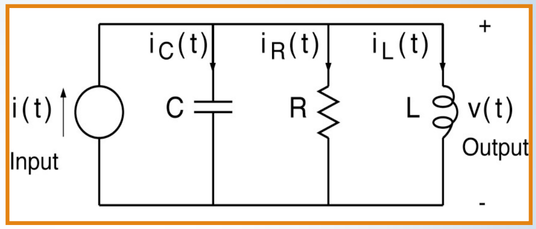
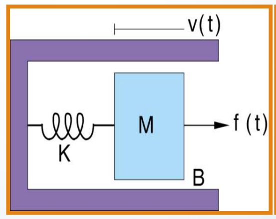
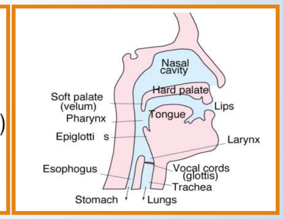
Mg.02: Continuous LTI system

Objectives:

- System
- Linear Time Invariant System
- Convolution Integral
- Properties of Convolution
- Interconnections of Systems







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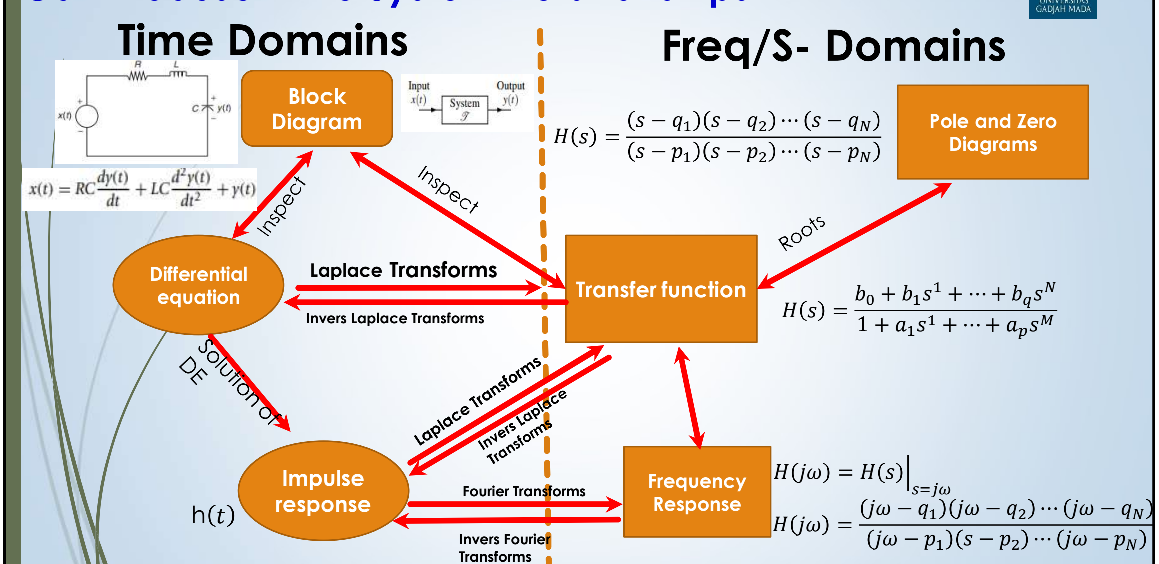
Continuouse-Time System Relationships

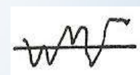


➢ A Continuous System

- An entity that manipulates one or more signals to accomplish some function, including yielding some new signals.

input signal → System → output signal

$x(t)$ → $h(t)$ → $y(t)$



- Examples
  - an electric circuit
  - a telephone handset
  - a PC software receiving pictures from Internet
  - a TV set
  - a computer with some software handling some data

Discrete Signal and System

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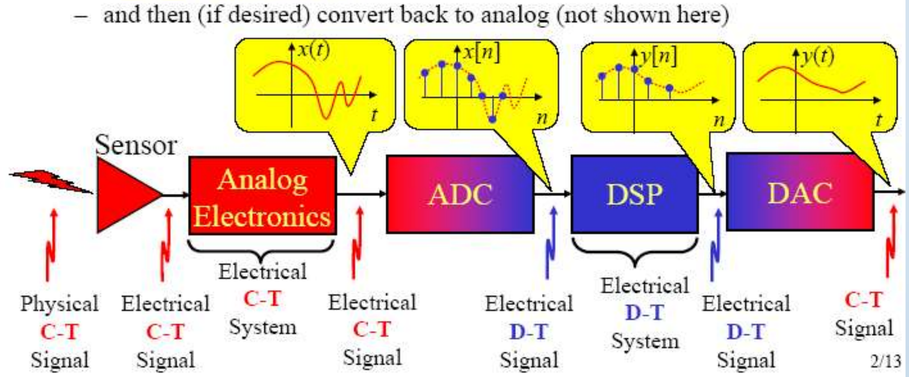
- Modern systems generally…
  - get a **continuous-time signal** from a sensor
  - a **cont.-time system** modifies the signal
  - an "analog-to-digital converter" (ADC or A-to-D) sample the signal to create a **discrete-time signal** … a "stream of numbers"
  - A **discrete-time system** to do the processing
  - and then (if desired) convert back to analog (not shown here)



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➢ Systems From An Input/Output Perspective

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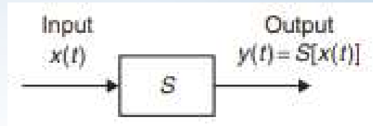
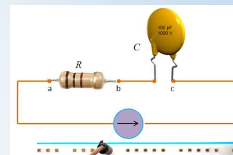
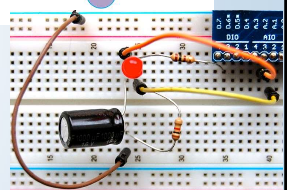
- **Systems** can be described by their **input/output** behavior.
- The **input**, *x*, causes the **output**, *y*.
- The form of the internal system can vary, and is often modeled by a **differential equation** or a **transfer function**.



