Introduction:

An insurance company operates in three states: A, B and C. The state-wise demand for insurance for the year is shown in the table provided below:

States	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
A	5240	4878	5942	2297	1992	2275	5334	3371	3759	3529	4284	5183
B	4927	2628	2974	2338	4020	3147	4271	2620	4517	4155	3137	4227
C	1162	1967	1898	2261	2030	1642	2489	2496	922	2421	963	1998

- The company can either handle an application with the staff that they hire or outsource it to a vendor. Assume that there is no capacity limitation to outsourcing.
- If they hire staff, he/she can handle 40 insurance applications per month when he/she works 100% of the workdays. However, there are days that he/she will be unavailable to process applications due to training, off days, etc.
- A staff member's availability (in percentage) to work on processing the insurance applications for each month is shown in the table given below. As mentioned before, with 100% availability, each member can handle 40 applications.

States	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
A	81%	76%	75%	80%	78%	73%	68%	76%	81%	73%	68%	65%
B	81%	76%	75%	80%	78%	73%	68%	76%	81%	73%	68%	65%
C	81%	76%	75%	80%	78%	73%	68%	76%	81%	73%	68%	65%

- States A and B have a regulatory restriction that the outsourced insurance applications cannot be more than 30% and 40% of the total number of applications for each month, respectively. The table given below shows the cost of the staff vs external resources:

State	Annual Salary of Staff	Outsourcing Cost per Application
A	$60,000	$180
B	$55,000	$150
C	$53,000	$160

The **Decision variables** for this problem are as follows:

1. <u>Count of internal staff members</u>:

Denoting count of internal staff members using $X_{s,m}$
where s = State and m = month
The count of internal staff members should be a <u>non-negative real number</u>

Matrix of count of internal staff members:

State/Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
A	x(Jan, A)	x(Feb, A)	x(Mar, A)	x(Apr, A)	x(May, A)	x(Jun, A)	x(Jul, A)	x(Aug, A)	x(Sep, A)	x(Oct, A)	x(Nov, A)	x(Dec, A)
B	x(Jan, B)	x(Feb, B)	x(Mar, B)	x(Apr, B)	x(May, B)	x(Jun, B)	x(Jul, B)	x(Aug, B)	x(Sep, B)	x(Oct, B)	x(Nov, B)	x(Dec, B)
C	x(Jan, C)	x(Feb, C)	x(Mar, C)	x(Apr, C)	x(May, C)	x(Jun, C)	x(Jul, C)	x(Aug, C)	x(Sep, C)	x(Oct, C)	x(Nov, C)	x(Dec, C)

2. <u>Number of applications processed by vendors:</u>

Denoting count of internal staff members using $y_{s,m}$
where s = State and m = month
The number of applications processed by outsourced members should be a <u>non-negative integer number</u>

Matrix of number of applications processed by outsourced vendors:

State/Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
A	y(Jan, A)	y(Feb, A)	y(Mar, A)	y(Apr, A)	y(May, A)	y(Jun, A)	y(Jul, A)	y(Aug, A)	y(Sep, A)	y(Oct, A)	y(Nov, A)	y(Dec, A)
B	y(Jan, B)	y(Feb, B)	y(Mar, B)	y(Apr, B)	y(May, B)	y(Jun, B)	y(Jul, B)	y(Aug, B)	y(Sep, B)	y(Oct, B)	y(Nov, B)	y(Dec, B)
C	y(Jan, C)	y(Feb, C)	y(Mar, C)	y(Apr, C)	y(May, C)	y(Jun, C)	y(Jul, C)	y(Aug, C)	y(Sep, C)	y(Oct, C)	y(Nov, C)	y(Dec, C)

The **Constraints** for this problem are as follows:

1. States A and B have a regulatory restriction that the outsourced insurance applications cannot be more than 30% and 40% of the total number of applications for each month, respectively. This constraint is represented mathematically as:

$$y_{s,m} <= ma_s * d_{s,m}$$

Where,

$y_{s,m}$ = Number of applications processed by vendors for each state per month

ma_s = Per state regulatory restriction [0.3 for State A, 0.4 for State B, 1 for State C]

$d_{s,m}$ = Demand for each state per month

2. For each month with respect to its State, the sum of total applications processed by vendors and total applications processed by the internal staff should be equal to the total number of applications.This constraint is represented mathematically as:

$$d_{s,m} = 40 * av_{s.m} * x_{s,m} + y_{s,m}$$

Where,

$d_{s,m}$ = Demand for each state per month

40 = Number of applications processed by staff when the availability is 100%

$av_{s,m}$ = Average availability of the internal staff for each state per month

$x_{s,m}$ = Count of internal staff members for each state per month

$y_{s,m}$ = Number of applications processed by outsourced vendors for each state per month

