Brandon M. Keltz
An Introduction to Computational Science by Allen Holder and Joseph Eichholz
Chapter 1 - Solving Single Variable Equations
September 7, 2019

Problem 1. Suppose the method of bisection is initialized with the interval $\left[-10^4, 10^4\right]$. How many iterations are needed to guarantee that the k-th interval is no wider than 10^{-8}?

We know that bisection converges

$$\frac{b-a}{2^k} \leq \epsilon,$$

where a and b are the left and right endpoints, k is the number of iterations, and ϵ is the tolerance. From the problem statement, we have

$$\frac{10^4 + 10^4}{2^k} \leq 10^{-8} \iff \frac{10^4}{2^{k-1}} \leq 10^{-8} \iff 10^{12} \leq 2^{k-1}.$$

Since $2^{k-1} > 0$ for all $k \in \mathbb{N}$, we have

$$\begin{aligned} 12\ln(10) &\leq (k-1)\ln(2) \\ \frac{12\ln(2) + 12\ln(5)}{\ln(2)} + 1 &\leq k \\ \frac{13\ln(2) + 12\ln(5)}{\ln(2)} &\leq k. \end{aligned}$$

We have

$$k \geq \frac{13\ln(2) + 12\ln(5)}{\ln(2)} \approx 40.8631,$$

which means that we need 41 iterations to guarantee the k-th interval is no wider than 10^{-18}.