Definitions

vector space: A vector space V over a field F consists of a set on which two operations (called **addition** and **scalar multiplication**) are defined so that for each pair of elements $x, y \in V$ there is a unique element $x + y$ in V, and for each element a in F and each element x in V there is a unique elemnt ax in V, such that the following conditions hold: ...

subspace: A subset W of a vector space V over a field F is called a **subspace** of V if W is a vector space over F with the operations of addition and scalar multiplication defined on V.

linear combination: Let V be a vector space and S a nonempty subset of V. A vector $v \in V$ is called a **linear combination** of vectors of S if there exist a finite number of vectors $u_1, u_2, .., u_n$ in S and scalars $a_1, a_2, ..., a_n$ in F such that $v = a_1u_1 + a_2u_2 + \cdots + a_nu_n$. In this case we also say that v is a linear combination of $u_1, u_2, ..., u_n$ and call $a_1, a_2, ..., a_n$ the coefficients of the linear combination.

span: Let S be a nonempty subset of a vector space V. The **span** of S, denoted $span(S)$, is the set consisting of all linear combinations of the vectors in S. For convenience, we define $span(0) = 0$.

linearly dependent: A subset S of a vector space V is called **linearly depenedent** if there exist a finite number of distinct vectors $u_1, u_2, ..., u_n$ in S and scalars $a_1, a_2, ..., a_n$, not all zero such that

$$a_1u_1 + a_2u_2 + \cdots + a_nu_n = 0$$

basis: A **basis** β for a vector space V is a linearly indepenedent subset of V that generates V. If β is a basis for V, we also say that the vectors of β form a basiss for V.

finite-dimensional: A vector space is called **finite-dimensional** if it has basis consisting of a finite number of vectors. The unique number of vectors in each basis for V is called the **dimension** of V and is denoted by $dim(V)$. A vector space that is not finite-dimensional is called **inifinite-dimensional**.

maximal: Let $\mathcal{F}$ be a family of sets. A member M of $\mathcal{F}$ (with respect to set inclusion) if M is contained in no member of $\mathcal{F}$ other than M itself.

linear transformation: Let V and W be vector spaces (over F). We call a function $T : V \to W$ a **linear transformation** from V to W if, for all $x, y \in V$ and $c \in F$, we have

- $T(x + y) = T(x) + T(y)$
- $T(cx) = xT(x)$

null space: Let V and W be vector spaces, and let $T : V \to W$ be linear. We define the **null space** $N(T)$ of T to be the set of all vectors in V such that $T(x) = 0$; that is, $N(T)$; that is, $N(T) = x \in V : T(x) = 0$.

range: We define the **range** $R(T)$ of T to be the subset of W consisting of all images (under T) of vectors in V; that is , $R(T) = T(x) : x \in V$.

nullity: $dim(N(T))$

rank: $dim(R(T))$