Homework 3

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In [2]:
```
%matplotlib inline
import numpy as np
import matplotlib.pyplot as plt
```

In [3]:
```
import warnings
warnings.simplefilter('ignore')
```

1. Math Warm-up

Given that our multivariate Gaussian class-conditional distributions $p(x|y)$ is defined by the equation

$$p(x|y) = \frac{1}{|2\pi C_y|^{m/2}} exp\left(-\frac{1}{2}(x-\mu_y)^T C_y^{-1}(x-\mu_y)\right)$$

Note: Worked problem with Joaquin from class on this problem.

(a)

If we have an equal class variance case (i.e. $C_0 = C_1 = C$), then we write our log ratio as follows:

$$\log\frac{p(x_i|y=1)}{p(x_i|y=0)} + \log\frac{p(y=1)}{p(y=0)} = \log\frac{p(x_i|y=1)}{p(x_i|y=0)}$$

as we know that $p(y=0) = p(y=1)$, and $\log(1) = 0$. Now applying our relationship for $p(x|y)$,

$$\log\frac{p(x_i|y=1)}{p(x_i|y=0)} = \log\frac{\left(\frac{1}{|2\pi C_1|^{m/2}} exp\left(\ldots\right)\right)}{\left(\frac{1}{|2\pi C_0|^{m/2}} exp\left(\ldots\right)\right)}$$

where $\ldots$ represents the value inside the numerator. Given that $C_0 = C_1 = C$ and it is just a constant, the term $\frac{1}{|2\pi C_y|^{m/2}}$ will cancel out from the top and bottom leaving us with,

$$= \log\frac{\left(exp\left(-\frac{1}{2}(x_i-\mu_1)^T C_1^{-1}(x_i-\mu_1)\right)\right)}{\left(exp\left(-\frac{1}{2}(x_i-\mu_0)^T C_0^{-1}(x_i-\mu_0)\right)\right)}$$

This can be simplified further to,

$$= \log\left(exp\left(-\frac{1}{2}(x_i-\mu_1)^T C_1^{-1}(x_i-\mu_1)\right)\right) - \log\left(exp\left(-\frac{1}{2}(x_i-\mu_0)^T C_0^{-1}(x_i-\mu_0)\right)\right)$$

$$= \left(-\frac{1}{2}(x_i - \mu_1)^T C_1^{-1}(x_i - \mu_1)\right) - \left(-\frac{1}{2}(x_i - \mu_0)^T C_0^{-1}(x_i - \mu_0)\right)$$

Again, given that $C_0 = C_1 = C$ and combining similar terms we find,

$$= -\frac{1}{2}(x_i - \mu_1)^T C^{-1}(x_i - \mu_1) + \frac{1}{2}(x_i - \mu_0)^T C^{-1}(x_i - \mu_0)$$

$$= -\frac{1}{2}\left(x_i^T C^{-1} x_i - x_i^T C^{-1}\mu_1 - \mu_1^T C^{-1} x_i + \mu_1^T C^{-1}\mu_1\right) + \frac{1}{2}\left(x_i^T C^{-1} x_i - x_i^T C^{-1}\mu_0 - \mu_0^T C^{-1} x_i +\right.$$

$$= -\frac{1}{2}\left(-\mu_1^T C^{-1} x_i - \mu_1^T C^{-1} x_i + \mu_1^T C^{-1}\mu_1\right) + \frac{1}{2}\left(-\mu_0^T C^{-1} x_i - \mu_0^T C^{-1} x_i + \mu_0^T C^{-1}\mu_0\right)$$

$$= (C^{-1}\mu_1 - C^{-1}\mu_0)^T x_i + \mu_0^T C^{-1}\mu_0 - \mu_1^T C^{-1}\mu_1$$

From this form of the log ratio, we can now equate $b_i = (C^{-1}\mu_1 - C^{-1}\mu_0)^T$ and $c_i = \mu_0^T C^{-1}\mu_0 - \mu_1^T C^{-1}\mu_1$ such that the log ratio is $b_i x_i + c_i$ (which can be written as $b_i x + c_i$ when we include the summation).

(b)

If we have an equal class variance case (i.e. $C_0 = C_1 = C$), then we write our log ratio as follows:

$$\log\frac{p(x_i|y=1)}{p(x_i|y=0)} + \log\frac{p(y=1)}{p(y=0)} = \log\frac{p(x_i|y=1)}{p(x_i|y=0)}$$

as we know that $p(y=0) = p(y=1)$, and $\log(1) = 0$. Now applying our relationship for $p(x|y)$,

$$\log\frac{p(x_i|y=1)}{p(x_i|y=0)} = \log\left(\frac{\left(\frac{1}{|2\pi C_1|^{m/2}} exp\left(-\frac{1}{2}(x-\mu_1)^T C_1^{-1}(x-\mu_1)\right)\right)}{\left(\frac{1}{|2\pi C_0|^{m/2}} exp\left(-\frac{1}{2}(x-\mu_0)^T C_0^{-1}(x-\mu_0)\right)\right)}\right)$$

We can isolate the first term from both the top and the bottom and apply our logarithimic properties such that,

$$\log\frac{p(x_i|y=1)}{p(x_i|y=0)} = \log\left(\frac{|2\pi C_0|^{m/2}}{|2\pi C_1|^{m/2}}\right) + \log\left(\frac{\left(exp\left(-\frac{1}{2}(x-\mu_1)^T C_1^{-1}(x-\mu_1)\right)\right)}{\left(exp\left(-\frac{1}{2}(x-\mu_0)^T C_0^{-1}(x-\mu_0)\right)\right)}\right)$$

$$\log\frac{p(x_i|y=1)}{p(x_i|y=0)} = \log\left(\frac{|2\pi C_0|^{m/2}}{|2\pi C_1|^{m/2}}\right) + \log\left(exp\left(-\frac{1}{2}(x-\mu_1)^T C_1^{-1}(x-\mu_1)\right)\right) - \log \left(exp\left(-\frac{1}{2}(x-\mu_0)^T C_0^{-1}(x-\mu_0)\right)\right)$$

$$\log\frac{p(x_i|y=1)}{p(x_i|y=0)} = \log\left(\frac{|2\pi C_0|^{m/2}}{|2\pi C_1|^{m/2}}\right) - \frac{1}{2}(x-\mu_1)^T C_1^{-1}(x-\mu_1) + \frac{1}{2}(x-\mu_0)^T C_0^{-1}(x-\mu_0)$$

Given that $m = 2$ is this situation, we can simply further and expand our quadratic relationships such that,

$$\log \frac{p(x_i|y=1)}{p(x_i|y=0)} = 2\log\left(\frac{C_0^2}{C_2^2}\right) - x^T C_1^{-1} x + \mu_1^T C_1^{-1} x + x^T C_1^{-1} \mu_1 -$$

$$\mu_1^T C_0^{-1} \mu_1 + x^T C_0^{-1} x - \mu_0^T C_0^{-1} x - x^T C_0^{-1} \mu_0 + \mu_0^T C_0^{-1} \mu_0$$

Combining like terms,

$$\log \frac{p(x_i|y=1)}{p(x_i|y=0)} = x^T (C_0^{-1} - C_1^{-1}) x + (C_1^{-1}\mu_1 - C_0^{-1}\mu_0)^T x + (\mu_0^T C_0^{-1} \mu_0 - \mu_1^T C_1^{-1} \mu_1) + 2\log\Bigg(\cdot$$

From this form of the log ratio, we can now equate $a_i = C_0^{-1} - C_1^{-1}$, $b_i = C_1^{-1}\mu_1 - C_0^{-1}\mu_0)^T$ and $c_i = (\mu_0^T C_0^{-1} \mu_0 - \mu_1^T C_1^{-1} \mu_1) + 2\log\left(\frac{C_0^2}{C_2^2}\right)$ such that the log ratio is $a_i x_i^2 + b_i x_i + c_i$.

(c)

A logistic regression and Gaussian class conditional distributions are similar and can be considered identical in the naive Bayes case. Assumptions need to be made to prove this relationship, as outlined in the article linked here: https://appliedmachinelearning.blog/2019/09/30/equivalence-of-gaussian-naive-bayes-and-logistic-regression-an-explanation/.

2. Gaussian Distribution and the Curse of Dimensionality

(a)

When $m = 2$, we know that a "sphere" in 2D space is actually a circle - thus we can define $S_{2-1}(r) = S_1(r)$ and $V_2(r)$ based on the known surface area and area (i.e. volume) equations for a circle,

$$S_1(r) = 2\pi r$$

$$V_2(r) = \pi r^2$$

When $m = 3$, a sphere follows the standard sphere equations - thus we can define $S_{3-1}(r) = S_2(r)$ and $V_3(r)$ based on the known surface area and volume equations for a sphere,

$$S_2(r) = 4\pi r^2$$

$$V_3(r) = \frac{4}{3}\pi r^3$$

(b)

The equation $S_{m-1} = \frac{d}{dr} V_m(r)$ states that the derivative of the volume of a m-dimensional sphere is its surface area (i.e. $S_{m-1}). We can see that this realtionship holds true from our derived equations from part (a) such that,

$$\frac{d}{dr}V_2(r) = \frac{d}{dr}(\pi r^2) = 2\pi r = S_1(r)$$

$$\frac{d}{dr}V_3(r) = \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right) = 4\pi r^2 = S_2(r)$$

(c)

Intuitively, we can see that the relationship for S_{m-1} can be defined as,

$$S_{m-1}(r) = \bar{S}_{m-1}r^{m-1}$$

(d)

Note: Worked problem with Tess, Charlie, Dean, and Steve from class on this problem.

