

# Discrete HW2 Nguyen

Khoi Nguyen Do

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## 1 Chapter 1.3

### 1.1 Question 2

Let  $C = D = \{-3, -2, -1, 1, 2, 3\}$  and define a relation  $S$  from  $C$  to  $D$  as follows: For all  $(x, y) \in C \times D$ ,  $(x, y) \in S$  means that  $\frac{1}{x} - \frac{1}{y}$  is an integer.

**a . Is  $2 \in S$ ? YES**

Is  $-1 \in S$ ? **YES**

Is  $(3, 3) \in S$ ? **YES**

Is  $(3, -3) \in S$ ? **NO**

**b . Write  $S$  as a set of ordered pairs.**

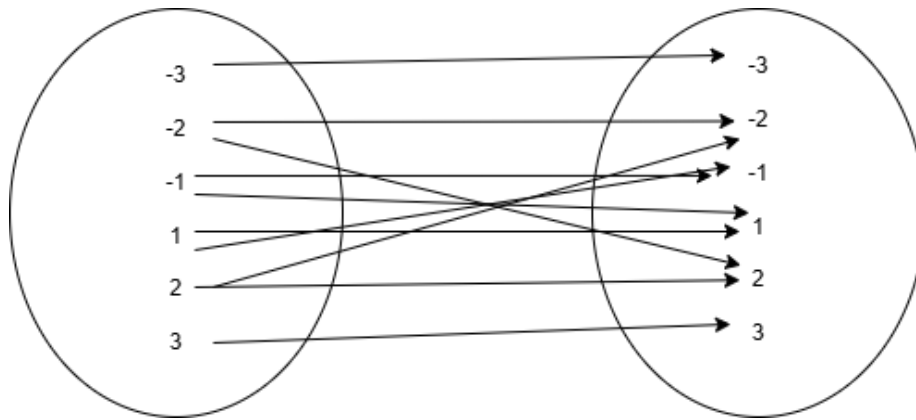
$$S = \{(-3, -3), (-2, -2), (-2, 2), (-1, -1), (-1, 1), (1, -1), (1, 1), (2, -2), (2, 2), (3, 3)\}$$

**c .**

Domain of  $S$ :  $\{-3, -2, -1, 1, 2\}$

Co-domain of  $S$ :  $\{-3, -2, -1, 1, 2\}$

**d . Draw an arrow diagram for  $S$ .**



## 1.2 Question 4

. Let  $G = \{-2, 0, 2\}$  and  $H = \{4, 6, 8\}$  and define a relation  $V$  from  $G$  to  $H$  as follows: For all  $(x, y) \in G \times H$ ,  $(x, y) \in V$  means that  $\frac{x-y}{4}$  is an integer.

a. Is  $2V6$ ? **YES**

Is  $(-2)V(-6)$ ? **YES**

Is  $(0, 6) \in V$ ? **NO**

Is  $(2, 4) \in V$ ? **NO**

b. Write  $V$  as a set of ordered pairs.

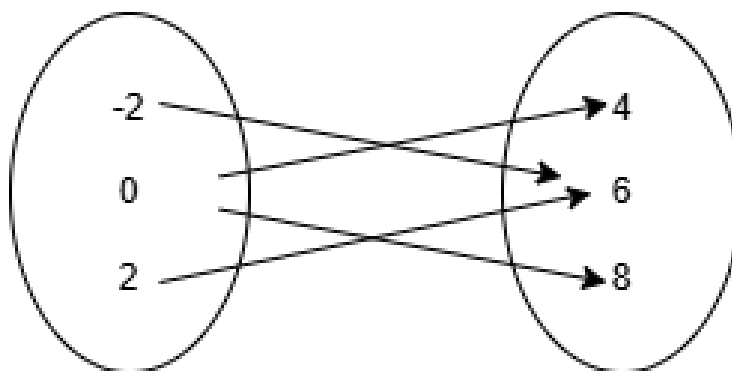
$$V = \{(-2, 6), (0, 4), (0, 8), (2, 6)\}$$

c. Write the domain and co-domain of  $V$ .

Domain of  $V$ :  $\{-2, 0, 2\}$

Co-domain of  $V$ :  $\{4, 6, 8\}$

d. Draw an arrow diagram for  $V$ .



### 1.3 Question 10

Find four relations from  $\{a, b\}$  to  $\{x, y\}$  that are not functions from  $\{a, b\}$  to  $\{x, y\}$ .

Four relations:

$$R1 = \{(a, x), (a, y)\}$$

$$R2 = \{(b, x), (b, y)\}$$

$$R3 = \{(a, x), (a, y), (b, x)\}$$

$$R4 = \{(a, x), (a, y), (b, y)\}$$

### 1.4 Question 12

Define a relation  $T$  from  $\mathbb{R}$  to  $\mathbb{R}$  as follows: For all real numbers  $x$  and  $y$ ,  $(x, y) \in T$  means that  $y^2 - x^2 = 1$ .

$$T = \{(x, y) \in \mathbb{R} \times \mathbb{R} : y^2 - x^2 = 1\}$$

### 1.5 Question 14

Let  $C = \{1, 2, 3, 4\}$  and  $D = \{a, b, c, d\}$ .

Define a function  $G: C \rightarrow D$  by the following arrow diagram: (image of a diagram)

a. Write the domain and co-domain of  $G$ .

Domain  $G: \{1, 2, 3, 4\}$

Co-domain  $G: \{c\}$

b. Find  $G(1)$ ,  $G(2)$ ,  $G(3)$ , and  $G(4)$ .

$G(1): c$

$G(2): c$

$G(3): c$

$G(4): c$

## 1.6 Question 20

Define functions  $H$  and  $K$  from  $\mathbb{R}$  to  $\mathbb{R}$  by the following formulas: For all  $x \in \mathbb{R}$ ,  
 $H(x) = (x - 2)^2$  and  $K(x) = (x - 1)(x - 3) + 1$ .  
Does  $H = K$ ? Explain.

*Proof.* After some expansion, we have:  $(x - 2)^2 = x^2 - 4x + 4$  and  
 $(x - 1)(x - 3) + 1 = x^2 - 4x + 4$

Looking at the expanded version, we can see that:

- All the coefficients are equal.
- All the Constants are equal.

Therefore,  $H(x) = K(x)$  for all  $x \in \mathbb{R}$

□