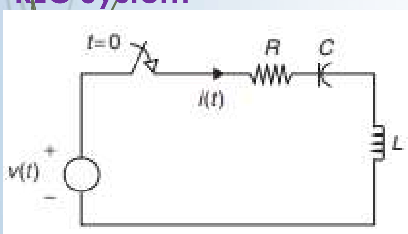




Representation of Systems Model by Differential Equations

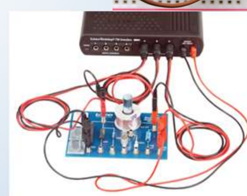
$$a_0 y(t) + a_1 \frac{dy(t)}{dt} + \cdots + \frac{d^N y(t)}{dt^N} = b_0 x(t) + b_1 \frac{dx(t)}{dt} + \cdots + b_M \frac{d^M x(t)}{dt^M} \qquad t \geq 0$$

RLC System

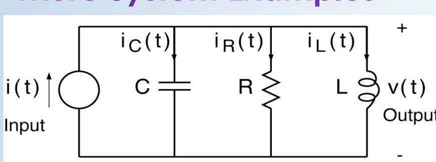


$$v(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(\tau)d\tau$$

$$\frac{dv(t)}{dt} = R\frac{di(t)}{dt} + L\frac{d^2 i(t)}{dt^2} + \frac{1}{C}i(t)$$



More System Examples

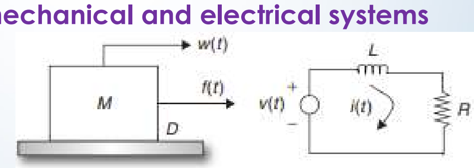


$$i(t) = \underbrace{C\frac{dv(t)}{dt}}_{\text{capacitance}} + \underbrace{\frac{v(t)}{R}}_{\text{resistance}} + \underbrace{\frac{1}{L}\int_{-\infty}^{t} v(\tau)d\tau}_{\text{inductance}}$$

- An example of a system you have studied extensively is an **RLC circuit.**
- You have learned how to compute voltages, currents, transient response, steady-state response, and the transfer function.
- In this course we will generalize these tools to any type of **linear system**.

Equivalences in Analog mechanical and electrical systems

Mechanical System	Electrical System
force f(t)	voltage v(t)
velocity w(t)	current i(t)
mass M	inductance L
damping D	resistance R
compliance K	capacitance C



$$f(t) = M\frac{dw(t)}{dt} + Dw(t)$$

$$v(t) = L\frac{di(t)}{dt} + Ri(t)$$

Using the equivalences R = D, L = M, v(t) = f(t), and i(t) = w(t), the two systems are represented by identical differential equations.

More System Examples

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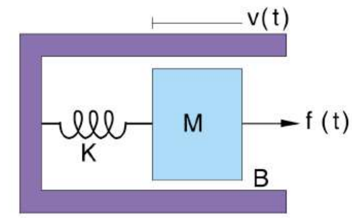
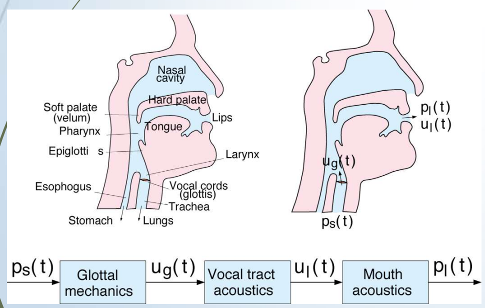
- **Physical systems** can be modeled by **differential equations** so that their input/output behavior can be studied using **transfer functions, Laplace transforms, etc.**
- This physical system is **identical** to the previous circuit from a mathematical point of view.



Force Balance: f — input, v — output

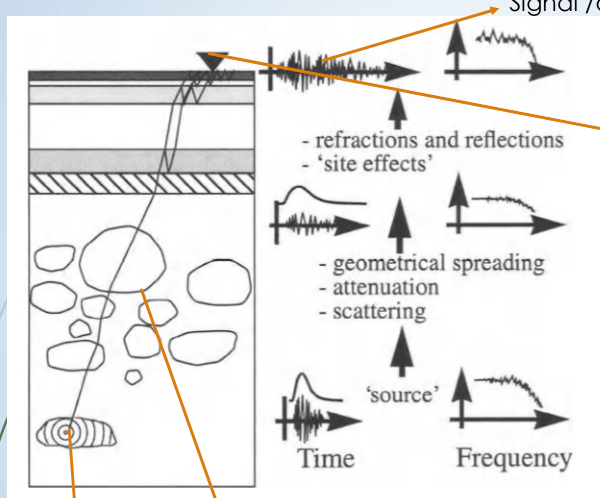
$$f(t) = \underbrace{M\frac{dv(t)}{dt}}_{inertial\ force} + \underbrace{Bv(t)}_{friction} + \underbrace{K\int_{-\infty}^{t} v(\tau)d\tau}_{spring\ force}.$$



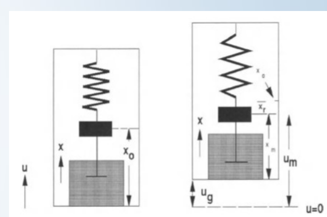
- More complex systems are often modeled as a **cascading of components**.
- Each component is often approximated by a **linear system**.
- DT models of such systems are an integral part of modern computer and information technology.