We can solve for $\rho_m(r) = \int_{x\in\delta_{m-1}(r)} p(x)dx$ by using our known equation for $p(x)$ first; thus,

$$\rho_m(r) = \int_{x\in\delta_{m-1}(r)} \frac{1}{(2\pi\sigma^2)^{m/2}} exp\left(-\frac{\|x\|_2^2}{2\sigma^2}\right) dx$$

We can extract our first term as it is not a function of x such that

$$\rho_m(r) = \frac{1}{(2\pi\sigma^2)^{m/2}} \int_{x\in\delta_{m-1}(r)} exp\left(-\frac{\|x\|_2^2}{2\sigma^2}\right) dx$$

We can now substitute the relationship defined by $\delta_{m-1}(r)$ in which $\|x\|_2 = r$; thus,

$$\rho_m(r) = \frac{1}{(2\pi\sigma^2)^{m/2}} \int_{x\in\delta_{m-1}(r)} exp\left(\frac{-r^2}{2\sigma^2}\right) dx$$

$$\rho_m(r) = \frac{exp\left(\frac{-r^2}{2\sigma^2}\right)}{(2\pi\sigma^2)^{m/2}} \int_{x\in\delta_{m-1}(r)} dx$$

From part (c), we know that the integral of all points about the surface of the m-sphere is just the surface area of the m-sphere itself (i.e. $\int_{x\in\delta_{m-1}(r)} dx = \bar{S}_{m-1}r^{m-1}$); thus,

$$\rho_m(r) = \frac{exp\left(\frac{-r^2}{2\sigma^2}\right)}{(2\pi\sigma^2)^{m/2}} \bar{S}_{m-1}r^{m-1}$$

(e)

Note: Worked problem with Tess, Charlie, Dean, and Steve from class on this problem.

We can show that for large m, that $\rho_m(r)$ has a single maximum value $\hat{r}$ such that $\hat{r} \approx \sqrt{m}\sigma$, by taking the derivative of $\rho_m(r)$ with respect to r such that,

$$\frac{d}{dr}\rho_m(r) = \frac{d}{dr}\left(\frac{exp\left(\frac{-r^2}{2\sigma^2}\right)}{(2\pi\sigma^2)^{m/2}}\bar{S}_{m-1}r^{m-1}\right) = \frac{\bar{S}_{m-1}}{(2\pi\sigma^2)^{m/2}}\frac{d}{dr}\left(exp\left(\frac{-r^2}{2\sigma^2}\right)r^{m-1}\right)$$

Once we've taken the derivative, we will be setting it equal to zero to find our maximum value (at $\hat{r}$), so we can eliminate the constant term $\frac{\bar{S}_{m-1}}{(2\pi\sigma^2)^{m/2}}$ as they will be divided out. Thus, our resulting equation when set equal to zero is:

$$\frac{d}{dr}\left(exp\left(\frac{-r^2}{2\sigma^2}\right)r^{m-1}\right) = 0$$

Applying the product rule,

$$\frac{d}{dr}\left(exp\left(\frac{-r^2}{2\sigma^2}\right)\right)r^{m-1} + exp\left(\frac{-r^2}{2\sigma^2}\right)\frac{d}{dr}\left(r^{m-1}\right) = 0$$

$$exp\left(\frac{-r^2}{2\sigma^2}\right)\frac{-2r}{2\sigma^2}r^{m-1} + exp\left(\frac{-r^2}{2\sigma^2}\right)(m-1)r^{m-2} = 0$$

Simplifying this,

$$\frac{-r^m}{\sigma^2} + (m-1)r^{m-2} = 0$$

$$\frac{-1}{\sigma^2} + (m-1)r^{-2} = 0$$

$$(m-1)r^{-2} = \sigma^{-2}$$

$$r^2 = (m-1)\sigma^2$$

$$r = \sqrt{(m-1)}\sigma$$

With our assumption, when we have a large m we can assume that $m-1 \approx m$; thus,

$$\hat{r} \approx \sqrt{m}\sigma$$

(f)

We can show that for large m and a small value ϵ such that $\epsilon \ll \hat{r}$, that $\rho(\hat{r}+\epsilon) \approx \rho(\hat{r})e^{-\frac{\epsilon^2}{\sigma^2}}$, by first relating $\rho(\hat{r}+\epsilon)$ and $\rho(\hat{r})$ through a fractional relationship such that,

$$\frac{\rho(\hat{r}+\epsilon)}{\rho(\hat{r})} = \frac{\frac{\bar{S}_{m-1}}{(2\pi\sigma^2)^{m/2}}exp\left(\frac{-(\hat{r}+\epsilon)^2}{2\sigma^2}\right)(\hat{r}+\epsilon)^{m-1}}{\frac{\bar{S}_{m-1}}{(2\pi\sigma^2)^{m/2}}exp\left(\frac{-\hat{r}^2}{2\sigma^2}\right)\hat{r}^{m-1}}$$

Simplifying by eliminating common terms from top and bottom we're left with,

$$\frac{\rho(\hat{r}+\epsilon)}{\rho(\hat{r})}=\frac{exp\left(\frac{-(\hat{r}+\epsilon)^2}{2\sigma^2}\right)(\hat{r}+\epsilon)^{m-1}}{exp\left(\frac{-\hat{r}^2}{2\sigma^2}\right)\hat{r}^{m-1}}$$

We can combine all terms into the exponential such that,

$$\frac{\rho(\hat{r}+\epsilon)}{\rho(\hat{r})}=\frac{exp\left(\frac{-(\hat{r}+\epsilon)^2}{2\sigma^2}\right)exp(\ln((\hat{r}+\epsilon)^{m-1})}{exp\left(\frac{-\hat{r}^2}{2\sigma^2}\right)exp(\ln(\hat{r}^{m-1}))}$$

$$\frac{\rho(\hat{r}+\epsilon)}{\rho(\hat{r})}=exp\left(\frac{-(\hat{r}+\epsilon)^2}{2\sigma^2}+\ln((\hat{r}+\epsilon)^{m-1})+\frac{\hat{r}^2}{2\sigma^2}-\ln(\hat{r}^{m-1})\right)$$

Combining like terms we find,

$$\frac{\rho(\hat{r}+\epsilon)}{\rho(\hat{r})}=exp\left(\frac{-(\hat{r}+\epsilon)^2+\hat{r}^2}{2\sigma^2}+\ln((\hat{r}+\epsilon)^{m-1})-\ln(\hat{r}^{m-1})\right)$$

$$\frac{\rho(\hat{r}+\epsilon)}{\rho(\hat{r})}=exp\left(\frac{-2\hat{r}\epsilon-\epsilon^2}{2\sigma^2}+\ln((\hat{r}+\epsilon)^{m-1})-\ln(\hat{r}^{m-1})\right)$$

$$\frac{\rho(\hat{r}+\epsilon)}{\rho(\hat{r})}=exp\left(\frac{-2\hat{r}\epsilon-\epsilon^2}{2\sigma^2}+\ln\left(\frac{(\hat{r}+\epsilon)^{m-1}}{\hat{r}^{m-1}}\right)\right)$$

$$\frac{\rho(\hat{r}+\epsilon)}{\rho(\hat{r})}=exp\left(\frac{-2\hat{r}\epsilon-\epsilon^2}{2\sigma^2}+(m-1)\ln\left(1+\frac{\epsilon}{\hat{r}}\right)\right)$$

We can apply a 2-term Maclauren expansion approximation of the form $ln\,(1+x)$ such that,

$$\frac{\rho(\hat{r}+\epsilon)}{\rho(\hat{r})}\approx exp\left(\frac{-2\hat{r}\epsilon-\epsilon^2}{2\sigma^2}+(m-1)\left(\frac{\epsilon}{\hat{r}}-\frac{1}{2}\left(\frac{\epsilon}{\hat{r}}\right)^2\right)\right)$$

Substituting $\hat{r}\approx\sqrt{m}\sigma$ as found in part (e),

$$\frac{\rho(\hat{r}+\epsilon)}{\rho(\hat{r})}\approx exp\left(\frac{-2\sqrt{m}\sigma\epsilon-\epsilon^2}{2\sigma^2}+(m-1)\left(\frac{\epsilon}{\sqrt{m}\sigma}-\frac{\epsilon^2}{2m\sigma^2}\right)\right)$$

With our assumption, when we have a large m we can assume that $m-1\approx m$; thus,

$$\frac{\rho(\hat{r}+\epsilon)}{\rho(\hat{r})}\approx exp\left(\frac{-2\sqrt{m}\sigma\epsilon-\epsilon^2}{2\sigma^2}+m\left(\frac{\epsilon}{\sqrt{m}\sigma}-\frac{\epsilon^2}{2m\sigma^2}\right)\right)$$

$$\frac{\rho(\hat{r}+\epsilon)}{\rho(\hat{r})}\approx exp\left(\frac{-2\sqrt{m}\sigma\epsilon-\epsilon^2}{2\sigma^2}+\frac{\sqrt{m}\epsilon}{\sigma}-\frac{\epsilon^2}{2\sigma^2}\right)$$

Simplifying we find,

$$\frac{\rho(\hat{r}+\epsilon)}{\rho(\hat{r})} \approx exp\left(\frac{-2\sqrt{m}\sigma\epsilon - \epsilon^2 + 2\sqrt{m}\sigma\epsilon - \epsilon^2}{2\sigma^2}\right)$$

$$\frac{\rho(\hat{r}+\epsilon)}{\rho(\hat{r})} \approx exp\left(\frac{-2\epsilon^2}{2\sigma^2}\right)$$

$$\frac{\rho(\hat{r}+\epsilon)}{\rho(\hat{r})} \approx exp\left(\frac{-\epsilon^2}{\sigma^2}\right)$$

$$\rho(\hat{r}+\epsilon) \approx \rho(\hat{r}) exp\left(\frac{-\epsilon^2}{\sigma^2}\right)$$

Thus, we can see that for large m and a small value $\epsilon \ll \hat{r}$, $\rho(\hat{r}+\epsilon) \approx \rho(\hat{r}) exp\left(\frac{-\epsilon^2}{\sigma^2}\right)$.

(g)

For the high dimensional Gaussian distribution (m is large), the majority of our points will reside at our value $\sqrt{m}$ as we see in our equation $\hat{r} \approx \sqrt{m}\sigma$ where σ is small compared to $\hat{r}$.

For a low dimensional Guassian distribution, most points will reside around the origin $x = 0$ because most of the mass is centered at the origin within the window of roughly σ.

