



## Practice quiz on Bayes Theorem and the Binomial Theorem

GESAMTPUNKTZAHL 9

1. A jewelry store that serves just one customer at a time is concerned about the safety of its isolated customer.

1 / 1 Punkten



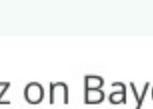
### Practice quiz on Bayes Theorem and the Binomial Theorem

Übungsquiz • 25 min

- 10% of the times that a jewelry store is robbed, a customer is in the store.
- A jewelry store has a customer on average 20% of each 24-hour day.
- The probability that a jewelry store is being robbed (anywhere in the world) is 1 in 2 million.

What is the probability that a robbery will occur while a customer is in the store?

- ☐  $\frac{1}{500000}$
- ☐  $\frac{1}{2000000}$
- ☒  $\frac{1}{4000000}$
- ☐  $\frac{1}{5000000}$



Richtig  
What is known is:



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$B$ : "a robbery is occurring,"  $P(B) = \frac{1}{2,000,000}$

$P(\text{a customer is in the store} \mid \text{a robbery occurs}) = P(A \mid B)$

$P(A \mid B) = 10\%$

What is wanted:

$P(\text{a robbery occurs} \mid \text{a customer is in the store}) = P(B \mid A)$

By the product rule:

$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$

and  $P(A, B) = P(A \mid B)P(B)$



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$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A \mid B)P(B)}{P(A)} = \frac{0.1 \cdot \frac{1}{2000000}}{0.2} = \frac{1}{4000000}$

2. If I flip a fair coin, with heads and tails, ten times in a row, what is the probability that I will get exactly six heads?

1 / 1 Punkten

- ☐ 0.021
- ☐ 0.187
- ☒ 0.2051
- ☐ 0.305



Richtig  
By Binomial Theorem: equals



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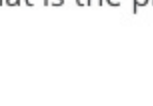
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$\frac{1}{1024} \cdot \frac{1}{4} \cdot 6^9 = 0.2051$

3. If a coin is bent so that it has a 40% probability of coming up heads, what is the probability of getting exactly 6 heads in 10 throws?

1 / 1 Punkten

- ☐ 0.0974
- ☐ 0.1045
- ☒ 0.1115
- ☐ 0.1219



Richtig  
 $\binom{10}{6} \times 0.4^6 \times 0.6^4 = 0.1115$



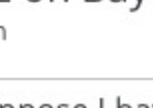
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4. A bent coin has a 40% probability of coming up heads on each independent toss. If I toss the coin ten times, what is the probability that I get at least 8 heads?

1 / 1 Punkten

- ☐ 0.0312
- ☐ 0.0132
- ☐ 0.0213
- ☒ 0.0123



Richtig  
The answer is the sum of three binomial probabilities:

$\left(\binom{10}{8} \times (0.4)^8 \times (0.6)^2\right) + \left(\binom{10}{9} \times (0.4)^9 \times (0.6)^1\right) +$

$\left(\binom{10}{10} \times (0.4)^{10} \times (0.6)^0\right)$



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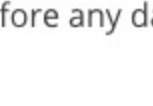
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5. Suppose I have a bent coin with a 60% probability of coming up heads. I throw the coin ten times and it comes up heads 8 times.

1 / 1 Punkten

What is the value of the "likelihood" term in Bayes' Theorem -- the conditional probability of the data given the parameter.

- ☒ 0.120932
- ☐ 0.122885
- ☐ 0.043945
- ☐ 0.168835



Richtig  
Bayesian "likelihood" --- the  $p(\text{observed data} \mid \text{parameter})$  is

$p(8 \text{ of } 10 \text{ heads} \mid \text{coin has } 60\% \text{ coming up heads})$



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$p(8 \text{ of } 10 \text{ heads} \mid \text{coin has } 60\% \text{ coming up heads}) = \binom{10}{8} (0.6)^8 (0.4)^2 = 0.120932$

6. We have the following information about a new medical test for diagnosing cancer.

1 / 1 Punkten

Before any data are observed, we know that 5% of the population to be tested actually have Cancer.