Earth quake system Recording

Signal /output



Seismometer (system)



$$\ddot{x}_r(t) + 2\varepsilon\dot{x}_r(t) + \omega_0^2 x_r(t) = \omega^2 A_i e^{j\omega t}$$

$$T(s) = \frac{X_r(s)}{U_g(s)} = \frac{-s^2}{s^2 + 2\varepsilon s + \omega_0^2}$$

$$|T(j\omega)| = \frac{\omega^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4h^2\omega_0^2\omega^2}}$$

Source / Input

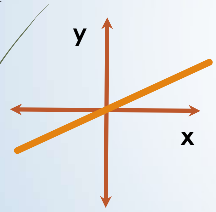
Earth (system)

➢ Linear Time-invariant Systems (LTI)

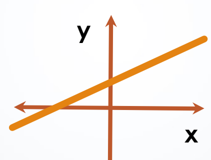
a. Linearity
b. Time invariance
c. Causality
d. Stability

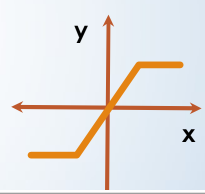
1. Linearity

- A system is **linear** if it obeys the **principle of superposition**:
  - If: $x_1(t) \rightarrow y_1(t)$ and $x_2(t) \rightarrow y_2(t)$
  - Then: $a\,x_1(t) + b\,x_2(t) \rightarrow a\,y_1(t) + b\,y_2(t)$
- Question: Which of these systems are linear?



$$y(t) = x(t)\cos(\omega t + \theta)$$





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2. Time-Invariance

- Informally, a system is **time-invariant (TI)** if **its behavior does not depend on the choice of $t = 0$**. *Then two identical* experiments will yield the same results, regardless the starting time.
- Mathematically CT time-invariant system:

  if $x(t) \to y(t)$, then $x(t - t_0) \to y(t - t_0)$.
- Examples:

$$y(t) = 1 + x(t)\cos(\omega t + \theta)$$

$$y(t) = \int_0^{\infty} x(t-\tau)e^{-\beta\tau}d\tau$$

$$y(t) = \int_{-\infty}^{\infty} \sin(t+\tau)x(\tau)d\tau$$

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Time-Invariance and Periodicity

- Fact: If the input to a TI system is **periodic**, then the output is also periodic with **the same period**.
- "Proof":

Suppose:	$x(t + T) = x(t)$
and:	$x(t) \to y(t)$
Then by TI:	$x(t + T) \to y(t+T)$

$\uparrow$ But these are the same input! $\uparrow$ So these must be the same output.

Therefore: $y(t) = y(t+T)$.

- A basic fact: **If we know the response of an LTI system to some inputs (e.g., sinewaves), we actually know the response to many inputs**.

  Why? Because we can build complex signals out of simple signals, and we can use the principle of linearity to compute the output of the complex signals by summing the outputs from the simpler signals.

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3. Causality: $h(t)=0 \quad n<0$ which implies: $\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{t} x(\tau)h(t-\tau)d\tau$

This means $y(t)$ only depends on $x(\tau < t)$.

✓ **A system is causal** if the output does not depend on future values of the input, i.e., if the output at any time depends only on values of the input up to that time.

4. Stability: $\int_{-\infty}^{\infty} |h(t)| < \infty$

Bounded Input ↔ Bounded Output

✓ **stable** if bounded input gives bounded output

✓ Stable if the impulse response is absolutely summable, $\int_{-\infty}^{\infty} |\,h(t)\,|\; dt < \infty$

or absolutely integrable,

Sufficient Condition:

for $|x(t)| \le x_{\max} < \infty$

$$|y(t)| = \left|\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau\right| \le x_{\max}\left|\int_{-\infty}^{\infty} h(t-\tau)\right|d\tau < \infty$$

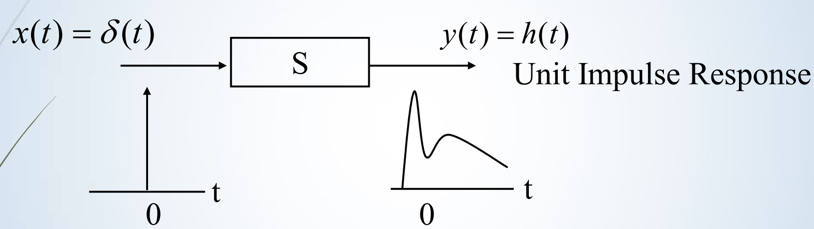
Necessary Condition: if $\left|\int_{-\infty}^{\infty} h(t-\tau)\right| = \infty$

Let $x(t) = h^*(-t)/|h(-t)|$, then $|x(t)| = 1$ (bounded)

$$\text{But } y(0) = \int_{-\infty}^{\infty} x(\tau)h(0-\tau)d\tau = \int_{-\infty}^{\infty} \frac{h^*(-\tau)}{|h(-\tau)|}h(0-\tau)d\tau = \int_{-\infty}^{\infty} |h(0-\tau)|d\tau = \infty$$

➢ Impulse Response

- Defining the **output** for an **unit impulse input** as the **unit impulse response**



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$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t) = x(t) * h(t)$$

$$h(t) \;\rightarrow\; x(t) = \delta(t)$$

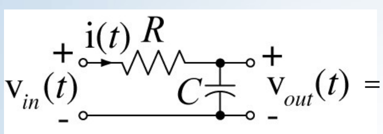
- Denote the **system impulse response**, $h(t)$, as the output produced when the input is **a unit impulse function**, $\delta(t)$.
- From **time-invariance**: $\delta(t-\tau) \;\rightarrow\; h(t-\tau)$
- From **linearity**:

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau \;\rightarrow\; y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = x(t) * h(t)$$

