Mg.01: Introduction to Signal & System (Continuous and Discrete)

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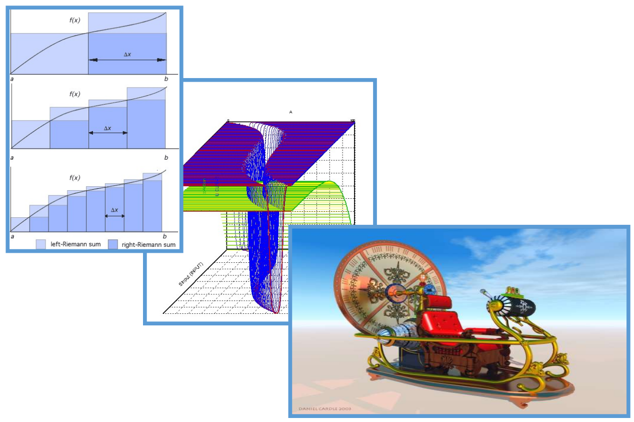

Introduction

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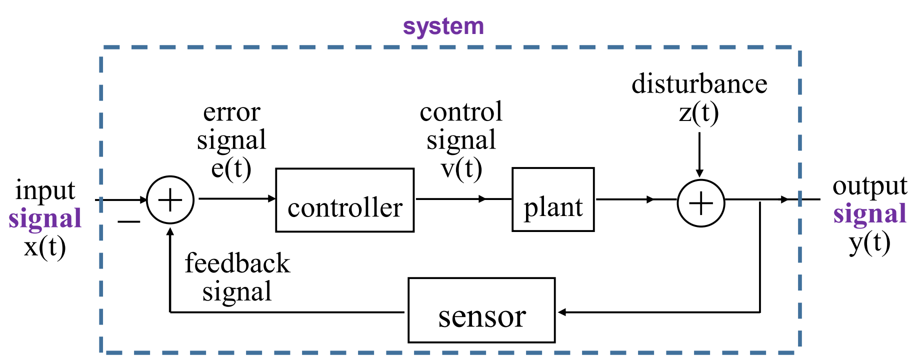
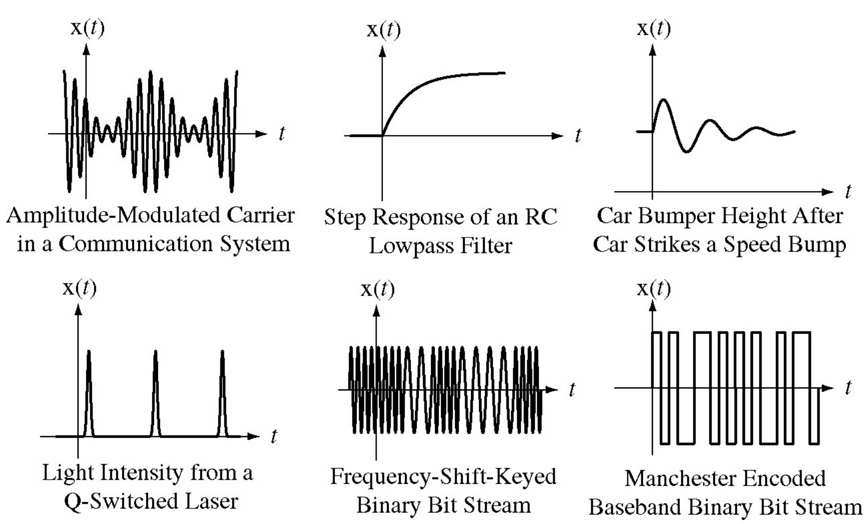


– example: aircraft landing systems, satellite stabilization systems, robot arm control systems, etc.

A Continuous Signal

- A **signal** is a function of one or more variables, which conveys information on the nature of some physical phenomena.
- Examples
  - f (t) : a voice signal, a music signal
  - f (x , y) : an image signal, a picture
  - f (x , y , t) : a video signal
  - x_n : a sequence of data (n: integer)
  - b_n : a bit stream (b:1 or 0)
  - continuous-time, discrete-time
  - analog, digital
- Human Perceptible/Machine Processed

Continuous-Time Signals



Combinations of Signals

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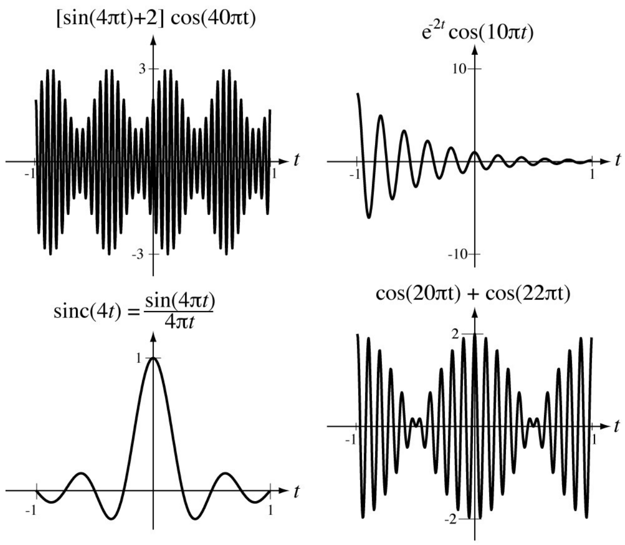

