Brandon M. Keltz An Introduction to Computational Science by Allen Holder and Joseph Eichholz Chapter 1 - Solving Single Variable Equations September 7, 2019

Problem 1. Suppose the method of bisection is initialized with the interval $[-10^4, 10^4]$. How many iterations are needed to guarantee that the k-th interval is no wider than 10^{-8} ?

We know that bisection converges

$$\frac{b-a}{2^k} \le \epsilon,$$

where a and b are the left and right endpoints, k is the number of iterations, and ϵ is the tolerance. From the problem statement, we have

$$\frac{10^4 + 10^4}{2^k} \le 10^{-8} \iff \frac{10^4}{2^{k-1}} \le 10^{-8} \iff 10^{12} \le 2^{k-1}.$$

Since $2^{k-1} > 0$ for all $k \in \mathbb{N}$, we have

$$12 \ln (10) \le (k-1) \ln (2)$$

$$\frac{12 \ln (2) + 12 \ln (5)}{\ln (2)} + 1 \le k$$

$$\frac{13 \ln (2) + 12 \ln (5)}{\ln (2)} \le k.$$

We have

$$k \ge \frac{13\ln(2) + 12\ln(5)}{\ln(2)} \approx 40.8631,$$

which means that we need 41 iterations to guarantee the k-th interval is no wider than 10^{-18} .