- This is referred to as **the convolution integral** for CT signals and systems.
- Its computation is completely analogous to the DT version:

$$h(\tau) \xrightarrow{\text{Flip}} h(-\tau) \xrightarrow{\text{Slide}} h(t-\tau) \xrightarrow{\text{Multiply}}$$
$$x(\tau)h(t-\tau) \xrightarrow{\text{Integrate}} \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau$$

➢ The RC Circuit (LP Filter)

$x(t) =$ 

$= y(t)$

Voltage Balance :

$$RI(t) + y(t) = x(t)$$

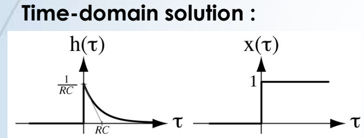
Current controlled by Capacitance (C):

$$I(t) = C\frac{dy(t)}{dt}$$

Differential equation of RC filters system:

$$\boxed{x(t) = RC\frac{dy(t)}{dt} + y(t)}$$

Time-domain solution :



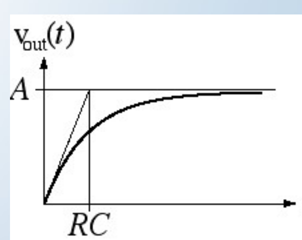
$t<0: \quad v_{out}(t) = 0$

$$t>0: \quad v_{out}(t) = \int_{-\infty}^{\infty} u(\tau)\frac{e^{-(t-\tau)/RC}}{RC}u(t-\tau)d\tau$$

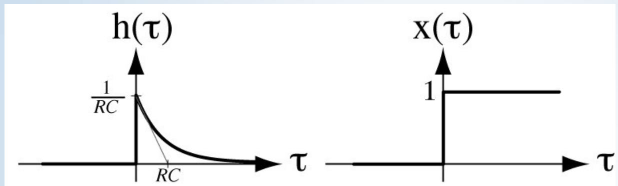
$$v_{out}(t) = \frac{1}{RC}\int_0^t e^{-(t-\tau)/RC}d\tau = \frac{1}{RC}\left[\frac{e^{-(t-\tau)/RC}}{-1/RC}\right]_0^t = \left[-e^{-(t-\tau)/RC}\right]_0^t = 1 - e^{-t/RC}$$

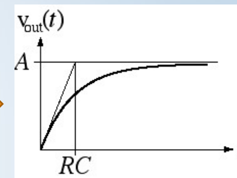
For all time, t:

$$v_{out}(t) = \left(1 - e^{-t/RC}\right)u(t)$$

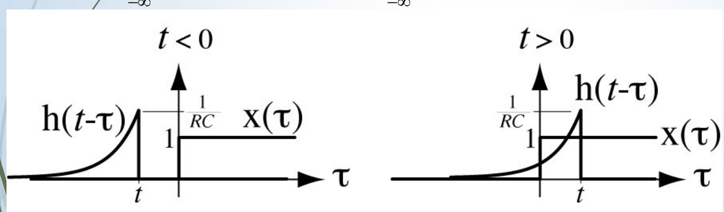


the Convolution Integral of RC Filter System





$$v_{out}(t)=\int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau=\int_{-\infty}^{\infty} u(\tau)\frac{e^{-(t-\tau)/RC}}{RC}u(t-\tau)\,d\tau$$



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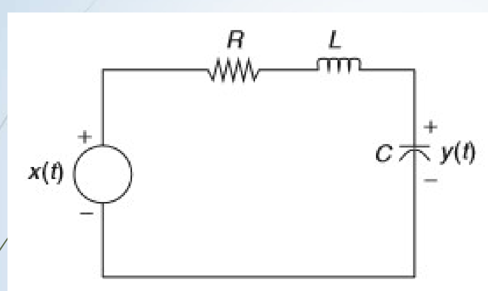
➢ The RLC Circuit



$$x(t)=Ri(t)+L\frac{di(t)}{dt}+y(t)$$

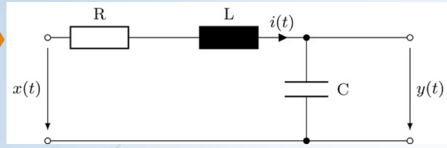
$$y(t)=\frac{1}{C}\int_0^t i(\sigma)d\sigma+y(0)$$

$$\frac{dy(t)}{dt}=\frac{1}{C}i(t)\quad\Rightarrow\quad i(t)=C\frac{dy(t)}{dt}$$

$$\frac{d^2y(t)}{dt^2}=\frac{1}{C}\frac{di(t)}{dt}\Rightarrow L\frac{di(t)}{dt}=LC\frac{d^2y(t)}{dt^2}$$

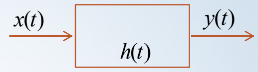
$$\boxed{x(t)=RC\frac{dy(t)}{dt}+LC\frac{d^2y(t)}{dt^2}+y(t)}$$

➢ Case Example: RLC Filters



Block Diagram

CT LTI



Differential Equation (Second Order):

$$\boxed{CL\frac{d^2y(t)}{dt^2}+CR\frac{dy(t)}{dt}+y(t)=x(t)}$$

$$\boxed{H(s)=\frac{1}{LCs^2+RCs+1}}$$

Impulse Response :

$C:\frac{2}{5},\quad L:\frac{1}{2},\quad R:1$

$$h(t)=C_1e^{\frac{t\left(-R-\frac{\sqrt{C(CR^2-4L)}}{C}\right)}{2L}}+C_2e^{\frac{t\left(-R+\frac{\sqrt{C(CR^2-4L)}}{C}\right)}{2L}}-\frac{e^{\frac{t\left(-R-\frac{\sqrt{C(CR^2-4L)}}{C}\right)}{2L}}\theta(t)}{\sqrt{C(CR^2-4L)}}+\frac{e^{\frac{t\left(-R+\frac{\sqrt{C(CR^2-4L)}}{C}\right)}{2L}}\theta(t)}{\sqrt{C(CR^2-4L)}}$$