Recorded Sound As A Signal Example

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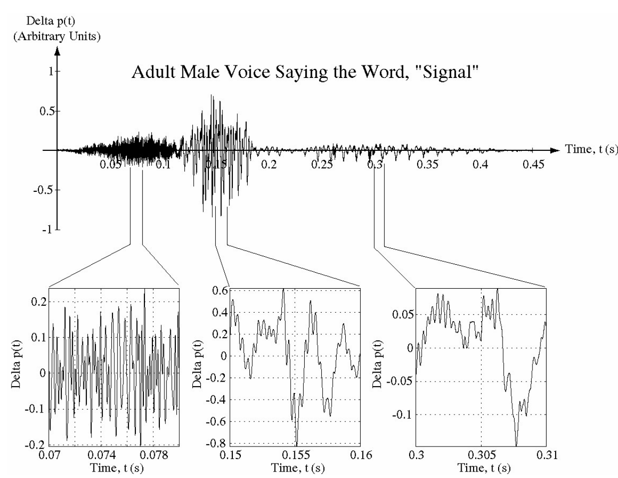
- "s" "i" "gn" "al"

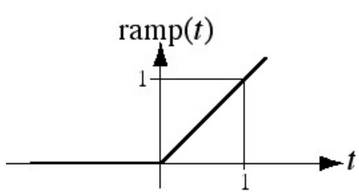


Basic Signal Operations

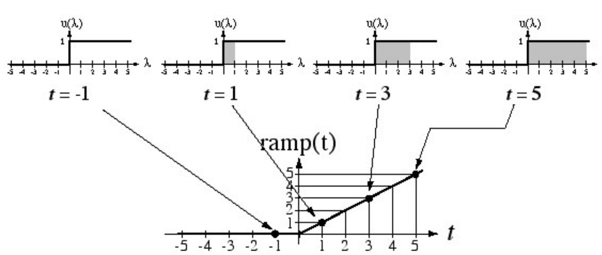
Table 1.1 Basic Signals

Signal	Definition
Complex exponential	$\lvert A\rvert e^{rt}[\cos(\Omega_0 t+\theta)+j\sin(\Omega_0 t+\theta)] \quad -\infty<t<\infty$
Sinusoid	$A\cos(\Omega_0 t+\theta)=A\sin(\Omega_0 t+\theta+\pi/2) \quad -\infty<t<\infty$
Unit impulse	$\delta(t)=0 \;\; t\neq 0$, undefined at $t=0$ $\int_{-\infty}^{t}\delta(\tau)d\tau=1, \quad t>0$ $\int_{-\infty}^{\infty}f(\tau)\delta(t-\tau)d\tau=f(t)$
Unit step	$u(t)=\begin{cases}1 & t>0\\ 0 & t<0\end{cases}$
Ramp	$r(t)=tu(t)=\begin{cases}t & t>0\\ 0 & t<0\end{cases}$ $\delta(t)=du(t)/dt$ $u(t)=\int_{-\infty}^{t}\delta(\tau)d\tau$ $r(t)=\int_{-\infty}^{t}u(\tau)d\tau$
Rectangular pulse	$p(t)=A[u(t)-u(t-1)]=\begin{cases}A & 0\le t\le 1\\ 0 & \text{otherwise}\end{cases}$
Triangular pulse	$\Lambda(t)=A[r(t)-2r(t-1)+r(t-2)]=\begin{cases}At & 0\le t\le 1\\ A(2-t) & 1<t\le 2\\ 0 & \text{otherwise}\end{cases}$
Sampling	$\delta_{T_s}(t)=\sum_k \delta(t-kT_s)$
Sinc	$S(t)=\sin(\pi t)/(\pi t)$ $S(0)=1$ $S(k)=0 \;\; k\neq 0$ integer $\int_{-\infty}^{\infty}S^2(t)dt=1$

The Unit Ramp Function

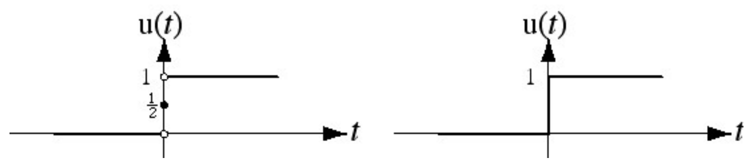


$$\text{ramp}(t)=\begin{Bmatrix}t & , & t>0\\ 0 & , & t\le 0\end{Bmatrix}=\int_{-\infty}^{t}\text{u}(\lambda)d\lambda=t\,\text{u}(t)$$



The Unit Step Function

$$\mathrm{u}(t) = \begin{cases} 1 & , \ t > 0 \\ 1/2 & , \ t = 0 \\ 0 & , \ t < 0 \end{cases}$$



The product signal $\mathrm{g}(t)\mathrm{u}(t)$ can be thought of as the signal $\mathrm{g}(t)$ "turned on" at time $t = 0$.

