Math 211 Homework 10 Kelvin Abrokwa-Johnson

1. (10 points)

$$0 = \gamma_1(1) + \gamma_2(1-t) + \gamma_3(2-4t+t^2) + \gamma_4(6-18t+9t^2+t^3)$$

$$0 = (-\gamma_4)t^3 + (9\gamma_4+\gamma_3)t^2 + (-18\gamma_4 - 4\gamma_3 - \gamma_2)t + (\gamma_4 - 2\gamma_3 + \gamma_2 + \gamma_1)$$

Since we know $b \neq 0$ then we must show that $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0$. When we solve the system composed of the above coefficients set to 0 we can clearly see below that the matrix has 4 pivoting columns so the system must only have one solution, the trivial solution.

$$\begin{bmatrix} 1 & 1 & 2 & 6 \\ 0 & -1 & -4 & -18 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Since we have shown that $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0$ we conclude that the set is also a basis.
The change of basis matrix from $\mathbb{C}$ to $\mathbb{B}$ is:

$$\begin{bmatrix} 1 & 1 & 2 & 6 \\ 0 & -1 & -4 & -18 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$[u]_{\mathbb{B}} = \begin{bmatrix} 1 & 1 & 2 & 6 \\ 0 & -1 & -4 & -18 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ 9 \\ -1 \end{bmatrix} = \begin{bmatrix} 10 \\ -15 \\ 0 \\ 1 \end{bmatrix}$$

2. (5 points)
rank $A = 5$
dim Nul $A = n - rankA = 1$
Basis Col $A = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ -13 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$
Basis for row space $= \{(1,1,-2,0,1,0), (0,1,-1,0,-3,0), (0,0,1,1,-13,0)(0,0,0,0,1,0)(0,0,0,0,0,1)\}$

3. To find the characteristic polynomial we simply set the determinant of $\begin{bmatrix} 4-\lambda & 4 \\ 1 & 4-\lambda \end{bmatrix}$ equal to 0 and solve. We find that it is $(4-\lambda)^2 - 4 = (\lambda-2)(\lambda-6) = 0$. We plug each of the solutions into the matrix $A - \lambda I$ and solve for 0 to get $Span\left\{ \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\}$ when $\lambda = 2$ and $Span\left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$ when $\lambda = 6$. So two linearly independent eigenvectors are $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

4. $A - \lambda I = \begin{bmatrix} 1-\lambda & 2 & 0 \\ 0 & 1-\lambda & 3 \\ 0 & 0 & 1-\lambda \end{bmatrix}$ so $detA = (1-\lambda)^3$, and the matrix is singular when $\lambda = 1$.

We substitute this eigenvalue back into $A - \lambda I$ to get $\begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$. We set this matrix to 0 and get the solution $Span\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}\right\}$ and this is the only eigenvector.

5. Since this is a triangular matrix we can immediately see that the determinant is the product is the product of the diagonal entries. So the characteristic polynomial is $(4-\lambda)(3-\lambda)(2-\lambda)=0$. When we substitute each of these into the matrix $A-\lambda I$ we get the solutions $Span\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}\right\}$, $Span\left\{\begin{bmatrix}0\\1\\0\end{bmatrix}\right\}$, and $Span\left\{\begin{bmatrix}1\\-8\\2\end{bmatrix}\right\}$ so three linearly indepenedent eigenvectors are $\begin{bmatrix}1\\0\\0\end{bmatrix}$, $\begin{bmatrix}0\\1\\0\end{bmatrix}$, and $\begin{bmatrix}1\\-8\\2\end{bmatrix}$.

6. $A-4I_4=\begin{bmatrix}0&2&3&3\\0&-2&h&3\\0&0&0&14\\0&0&0&-2\end{bmatrix}$. After row reduction, we have $\begin{bmatrix}0&0&h+3&0\\0&1&h&0\\0&0&0&0\\0&0&0&1\end{bmatrix}$ and we see that the null space of $A-4I_4$ has nullity 2 when $h=-3$.

7. We know that $(A+B)^T=A^T+B^T$ and $\det A=\det A^T$. So $det(A-\lambda I)=det(A-\lambda I)^T=det(A^T-(\lambda I)^T)=det(A^T-\lambda I)$.