(h)

Calculating the probability density at the origin, we find that

$$\rho_m(0) = \frac{exp\left(\frac{-(0)^2}{2\sigma^2}\right)}{(2\pi\sigma^2)^{m/2}} \bar{S}_{m-1}(0)^{m-1} = 0$$

Calculating the probability density at a point on the sphere,

$$\rho_m(\hat{r}) = \frac{exp\left(\frac{-(\hat{r})^2}{2\sigma^2}\right)}{(2\pi\sigma^2)^{m/2}} \bar{S}_{m-1}(\hat{r})^{m-1}$$

For large m, i.e. high dimensionality, we can approximate $\hat{r}$ as $\sqrt{m}\sigma$, such that

$$\rho_m(\hat{r}) = \frac{exp\left(\frac{-(\sqrt{m}\sigma)^2}{2\sigma^2}\right)}{(2\pi\sigma^2)^{m/2}} \bar{S}_{m-1}(\sqrt{m}\sigma)^{m-1}$$

$$\rho_m(\hat{r}) = \frac{exp\left(\frac{-m}{2}\right)}{(2\pi\sigma^2)^{m/2}} \bar{S}_{m-1}(\sqrt{m}\sigma)^{m-1}$$

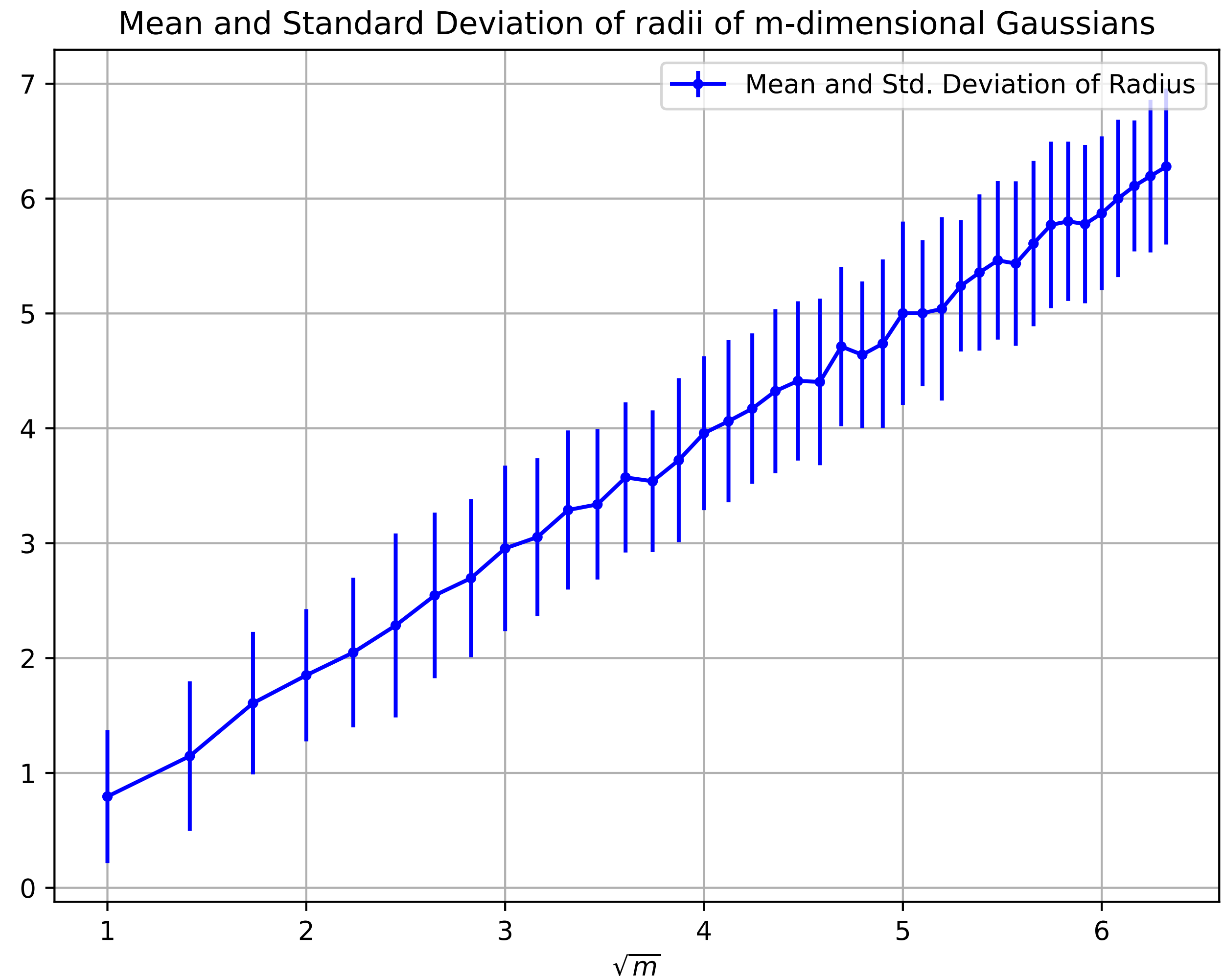
(i)

From the plot generated by the plot below, we can see that many of the relationships we calculated above hold true. We can see that there is a linear relationship between mean of our radii and the root of m.

This aligns with our logic as we defined $\hat{r} \approx \sqrt{m}\sigma$ as m increases. In the case outlined below, our σ was set at a value of 1 such that our $\hat{r}$ is directly a function of $\sqrt{m}$. It is expected that $\hat{r}$ should align with where the mean of our radii as this is the "maximum" of our distribution for the radii. This maximum should occur where the mean of the distribution occurs.

In [23]:

```
m = np.arange(40) + 1
sample_size = 100

# Set an arbitrary mean and std. deviation value for our m-dimensional Gaussian
mean_G = 0
sigma_G = 1

mean = np.empty(shape=(40))
std_dev = np.empty(shape=(40))

for m_i in m:
    # For each value of m (i.e. each m-dimensional Gaussian), generate a random sample
    cov_i = np.identity(m_i) * sigma_G**2
    mean_i = np.ones(m_i) * mean_G
    x_i = np.random.multivariate_normal(mean_i, cov_i, size=100)

    # Calculate the radii of the samples (i.e. the sqrt(x^2) = abs(x))
    r_i = np.sum(x_i**2, axis=1)**(1/2)
    #print('\n', x_i.shape, r_i.shape)

    # Calculate the mean and std deviation
    mean[m_i - 1] = np.mean(r_i)
    std_dev[m_i - 1] = np.std(r_i)

#print(mean.shape, std_dev.shape)

# Generate a 2D plot of mean/std dev. vs. m
fig, ax = plt.subplots(figsize=(8, 6))
#plt.plot(m, mean, ls = '-', marker = '.', c = 'blue', label = 'Mean')
#plt.plot(m, std_dev, ls = '-', marker = '.', c='red', label = 'Standard Deviation')
plt.errorbar(np.sqrt(m), mean, yerr=std_dev, marker='.', c='blue', label='Mean and Std.
#plt.errorbar(m, mean, yerr=std_dev, marker='.', c='red', label='Mean and Std. Deviatio
plt.grid()
plt.legend()
plt.xlabel('$\sqrt{m}$')
plt.title('Mean and Standard Deviation of radii of m-dimensional Gaussians')
fig.show()
plt.savefig('Problem_2_i.png') # If saving a file
```



3. Programming Problem: Lasso

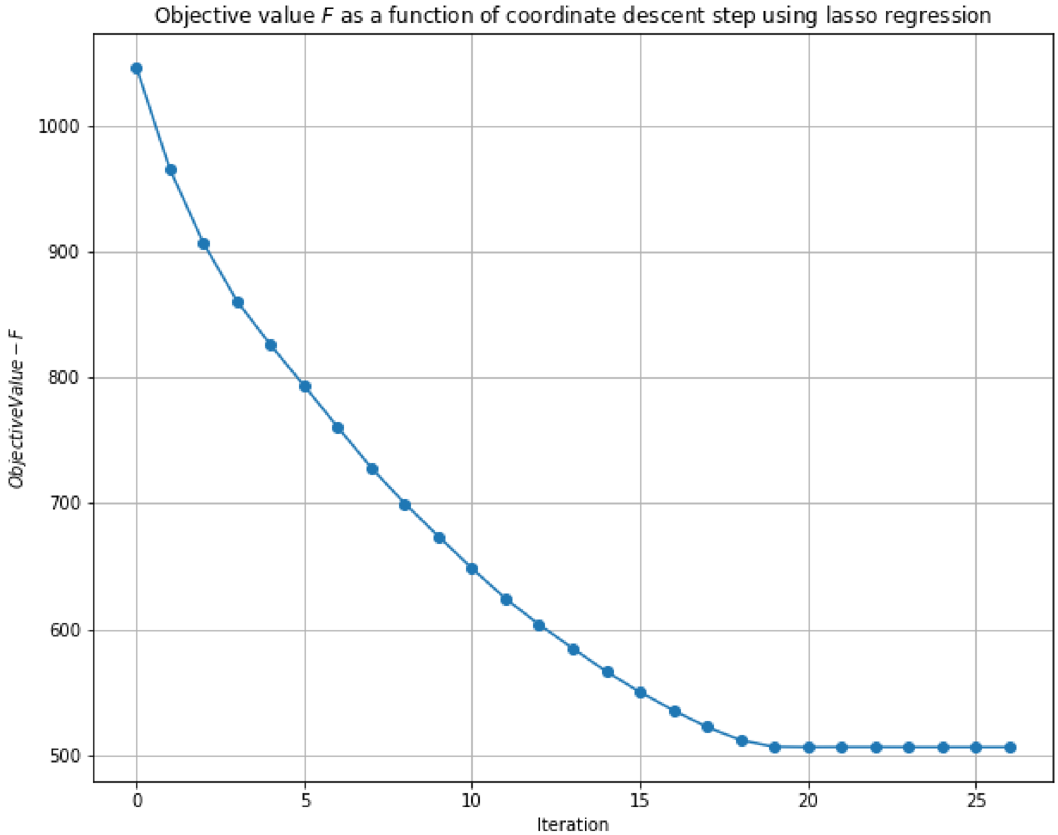
(a)

Captured within the *lass.ipynb* file is the coordinate descent algorithim.

