MODULE EWD998_stat

EXTENDS Integers, FiniteSets, Functions, SequencesExt, Randomization, CSV

CONSTANT

N
ASSUME NAssumption $\triangleq$ $N \in Nat \setminus \{0\}$ At least one node.

$Node \triangleq 0 \,..\, N-1$
$Color \triangleq \{\text{“white”}, \text{“black”}\}$
$Token \triangleq [pos : Node, q : Int, color : Color]$

VARIABLES

active, activation status of nodes

color, color of nodes

counter, nb of sent messages − nb of rcvd messages per node

pending, nb of messages in transit to node

token token structure
VARIABLES hasEmittedCSV, c

$vars \triangleq \langle active, color, counter, pending, token, hasEmittedCSV, c \rangle$

$TypeOK \triangleq$
$\land\ active \in [Node \to BOOLEAN\ ]$
$\land\ color \in [Node \to Color]$
$\land\ counter \in [Node \to Int]$
$\land\ pending \in [Node \to Nat]$
$\land\ token \in Token$

$Sum(f, S) \triangleq FoldFunctionOnSet(+, 0, f, S)$

B $\triangleq$ Sum(pending, Node)
Termination $\triangleq$
$\land \forall$ i $\in$ Node : $\neg$active[i]
$\land$ B = 0

terminationDetected $\triangleq$
$\land$ token.pos = 0
$\land$ token.color = "white"
$\land$ token.q + counter[0] = 0
$\land$ color[0] = "white"
$\land \neg$active[0]

Init $\triangleq$

$\land$ active $\in$ [Node $\to$ BOOLEAN]
$\land$ color $\in$ [Node $\to$ Color]

$\land$ counter = [i $\in$ Node $\mapsto$ 0] c properly initialized
$\land$ pending = [i $\in$ Node $\mapsto$ 0]
$\land$ token $\in$ [pos : {0}, q : {0}, color : {"black"}]
$\land$ hasEmittedCSV = FALSE
$\land$ c = [temp $\mapsto$ [counter $\mapsto$ 0, condition $\mapsto$ terminationDetected, hasAntecedent $\mapsto$ Termination]]

InitiateProbe $\triangleq$

$\land$ token.pos = 0
$\land$ previous round not conclusive if:
$\lor$ token.color = "black"
$\lor$ color[0] = "black"
$\lor$ counter[0] + token.q > 0
$\land$ token$'$ = [pos $\mapsto$ N $-$ 1, q $\mapsto$ 0, color $\mapsto$ "white"]
$\land$ color$'$ = [color EXCEPT ![0] = "white"]

$\land$ UNCHANGED $\langle$active, counter, pending$\rangle$
PassToken(i) $\triangleq$

$\land \neg$active[i] If machine i is active, keep the token.
$\land$ token.pos = i
$\land$ token$'$ = [pos $\mapsto$ token.pos $-$ 1,
q $\mapsto$ token.q + counter[i],

color $\mapsto$ IF color[i] = "black" THEN "black" ELSE token.color]

$\land$ color$'$ = [color EXCEPT ![i] = "white"]

$\land$ UNCHANGED $\langle$active, counter, pending$\rangle$

System $\triangleq$ $\lor$ InitiateProbe
$\lor \exists$ i $\in$ Node $\setminus$ {0} : PassToken(i)

SendMsg(i) $\triangleq$

$\land$ active[i]

$\land$ counter$'$ = [counter EXCEPT ![i] = @ + 1]

$\land \exists$ j $\in$ Node $\setminus$ {i} : pending$'$ = [pending EXCEPT ![j] = @ + 1]

$\land$ UNCHANGED $\langle$active, color, token$\rangle$

RecvMsg(i) $\triangleq$
$\land$ pending[i] $>$ 0
$\land$ pending$'$ = [pending EXCEPT ![i] = @ $-$ 1]

$\land$ counter$'$ = [counter EXCEPT ![i] = @ $-$ 1]

$\land$ color$'$ = [color EXCEPT ![i] = "black"]

$\land$ active$'$ = [active EXCEPT ![i] = TRUE]
$\land$ token$'$ = token
$\land$ hasEmittedCSV$'$ = hasEmittedCSV

Deactivate(i) $\triangleq$
$\land$ active[i]
$\land$ active$'$ = [active EXCEPT ![i] = FALSE]
$\land$ UNCHANGED $\langle$color, counter, pending, token$\rangle$

Environment $\triangleq$ $\exists$ i $\in$ Node : SendMsg(i) $\lor$ RecvMsg(i) $\lor$ Deactivate(i)

Next $\triangleq$
$\land$ (System $\lor$ Environment)
$\land$ c′ = [temp $\mapsto$ [condition $\mapsto$ (c[“temp”][“condition”] $\lor$ terminationDetected), counter $\mapsto$ (IF ¬(c[“temp”][“hasA
$\land$ hasEmittedCSV′ = ¬(((c′[“temp”][“counter”] $\leq$ 8) $\lor$ c′[“temp”][“condition”]))
$\land$ (IF (hasEmittedCSV′ $\land$ ¬(hasEmittedCSV)) THEN CSVWrite(“%1$s”, ⟨TRUE⟩, “statPropsRes.csv”) ELSE T

Spec $\triangleq$ Init $\land$ $\Box$[Next]$_{\text{vars}}$ $\land$ WF$_{\text{vars}}$(System)

StateConstraint $\triangleq$
$\land$ $\forall$ i $\in$ Node : counter[i] $\leq$ 3 $\land$ pending[i] $\leq$ 3
$\land$ token.q $\leq$ 9

TerminationDetection $\triangleq$
terminationDetected $\Rightarrow$ Termination

Rng(a, b) $\triangleq$ {i $\in$ Node : a $\leq$ i $\land$ i $\leq$ b}

Inv $\triangleq$
$\land$ P0:: B = Sum(counter, Node)

$\land\ \lor$ P1:: $\land \forall$ i $\in$ Rng(token.pos + 1, N − 1) : active[i] = FALSE machine nr.i is passive
$\land$ IF token.pos = N − 1
THEN token.q = 0
ELSE token.q = Sum(counter, Rng(token.pos + 1, N − 1))

$\lor$ P2:: Sum(counter, Rng(0, token.pos)) + token.q > 0

$\lor$ P3:: $\exists$ i $\in$ Rng(0, token.pos) : color[i] = "black"

$\lor$ P4:: token.color = "black"

TypedInv $\triangleq$
$\land$ TypeOK
$\land$ Inv

Liveness $\triangleq$
$\Box$(Termination $\Rightarrow$ $\Diamond\Box$terminationDetected)

TD $\triangleq$ INSTANCE AsyncTerminationDetection

TDSpec $\triangleq$ TD!Spec

THEOREM Spec $\Rightarrow$ TDSpec