Math 211 Homework 14 Kelvin Abrokwa-Johnson

1. We know that $A^TAx = Ab$.

$$A^TA = \begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 1 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ -3 & 11 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ -16 \end{bmatrix}$$

So, solving for x we find our solution:

$$\hat{x} = \begin{bmatrix} \frac{3}{4} \\ -\frac{5}{4} \end{bmatrix}$$

2. We know that the orthogonal projection $\hat{b} = \frac{b \cdot a_1}{a_1 \cdot a_1} a_1 + \frac{b \cdot a_2}{a_2 \cdot a_2} a_2 + \frac{b \cdot a_2}{a_2 \cdot a_2} a_2$. So,

$$\hat{b} = \begin{bmatrix} 5 \\ 2 \\ 3 \\ 6 \end{bmatrix}, \hat{x} = \begin{bmatrix} 1/3 \\ 14/3 \\ 5/3 \end{bmatrix}$$

3. Since we have the QR factorizationn of A the solution is immediate. We can simply calculate $\hat{x} = R^{-1}Q^Tb$ to get:

$$\hat{x} = \begin{bmatrix} 2.9 \\ 0.9 \end{bmatrix}$$

4. We know that $\text{Col}(A)^\perp$ is Nul A^T, so:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 4 & -1 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

So we have $\text{Col}(A)^\perp = \text{Nul } A^T = \text{Span} \left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

5.

$$X = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 5 \\ 1 & 6 \end{bmatrix}, y = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

$$X^TX = \begin{bmatrix} 4 & 16 \\ 16 & 74 \end{bmatrix}$$

$$X^Ty = \begin{bmatrix} 6 \\ 17 \end{bmatrix}$$

Hence. $\beta = \begin{bmatrix} 4.3 \\ -0.7 \end{bmatrix}$