The **Objective** for this problem is as follows:

The objective is to optimise the total cost for the application approval process by distributing the right number of applications between the internal staff and the vendors while meeting the constraints of the monthly demand for each state at the same time..This objective is represented mathematically as:

$$tc = \sum_{s}^{A,B,C} \sum_{m}^{1-12} (ms_{s,m} * x_{s,m}) + (uc_{s,m} * y_{s,m})$$

Where,

tc = Total cost

s = States

m = Months

$ms_{s,m}$ = Monthly salary for each state per month

$x_{s,m}$ = Count of internal staff members for each state per month

$uc_{s,m}$ = Unit cost of applications processed by vendors for each state per month

$y_{s,m}$ = Number of applications processed by vendors for each state per month

The value of the objective function comes up to be 17962336.45.This means that the company has to spend around **$17.9 million** in total for the application approval process.
The following is the result for given scenario:

	State	Month	staff_count	num_app_staff	num_app_outsourced	total_applications	staff_cost	outsourcing_cost	total_cost
0	A	Jan	161.73	5240.0	0.0	5240	808641.98	0.0	808641.98
1	A	Feb	160.46	4878.0	0.0	4878	802302.63	0.0	802302.63
2	A	Mar	198.07	5942.0	0.0	5942	990333.33	0.0	990333.33
3	A	Apr	71.78	2297.0	0.0	2297	358906.25	0.0	358906.25
4	A	May	63.85	1992.0	0.0	1992	319230.77	0.0	319230.77
5	A	Jun	77.91	2275.0	0.0	2275	389554.79	0.0	389554.79
6	A	Jul	137.28	3734.0	1600.0	5334	686397.06	288000.0	974397.06
7	A	Aug	110.89	3371.0	0.0	3371	554440.79	0.0	554440.79
8	A	Sep	116.02	3759.0	0.0	3759	580092.59	0.0	580092.59
9	A	Oct	120.86	3529.0	0.0	3529	604280.82	0.0	604280.82

- 18.31% of the applications is done by the vendors
- The average cost per application is $158.55

Worst-case and best-case analysis based on the staffs' availability

Worst case scenario:

The value of the objective function comes up to be 19599482.52.This means that the company has to spend around **$19.6 million** in total for the worst case scenario application approval process.

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worst_case_output.head(10)
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	State	Month	staff_count	num_app_staff	num_app_outsourced	total_applications	staff_cost	outsourcing_cost	total_cost
0	A	Jan	187.14	5240.0	0.0	5240	935714.29	0.0	935714.29
1	A	Feb	131.35	3415.0	1463.0	4878	656730.77	263340.0	920070.77
2	A	Mar	212.21	5942.0	0.0	5942	1061071.43	0.0	1061071.43
3	A	Apr	76.57	2297.0	0.0	2297	382833.33	0.0	382833.33
4	A	May	71.14	1992.0	0.0	1992	355714.29	0.0	355714.29
5	A	Jun	61.27	1593.0	682.0	2275	306346.15	122760.0	429106.15
6	A	Jul	155.58	3734.0	1600.0	5334	777916.67	288000.0	1065916.67
7	A	Aug	90.77	2360.0	1011.0	3371	453846.15	181980.0	635826.15
8	A	Sep	134.25	3759.0	0.0	3759	671250.00	0.0	671250.00
9	A	Oct	95.04	2471.0	1058.0	3529	475192.31	190440.0	665632.31

- 35.14% of the applications is processed by the vendors for the worst case scenario
- The average cost per application for the worst case scenario is $173

Best case scenario:

The value of the objective function comes up to be 16527535.64.This means that the company has to spend around **$16.5 million** in total for the best case scenario application approval process.

