CSC231 Homework 1

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Homework 1/Problem 2

Prove the following lemma by induction:

$2 \cdot n + 1 \leq 2^n \ \forall n \geq 5, n \in \mathbb{N}$

Proof.

Base case

Let n = 5. Then $2 \cdot 5 + 1 = 11 \leq 2^5 = 32$.

Thus the statement holds for n = 5

Inductive Hypothesis

Suppose $2 \cdot n + 1 \leq 2^n \ \forall n \geq 5, n \in \mathbb{N}$ is true for some n = k, we have:

$$2 \cdot k + 1 \leq 2^k$$

Inductive Step

We will show that $2 \cdot (k + 1) + 1 \leq 2^{k+1}$.

By the inductive hypothesis, $2 \cdot k + 1 \leq 2^k$.

Therefore, $2 \cdot (k + 1) + 1 \leq 2^k + 2$. (1)

Since $k \geq 5$, we know that $2^k \geq 2$ (as $2^5 = 32$). (2)

Using the two inequalities (1) and (2), we get

$$2 \cdot (k+1) + 1 \leq 2^k + 2 \leq 2^k + 2^k = 2^{k+1}$$

Thus the statement holds for all n = k

Conclusion

By the principle of mathematical induction, the statement $2 \cdot n + 1 \leq 2^n$ holds true for all integers $n \geq 5$. '

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