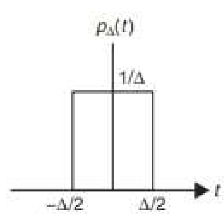
The Impulse Function

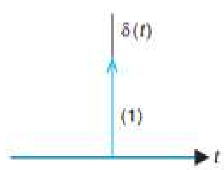
$$p_\Delta(t) = \begin{cases} \dfrac{1}{\Delta} & -\Delta/2 \le t \le \Delta/2 \\ 0 & t < -\Delta/2 \text{ and } t > \Delta/2 \end{cases}$$

$$\delta(t) = \lim_{\Delta \to 0} p_\Delta(t)$$

$$\delta(t) = \begin{pmatrix} 0 & for\ t \neq 0 \\ \infty & for\ t = 0 \end{pmatrix}$$

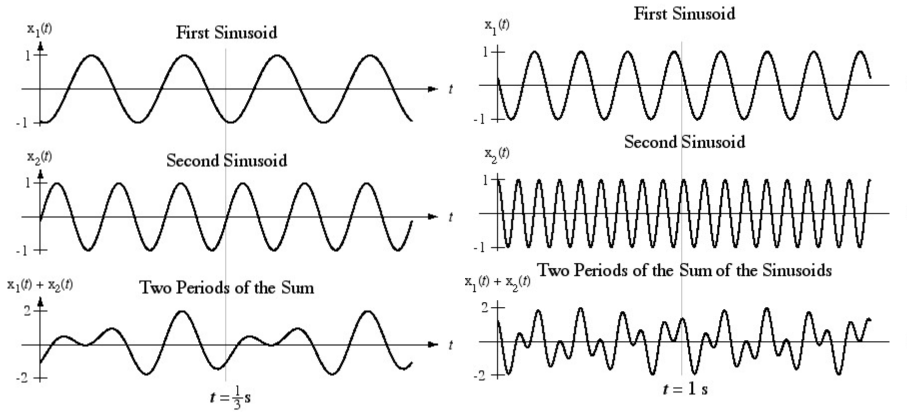
$$\int_{0^-}^{0^+} \delta(t)dt = 1$$





Sums Of Periodic Functions

The period of the sum of periodic functions is the **least common multiple** of the periods of the individual functions summed. If the least common multiple is infinite, the sum function is aperiodic.



Exponential/Sinusoidal Signals

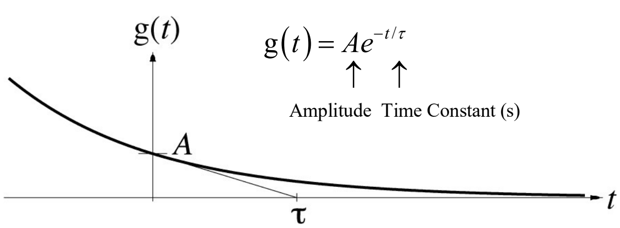
- Basic Building Blocks from which one can construct many different signals and define frameworks for analyzing many different signals efficiently

$$x(t) = e^{j\omega_0 t},$$

fundamental period $T_0 = \dfrac{2\pi}{|\omega_0|}$

fundamental frequency $\omega_0 = \dfrac{2\pi}{T_0}$

$\omega_0 : rad$ / sec



REAL AND COMPLEX SIGNALS

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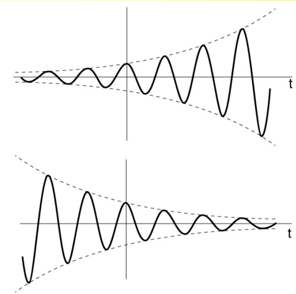
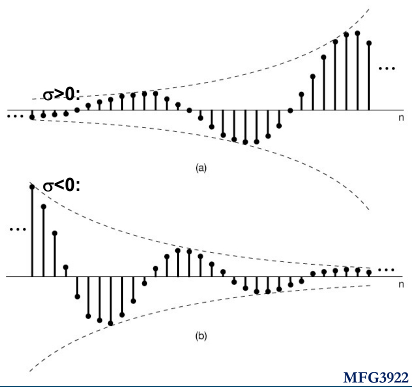
For example, suppose $s = \sigma + j\omega$ for a CT signal:

$$\Re\{x(t) = e^{st}\} = \Re\{e^{(\sigma + j\omega)t}\} = e^{\sigma t}\cos(j\omega t)$$

σ>0:

σ<0:





For example, suppose $z = e^{(\sigma + j\omega)}$ for a DT signal:

$$\Re\{x(t) = z^{n}\} = \Re\{z^{(\sigma + j\omega)n}\} = e^{\sigma n}\cos(\omega n)$$

Basic Operation for Continuous-Time Signals

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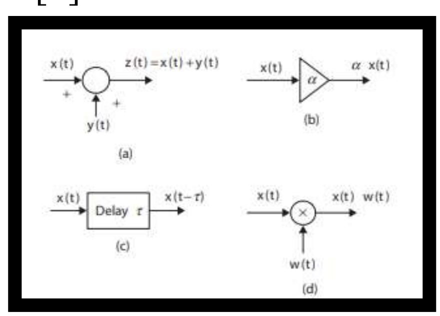
- Time Shift $x(t) \to x(t - t_0)$, $x[n] \to x[n - n_0]$
- Time Reversal (folding) $x(t) \to x(-t)$, $x[n] \to x[-n]$
- Time Scaling $x(t) \to x(at)$, $x[n] \to ?$
- Combination (additian) $x(t) \to x(at + b)$, $x[n] \to ?$
- Integration
- Deferensiation



Time Scaling

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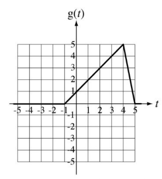
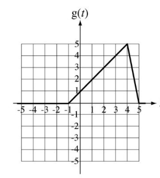
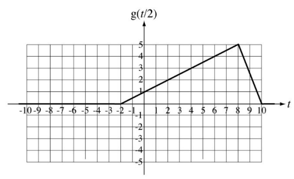
Time scaling, $t \to t/a$