From this algorithim, the reported non-zero weight entries are

$$\{\text{Non-zero indices of } \theta\} = \{0, 1, 2, 3, 4, 7, 11, 12, 14, 16, 21, 22, 23, 25, 34, 36, 48, 52, 54, 61, 63, 71\}$$

In addition, the plot below showing the objective value during gradient descent is captured below.

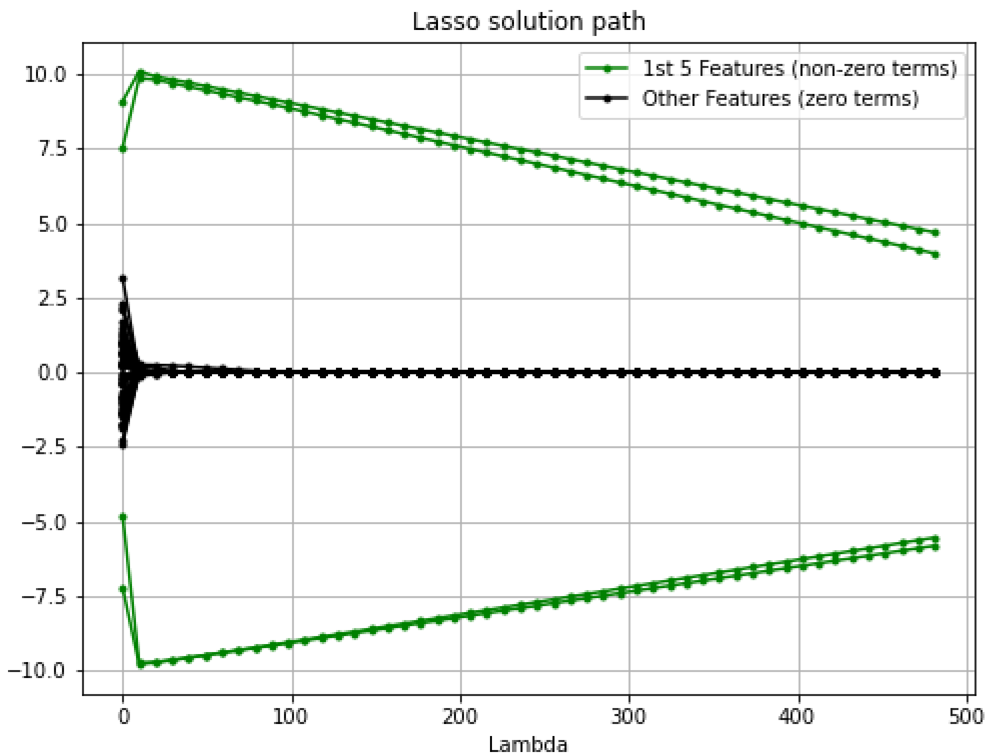


(b)

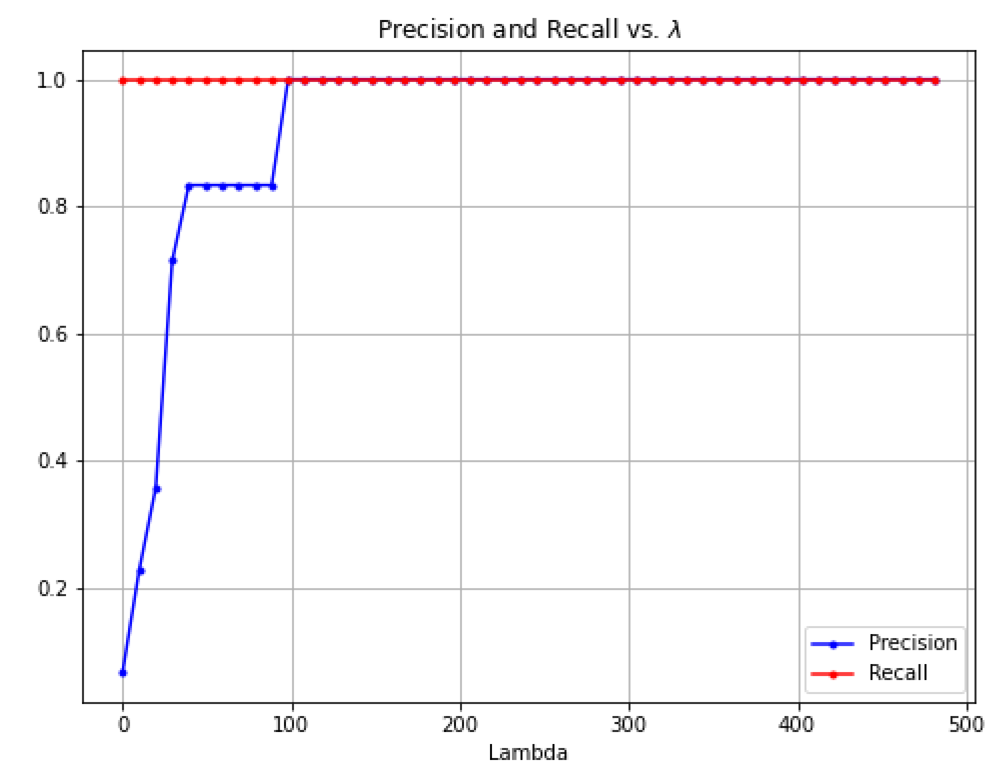
From the *lasso.ipynb*, $\text{precision} = 0.22727272727272727$ and $\text{recall} = 1.0$ for the lasso solution in part (a).

(c)

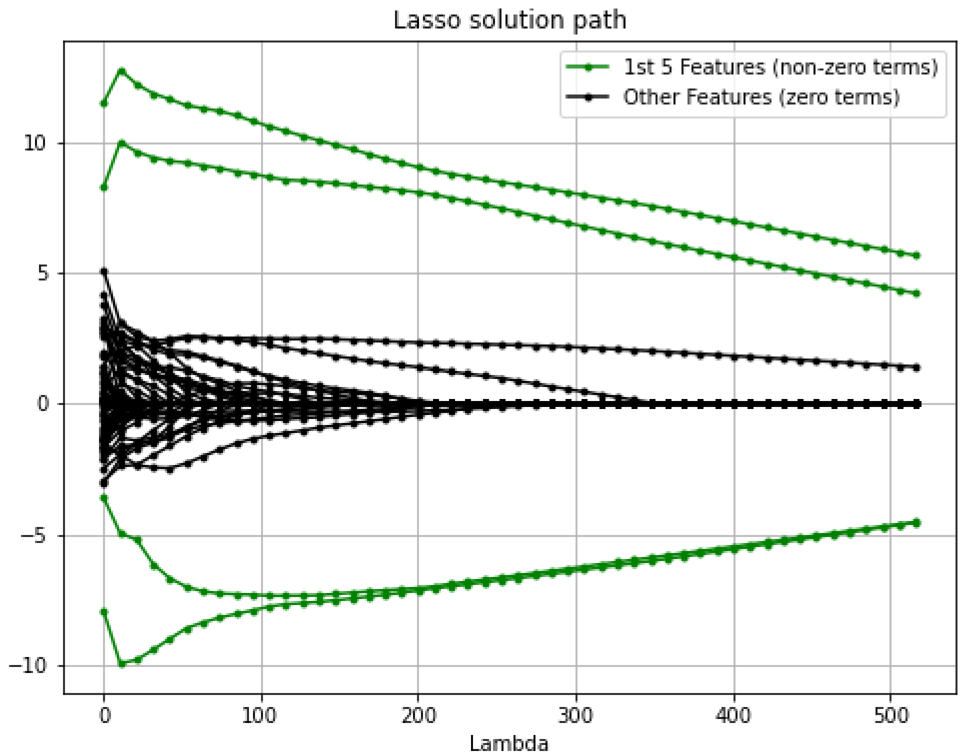
From the *lasso.ipynb*, we can plot the precision and recall vs. λ as shown in the plot below.



We can also plot the lasso solution path for the varying values of λ as shown in the plot below.



Varying our noise's standard deviation to $\sigma = 10$ through the DataGenerator function, we find the following results.



Comparing the two lasso solution paths, we can see the impact noise has on calculating our weights for our lasso solution. With an increases variance in the induced noise, we are more likely to see more impactful incorrect weights (color coded by black lines) at lower lambdas. For our lower noise std. deviation, these incorrect weights were nearly zero for the majority of our values for lambda. This was not the case when we increase the noise std. deviation to 10 as we see that multiply incorrect weights are non-zero for all lambdas.

