Math 211 Homework 7 Kelvin Abrokwa-Johnson

1.

$$det(A) = \begin{vmatrix} -5 & 3 \\ 3 & -1 \end{vmatrix} = -4$$

$$x_1 = \frac{\begin{vmatrix} 9 & 3 \\ -5 & -1 \end{vmatrix}}{det(A)} = \frac{6}{-4} = -\frac{3}{2}$$

$$x_1 = \frac{\begin{vmatrix} -5 & 9 \\ 3 & -5 \end{vmatrix}}{det(A)} = \frac{-2}{-4} = \frac{1}{2}$$

2.

$$det(A) = \begin{vmatrix} 2 & 1 & 1 \\ -1 & 0 & 3 \\ 3 & 1 & 3 \end{vmatrix} = 5$$

$$det(A_1(b)) = \begin{vmatrix} 4 & 1 & 1 \\ 2 & 0 & 3 \\ 2 & 1 & 3 \end{vmatrix} = -10, x_1 = \frac{-10}{5} = -2$$

$$det(A_1(b)) = \begin{vmatrix} 2 & 4 & 1 \\ -1 & 2 & 3 \\ 2 & 2 & 3 \end{vmatrix} = 40, x_2 = \frac{40}{5} = 8$$

$$det(A_1(b)) = \begin{vmatrix} 2 & 1 & 4 \\ -1 & 0 & 2 \\ 3 & 1 & 2 \end{vmatrix} = 0, x_3 = \frac{0}{5} = 0$$

3.

$$A_{11} = 0, A_{12} = -3, A_{13} = 3, A_{21} = 1, A_{22} - 1, A_{23} = 2, A_{31} = 0, A_{32} = -3, A_{33} = 6$$

$$det(A) = \begin{vmatrix} 0 & -2 & -1 \\ 3 & 0 & 0 \\ -1 & 1 & 1 \end{vmatrix} = 3$$

$$Adj(A) = \begin{bmatrix} 0 & 1 & 0 \\ -3 & -1 & -3 \\ 3 & 2 & 6 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & \frac{1}{3} & 0 \\ -1 & -\frac{1}{3} & -1 \\ 1 & \frac{2}{3} & 2 \end{bmatrix}$$

4. We translate the parallelogram to the origin by adding $(0, 2)$ to all of the vertices to get:

$$(0, 0), (6, 1), (-3, 3), (3, 4)$$

So our matrix $A = \begin{bmatrix} 6 & 1 \\ -3 & 3 \end{bmatrix}$. The area of the parrallelogram $A = det(A) = 21$.

5.

1. If $xy \geq 0$ then x and y are both positive or both negative (or one or both is zero), that is, x and y have the same sign for any $u \in W$. So, the signs of x and y will always remain the same when multiplied by some $c \in \mathbb{R}$ therefore xy will be greater than or equal to 0.

2.
$$u = \begin{bmatrix} -1 \\ -2 \end{bmatrix}, v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, u + v = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \notin W, 1 \cdot -1 \leq 0$$

6. W can be written as $s \begin{bmatrix} 2 \\ 2 \\ 2 \\ 0 \end{bmatrix} + t \begin{bmatrix} 4 \\ 0 \\ -3 \\ 5 \end{bmatrix}$ so $W = Span \left\{ \begin{bmatrix} 2 \\ 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ -3 \\ 5 \end{bmatrix} \right\}$. Therefore, W is a subspace by Theorem 1.

7. We know that $0 \notin W$ because $0 + 0 + 0 \neq 1$ so W is not a vector space.

8. (a) It is clear what $F0_{2x4} = 0$ so $0_{2x4} \in W$.
(b) $X, Y \in W \rightarrow FX = 0, FY = 0$. So, $F(X + Y) = FX + FY = 0 + 0 = 0 \in W$. Therefore $X + Y \in W$.
(c) If $X \in W$ then $FX = 0$, so to show $cX \in W$ we simply evaluate $FcX = c(FX) = c(0) = 0 \in W$. Therefore $cX \in W$.

9. (a) H and K are subspaces implies that $\exists x \in H, \exists y \in K : x = 0, y = 0$ therefore $x + y = 0 + 0 = 0$. So $0 \in H + K$.
(b) Say $u = h_1 + k_1, v = h_2 + k_2$ for $h_1, h_2 \in H, k_1, k_2 \in K$, then $u + v = (h_1 + h_2) + (k_1 + k_2)$. Since H, K are subspaces, we know that $h_1 + h_2 = h \in H$ and $k_1 + k_2 = k \in K$, so $u + v = h + k \in H + K$.
(c) Say $u = h_1 + k_1 \in H + K$, then $cu = c(h_1 + k_1) = ch_1 + ck_1$. Since $h_1 \in H \rightarrow ch_1 = h \in H$ and $k_1 \in K \rightarrow ck_1 = k \in K$, it is clear that $cu = h + k \in H + K$.