Math 211 Homework 8 Kelvin Abrokwa-Johnson

1. Find an explicit description of Nul$A$ where

$$A = \begin{bmatrix} 1 & -3 & 2 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix}.$$

[Find vectors such that Nul$A$ is the set of linear combinations of the vectors.]

$$NulA = t\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + s\begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

2. Let

$$A = \begin{bmatrix} 10 & -8 & -2 & -2 \\ 0 & 2 & 2 & -2 \\ 1 & -1 & 6 & 0 \\ 1 & 1 & 0 & -2 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 2 \\ 2 \\ 0 \\ 2 \end{bmatrix}.$$

1. Determine whether $\mathbf{w}$ is in Col$A$.
2. Determine whether $\mathbf{w}$ is in Nul$A$.

1. Yes, $Ax = w$, for $x = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}$.

2. Yes, $Aw = \begin{bmatrix} 10 & -8 & -2 & -2 \\ 0 & 2 & 2 & -2 \\ 1 & -1 & 6 & 0 \\ 1 & 1 & 0 & -2 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

3. Let $T : \mathbb{P}_2(t) \to \mathbb{R}^2$ be a linear transformation defined as $T(\mathbf{p}) = \begin{bmatrix} \mathbf{p}(0) \\ \mathbf{p}(0) \end{bmatrix}$. Find two polynomials in $\mathbb{P}_2(t)$ that span the kernel of T, and describe the range of T.

[Hint: Determine the matrix A of transformation of T. Find the kernel and range space using Nul$A$ and Col$A$.]

4. Let $A = \begin{bmatrix} -2 & 4 & -2 & -4 \\ 2 & -6 & -3 & 1 \\ -3 & 8 & 2 & -3 \end{bmatrix}$. Fine bases for Nul$A$ and Col$A$.

$$A = \begin{bmatrix} -2 & 4 & -2 & -4 \\ 2 & -6 & -3 & 1 \\ -3 & 8 & 2 & -3 \end{bmatrix} \to \begin{bmatrix} -2 & 4 & -2 & -4 \\ 0 & -2 & -5 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$NulA = \left\{ \begin{bmatrix} -11 \\ -5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ -3 \\ 0 \\ 1 \end{bmatrix} \right\}, Basis = \left\{ \begin{bmatrix} -2 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \\ 8 \end{bmatrix} \right\}$$

5. Find a basis for the space spanned by the following vectors.

$$\begin{bmatrix}1\\0\\0\\1\end{bmatrix}, \begin{bmatrix}-2\\0\\0\\2\end{bmatrix}, \begin{bmatrix}3\\-1\\1\\-1\end{bmatrix}, \begin{bmatrix}5\\-3\\3\\-4\end{bmatrix}, \begin{bmatrix}2\\-1\\1\\0\end{bmatrix}.$$

We remove the second colum since it is clearly a multiple of the first. And then we row reduce:

$$\begin{bmatrix}1 & -2 & 3 & 5 & 2\\0 & 0 & -1 & -3 & -1\\0 & 0 & 1 & 3 & 1\\1 & 2 & -1 & -4 & 0\end{bmatrix} \to \begin{bmatrix}1 & -2 & 3 & 5 & 2\\0 & 0 & -1 & -3 & -1\\0 & 0 & 0 & 0 & 0\\0 & 4 & -4 & -9 & -2\end{bmatrix} \to \begin{bmatrix}1 & -2 & 3 & 5 & 2\\0 & 0 & -1 & -3 & -1\\0 & 0 & 0 & 0 & 0\\0 & 0 & 0 & -5 & -2\end{bmatrix}$$

$$basis = \left\{ \begin{bmatrix}1\\0\\0\\1\end{bmatrix}, \begin{bmatrix}3\\-1\\1\\-1\end{bmatrix}, \begin{bmatrix}5\\-3\\3\\-4\end{bmatrix} \right\}$$

6. Let

$$\mathbf{v_1} = \begin{bmatrix}3\\4\\-2\\-5\end{bmatrix}, \mathbf{v_2} = \begin{bmatrix}4\\3\\2\\4\end{bmatrix}, \mathbf{v_3} = \begin{bmatrix}2\\5\\-6\\-14\end{bmatrix}.$$

It can be verified that $2\mathbf{v_1} - \mathbf{v_2} - \mathbf{v_3} = \mathbf{0}$. Find a basis for $H = \text{Span}\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$.
$basis = v_1, v_2$

7. Find a basis for the vector space H of continuous functions spanned by the set

$$\{\cos t, \sin t, \sin 2t, \sin t \cos t\}.$$

$basis = \{cost, sint, sin2t\}$