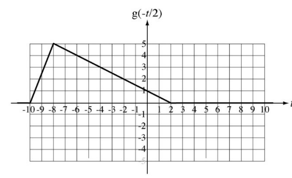
t	−5	−4	−3	−2	−1	0	1	2	3	4	5
$g(t)$	0	0	0	0	0	1	2	3	4	5	0



t	−5	−4	−3	−2	−1	0	1	2	3	4	5
$g(t)$	0	0	0	0	0	1	2	3	4	5	0



t	−10	−8	−6	−4	−2	0	2	4	6	8	10
$t/2$	−5	−4	−3	−2	−1	0	1	2	3	4	5
$g(t/2)$	0	0	0	0	0	1	2	3	4	5	0



t	−10	−8	−6	−4	−2	0	2	4	6	8	10
$-t/2$	5	4	3	2	1	0	−1	−2	−3	−4	−5
$g(-t/2)$	0	5	4	3	2	1	0	0	0	0	0

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Amplitudo Scaling

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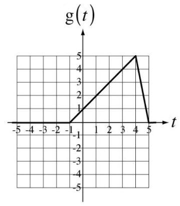
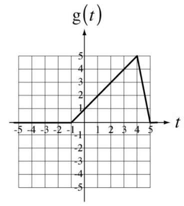
Amplitude Scaling, $g(t) \to A\,g(t)$

$g(t)$



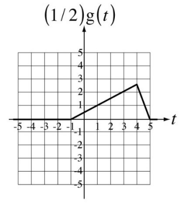
t	−5	−4	−3	−2	−1	0	1	2	3	4	5
$g(t)$	0	0	0	0	0	1	2	3	4	5	0

$g(t)$



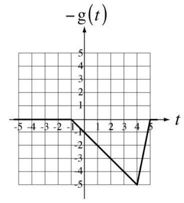
t	−5	−4	−3	−2	−1	0	1	2	3	4	5
$g(t)$	0	0	0	0	0	1	2	3	4	5	0

$(1/2)g(t)$



t	−5	−4	−3	−2	−1	0	1	2	3	4	5
$(1/2)g(t)$	0	0	0	0	0	1/2	1	3/2	2	5/2	0

$-g(t)$



t	−5	−4	−3	−2	−1	0	1	2	3	4	5
$-g(t)$	0	0	0	0	0	−1	−2	−3	−4	−5	0

16

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Shifting and Scaling Functions

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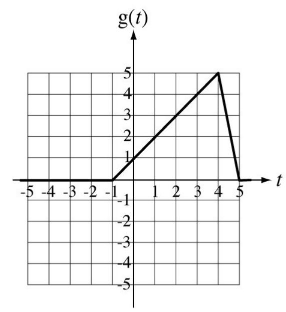
Let a function be defined graphically by



t	-5	-4	-3	-2	-1	0	1	2	3	4	5
$g(t)$	0	0	0	0	0	1	2	3	4	5	0

and let $g(t) = 0 \quad , \quad |t| > 5$

17

Shifting And Scaling Functions

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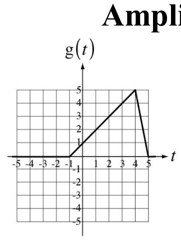
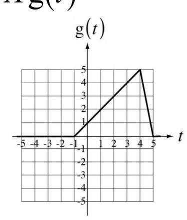
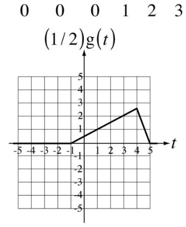
Amplitude Scaling, $g(t) \rightarrow A\,g(t)$



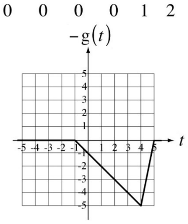
t	-5	-4	-3	-2	-1	0	1	2	3	4	5
$g(t)$	0	0	0	0	0	1	2	3	4	5	0



t	-5	-4	-3	-2	-1	0	1	2	3	4	5
$g(t)$	0	0	0	0	0	1	2	3	4	5	0



t	-5	-4	-3	-2	-1	0	1	2	3	4	5
$(1/2)g(t)$	0	0	0	0	0	$1/2$	1	$3/2$	2	$5/2$	0



t	-5	-4	-3	-2	-1	0	1	2	3	4	5
$-g(t)$	0	0	0	0	0	-1	-2	-3	-4	-5	0

18

Shifting And Scaling Functions

Multiple transformations $\mathrm{g}(t) \to A\,\mathrm{g}\left(\frac{t-t_0}{a}\right)$