The integration constants C_1 and C_2 have to be determined from the initial conditions $y(t)=0$ and $dy(t)/dt=0$ for $t<0$

$C_1:0,\quad C_2:0$

$$\boxed{h(t)=-\frac{e^{\frac{t\left(-R-\frac{\sqrt{C(CR^2-4L)}}{C}\right)}{2L}}\theta(t)}{\sqrt{C(CR^2-4L)}}+\frac{e^{\frac{t\left(-R+\frac{\sqrt{C(CR^2-4L)}}{C}\right)}{2L}}\theta(t)}{\sqrt{C(CR^2-4L)}}}$$

➢The Convolution Integral

- Representing an arbitrary signal as an integral of impulses

$$x(t)=\lim_{\Delta\to 0}\sum_{k=-\infty}^{\infty}x(k\Delta)\delta_\Delta(t-k\Delta)\Delta$$

$$x(t)=\int_{-\infty}^{\infty}x(\tau)\delta(t-\tau)d\tau$$

an impulse located at $t=\tau$ whose value is $x(\tau)$

$$u(t)=\int_0^{\infty}\delta(t-\tau)d\tau \qquad \text{a special case}$$

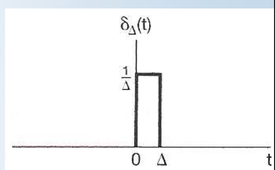


Figure 1.34 Derivative of $u_\Delta(t)$.

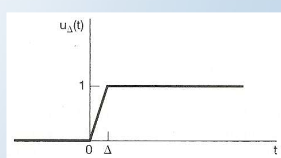


Figure 1.33 Continuous approximation to the unit step, $u_\Delta(t)$.

By Linearity (Superposition Property)

– The output for an arbitrary input signal is the superposition of a series of "**shifted, scaled unit impulse response**"

$$\sum_k a_k x_k(t) \rightarrow \sum_k a_k y_k(t)$$

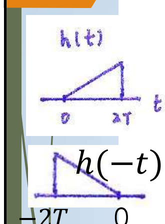
$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\, d\tau$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\, d\tau$$

$$\boxed{y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \equiv x(t) * h(t)}$$

Convolution Integral

Examples of Convolution Integral



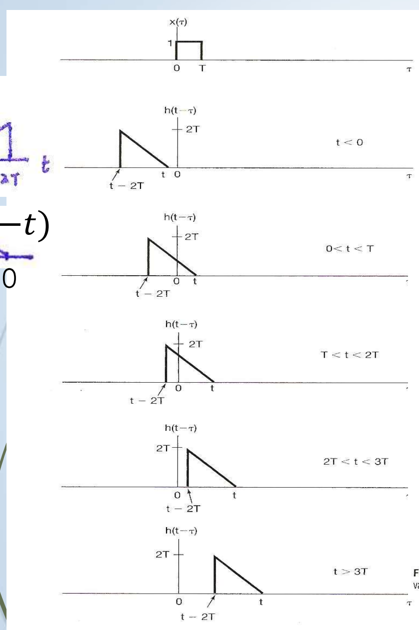


Figure 2.19 Signals $x(\tau)$ and $h(t-\tau)$ for different values of t for Example 2.7.

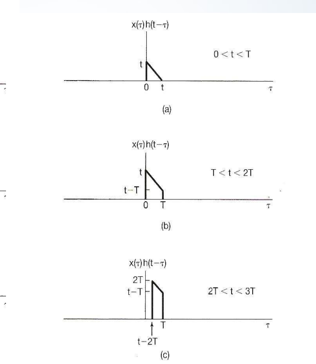


Figure 2.20 Product $x(\tau)h(t-\tau)$ for Example 2.7 for the three ranges values of t for which this product is not identically zero. (See Figure 2.19.)

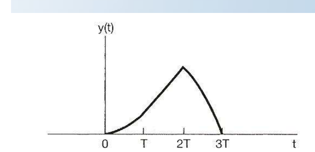
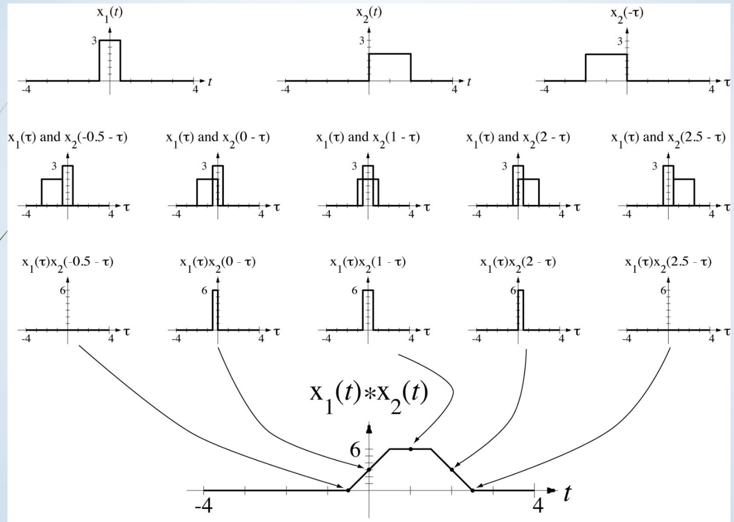
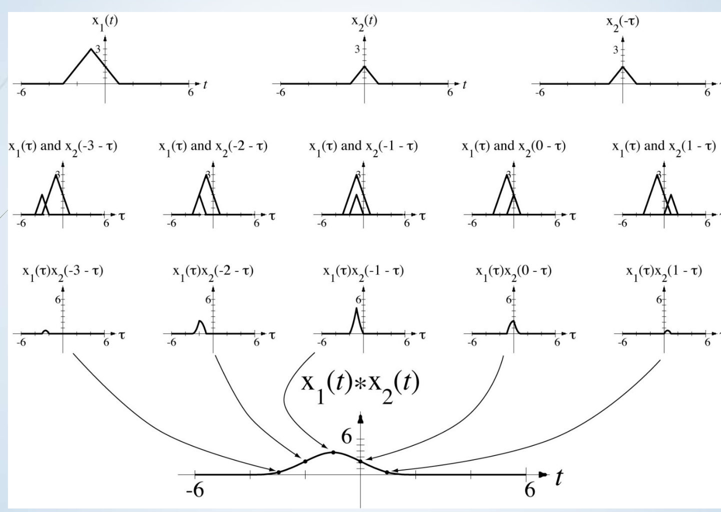