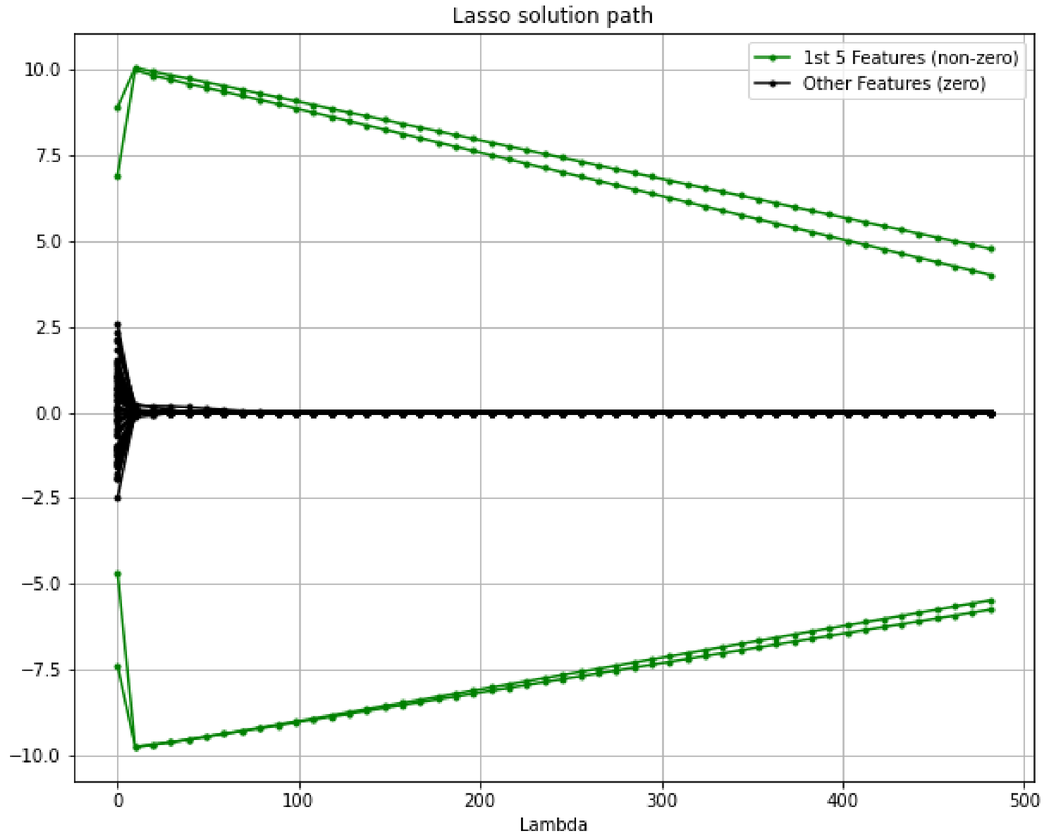
(d)

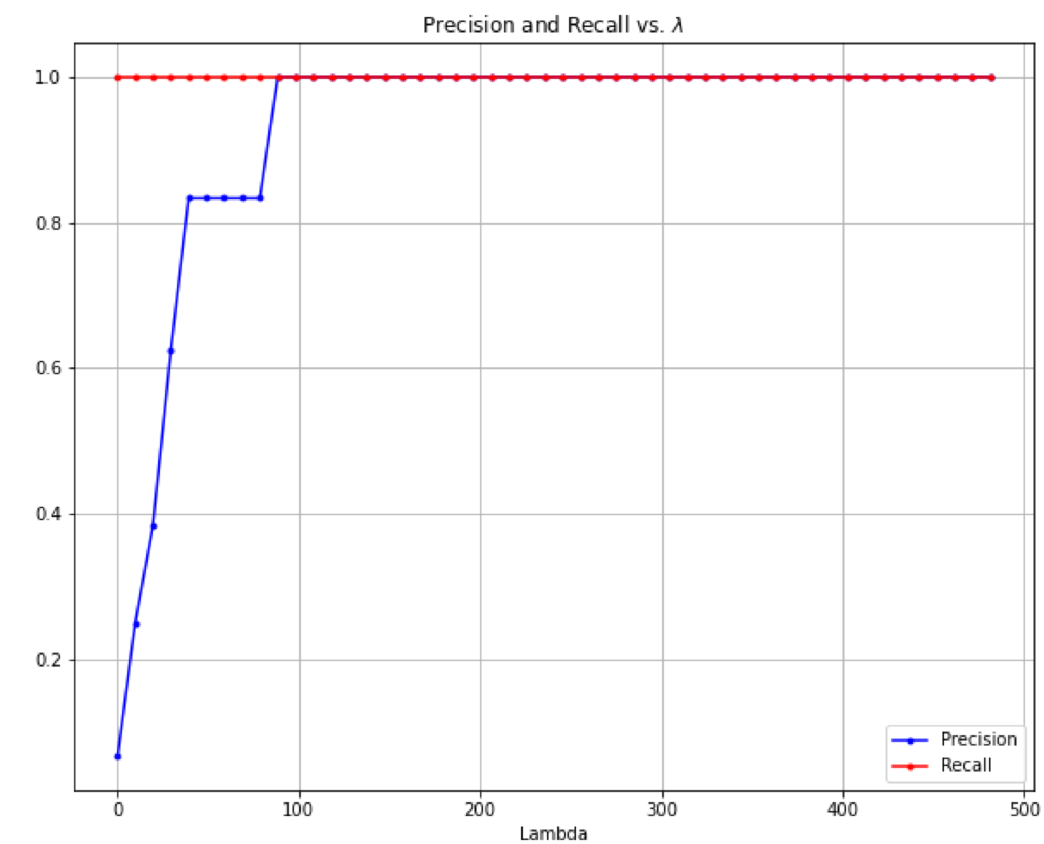
With varying input parameters for DataGenerator, below are the plots of both (1) precision and recall vs. λ and (2) the lasso solution path.

From the graphs below, we can surmise that an ideal λ for these various input parameters we when we can maximize both precision and recall.

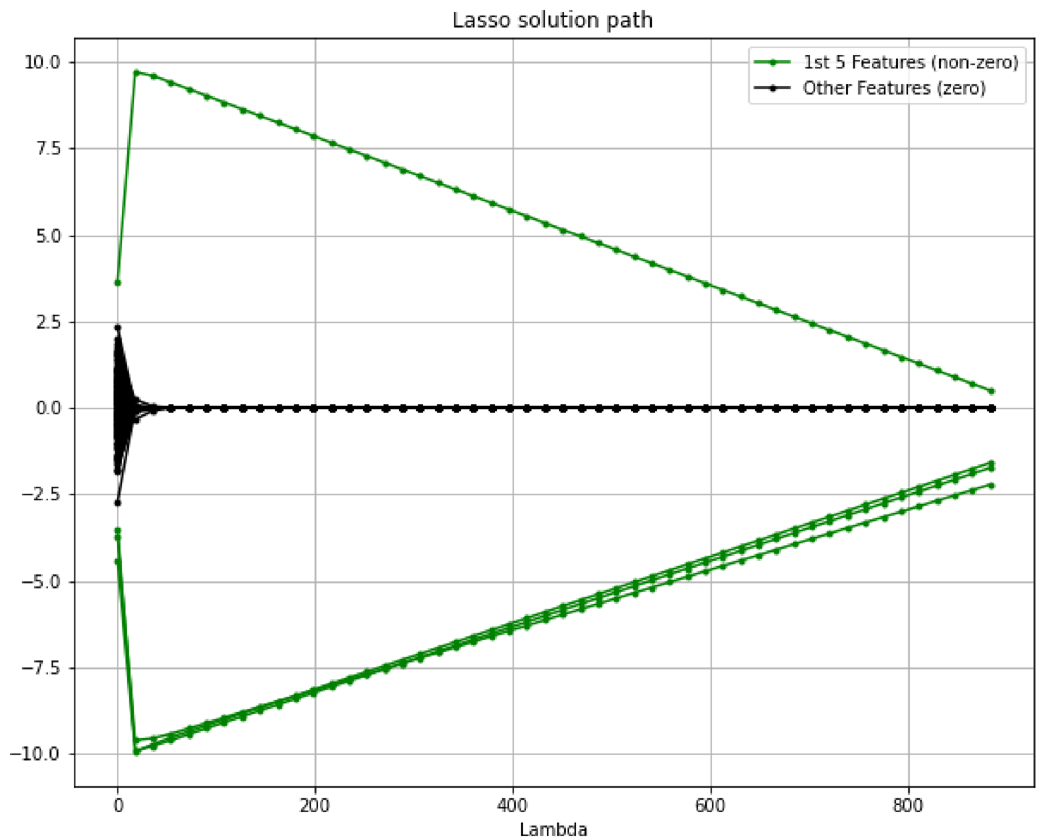
Based on the graphs below, it seems that a good precision and recall value is obtained when $n = O(log(m))$. It seems that when $n = O(m^2)$ the precision and recall can exhibit strange behavior (as seen in our $(n = 50, m = 1000)$), and $n = O(m)$ yields precision and recall values that are undesirable when we have more accurate weight values (i.e. smaller lambda). Thus it seems that $n = O(log(m))$ is an desirable relationship.

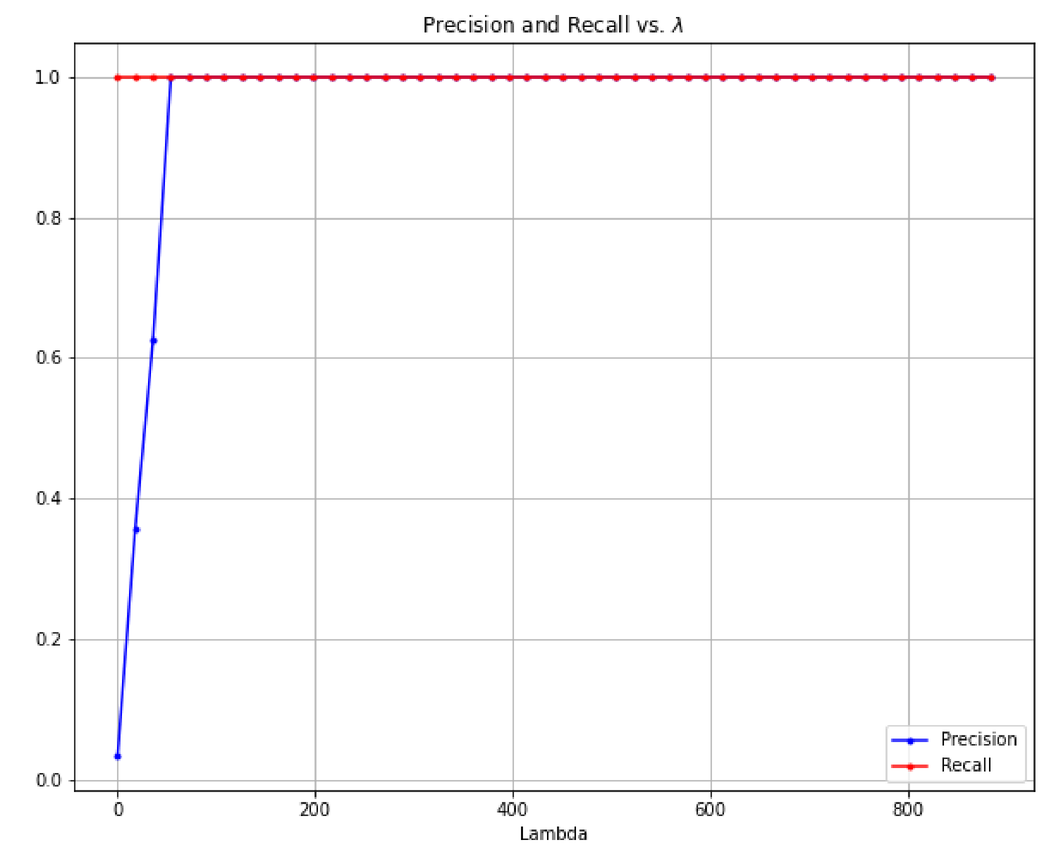
For $(n = 50, m = 75)$,


Lasso solution path
1st 5 Features (non-zero)
Other Features (zero)
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
0
100
200
300
400
500
Lambda