A multiple transformation can be done in steps

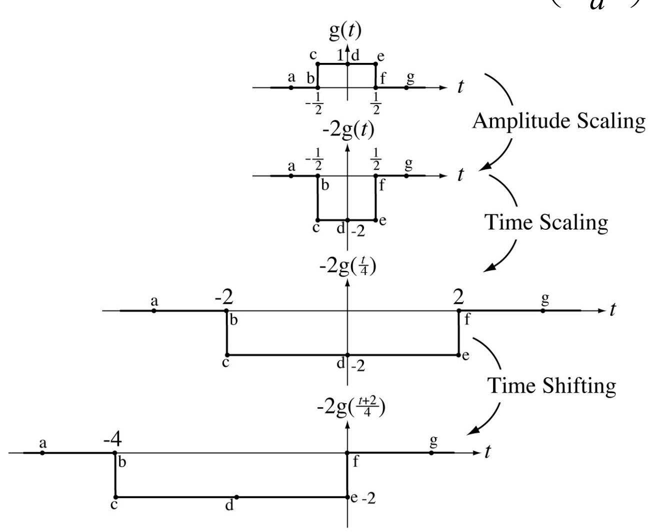
$$\mathrm{g}(t) \xrightarrow[\text{scaling, } A]{\text{amplitude}} A\,\mathrm{g}(t) \xrightarrow{t \to t/a} A\,\mathrm{g}\left(\frac{t}{a}\right) \xrightarrow{t \to t-t_0} A\,\mathrm{g}\left(\frac{t-t_0}{a}\right)$$

The sequence of the steps is significant

$$\mathrm{g}(t) \xrightarrow[\text{scaling, } A]{\text{amplitude}} A\,\mathrm{g}(t) \xrightarrow{t \to t-t_0} A\,\mathrm{g}(t-t_0) \xrightarrow{t \to t/a} A\,\mathrm{g}\left(\frac{t}{a}-t_0\right) \neq A\,\mathrm{g}\left(\frac{t-t_0}{a}\right)$$

Simultaneous Shifting And Scaling Functions

Simultaneous scaling and shifting $\mathrm{g}(t) \to A\,\mathrm{g}\left(\frac{t-t_0}{a}\right)$



Simultaneous Shifting And Scaling Functions

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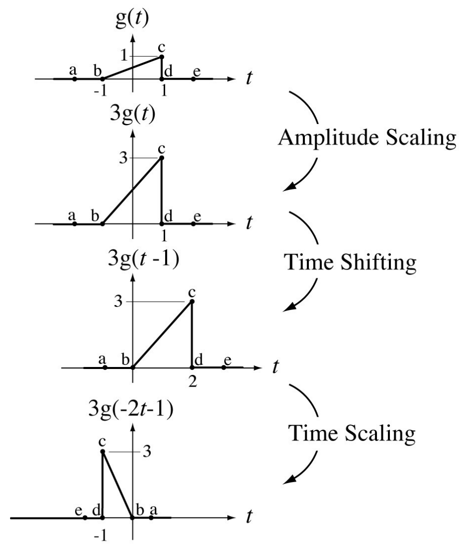
Simultaneous scaling and shifting, $A\,\mathrm{g}\left(bt - t_0\right)$



Shifting and Scaling Functions

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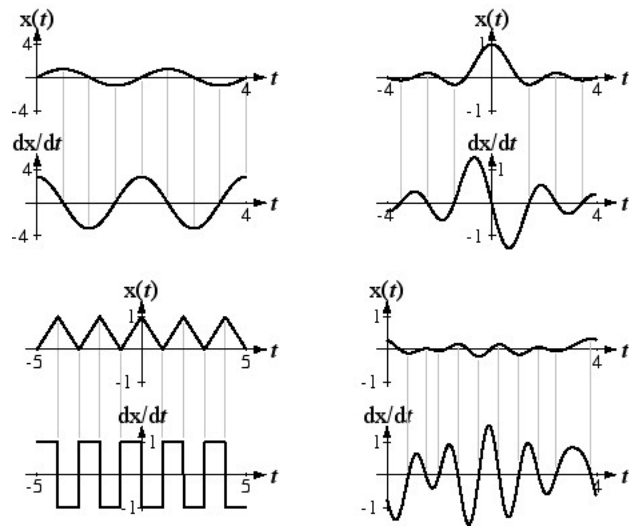
$$\text{Height } +5 \;\rightarrow -2 \Rightarrow A = -0.4 \Rightarrow \mathrm{g}_1\left(t\right) \rightarrow -0.4\,\mathrm{g}_1\left(t\right)$$

$$\text{Shift left } 5 \Rightarrow t_0 = -5 \Rightarrow -0.4\,\mathrm{g}_1\left(t\right) \rightarrow -0.4\,\mathrm{g}_1\left(t+5\right)$$

$$\text{Width } +6 \text{ to } +2 \Rightarrow w = 3 \Rightarrow -0.4\,\mathrm{g}_1\left(t+5\right) \rightarrow -0.4\,\mathrm{g}_1\left(3t+5\right)$$

Differentiation



Integration

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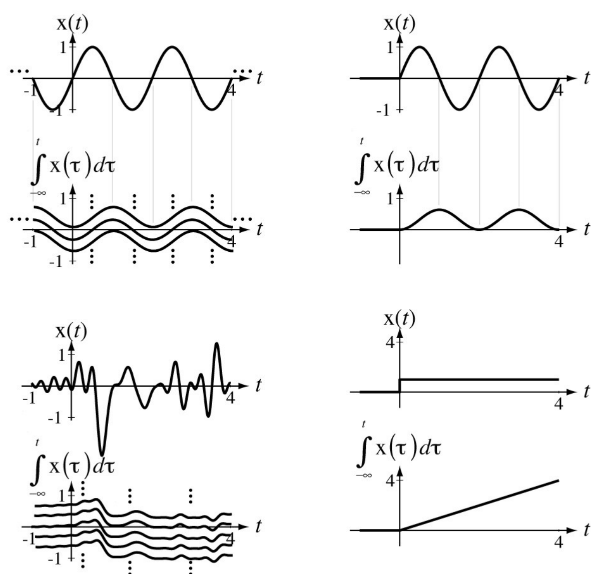

