Math 211 Homework 11 Kelvin Abrokwa-Johnson

1. When $\lambda = 2$ our solution set to solving the system set equal to 0 is $Span\left\{\begin{bmatrix}-1\\1\\0\end{bmatrix}\right\}$ and when $\lambda = 3$ we have $Span\left\{\begin{bmatrix}-2\\0\\1\end{bmatrix},\begin{bmatrix}0\\1\\0\end{bmatrix}\right\}$. So we have:

$$P = \begin{bmatrix}-1 & -2 & 0\\1 & 0 & 1\\0 & 1 & 0\end{bmatrix}, D = \begin{bmatrix}2 & 0 & 0\\0 & 3 & 0\\0 & 0 & 3\end{bmatrix}$$

2. (a) We know that the dimension of Nul$(A - \lambda_3 I) = 2$ because, by Theorem 6, an nxn matrix with n distinct eigenvalues is diagonalizable. Since, $A - \lambda_1 I$ and $A - \lambda_2 I$ produce 5 eigenvalues, $A - \lambda_3 I$ must produce 2.

(b) If A is not diagonalizable then dim Nul$(A - \lambda_3 I) = 1$. (c)

3. Let $\mathcal{D} = \{\mathbf{d}_1, \mathbf{d}_2\}$ and $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ be the bases for vector spaces W and V respectively. Let $T : W \to V$ with

$$T(\mathbf{d}_1) = 3\mathbf{b}_1 - 3\mathbf{b}_2 \ \ and \ \ T(\mathbf{d}_2) = -2\mathbf{b}_1 + 5\mathbf{b}_2.$$

Find the matrix for T relative to $\mathcal{D}$ and $\mathcal{B}$.

We see immediately that $[T(\mathbf{d}_1)] = \begin{bmatrix}3\\-3\end{bmatrix}$ and $[T(\mathbf{d}_2)] = \begin{bmatrix}-2\\5\end{bmatrix}$. So the matrix for T relative to $\mathcal{D}$ and $\mathcal{B}$ is $\begin{bmatrix}3 & -2\\-3 & 5\end{bmatrix}$.

4. (a)

$$T(3 - 2t + t^2) = (3 - 2t + t^2) + 2t^2(3 - 2y + t^2) = 3 - 2t + 7t^2 - 4t^3 + 2t^4$$

(b) First we show that the transformation is closed under addition. Take $p(t) = a_1 + b_1 t + c_1 t^2$ and $q(t) = a_2 + b_2 t + c_2 t^2$

$$T(p(t) + q(t)) = T((a_1 + a_2) + (b_1 + b_2)t + (c_1 + c_2)t^2)$$

$$= ((a_1 + a_2) + (b_1 + b_2)t + (c_1 + c_2)t^2) + 2t^2((a_1 + a_2) + (b_1 + b_2)t + (c_1 + c_2)t^2)$$

$$= (a_1 + b_1 t + c_1 t^2 + 2t^2 a_1 + 2t^3 b_1 + 2t^4 c_1) + (a_2 + b_2 t + c_2 t^2 + 2t^2 a_2 + 2t^3 b_2 + 2t^4 c_2)$$

$$= T(p(t)) + T(q(t))$$

Now we show that it is closed under scalar multiplication. Take $p(t) = a + bt + ct^2$ and some scalar k:

$$T(kp(t)) = T(ka + kbt + kct^2)$$

$$= ka + kbt + kct^2 + 2kat^2 + 2kbt^3 + 2kct^4$$

$$= k(a + bt + ct^2 + 2at^2 + 2bt^3 + 2ct^4)$$

$$= kT(p(t))$$

(c) We simply define the transformation on the standard polynomials to get:

$$T(1) = 1 + 2t^2$$

$$T(t) = t + 2t^3$$

$$T(t^2) = t^2 + 2t^3$$

so the matrix T relative to the bases is:

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

5.

$$T(4b_1 - 3b_2) = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 1 & -2 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ -3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ -5 \end{bmatrix}$$

6. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(\mathbf{x}) = A\mathbf{x}$, where

$$A = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}.$$

Find a basis $\mathcal{B}$ with the property that $[T]_{\mathcal{B}}$ is diagonal.

Since this is a $2x2$ matrix we can immediately see that $det(A - \lambda I) = (\lambda - 5)(\lambda + 1)$. We solve for 0 to get $\lambda = 5, -1$. So we have $[T]_{\mathcal{B}} = \begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix}$

7. Let $T : \mathbb{P}_1(t) \to \mathbb{P}_1(t)$ be defined by $T(a + bt) = (2a + 3b) + (3a + 2b)t$.

Find a basis $\mathcal{B}$ of $\mathbb{P}_1(t)$ with the property that $[T]_{\mathcal{B}}$ is diagonal.