Math 211 Homework 12 Sample Texfile

1.

$$x \cdot w = (6)(3) + (-2)(-1) + (3)(-3) = 5$$

$$x \cdot x = 6^2 + 2^2 + 3^2 = 49$$

So

$$\left(\frac{\mathbf{x} \cdot \mathbf{w}}{\mathbf{x} \cdot \mathbf{x}}\right)\mathbf{x} = \begin{bmatrix} \frac{30}{49} \\ \frac{-10}{49} \\ \frac{15}{49} \end{bmatrix}$$

2.

$$||u|| = \sqrt{6^2 + 4^2 + 3^2} = \sqrt{61}$$

So the unit vector is:

$$\begin{bmatrix} \frac{-6}{\sqrt{61}} \\ \frac{4}{\sqrt{61}} \\ \frac{-3}{\sqrt{61}} \end{bmatrix}$$

3.

$$u \cdot v = (12)(2) + (3)(-3)|(-5)(3) = 0$$

.

So u and v are orthogonal.

4. (a)

$$\begin{bmatrix} 2 \\ -5 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix} = 8 + 10 - 18 = 0$$

Since the only remaining vector is 0 its dot product with the other vectors is also 0, so the set is orthogonal.

(b) Find a nonzero vector orthogonal to the span of the set.

A vector orthogonal to the span of the set can be found by applying the cross product:

$$\begin{bmatrix} 2 \\ -5 \\ -3 \end{bmatrix} \times \begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix} = \begin{bmatrix} -9 \\ -6 \\ 4 \end{bmatrix}$$

5. We know the set is orthogonal because we can reduce the matrix whos columns are composed of the set to row echelon form to get 3 pivoting columns (so the set is linearly independent):

$$\begin{bmatrix} 3 & 2 & 1 \\ -3 & 2 & 1 \\ 0 & -1 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 5 \end{bmatrix}$$

We can express $\mathbf{x} = \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}$ as a linear combination of set by simly augementing the matrix and solving:

$$\begin{bmatrix} 3 & 2 & 1 & 5 \\ -3 & 2 & 1 & -3 \\ 0 & -1 & 4 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 0 & 0 & 4 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 5 & 3 \end{bmatrix}$$

So $\mathbf{x} = \frac{4}{3}\mathbf{u}_1 + \frac{1}{3}\mathbf{u}_2 + \frac{1}{3}\mathbf{u}_3$

6. Let $\mathbf{y} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$. Express $\mathbf{y} = a\mathbf{u} + b\mathbf{z}$ such that $\mathbf{z}$ is orthogonal to $\mathbf{u}$.

7. Let $\mathbf{y} = \begin{bmatrix} -3 \\ 9 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Find the distance from $\mathbf{y}$ to the line passing through $\mathbf{u}$.

8. In order for the vectors to be orthogonal $\frac{-2}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot h = 0$ must hold. A quick observation shows that h must equal $\frac{2}{3}$.

For the set to be orthonormal, we must divide each entry by its magnitude. So $a = 1/\sqrt{\left(\frac{-2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2}$ and $b = 1/\sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + (0)^2}$ so $a = 1$ and $b = \frac{3}{\sqrt{5}}$.