Of those tested who do have cancer, 90% of them get an accurate test result of "Positive" for cancer. The other 10% get a false test result of "Negative" for Cancer.

Of the people who do not have cancer, 90% of them get an accurate test result of "Negative" for cancer. The other 10% get a false test result of "Positive" for cancer.

**What is the conditional probability that I have Cancer, if I get a "Positive" test result for Cancer?**

**\*\*Formulas in the feedback section are very long, and do not fit within the standard viewing window. Therefore, the font is a bit smaller and the word "positive test" has been abbreviated as PT.**



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- ☐ 4.5%
- ☒ 32.1% probability that I have cancer



Richtig  
I still have a more than  $\frac{2}{3}$  probability of not having cancer

Posterior probability:

$p(\text{I actually have cancer} \mid \text{receive a "positive" Test})$

By Bayes Theorem:

$$= \frac{(\text{chance of observing a PT if I have cancer})(\text{prior probability of having cancer})}{(\text{marginal likelihood of the observation of a PT})}$$



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$= \frac{(90\%)(5\%)}{(90\%)(5\%) + (10\%)(95\%)}$

$= 32.1\%$

7. We have the following information about a new medical test for diagnosing cancer.

1 / 1 Punkten

Before any data are observed, we know that 8% of the population to be tested actually have Cancer.

Of those tested who do have cancer, 90% of them get an accurate test result of "Positive" for cancer.

The other 10% get a false test result of "Negative" for Cancer.

Of the people who do not have cancer, 95% of them get an accurate test result of "Negative" for cancer.

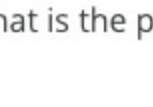


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What is the conditional probability that I have cancer, if I get a "Negative" test result for Cancer?

- ☒ 0.9%
- ☐ .80%
- ☐ 88.2%
- ☐ 99.1%



Richtig  
 $p(\text{cancer} \mid \text{negative test}) =$

$$\frac{p(\text{negative test} \mid \text{Cancer}) p(\text{Cancer})}{p(\text{negative test} \mid \text{cancer}) p(\text{cancer}) + p(\text{negative test} \mid \text{no cancer}) p(\text{no cancer})}$$

$$\frac{(10\%)(8\%)}{(10\%)(8\%) + (95\%)(92\%)}$$



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0.8%

88.2%

$= 0.9\%$

8. An urn contains 50 marbles – 40 blue and 10 white. After 50 draws, exactly 40 blue and 10 white are observed.

1 / 1 Punkten

You are not told whether the draw was done "with replacement" or "without replacement."

What is the probability that the draw was done with replacement?

- ☒ 12.27%



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- ☐ 13.98%



Richtig  
 $p(40 \text{ blue and } 10 \text{ white} \mid \text{draws without replacement}) = 1$  [this is the only possible outcome when 50 draws are made without replacement]

$p(40 \text{ blue and } 10 \text{ white} \mid \text{draws with replacement})$

$S = 40$

$N = 50$

$P = .8$  [for draws with replacement] because 40 blue of 50 total means  $p(\text{blue}) = 40/50 = .8$

$\left(\binom{50}{s}\right)(0.8^s)(0.2^{10-s})$



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The majority of all Smugglers at the border (65%) appear nervous and sweaty.

Only 8% of innocent people at the border appear nervous and sweaty.

If someone at the border appears nervous and sweaty, what is the probability that they are a Smuggler?

- ☒ 7.58%
- ☐ 8.57%
- ☐ 92.42%
- ☐ 7.92%



Richtig  
By Bayes' Theorem, the answer is

$$\frac{p(\text{nervous and sweaty} \mid \text{Smuggler}) p(\text{Smuggler})}{p(\text{nervous and sweaty} \mid \text{Smuggler}) p(\text{Smuggler}) + p(\text{nervous and sweaty} \mid \text{innocent}) p(\text{innocent})}$$