1. Roll rotation is represented by:

$$R_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

And this matches the rotation part of T1:

$$T_1 = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 & -1 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So:

$$\cos\gamma = 1/\sqrt{2}$$

$$\gamma = \cos^{-1}(1/\sqrt{2})$$

$$\gamma = \pi/4 = 45^\circ$$

Translation with no rotation is represented by:

$$\begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

And from T1, we get:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$$

Answer: T1 is a roll by π/4 (45 degrees) followed by a translation -1 on the x-axis and 2 on the y-axis

2. First we need to address the translation by the point (3,0, 2):

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then the pitch rotation by π/4 (45 degrees)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} & 0 \\ 0 & \sin\frac{\pi}{4} & \cos\frac{\pi}{4} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

And since combining them would incorrectly order the transformation as rotation first and translation second, we will instead use a product of two matrices to control the order (translation first, rotation second).

Answer:

$$\boxed{T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

3. To calculate the reverse of T2T1, we need need to first get the inverse of each transformation.

   T2 inverse can be put into one matrix because the order needs to be rotation first and translation second. Then we find the transpose of T2 rotation and the negatives of T2 translation.

   T1 now has to be split into two matrices because the order needs to be translation first and rotation second which can not be represented by one matrix. Then we find the transpose of T1 rotation and the negatives of T1 translation.

Now that we have the inverse of each translation, we take the inverse product of them. Since the product of two translations is not commutative, we have to swap the order of these as well.

Answer:

$$\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. Work:

$$q_1 * q_2 = (a_3, b_3, c_3, d_3)$$

$$\begin{aligned} a_3 &= a_1a_2 - b_1b_2 - c_1c_2 - d_1d_2 \\ &= \cos\tfrac{\pi}{8}\cdot\cos\tfrac{\pi}{8} - 0\cdot\sin\tfrac{\pi}{8} - 0\cdot 0 - \sin\tfrac{\pi}{8}\cdot 0 \\ &= \boxed{\cos\tfrac{\pi}{8}\cdot\cos\tfrac{\pi}{8}} \end{aligned}$$

$$\begin{aligned} b_3 &= a_1b_2 + a_2b_1 + c_1d_2 - c_2d_1 \\ &= \cos\tfrac{\pi}{8}\sin\tfrac{\pi}{8} + \cos\tfrac{\pi}{8}\cdot 0 + 0\cdot 0 - 0\cdot\sin\tfrac{\pi}{8} \\ &= \boxed{\cos\tfrac{\pi}{8}\sin\tfrac{\pi}{8}} \end{aligned}$$

$$\begin{aligned} c_3 &= a_1c_2 + a_2c_1 + b_2d_1 - b_1d_2 \\ &= \cos\tfrac{\pi}{8}\cdot 0 + \cos\tfrac{\pi}{8}\cdot 0 + \sin\tfrac{\pi}{8}\sin\tfrac{\pi}{8} - 0\cdot 0 \\ &= \boxed{\sin\tfrac{\pi}{8}\sin\tfrac{\pi}{8}} \end{aligned}$$

$$\begin{aligned} d_3 &= a_1d_2 + a_2d_1 + b_1c_2 - b_2c_1 \\ &= \cos\tfrac{\pi}{8}\cdot 0 + \cos\tfrac{\pi}{8}\sin\tfrac{\pi}{8} + 0\cdot 0 - \sin\tfrac{\pi}{8}\cdot 0 \\ &= \boxed{\cos\tfrac{\pi}{8}\sin\tfrac{\pi}{8}} \end{aligned}$$

Answer:

$$q_1 * q_2 = \left(\cos\frac{\pi}{8}\cos\frac{\pi}{8},\ \cos\frac{\pi}{8}\sin\frac{\pi}{8},\ \sin\frac{\pi}{8}\sin\frac{\pi}{8},\ \cos\frac{\pi}{8}\sin\frac{\pi}{8} \right)$$