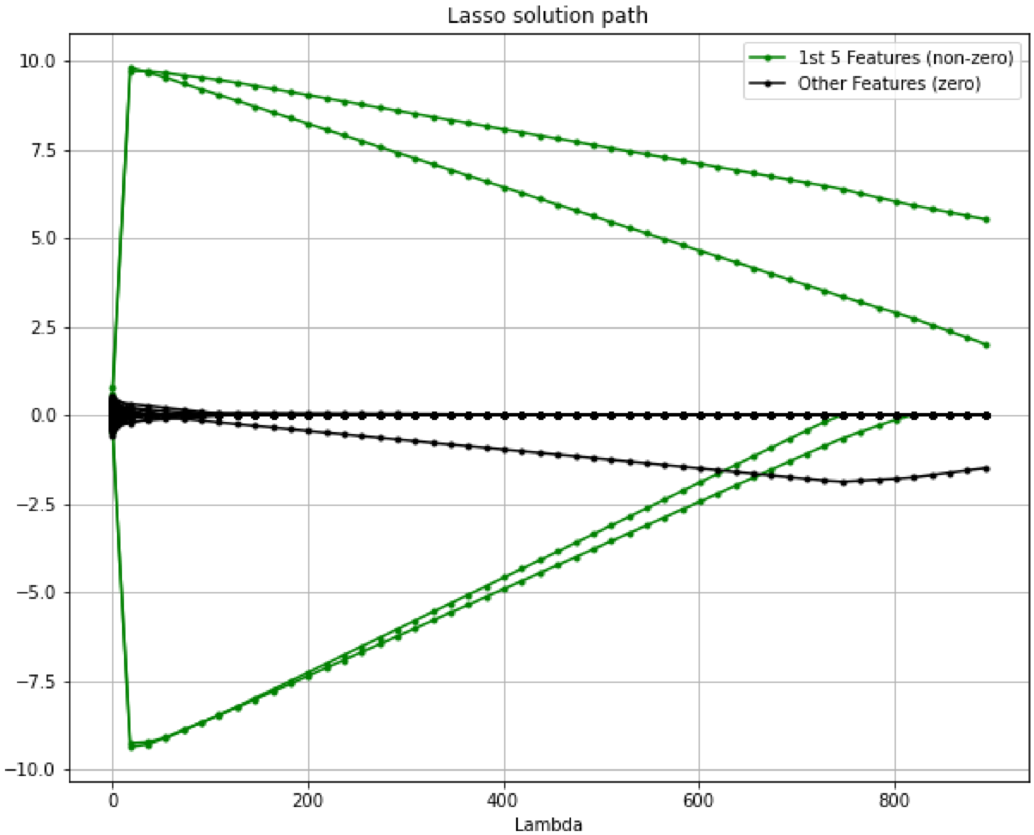

Precision and Recall vs. λ
Precision
Recall
1.0
0.8
0.6
0.4
0.2
0
100
200
300
400
500
Lambda

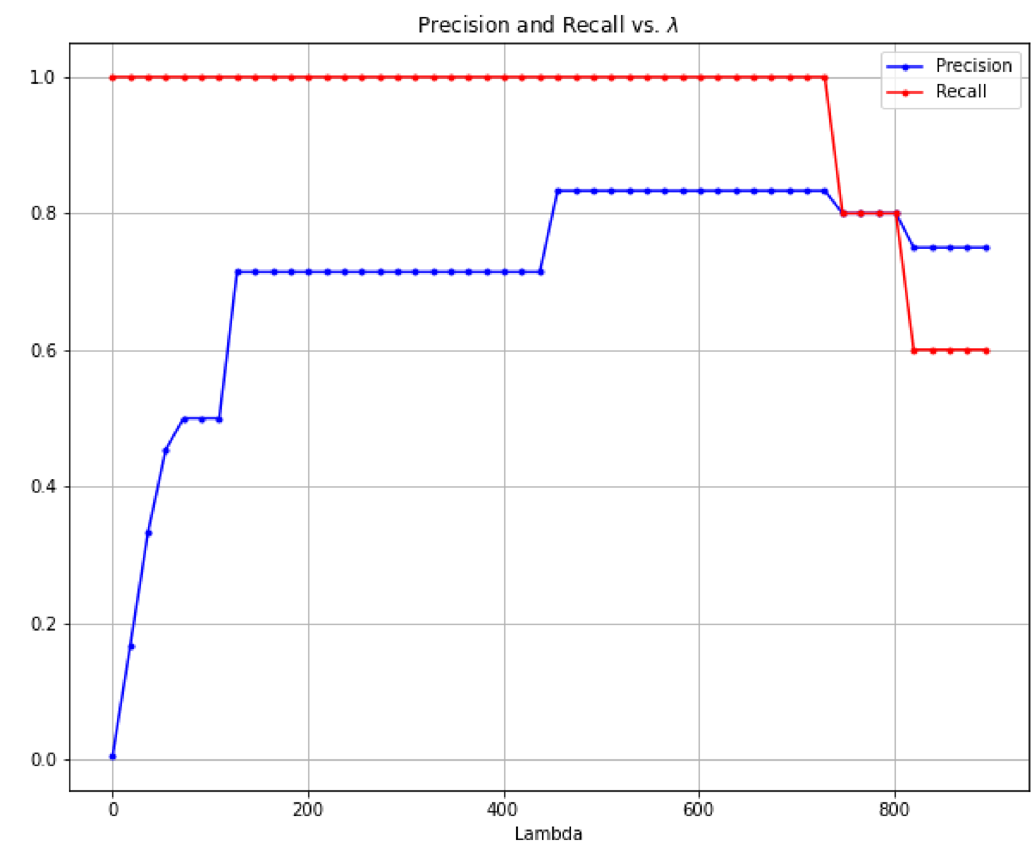
For $(n = 50, m = 150)$,


Lasso solution path
1st 5 Features (non-zero)
Other Features (zero)
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
0
200
400
600
800
Lambda

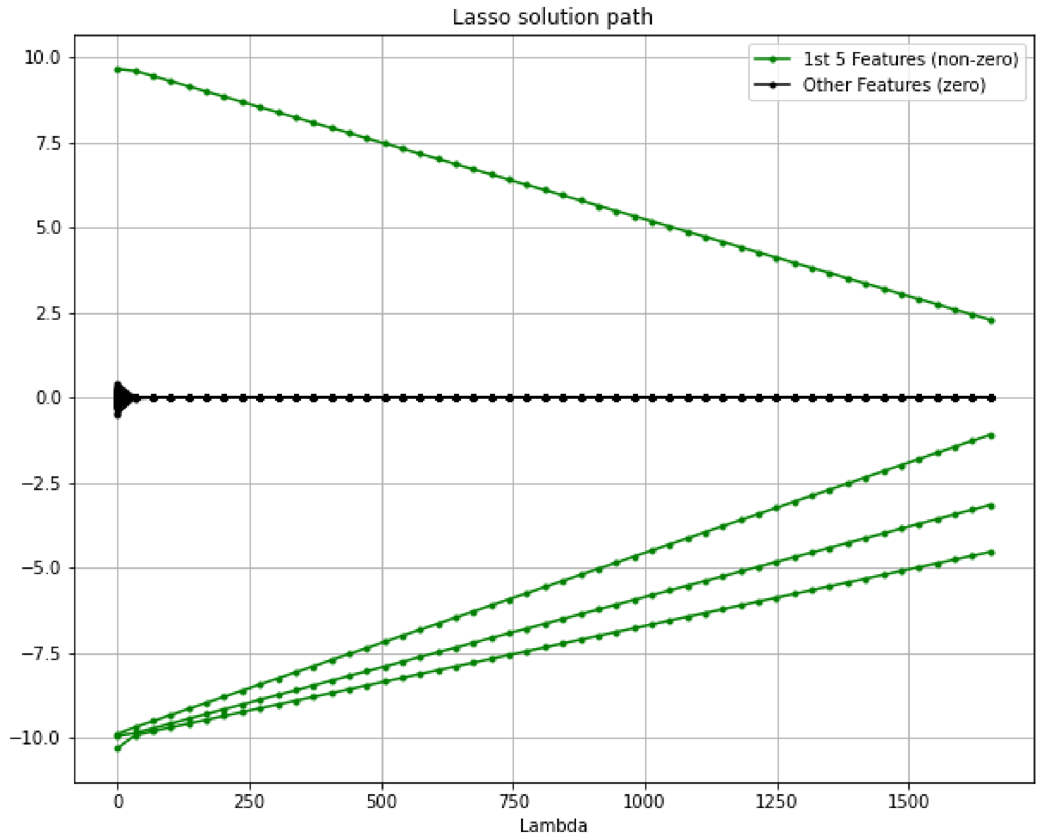

Precision and Recall vs. λ
Precision
Recall
1.0
0.8
0.6
0.4
0.2
0.0
0
200
400
600
800
Lambda