Unit Impulse and Unit Step Functions

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- Continuous-time

$$\delta(t) \quad , \quad u(t)$$

  - **First Derivative**

$$\boxed{\delta(t) = \frac{du(t)}{dt}}$$

  - **Running Integral**

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau$$

  - **Sampling property**

$$x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$$

Continuouse-Time System Relationships

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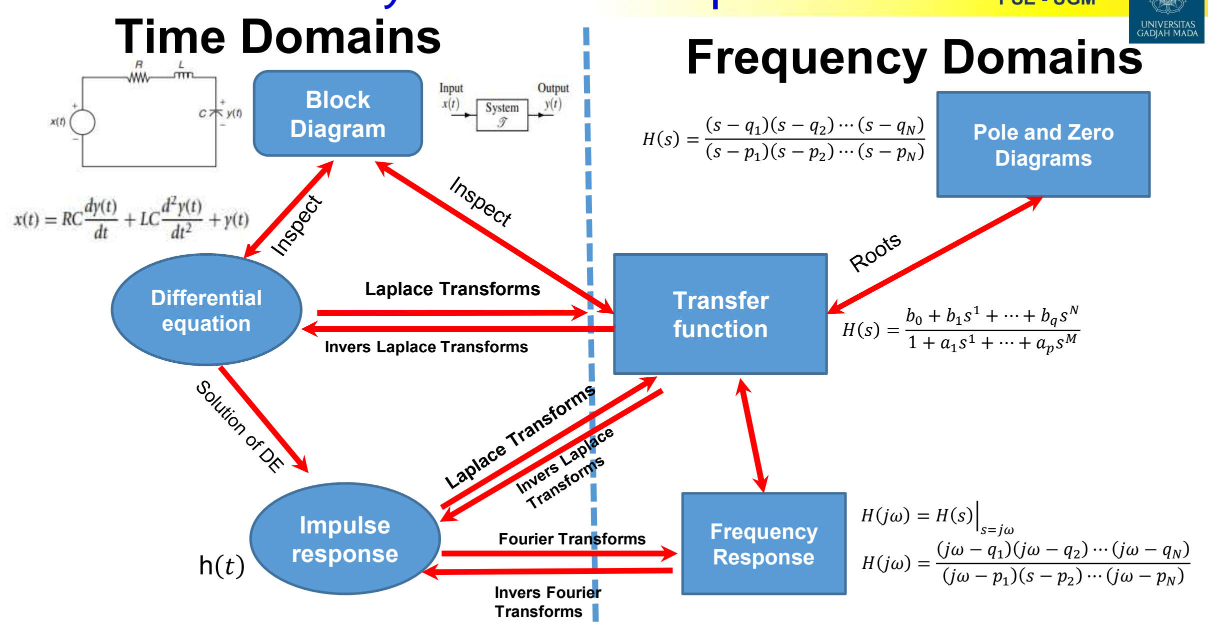

Discrete-Time Signals and Systems

- Quote of the Day
- Mathematics is the tool specially suited for dealing with abstract concepts of any kind and there is no limit to its power in this field.

  - Paul Dirac

Discrete Signal and System

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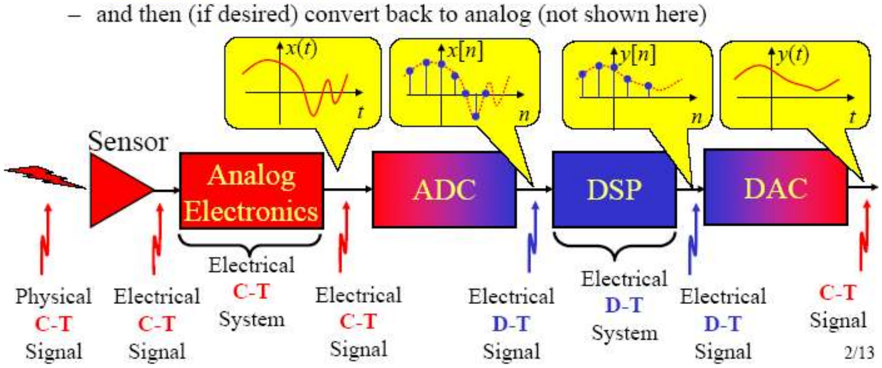
- Modern systems generally…
  - get a continuous-time signal from a sensor
  - a cont.-time system modifies the signal
  - an "analog-to-digital converter" (ADC or A-to-D) sample the signal to create a discrete-time signal … a "stream of numbers"
  - A discrete-time system to do the processing
  - and then (if desired) convert back to analog (not shown here)



