# Math 211    Homework 8                    Kelvin Abrokwa-Johnson

**1.** We see that $x_4$ and $x_2$ are free variables. Using this we can set them to $t$ and $s$ and solve for the null space:

$$NulA = t \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + s \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

**2.**

1. Yes, $Ax = w$, for $x = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}$.

2. Yes, $Aw = \begin{bmatrix} 10 & -8 & -2 & -2 \\ 0 & 2 & 2 & -2 \\ 1 & -1 & 6 & 0 \\ 1 & 1 & 0 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

**3.** In order to find the set of all functions for which $T(p) = \begin{bmatrix} p(0) \\ p(0) \end{bmatrix}$ we must simply ensure that the constant term of the polynomial is 0 (because all other terms will be multiplied by 0). So, the kernel can be defined as $Span\{t, t^2\}$.

The range given that the generic form of the polynomial is $p(t) = c_1 + c_2 t + c_3 t^2$ is $\left\{ \begin{bmatrix} c_1 \\ c_1 \end{bmatrix} : c_1 \in R \right\}$ because after the transformation all terms except for the constant will become 0.

**4.** We can find both the $NulA$ and $ColA$ by row reducing like so:

$$A = \begin{bmatrix} -2 & 4 & -2 & -4 \\ 2 & -6 & -3 & 1 \\ -3 & 8 & 2 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} -2 & 4 & -2 & -4 \\ 0 & -2 & -5 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$NulA = \left\{ \begin{bmatrix} -11 \\ -5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ -3 \\ 0 \\ 1 \end{bmatrix} \right\}, Basis = \left\{ \begin{bmatrix} -2 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \\ 8 \end{bmatrix} \right\}$$

**5.** We can find the basis by simply row reducing and choosing columns with leading non-zeros.

$$\begin{bmatrix} 1 & -2 & 3 & 5 & 2 \\ 0 & 0 & -1 & -3 & -1 \\ 0 & 0 & 1 & 3 & 1 \\ 1 & 2 & -1 & -4 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 3 & 5 & 2 \\ 0 & 0 & -1 & -3 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & -4 & -9 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 3 & 5 & 2 \\ 0 & 0 & -1 & -3 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 & -2 \end{bmatrix}$$

$$basis = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 5 \\ -3 \\ 3 \\ -4 \end{bmatrix} \right\}$$

**6.** Since $2v_1 - v_2 - v_3 = 0$ implied that $v_3$ can be expressed as a multiple of $v_1$ and $v_2$ because $2v_1 - v_2 = v_3$, we can eliminate it from our basis. It is clear that $v_1$ and $v_2$ are not multiples of one another so the basis $= \{v_1, v_2\}$

**7.** Find a basis for the vector space $H$ of continuous functions spanned by the set

$$\{\cos t, \sin t, \sin 2t, \sin t \cos t\}.$$

It is clear that $\sin \sin t$ can not be expressed as a multiple of $\cos t$, so those two are in the basis. We know, howerver, from trigonometric identities, that $\frac{1}{2} \sin 2t = \sin t \cos t$. Since $\sin t \cos t$ can be expressed as a multiple of the other functions we can disclude it from the basis.

So the basis $= \{\cos t, \sin t, \sin 2t\}$