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best_case_output.head(10)
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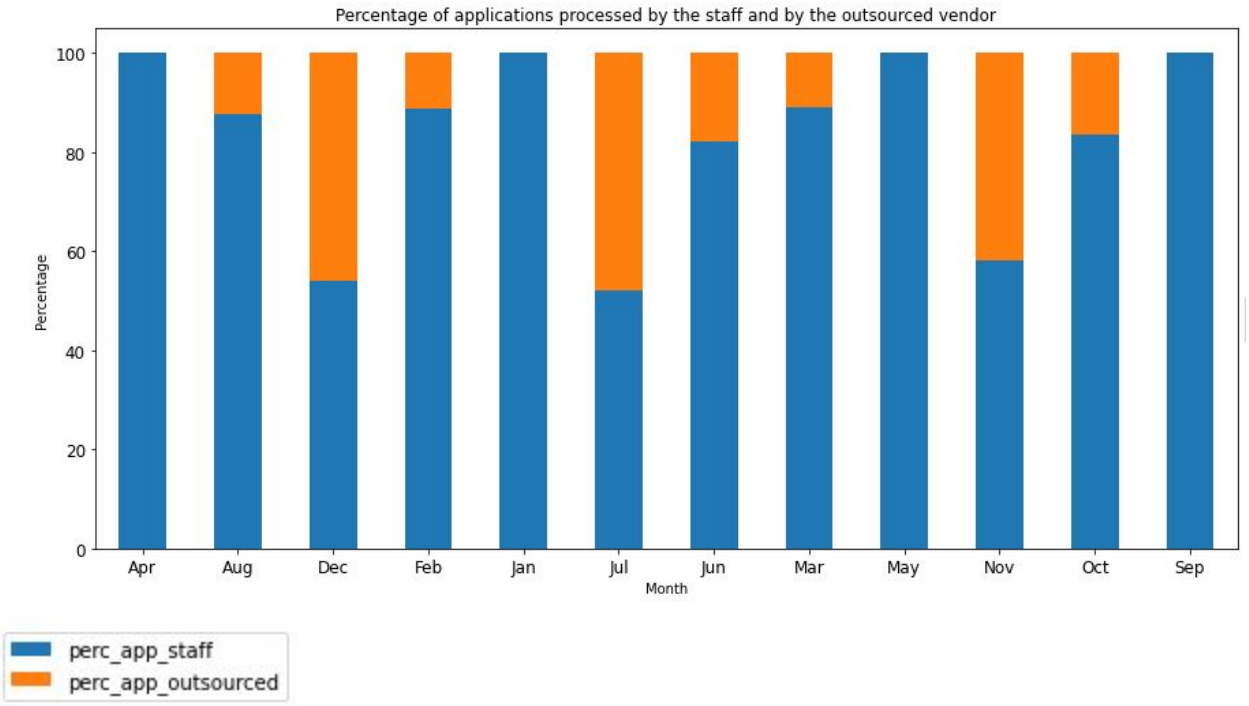
	State	Month	staff_count	num_app_staff	num_app_outsourced	total_applications	staff_cost	outsourcing_cost	total_cost
0	A	Jan	145.56	5240.0	0.0	5240	727777.78	0.0	727777.78
1	A	Feb	143.47	4878.0	0.0	4878	717352.94	0.0	717352.94
2	A	Mar	185.69	5942.0	0.0	5942	928437.50	0.0	928437.50
3	A	Apr	67.56	2297.0	0.0	2297	337794.12	0.0	337794.12
4	A	May	58.59	1992.0	0.0	1992	292941.18	0.0	292941.18
5	A	Jun	71.09	2275.0	0.0	2275	355468.75	0.0	355468.75
6	A	Jul	177.80	5334.0	0.0	5334	889000.00	0.0	889000.00
7	A	Aug	99.15	3371.0	0.0	3371	495735.29	0.0	495735.29
8	A	Sep	104.42	3759.0	0.0	3759	522083.33	0.0	522083.33
9	A	Oct	110.28	3529.0	0.0	3529	551406.25	0.0	551406.25

- 4.11% of the applications is processed by the vendors for the worst case scenario
- The average cost per application for the best case scenario is $145.88

