Special Matrices

Diagonally Dominant

A matrix is **diagonally dominant** when

$$|a_{ii}| \geq \sum_{j=1, j\neq i}^{n} |a_{ij}|$$

holes for each $i = 1, 2, ..., n$.
It is **strictly diagonally dominant** when,

$$|a_{ii}| > \sum_{j=1, j\neq i}^{n} |a_{ij}|$$

Symmetric Positive Definite Matrices (SPD)

A matrix A is **symmertric positive definite** if it is symmetric and if $x^t Ax > 0$ for every n-dimensional vector $x \neq 0$.
If A is an $n \times n$ SPD matrix, then

- A has an inverse
- $a_{ii} > 0$ for each $i = 1, 2, ..., n$
- $max_{1\leq k,j\leq n}|a_{kj}| \leq max_{1\leq i\leq n}|a_{ii}|$
- $(a_{ij})^2 < a_{ii}a_{jj}$, for each $i \leq j$

The matrix A is positive definite if and only if A can be factored in the form LDL^t, where L is lower triangular with 1s on its diagonal and D is a diagonal matrix with positive diagonal entries.
The matrix A is positive definite if and only if A can be factored in the form LL^t, where L is lower triangular with nonzero diagonal entries.

Cholesky (LL^T)

Tridiagonal LU Factorization

Vector Norms

A vector norm on R^n is a function, $||\cdot||$, from R^n into R with the following properties:

- $||x|| \geq 0$ for all $x \in R^n$
- $||x|| = 0$ if and only if $x = 0$
- $||\alpha x|| = |\alpha|||x||$ for all $\alpha \in R$, $x \in R^n$
- $||x + y|| \leq ||x|| + ||y||$ for all $x, y \in R^n$

The l_2 norm is defined as such:

$$||x||_2 = \left\{ \sum_{i=1}^{n} x_i^2 \right\}^{\frac{1}{2}}$$

The l_∞ norm is defined as such:

$$||x||_\infty = max_{1 \leq i \leq n} |x_i|$$