Figure 2.21 Signal $y(t) = x(t) * h(t)$ for Example 2.7

Convolution Example



Convolution Example



Convolution Example

Compute the convolution $y(t) = x(t) * h(t)$ where

$$x(t) = \begin{cases} 1 & \text{for } 0 \le t < 1 \\ 0 & \text{otherwise,} \end{cases} \quad \text{and}$$

$$h(t) = \begin{cases} t & \text{for } 0 \le t < 1 \\ 0 & \text{otherwise.} \end{cases}$$

$$x(t) * h(t) = \begin{cases} 0 & \text{for } t < 0 \\ \frac{1}{2}t^2 & \text{for } 0 \le t < 1 \\ -\frac{1}{2}t^2 + t & \text{for } 1 \le t < 2 \\ 0 & \text{for } t \ge 2. \end{cases}$$

$t < 0.$

$$\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = 0.$$

$0 \le t < 1.$

$$\begin{aligned} \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau &= \int_0^t (t-\tau)d\tau \\ &= [t\tau - \tfrac{1}{2}\tau^2]|_0^t \\ &= t^2 - \tfrac{1}{2}t^2 \\ &= \tfrac{1}{2}t^2. \end{aligned}$$

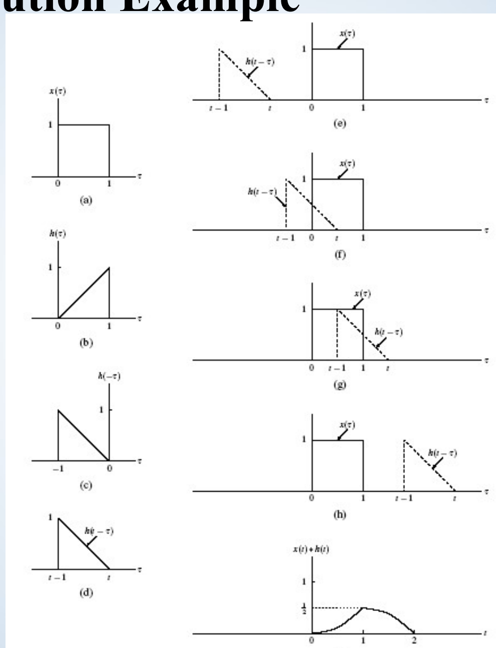
$1 \le t < 2.$

$$\begin{aligned} \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau &= \int_{t-1}^{1} (t-\tau)d\tau \\ &= [t\tau - \tfrac{1}{2}\tau^2]|_{t-1}^{1} \\ &= t - \tfrac{1}{2}(1)^2 - [t(t-1) - \tfrac{1}{2}(t-1)^2] \\ &= t - \tfrac{1}{2} - [t^2 - t - \tfrac{1}{2}(t^2 - 2t + 1)] \\ &= -\tfrac{1}{2}t^2 + t. \end{aligned}$$

$t \ge 2.$

$$\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = 0.$$

Convolution Example



Convolution Example

Compute the quantity $y(t) = x(t) * h(t)$, where

$$x(t) = \begin{cases} 0 & \text{for } t < 0 \\ t & \text{for } 0 \le t < 1 \\ -t+2 & \text{for } 1 \le t < 2 \\ 0 & \text{for } t > 2, \end{cases} \quad \text{and}$$

$$h(t) = u(t) - u(t-1).$$

$$x(t) * h(t) = \begin{cases} 0 & \text{for } t < 0 \\ \frac{1}{2}t^2 & \text{for } 0 \le t < 1 \\ -t^2 + 3t - \frac{3}{2} & \text{for } 1 \le t < 2 \\ \frac{1}{2}t^2 - 3t + \frac{9}{2} & \text{for } 2 \le t < 3 \\ 0 & \text{for } t \ge 3. \end{cases}$$

$t < 0$. $\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = 0.$

$0 \le t < 1$.

$$\begin{aligned} \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau &= \int_0^t \tau d\tau \\ &= [\tfrac{1}{2}\tau^2]_0^t \\ &= \tfrac{1}{2}t^2. \end{aligned}$$

$1 \le t < 2$.

