Math 211 Homework 1 **Name: Kelvin Abrokwa-Johnson**

1 (5 points) Find the point of intersection of the lines: $x_1 + 2x_2 = -13, 3x_1 - 2x_2 = 1$.

Solution: To find the point of intersection, we solve the system with augmented matrix

$$\begin{bmatrix} 1 & 2 & -13 \\ 3 & -2 & 1 \end{bmatrix}$$

First we row reduce:

$$\begin{bmatrix} 1 & 2 & -13 \\ 3 & -2 & 1 \end{bmatrix}$$

We subtract 3 times the first row from the second row to get:

$$\begin{bmatrix} 1 & 2 & -13 \\ 0 & -8 & 40 \end{bmatrix}$$

And then simplify:

$$\begin{bmatrix} 1 & 2 & -13 \\ 0 & 1 & -5 \end{bmatrix}$$

We can immediately see that $x_2 = -5$. We plug that back into the first equation to solve for x_1:

$$\begin{aligned} x_1 &= -13 - 2x_2 \\ &= -13 - 2(-5) \\ &= -3 \end{aligned}$$

And so our final solutions are:

$$\begin{aligned} x_1 &= -3 \\ x_2 &= -5 \end{aligned}$$

2 (5 points) Solve the linear system:

$$\begin{aligned} x_1 - 5x_2 + 4x_3 &= -3 \\ 2x_1 - 7x_2 + 3x_3 &= -2 \\ -2x_1 + x_2 + 7x_3 &= -1 \end{aligned}$$

Solution. Consider augmented matrix

$$\begin{bmatrix} 1 & -5 & 4 & -3 \\ 2 & -7 & 3 & -2 \\ -2 & 1 & 7 & -1 \end{bmatrix}.$$

We can row reduce here starting with the third equation. We can start by subtracting 2 times the first row from the second row and adding 2 times the first row to the the third row:

$$\begin{bmatrix} 1 & -5 & 4 & -3 \\ 0 & 3 & -5 & 4 \\ 0 & -9 & 15 & -7 \end{bmatrix}$$

Next, we add 3 times the second row to the third row:

$$\begin{bmatrix} 1 & -5 & 4 & -3 \\ 0 & 3 & -5 & 4 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

We can now see that the system is inconsistent. There is no solution.

3 (5 points) Solve the linear system with augmented matrix

$$\begin{bmatrix} 2 & 0 & -6 & -8 \\ 0 & 1 & 2 & 3 \\ 3 & 6 & -2 & -4 \end{bmatrix}.$$

Solution: We can use row reduction. We start by subtracting 3 times the first column from the third column to get:

$$\begin{bmatrix} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 6 & 7 & 8 \end{bmatrix}$$

And for the second column we can subtract 6 times the second row from the third row to get:

$$\begin{bmatrix} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & -5 & -10 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

We immediately see that $x_3 = 2$. We can then solve for x_2:

$$\begin{aligned} x_2 &= 3 - 2x_3 \\ &= 3 - 2(2) \\ &= -1 \end{aligned}$$

And for x_1:

$$\begin{aligned} x_1 &= -4 + 3x_3 \\ &= -4 + 3(2) \\ &= 2 \end{aligned}$$

Our solutions are:

$$\begin{aligned} x_1 &= 2 \\ x_2 &= -1 \\ x_3 &= 2 \end{aligned}$$

4 (5 points) Find all the values h so that the linear system with the following augmented matrix is consistent:Applying row reduction to:

$$\begin{bmatrix} 1 & h & -5 \\ 2 & -8 & 6 \end{bmatrix}.$$

Solution. To simplify our calculations a bit we can divide the second row by 2 and then flip the rows to get:

$$\begin{bmatrix} 1 & -4 & 3 \\ 1 & h & 6 \end{bmatrix}$$

Using row reduction, we get:

$$\begin{bmatrix} 1 & -4 & 3 \\ 0 & h+4 & 3 \end{bmatrix}$$

We can see that if $h = -4$ then we have $0 =$ a non-zero which would render the system inconsistent. So, the system is consistent exactly when $h \neq -4$.

5 (5 points) Row reduce the following augmented matrix. Determine the leading ones and pivot columns.

$$\begin{bmatrix} 1 & 2 & 4 & 5 \\ 2 & 4 & 5 & 4 \\ 4 & 5 & 4 & 3 \end{bmatrix}.$$

Solution: We begin our row reduction by subtracting 2 times the first frow from the second row and subtracting 4 times the first row from the third row to get:

$$\begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 0 & -4 & -6 \\ 0 & -3 & -12 & -17 \end{bmatrix}.$$

To simplify we can divide the second row by -3 and switch the second and third rows to get:

$$\begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & -3 & -12 & -17 \\ 0 & 0 & 1 & 2 \end{bmatrix}. = \begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 1 & 4 & \frac{17}{3} \\ 0 & 0 & 1 & 2 \end{bmatrix}.$$

With reference to the original matrix (before we swapped rows), the leading ones are at $(1,1)$, $(2,3)$, and $(3,2)$ and the pivot columns are 1, 2, and 3.

6 (5 points) Find the solutions of the system with augmented matrix:

$$\begin{bmatrix} 1 & -2 & -1 & 4 \\ -2 & 4 & -5 & 6 \end{bmatrix}$$

Solution: Using row reduction, we add 2 times the first column to the second column to get:

$$\begin{bmatrix} 1 & -2 & -1 & 4 \\ 0 & 0 & -7 & 14 \end{bmatrix} = \begin{bmatrix} 1 & -2 & -1 & 4 \\ 0 & 0 & 1 & -2 \end{bmatrix}.$$

We can immediately see that $x_3 = -2$ and since there is no leading one in for the second column x_2 is a free variable (we'll call it t). Solving for x_1 we get:

$$\begin{aligned} x_1 &= 4 + x_3 + 2x_2 \\ &= 4 + (-2) + t \\ &= 2 + t \end{aligned}$$

And so our final solution is:

$$
\begin{aligned}
x_1 &= 2 + t \\
x_2 &= t \\
x_3 &= -2
\end{aligned}
$$

7 (5 points) Find the solutions of the system with augmented matrix:

$$
\begin{bmatrix}
1 & 0 & -9 & 0 & 4 \\
0 & 1 & 3 & 0 & -1 \\
0 & 0 & 0 & 1 & -7 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}.
$$

Solution: We can immediately see that $x_4 = -7$ and that x_3 is a free variable. We plug these in to solve for x_2:

$$x_2 = -1 + t$$

and for x_1:

$$x_1 = 4 + 9t$$

And our final solution:

$$
\begin{aligned}
x_1 &= 4 + 9t \\
x_2 &= -1 + t \\
x_3 &= t \\
x_4 &= -7
\end{aligned}
$$

8 (5 points) Determine the conditions on (h, k) so that the following system has no solution, one solution, and infinitely many solutions.

$$x_1 - 3x_2 = 1, \quad 2x_1 + hx_2 = k.$$

Solution. The equations produce the following augmented matrix:

$$
\begin{bmatrix}
1 & -3 & 1 \\
2 & h & k
\end{bmatrix}
$$

To row reduce we subtract 2 times the first row from the second row to get:

$$
\begin{bmatrix}
1 & -3 & 1 \\
0 & h+6 & k-2
\end{bmatrix}
$$

From this reduction we can see that if $h = -6$ and $k \neq 2$, the leading one would be the last entry, which would make the system inconsistent. If $h = -6$ and $k = 2$ then there are infinite solutions because the second row would be all zeroes. And finally, if $h \neq -6$ there is one solution.