Five points for each question.

Math 211 Homework 6 Kelvin Abrokwa-Johnson

1.

$$det(A) = 8\begin{vmatrix} 0 & 3 \\ -2 & 5 \end{vmatrix} - 1\begin{vmatrix} 4 & 3 \\ 3 & 5 \end{vmatrix} + 6\begin{vmatrix} 4 & 0 \\ 3 & -2 \end{vmatrix} = 8(6) - 11 - 6(8) = -11$$

2.

$$det(A) = 4\begin{vmatrix} -1 & 0 & 0 \\ 6 & 3 & 0 \\ -8 & 4 & -3 \end{vmatrix} = 4(-1)\begin{vmatrix} 3 & 0 \\ 4 & -3 \end{vmatrix} = 4 \times -1 \times 3 \times |-3| = 36$$

3. We can first augment the matrix as a visual aid like so:

$$\begin{bmatrix} 1 & 3 & 5 & 1 & 3 \\ 2 & 1 & 1 & 2 & 1 \\ 3 & 4 & 2 & 3 & 4 \end{bmatrix}$$

Ans now we add and products of the downward diagonals and subtract the products of the upward diagonals. So,

$$det(A) = 40 + 2 + 9 - 15 - 4 - 12 = 51 - 31 = 20$$

4.

$$det(A) = 3\begin{vmatrix} 1 & 5 & -3 \\ 0 & -6 & 4 \\ 0 & 3 & -1 \end{vmatrix} = 3(2)\begin{vmatrix} 1 & 5 & -3 \\ 0 & -3 & 2 \\ 0 & 0 & 1 \end{vmatrix} = -18$$

5. We begin our row reduction with the first column. So,

$$det(A) = \begin{vmatrix} 1 & 3 & 3 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 5 & 4 & -3 \\ -3 & -7 & -5 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 3 & -4 \\ 0 & 1 & 2 & -5 \\ 0 & -1 & -2 & 5 \\ 0 & 2 & 4 & -10 \end{vmatrix}$$

We immediately see that the last three rows are multiples of each other, therefore, further reduction would result in 0 rows. We can conclude that the $det(A) = 0$.

6. $det(M) = 7$ because it takes at least 2 row swaps to reach M from the original matrix, call it A. so $(M) = -1(-1)det(A) = 7$.

Since N is obtained by adding a multiple of one row to another the determinant is the same as the original matrix. $det(N) = 7$.

7. (a) We know that an invertible matrix can be reduced to the form $\begin{bmatrix} a_{11} & * & * & * \\ 0 & a_{22} & * & * \\ 0 & 0 & \ddots & * \\ 0 & 0 & 0 & a_{nn} \end{bmatrix}$ and that $det(A)$ is the product of the diagonals $a_{11} \times a_{22} \times \cdots \times a_{nn}$. So, if we multiply the matrix by some scalar r then it becomes $\begin{bmatrix} ra_{11} & * & * & * \\ 0 & ra_{22} & * & * \\ 0 & 0 & \ddots & * \\ 0 & 0 & 0 & ra_{nn} \end{bmatrix}$. The determinant of this matrix is $ra_{11} \times ra_{22} \times \cdots \times ra_{nn} = r^n(a_{11} \times a_{22} \times \cdots \times a_{nn}) = r^n det(A)$.

(b) $det(A) \neq 0$ means that A is invertible, therefore, there exists a set of matrices $E_1 \cdots E_k$ such that $E_k \cdots E_1 A = I_n$. We know that $1 = det(I_n) = det(E_k) \cdots det(E_1) det(A)$. So $\frac{1}{det(A)} = det(E_k) \cdots det(E_1) = det(A^{-1})$. QED.

8. We start by expanding the expression to $det(PAP^{-1}) = det(P)det(A)det(P^{-1})$. Here there are two cases. If A is not invertible, then $det(A) = 0$ and so $det(P) \cdot 0 \cdot det(P^{-1}) = 0 = det(A)$. For second case, in which A is invertible, we can simple the expression as $det(P)det(A)\frac{1}{det(P)} = 1 \cdot det(A) = det(A)$.

9.

(a) $det(AB) = det(A)det(B) = -2$
(b) $det(B^5) = det(B)^5 = -32$
(c) $det(2A) = 2^4 det(A) = -16$
(d) $det(A^T A) = det(A)^2 = 1$
(e) $det(AB^{-1}) = \frac{det(A)}{det(B)} = -\frac{1}{2}$