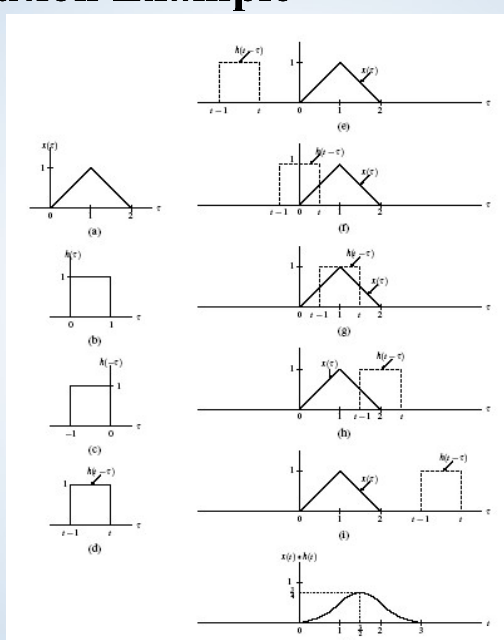
$$\begin{aligned} \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau &= \int_{t-1}^{1} \tau d\tau + \int_1^t (-\tau+2)d\tau \\ &= [\tfrac{1}{2}\tau^2]_{t-1}^1 + [-\tfrac{1}{2}\tau^2 + 2\tau]_1^t \\ &= \tfrac{1}{2} - [\tfrac{1}{2}(t-1)^2] - \tfrac{1}{2}t^2 + 2t - [-\tfrac{1}{2} + 2] \\ &= -t^2 + 3t - \tfrac{3}{2}. \end{aligned}$$

$2 \le t < 3$.

$$\begin{aligned} \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau &= \int_{t-1}^{2} (-\tau+2)d\tau \\ &= [-\tfrac{1}{2}\tau^2 + 2\tau]_{t-1}^2 \\ &= 2 - [-\tfrac{1}{2}t^2 + 3t - \tfrac{5}{2}] \\ &= \tfrac{1}{2}t^2 - 3t + \tfrac{9}{2}. \end{aligned}$$

$t \ge 3$. $\int_{-\infty}^{\infty} x(\tau)h(t-\tau) = 0.$

Convolution Example



Convolution Example

Compute the convolution $y(t) = x(t) * h(t)$ where

$$x(t) = e^{-at}u(t), \quad \text{and}$$

$$h(t) = u(t),$$

and a is a positive real constant.

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$
$$= \int_{-\infty}^{\infty} e^{-a\tau}u(\tau)u(t-\tau)d\tau.$$

$$y(t) = \begin{cases} \frac{1}{a}(1-e^{-at}) & \text{for } t > 0 \\ 0 & \text{otherwise} \end{cases}$$
$$= \frac{1}{a}(1-e^{-at})u(t).$$

convolution of a signal with a shifted impulse

– time shift of signals

$$x(t),\ \delta(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)=x(t-t_0),\ h(t)=\delta(t-t_0)$$

$$x(t-t_0) = x(t) * \delta(t-t_0)$$

– convolution of a signal with a shifted impulse is the **signal itself but shifted**

Properties of Convolution

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- **Commutative Property:**

$$x(t)*h(t)=h(t)*x(t)$$

Proof:

$$x(t)*h(t)=\int_{\tau=-\infty}^{\tau=+\infty}x(\tau)h(t-\tau)d\tau$$

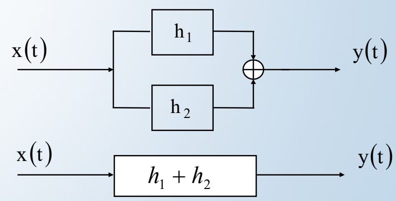
$$let\ \lambda=t-\tau,\ or\ \tau=t-\lambda,\ and\ d\tau=-d\lambda$$

$$x(t)*h(t)=\int_{\lambda=+\infty}^{\lambda=-\infty}x(t-\lambda)h(\lambda)(-d\lambda)=\int_{\lambda=-\infty}^{\lambda=+\infty}h(\lambda)x(t-\lambda)d\lambda=h(t)*x(t)$$

- **Distributive Property:**

$$x(t)*[h_1(t)+h_2(t)]=x(t)*h_1(t)+x(t)*h_2(t)$$

Proof:



$$x(t)*[h_1(t)+h_2(t)]=\int_{\tau=-\infty}^{\tau=+\infty}x(\tau)[h_1(t-\tau)+h_2(t-\tau)]d\tau$$

$$=\int_{\tau=-\infty}^{\tau=+\infty}x(\tau)h_1(t-\tau)d\tau+\int_{\tau=-\infty}^{\tau=+\infty}x(\tau)h_2(t-\tau)d\tau$$

$$=x(t)*h_1(t)+x(t)*h_2(t)$$

Properties of Convolution (Cont.)

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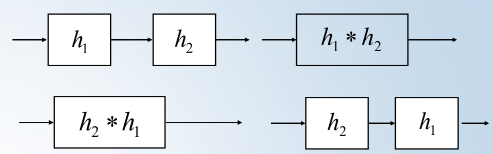
- **Associative Property:**

$$x(t)*h_1(t)*h_2(t)=(x(t)*h_1(t))*h_2(t)$$
$$=(x(t)*h_2(t))*h_1(t)$$



Proof:

$$[x(t)*h_1(t)]=\int_{\tau=-\infty}^{\tau=+\infty}x(\tau)h_1(t-\tau)d\tau$$

$$[x(t)*h_1(t)]*h_2(t)=\int_{\lambda=-\infty}^{\lambda=+\infty}[\int_{\tau=-\infty}^{\tau=+\infty}x(\tau)h_1(\lambda-\tau)d\tau]h_2(t-\lambda)d\lambda$$

$$=\int_{\tau=-\infty}^{\tau=+\infty}x(\tau)\int_{\lambda=-\infty}^{\lambda=+\infty}h_1(\lambda-\tau)h_2(t-\lambda)d\lambda d\tau$$

$$Let\ \alpha=\lambda-\tau\Rightarrow\lambda=\alpha+\tau\quad and\quad d\alpha=d\lambda$$

$$=\int_{\tau=-\infty}^{\tau=+\infty}x(\tau)\left[\int_{\alpha=-\infty}^{\alpha=+\infty}h_1(\alpha)h_2(t-(\alpha+\tau))d\alpha\right]d\tau$$

$$=\int_{\tau=-\infty}^{\tau=+\infty}x(\tau)\left[\int_{\alpha=-\infty}^{\alpha=+\infty}h_1(\alpha)h_2((t-\tau)-\alpha)d\alpha\right]d\tau$$