28

Discrete-Time Signals: Sequences

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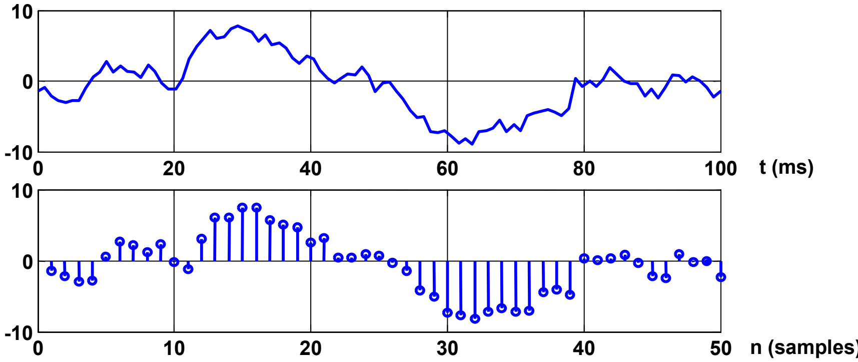
- **Discrete-time signals** are represented by sequence of numbers
  - The n^{th} number in the sequence is represented with $x[n]$
- Often times sequences are obtained by sampling of continuous-time signals
  - In this case $x[n]$ is value of the analog signal at $x_c(nT)$
  - Where T is the sampling period



Basic Sequences Types

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$$\delta[n] = \begin{cases} 0 & n \neq 0 \\ 1 & n = 0 \end{cases}$$

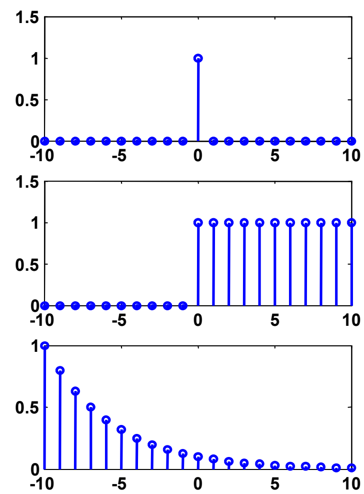
- Unit sample (**impulse**) sequence

$$u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$$

- Unit **step** sequence

$$x[n] = A\alpha^n$$

- **Exponential** sequences



Basic Sequences Types (cont')

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- **Complex-valued exponential sequence**:

$$x(n) = e^{(\sigma + j\omega_0)n}, \forall n$$

where σ produces an attenuation (if <0) or amplification (if >0) and $\omega 0$ is the frequency in radians

- **Sinusoidal sequence**:

$$x(n) = A\cos(\omega_0 n + \theta_0), \forall n$$

where A is an amplitude and $\vartheta 0$ is the phase in radians.

- **Random sequences**:
- **Periodic sequence**:

A sequence $x(n)$ is periodic if $x(n) = x(n + N)$, $\forall n$.

OPERATIONS ON SEQUENCES

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- **Delaying** (Shifting) a sequence

$$y[n] = x[n - n_o]$$

- **Signal addition**:

$$\{x_1(n)\} + \{x_2(n)\} = \{x_1(n) + x_2(n)\}$$

- **Signal multiplication**:

$$\{x_1(n)\} \cdot \{x_2(n)\} = \{x_1(n)x_2(n)\}$$

- **Scaling**: $\alpha\{x(n)\} = \{\alpha x(n)\}$
- **Shifting**: $y(n) = \{x(n-k)\}$
- **Folding** $y(n) = \{x(-n)\}$
- **Sample summation**: $\sum_{n=n_1}^{n_2} x(n) = x(n_1) + \cdots + x(n_2)$

OPERATIONS ON SEQUENCES (cont'

- **Sample products**:

$$\prod_{n_1}^{n_2} x(n) = x(n_1) \times \cdots \times x(n_2)$$

- **Signal energy**:

$$\mathcal{E}_x = \sum_{-\infty}^{\infty} x(n)x^*(n) = \sum_{-\infty}^{\infty} |x(n)|^2$$

- **Signal power**: The

$$\mathcal{P}_x = \frac{1}{N} \sum_{0}^{N-1} |\tilde{x}(n)|^2$$

Example: time sifting

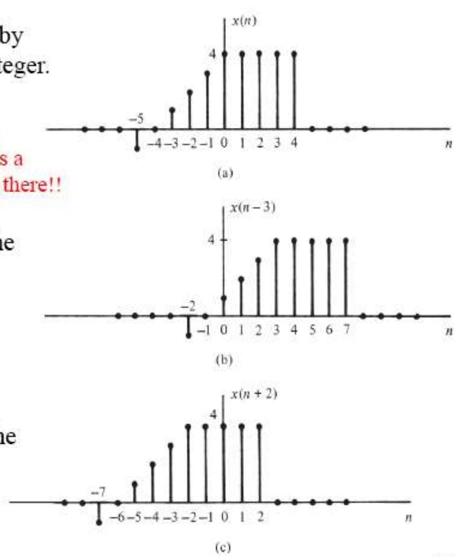
Simple Manipulation of DT Signals

Time Shift

A given signal $x[n]$ is shifted in time by replacing n by $n - k$, where k is an integer.

Don't forget there is a negative already in there!!

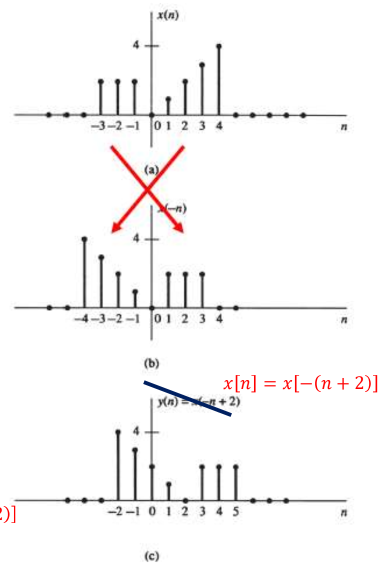
- If k is positive then the shift is to the right… which is a "delay"
- If k is negative then the shift is to the left… which is an "advance"



Example : time reversal

Time Reversal

A given signal $x[n]$ is reversed in time by replacing n by $-n$.



