Math 211 Homework 9 Kelvin Abrokwa-Johnson

**1.** Let

$$\mathcal{B} = \left\{ \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix} \right\}$$

be a basis for $\mathbb{R}^3$. (a) Find $\mathbf{x}$ if $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix}$. (b) Find $[\mathbf{y}]_{\mathcal{B}}$ if $\mathbf{y} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

(a)

$$x = -3 \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix} + 2 \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} - 1 \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ 1 \end{bmatrix}$$

(b)

$$\begin{bmatrix} -2 & 3 & 4 & 1 \\ 2 & 0 & -1 & 2 \\ 0 & 2 & 3 & 3 \end{bmatrix} \to \begin{bmatrix} -2 & 3 & 4 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$[y]_{\mathcal{B}} = \begin{bmatrix} \frac{3}{2} \\ 0 \\ 1 \end{bmatrix}$$

**2.** Let $\mathcal{B}$ be the basis in Question 1, and

$$\mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

be another basis basis for $\mathbb{R}^3$.

(a) Find the matrix $\mathcal{P}_{\mathcal{C}\leftarrow\mathcal{B}}$. (b) Find $[\mathbf{z}]_{\mathcal{C}}$ if $[\mathbf{z}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

**3.** Suppose $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ is a basis for $\mathbb{R}^3$. Show that $\mathcal{C} = \{\mathbf{b}_1, \mathbf{b}_1 + \mathbf{b}_2, \mathbf{b}_1 - \mathbf{b}_3\}$ is also a basis, and find the matrix $\mathcal{P}_{\mathcal{C}\leftarrow\mathcal{B}}$.

**4.** Find the dimension of the subspace $H$ of $\mathbb{R}^3$ spanned by $\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ -6 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} -3 \\ 5 \\ 5 \end{bmatrix}$.

**5.** Find the dimensions of Nul $A$ and Col $A$ for the matrix

$$A = \begin{bmatrix} 1 & 2 & -4 & 3 & -2 & 6 & 0 \\ 0 & 0 & 0 & 1 & 0 & -3 & 7 \\ 0 & 0 & 0 & 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Determine rank$A$, $\dim \text{Nul}A$, and a basis for the row space of $A$.

**6.** Let $A$ be a $7 \times 5$ matrix with rank 2. Determine $\dim \text{Nul}A$ and rank$A^T$.

**7.** Consider the linear system $Ax = b$ such that $A$ is $6 \times 8$. Suppose $A$ has rank 6.
(a) Is there any $b$ such that the system is inconsistent?
(b) If there any $b$ such that the system has a unique solution?

(Explain your answer.)

**8.** Let $H = \{(a, b, c, d) : a - 3b + c = 0\}$.
(a) Show that $H$ is a subspace of $\mathbb{R}^{1\times 4}$.
(b) Find a basis for $H$, and hence deduce the dimension of $H$.

**9.** Let $W = \{a + bt + ct^2 + dt^3 : a - 3b + c = 0\}$.
(a) Show that $W$ is a subspace of $\mathbb{P}_3(t)$.
(b) Find a basis for $W$.