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Math 309

Homework 1

7. Less $S = 0, 1$ and $F = R$. In $\mathcal{F}(S, R$, show that $f = g$ and $f + g = h$, where $f(t) = 2t + 1, g(t) = 1 + 4t + 2t^2$, and $h(t) = 5^t + 1$.

Solution:
For 0:

$$f0 = 2(0) + 1 = g(0) = 1 + 4(0) - 2(0)^2 = 1$$
$$h(0) = 5^0 + 1 = f(0) + g(0) = 1 + 1 = 2$$

For 1:

$$f(1) = 2(1) + 1 = g(1) = 1 + 4(1) - 2(1)^2 = 3$$
$$h(1) = 5^1 + 1 = f(1) + g(1) = 3 + 3 = 6$$

Thus, we have shown what was to be shown.

8. In any vector space V, show that $(a + b)(x + y) = ax + ay + bx + by$ for any $x, y \in V$ and any $a, b \in F$.

Solution:

12. A real-valued function f defined on the real line is called an even function if $f(-t) = f(t)$ for each real number t. Prove that the set of even functions defined on the real line with the operations of addition and scalar multiplication defined in Example 3 is a vector space.

Solution:

- Suppose E is the set of even functions. Take the function $f \in E$, . Since $f(-t) = f(t)$, we know that $af(-t) = af(t)$ for all $a \in \mathbf{R}$, so $(af)(t) \in E$, i.e. closed under scalar multiplication.
- Take $f, g \in E$. $(f + g)(t) = f(t) + g(t) = f(-t) + g(-t) = (f + g)(-t)$. In short $(f + g)(t) = (f + g)(-t)$ so $(f + g)(t) \in E$, i.e. closed under addition.
- Since $f(t) = f(-t)$ for $f \in E$ where $f(t) = 0$, $f(t) = 0 \in E$, i.e. has a 0 function.

18. Let $V = \{(a_1, a_2) : a_1, a_2 \in \mathbf{R}\}$. For $(a_1, a_2), (b_1, b_2) \in V$ adn $c \in \mathbf{R}$, define

$$(a_1, a_2) + (b_1, b_2) = (a_1 + 2b_1, a_2 + 3b_2) \text{ and } c(a_1, a_2) = (ca_1, ca_2)$$

Is V a vector space over $\mathbf{R}$ wit these operations?

Solution:
No, V is not a vector space over $\mathbf{R}$.
Take $a_1, a_2 \neq 0 \in \mathbf{R}$. Since $(a_1, a_2) + (0, 0) = (a_1, a_2)$ and $(0, 0) + (a_1, a_2) = (2a_1, 3a_2)$, i.e. $(a_1, a_2) + 0 \neq 0 + (a_1, a_2)$. The set does not have commutative addition so it is not a vector space.

21. Let V adn W be vector spaces over a field F. Let

$$Z = \{(v, w) : v \in V, w \in W\}$$

Prove that Z is a vector space over F with operations:

$$(v_1, w_1) + (v_2, w_2) = (v_1 + v_1, w_1 + w_2) \text{ and } c(v_1, w_1) = (cv_1, cw_1)$$

Solution:

- Take $v_1, v_2 \in V$, $w_1, w_2 \in W$. Since V and W are vector spaces, $v_1 + v_2 \in V$ and $w_1 + w_2 \in W$, so it follows that $(v_1, w_1) + (v_2, w_2) \in Z$, i.e. closed under addition.
- Take $v \in V, w \in W, c \in F$. Since V and W are vector spaces, $cv \in V$ and $cw \in W$. So, $c(v, w) = (cv, cw) \in Z$, i.e. closed under scalar multiplication.