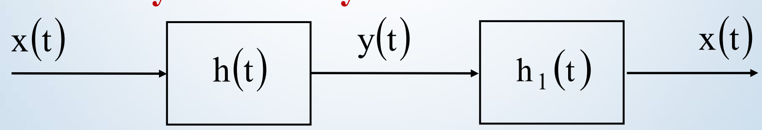
$$=x(t)*[h_1(t)*h_2(t)]$$

- **Memoryless / with Memory**
  - A linear, time-invariant, causal system is memory less only if $h(t)=K\delta(t)$ $\quad y(t)=Kx(t)$

    if k=1 further, they are identity systems

$$y(t)=x(t)=\int_{-\infty}^{\infty}x(\tau)\delta(t-\tau)d\tau=x(t)*\delta(t)$$

  - sifting property, i.e., convolution sum (or integral) with an unit impulse function gives the original signal
- **Invertibility / Inverse system**



$$h(t)*h_1(t)=\delta(t)$$

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- **Sifting Property:**

$$x(t)*\delta(t-t_0)=x(t-t_0)$$

Proof:

$$x(t)*\delta(t-t_0)=\int_{\tau=-\infty}^{\tau=+\infty}x(\tau)\delta(t-t_0-\tau)d\tau=\int_{\tau=t-t_0^-}^{t-t_0^+}x(\tau)\delta(t-t_0-\tau)d\tau=x(t-t_0)$$

- **Integration:**

$$x(t)*u(t)=\int_{-\infty}^{t}x(\tau)d\tau$$

Proof:

$$x(t)*u(t)=\int_{-\infty}^{\infty}x(\tau)u(t-\tau)d\tau=\int_{-\infty}^{t}x(\tau)d\tau \quad because \quad u(t-\tau)=0 \ for \ \tau>t$$

- **Step Response (follows from the integration property):**

$$u(t)*h(t)=h(t)*u(t)=\int_{-\infty}^{t}h(\tau)d\tau$$

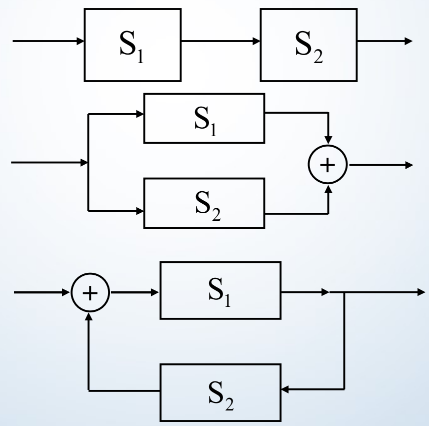
Comments:

- Requires proof of the commutative property.
- In practice, measuring the step response of a system is much easier than measuring the impulse response directly. How can we obtain the impulse response from the step response?

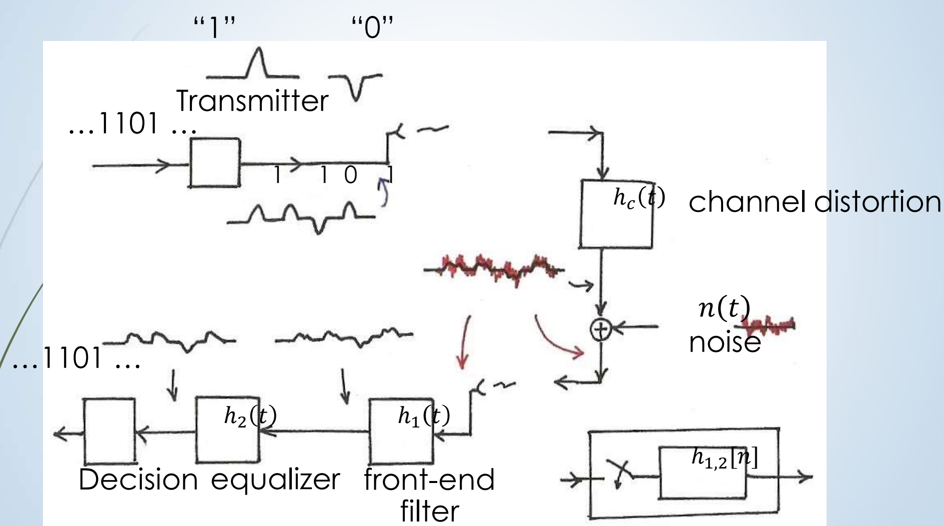
➢ Interconnections of Systems

- Continuous/Discrete-time Systems

- Interconnections of Systems
  - Series
  - Parallel
  - Feedback



Example of Data Transmission system



$h_{1,2}[t]$

$h_2[t] \approx h_c^{-1}[t]$

Finish

Home work -01

- Michael D. Adams, 2013, **Continuous-Time Signals and Systems,** University of Victoria, Canada

NIM : male

1. Find the solution of exercise no. 2.9 page. 41
2. Find the solution of exercise no. 2.13 page. 42
3. Find the solution of exercise no. 3.2 page 73

Nim : female

1. Find the solution of exercise no. 2.9 page. 41
2. Find the solution of exercise no. 2.16 page. 42
3. Find the solution of exercise no. 3.2 page 73

Home work must be submitted to google class room before next Friday class