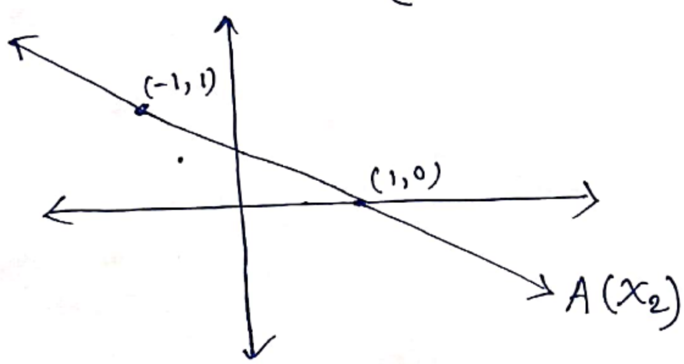
Now, for $A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ and $x \in \mathcal{X}_2$

$$\mathcal{X}_2 = \{x \mid x_1 - x_2 = 1, x \in \mathbb{R}^2\}$$
$$= \{x = (x_1, x_1 - 1) : x_1 \in \mathbb{R}\}$$

$\therefore$ ~~$A(\mathcal{X}_2) = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$~~

$$\therefore \quad Ax = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_1 - 1 \end{bmatrix} = \begin{bmatrix} x_1 + x_1 - 1 \\ 1 - x_1 \end{bmatrix} = \begin{bmatrix} 2x_1 - 1 \\ 1 - x_1 \end{bmatrix}, x \in \mathcal{X}_2$$

$$\Rightarrow \quad A(\mathcal{X}_2) = \{(2x_1 - 1, 1 - x_1) : x_1 \in \mathbb{R}\}$$
$$= \{(2t - 1, 1 - t) : t \in \mathbb{R}\}$$



Now, for $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ and $x \in \mathcal{X}_1$

$$\mathcal{X}_1 = \{x \mid x_1 + x_2 = 1, x \in \mathbb{R}^2\}$$
$$= \{x = (x_1, 1 - x_1) : x_1 \in \mathbb{R}\}$$

$$\therefore \quad Ax = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ 1 - x_1 \end{bmatrix} = \begin{bmatrix} x_1 - (1 - x_1) \\ x_1 + (1 - x_1) \end{bmatrix} = \begin{bmatrix} 2x_1 - 1 \\ 1 \end{bmatrix}$$

, for all $x \in \mathcal{X}_1$

$$\Rightarrow \quad A(\mathcal{X}_1) = \{(2x_1 - 1, 1) : x_1 \in \mathbb{R}\}$$
$$= \{(2t - 1, 1) : t \in \mathbb{R}\}$$