Percentage of applications processed by internal staff and vendors

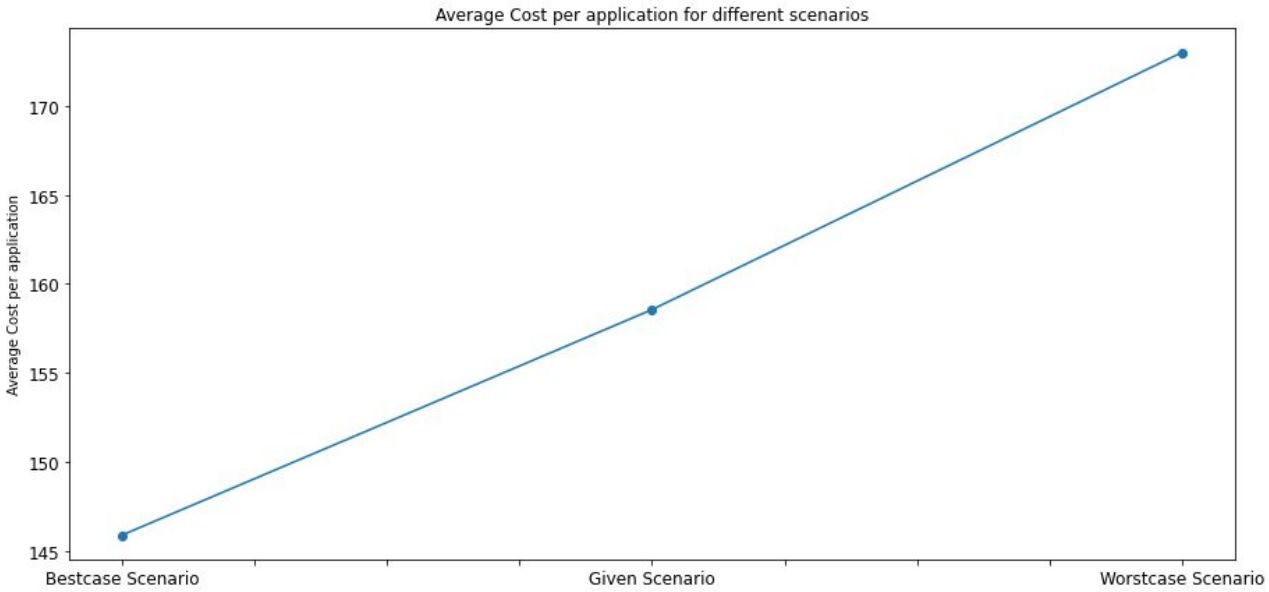


- 100% of the applications are processed by the internal staff for the months of January, April, May and September

- We can see that the months of July and December have round 50% applications processed by internal staff and 50% processed by vendors

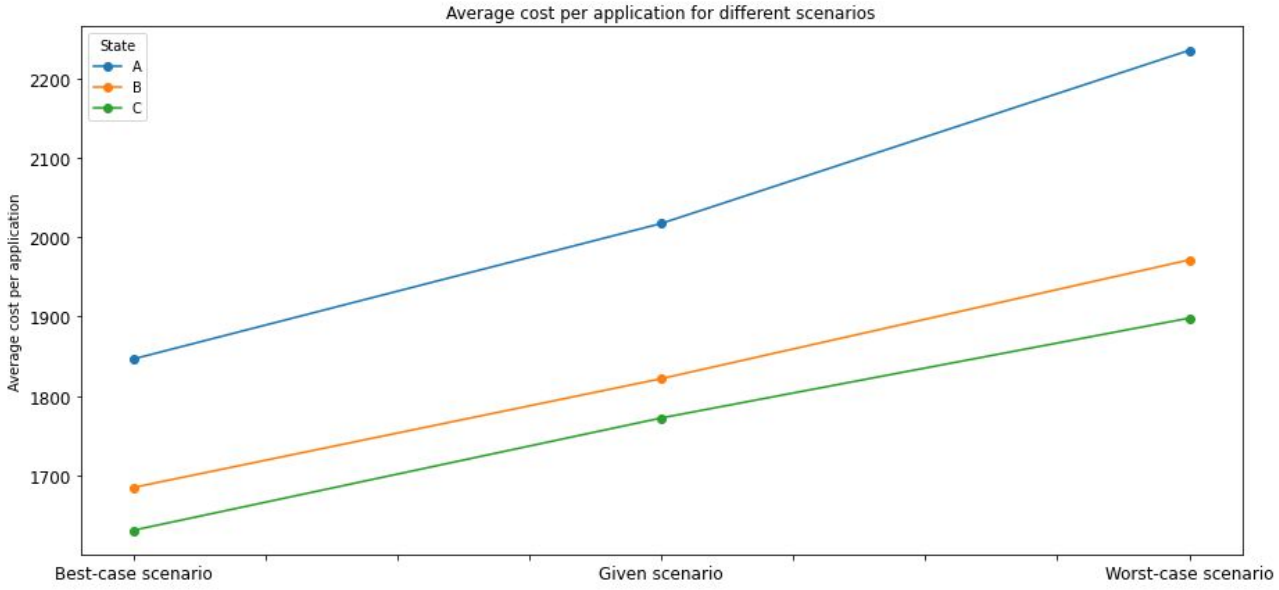
- February, March and August have the lowest percentages of applications processed by vendors

Average cost per application for different scenarios



- Average cost per application is the lowest for the “Best Case Scenario” ($145.88) whereas it is the highest for the “Worst Case Scenario” ($173), an increase of $ 27.12
- Average cost per application in the “Given Case Scenario” is $158.55

Average cost per application for different scenarios for each state



State A has the highest overall average cost per application for all the three scenarios, followed by State B and State C.