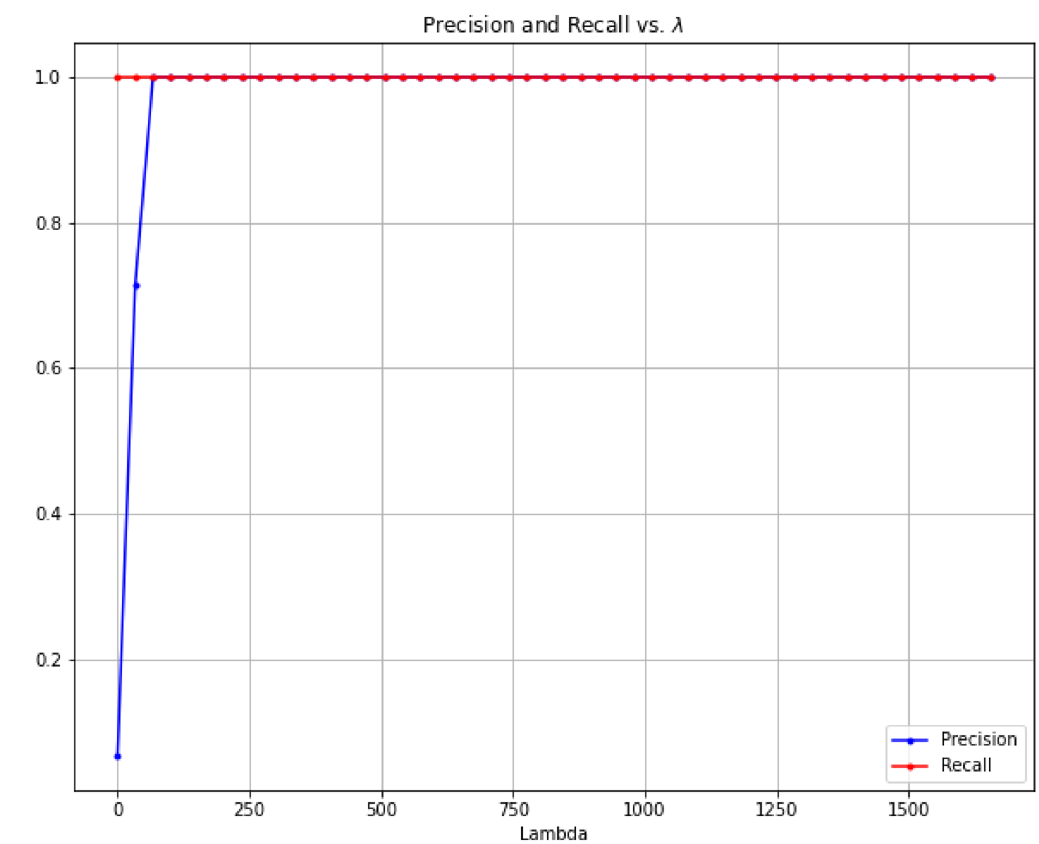
For $(n = 50, m = 1000)$,


Lasso solution path
1st 5 Features (non-zero)
Other Features (zero)
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
0
200
400
600
800
Lambda

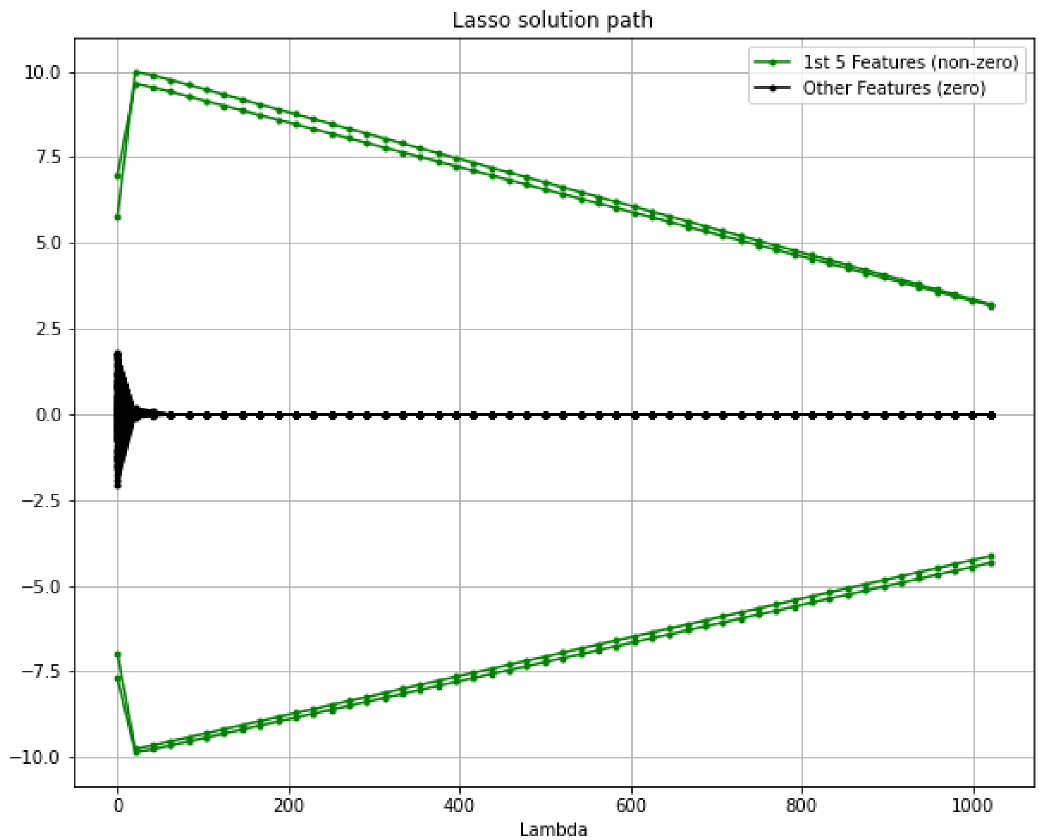

Precision and Recall vs. λ
Precision
Recall
1.0
0.8
0.6
0.4
0.2
0.0
0
200
400
600
800
Lambda

For $(n = 100, m = 75)$,

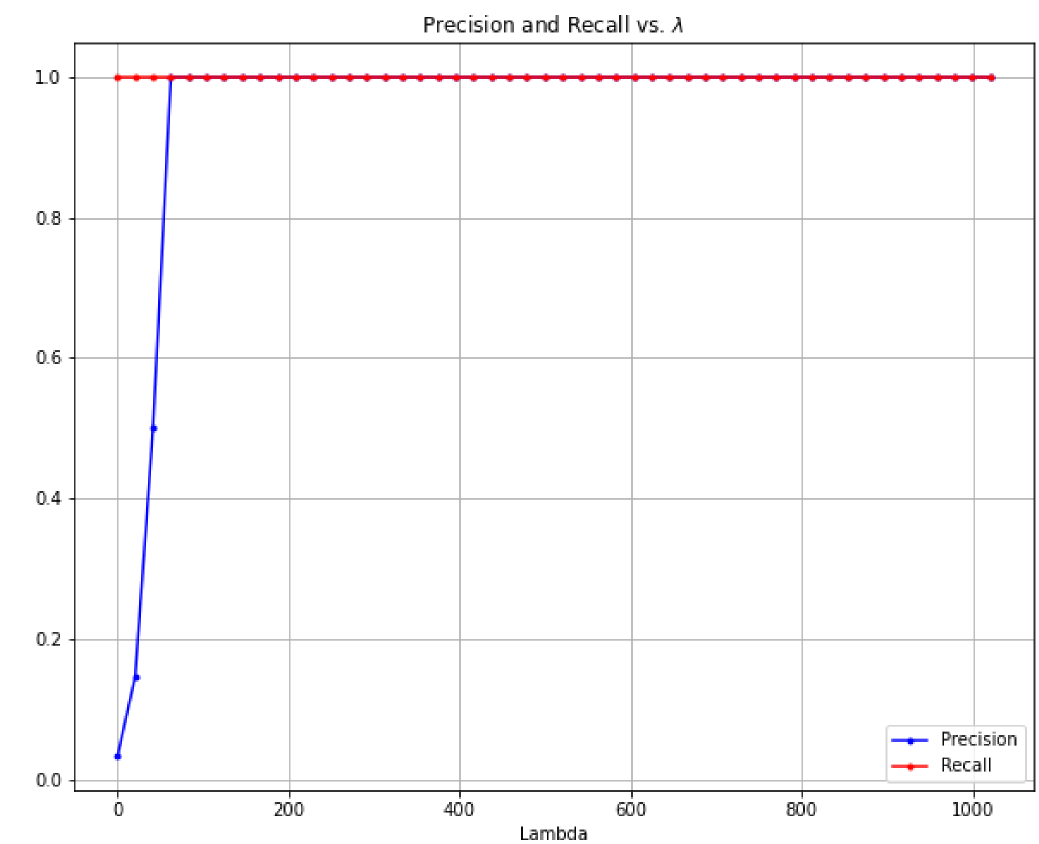

Lasso solution path
1st 5 Features (non-zero)
Other Features (zero)
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
0
250
500
750
1000
1250
1500
Lambda


Precision and Recall vs. λ
1.0
0.8
0.6
0.4
0.2
Precision
Recall
0
250
500
750
1000
1250
1500
Lambda

