\vdots \quad 1$$

$$2\cos\theta \quad 0$$

$$=2\cos\theta D_{k}-D_{k-1}$$

$$= 2\cos\theta\cos k\theta - \cos(k-1)\theta$$

$$= 2\cos k\theta\cos\theta - \cos(k-1)\theta$$

$$=\cos(k+1)\theta$$

$$2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$$

$$2\cos\alpha\cos\beta-\cos(\alpha-\beta)=\cos(\alpha+\beta)$$

∴ 对
$$\forall n, \not \in D_n = \cos n\theta$$

3(1) 
$$\begin{vmatrix} a_1 + \lambda_1 & a_2 & a_n \\ a_1 & a_2 + \lambda_2 & a_n \\ & & \ddots & \\ a_1 & a_2 & a_n + \lambda_n \end{vmatrix} (\lambda_i \neq 0, i = 1, 2, \dots, n)$$

\_\_\_\_\_\_

$$=\begin{vmatrix} a_1 + \lambda_1 & a_2 & & a_n \\ -\lambda_1 & \lambda_2 & & 0 \\ & & \ddots & \\ -\lambda_1 & 0 & & \lambda_n \end{vmatrix} = \left[ a_1 + \lambda_1 + \frac{\lambda_1 a_2}{\lambda_2} + \dots + \frac{\lambda_1 a_n}{\lambda_n} \right] \lambda_2 \cdots \lambda_n$$

$$= \left[\frac{a_1}{\lambda_1} + 1 + \frac{a_2}{\lambda_2} + \dots + \frac{a_n}{\lambda_n}\right] \lambda_1 \lambda_2 \cdots \lambda_n$$

$$= \left[1 + \sum_{i=1}^{n} \frac{a_i}{\lambda_i}\right] \lambda_1 \lambda_2 \cdots \lambda_n$$

$$3(2) \begin{vmatrix} a_1^n & a_1^{n-1}b_1 & \cdots & a_1b_1^{n-1} & b_1^n \\ a_2^n & a_2^{n-1}b_2 & & a_2b_2^{n-1} & b_2^n \\ & & \ddots & & \\ a_n^n & a_n^{n-1}b_1 & & a_nb_n^{n-1} & b_n^n \\ a_{n+1}^n & a_{n+1}^{n-1}b_1 & \cdots & a_{n+1}b_{n+1}^{n-1} & b_{n+1}^n \end{vmatrix}$$
  $(a_i \neq 0, i = 1, 2, \dots, n+1)$ 

$$= a_{1}^{n} a_{2}^{n} \cdots a_{n+1}^{n} \begin{vmatrix} 1 & b_{1} / a_{1} & \cdots & (b_{1} / a_{1})^{n-1} & (b_{1} / a_{1})^{n} \\ 1 & b_{2} / a_{2} & (b_{2} / a_{2})^{n-1} & (b_{2} / a_{2})^{n} \\ & & \ddots & \\ 1 & b_{n} / a_{n} & (b_{n} / a_{n})^{n-1} & (b_{n} / a_{n})^{n} \\ 1 & b_{n+1} / a_{n+1} & \cdots & (b_{n+1} / a_{n+1})^{n-1} & (b_{n+1} / a_{n+1})^{n} \end{vmatrix}$$

$$= a_1^n a_2^n \cdots a_{n+1}^n \prod_{1 \le j < i \le n+1} \left( b_i / a_i - b_j / a_j \right)$$

### 已知n阶矩阵A满足 $A^2 = A$ ,证明:A = I或|A| = 0

#### 错误解法

$$A^{2} = A$$

$$A(A - I) = 0$$

$$|A||A - I| = 0$$

$$|A| = 0 \text{ or } |A - I| = 0$$

A = I

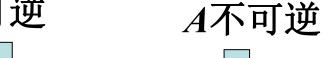
#### 正确解法

2.(2) 
$$A_{n \times n}$$
, 证明  $|A^*| = |A|^{n-1}$ 

$$AA^* = |A|I \Longrightarrow |A||A^*| = |A|^n$$

反证

$$A$$
可逆

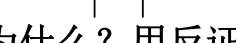


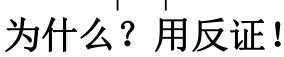


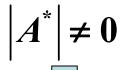
$$|A| = 0$$



$$\left|A^*\right| = \left|A\right|^{n-1} \qquad \left|A^*\right| = 0?$$











$$AA^{*}(A^{*})^{-1} = |A|(A^{*})^{-1}$$



$$A = |A|(A^*)^{-1} = 0$$



$$\left|A^*\right| \neq 0$$

 $A_{n\times n} \neq 0, A^* = A^T$ , 证明: A是可逆矩阵。

 $A_{n\times n}\neq 0, A^*=A^T,$  $A_{ii} = a_{ii}$  $|A| = \sum_{i=1}^{n} a_{1j} A_{1j} = \sum_{i=1}^{n} a_{1j} a_{1j} = \sum_{i=1}^{n} a_{1j}^{2} \neq 0$ 

### 复习题二

 $2.A_{n\times n}\neq 0,\exists k\in Z^+,s.t.A^k=0$ ,证明: A不可逆。

 $3.A_{3\times 3}, a_{ij} = A_{ij}, \Re |A|.$ 

$$A_{4\times3}$$
,  $\Re |AA^T|$ .

解:  $AA^Tx = 0$ 和 $A^Tx = 0$ 同解。

系数矩阵的秩相等  $R(AA^T) = R(A^T) = R(A) < 4$ 

$$\left| AA^T \right| = 0$$

下证  $AA^Tx = 0$ 和 $A^Tx = 0$ 同解。

$$A^T x = \mathbf{0} \Longrightarrow AA^T x = \mathbf{0}_{\circ}$$

$$AA^{T}x = 0 \Rightarrow x^{T}AA^{T}x = 0 \Rightarrow (A^{T}x)^{T}(A^{T}x) = 0 \Rightarrow A^{T}x = 0$$