- Time reversal just "anchors" the signal at $n = 0$ and flips it
- Flipping and shifting can be combined… Careful here: Do the shift first then the flip.

$x[n] = x[-(n+2)]$

$$x[n] \rightarrow \underbrace{x[n+2]}_{\text{Shift left by 2}} \rightarrow \underbrace{\cancel{x[-n+2]}}_{\text{Flip the shifted signal}}$$

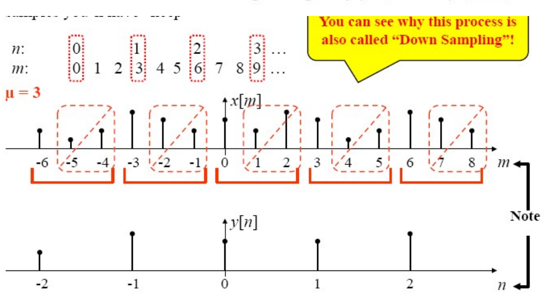
15

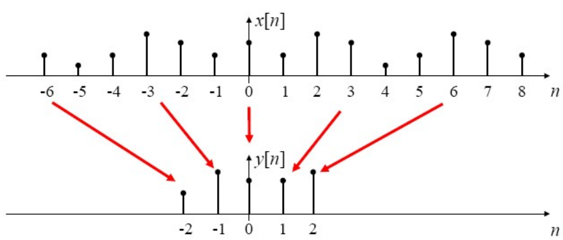
Example : Time Scaling

Time Scaling

A given signal $x[n]$ is scaled in time by replacing n by μn, where μ must be an integer.

$$\boxed{y[n] = x[\mu n]}$$





Sinusoidal Sequences

PSE - UGM

- Important class of sequences

$$x[n] = \cos(\omega_o n + \phi)$$

- An **exponential sequence** with complex $\alpha = |\alpha| e^{j\omega_o}$ and $A = |A| e^{j\phi}$

$$x[n] = A\alpha^n = |A| e^{j\phi} |\alpha|^n e^{j\omega_o n} = |A||\alpha|^n e^{j(\omega_o n + \phi)}$$

$$x[n] = |A||\alpha|^n \cos(\omega_o n + \phi) + j|A||\alpha|^n \sin(\omega_o n + \phi)$$

- x[n] is a sum of weighted sinusoids
- Different from continuous-time, discrete-time sinusoids
  - Have ambiguity of $2\pi k$ in frequency

$$\cos((\omega_o + 2\pi k)n + \phi) = \cos(\omega_o n + \phi)$$

  - Are not necessary periodic with $2\pi/\omega_o$

$$\cos(\omega_o n + \phi) = \cos(\omega_o n + \omega_o N + \phi) \text{ only if } N = \frac{2\pi k}{\omega_o} \text{ is an integer}$$

37

Discrete Signal Analysis

PSE - UGM

Transforms & Notation

Proakis & Manolakis don't use this superscript Notation. I borrowed it from Porat's DSP Book

Fourier Transform for CT Signals

$$X^F(F) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi Ft} dt$$

$$x(t) = \int_{-\infty}^{\infty} X^F(F) e^{j2\pi Ft} dF$$

Fourier Transform for DT Signals

$$X^f(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

Discrete Time Fourier Transform

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X^f(\omega) e^{j\omega n} d\theta$$

Set $z = e^{j\omega}$

Z Transform for DT Signals

$$X^z(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

Inverse ZT done using partial fractions & a ZT table

Discrete Fourier Transform for DT Signals

Discrete Fourier Transform

$$X^d[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N} \quad k = 0, 1, 2, \ldots, N-1$$

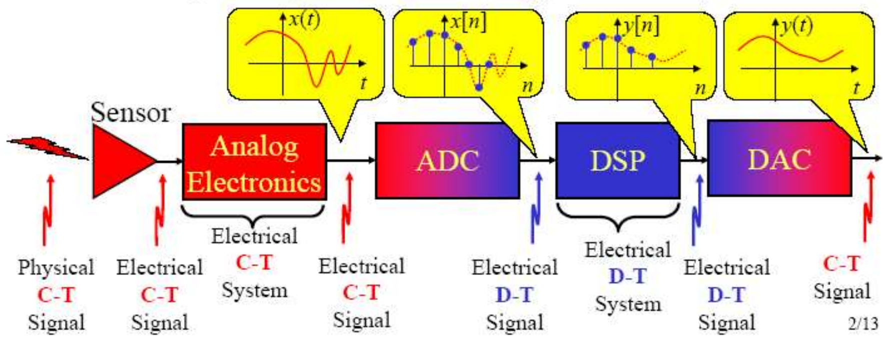
$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X^d[k] e^{j2\pi kn/N} \quad n = 0, 1, 2, \ldots, N-1$$

4/13

38

Discrete Signal and System

- Modern systems generally…
  - get a **continuous-time signal** from a sensor
  - a **cont.-time system** modifies the signal
  - an "analog-to-digital converter" (ADC or A-to-D) sample the signal to create a **discrete-time signal** … a "stream of numbers"
  - A **discrete-time system** to do the processing
  - and then (if desired) convert back to analog (not shown here)



Discrete-Time System Relationships