For $(n = 100, m = 150)$,

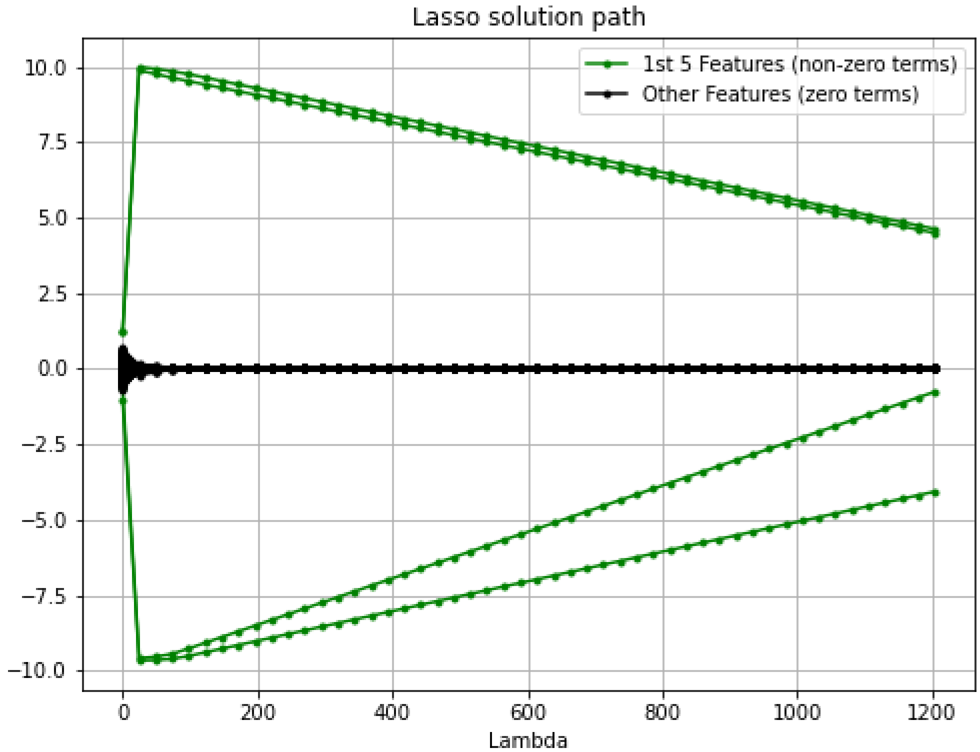


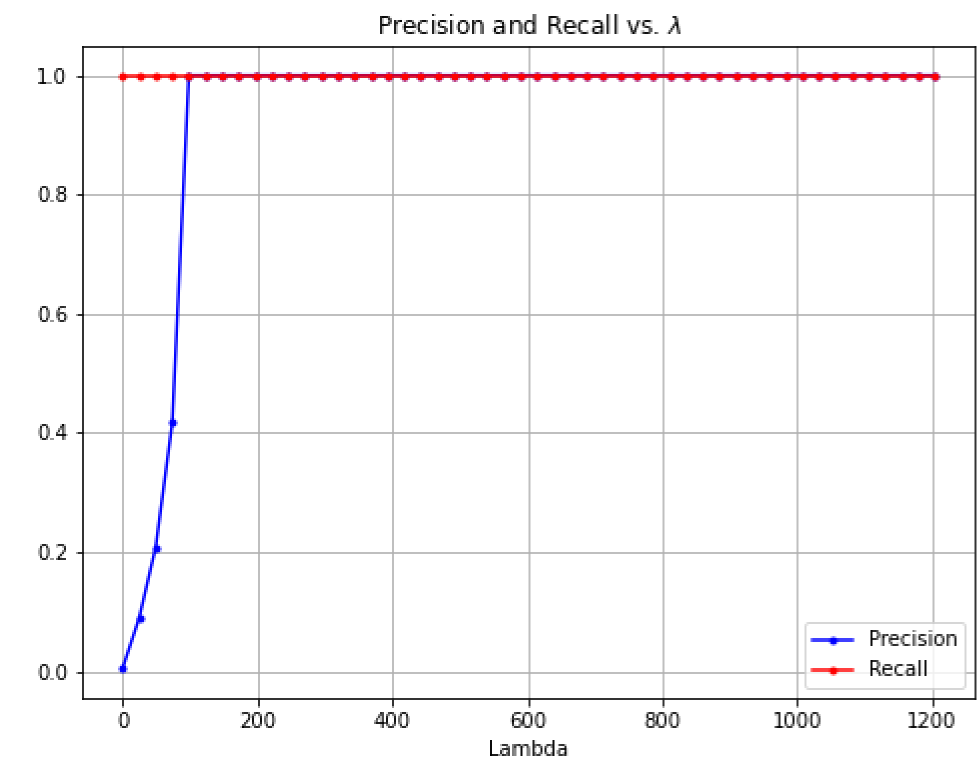
Lasso solution path
1st 5 Features (non-zero)
Other Features (zero)
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
0
200
400
600
800
1000
Lambda



Precision and Recall vs. λ
Precision
Recall
1.0
0.8
0.6
0.4
0.2
0.0
0
200
400
600
800
1000
Lambda

For $(n = 100, m = 1000)$,





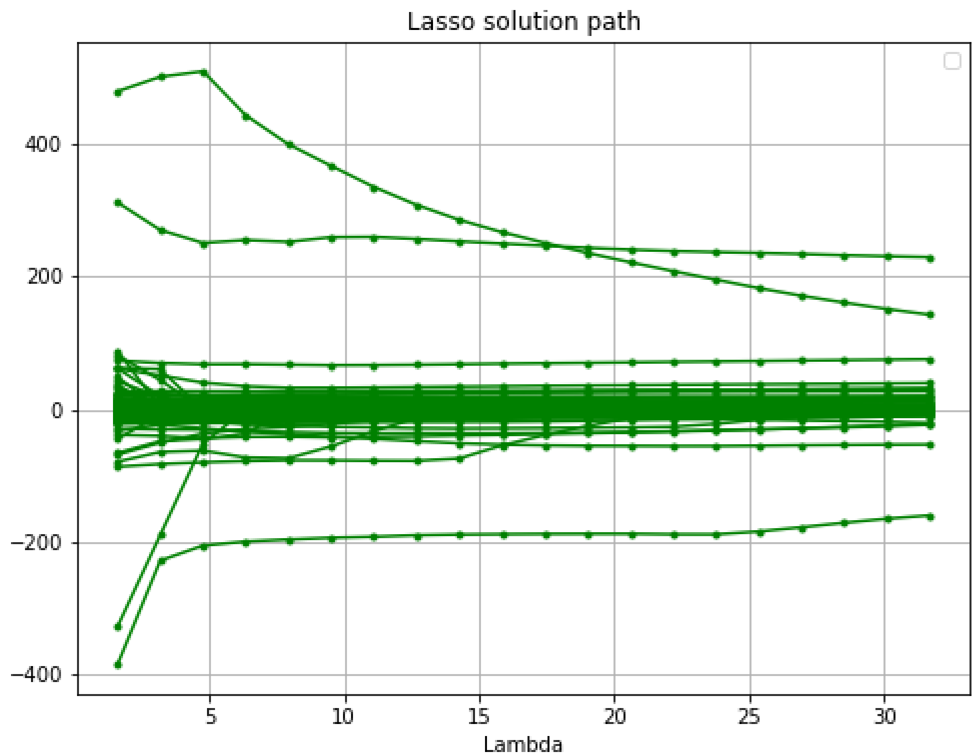
(e)

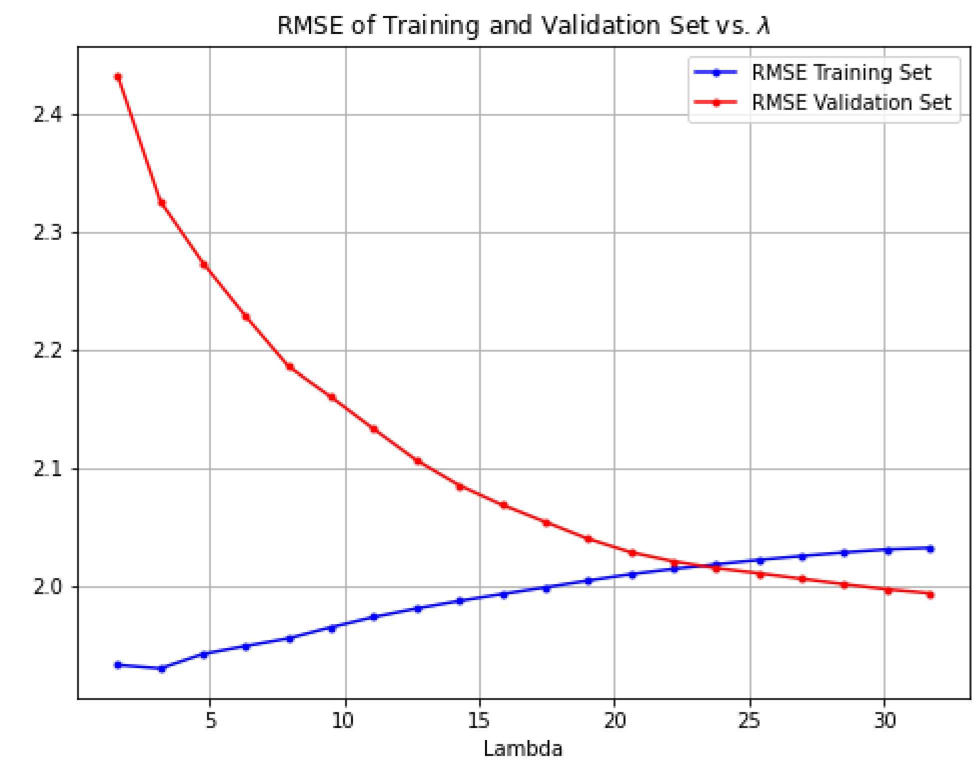
This portion was completed late and submitted on 2/23.

Below is a plot of the lasso solution path and a plot of RMSE (for both the validation and test set). In addition, the top-10 features with the largest magnitude are identified below.

Feature	Magnitude
and	228.92007242281383
were soaked in	-159.55591374549522
the	142.08381416889432
great	75.10155839764127
set	-52.662879026941425
best	39.82050044662048
love	32.49102678604067
amazing	30.24397828436301
delicious	27.33324807664297
of a	-22.706295466396767

Based on the results of the top-10 features, it makes sense that features relating to positive features ("great", "best", "love", "amazing") have a positive correlation to predicting the stars of a yelp review. In contrast, negative ("were soaked in") had a strong negative correlation to the stars yelp review. It is of importance to notice that many of the top features include conjunction terms like "and" and "of a" which can considered potential noise in our solution.


Lasso solution path
400
200
0
−200
−400
5
10
15
20
25
30
Lambda


RMSE of Training and Validation Set vs. λ
RMSE Training Set
RMSE Validation Set
2.4
2.3
2.2
2.1
2.0
5
10
15
20
25
30
Lambda

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In [ ]:
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