

From Markov's Inequality, we have 10-27-2020

$$\Pr(X > a) \leq \frac{E[X]}{a}.$$

Let $Y = (X - E[X])^2$. Then we have:

$$\Pr[(X - E[X])^2 > a] \leq \frac{E[(X - E[X])^2]}{a}$$

$$= \Pr[(X - E[X])^2 > a] \leq \frac{\text{var}(X)}{a}.$$

Let $a = \epsilon^2$. Then:

$$\Pr[(X - E[X])^2 > \epsilon^2] \leq \frac{\text{var}(X)}{\epsilon^2}$$

$$= \Pr[|X - E[X]| > \epsilon] \leq \frac{\text{var}(X)}{\epsilon^2}, \quad \epsilon > 0.$$

which is Chebyshev's inequality.