


PRACTICE MIDTERM

MA1521 CALCULUS FOR COMPUTING

Time allowed: 1 hour 15 mins.

Answer all 7 questions. Each question carries 10 marks.

1. Let a be a positive integer. Given $\lim_{x \rightarrow \infty} \left(\frac{2x-3}{2x+5} \right)^{2x+1} = \frac{1}{e^a}$, determine the value of a .
2. Let k be a positive number. The area of the region bounded by the curves $y = x^3 - x$ and $y = kx$ is 72. Determine the value of k .
3. An athlete leaves a given point O and runs north at 15 km per hour. One hour later, a car leaves the point O and travels east at 40 km per hour. At what rate in km per hour is the distance between the athlete and the car changing at the instant the car has been traveling for 1 hour?
4. Durian trees grown in Malaysia produce 120 durians per tree per year if no more than 20 tree are planted per acre. For each additional tree planted per acre, the yield per tree per year decreases by 4 durians. Assuming that the maximum capacity per acre is 40 durian trees, how many trees per acre should be planted to produce the greatest number of durians per acre per year? Your answer should be a positive integer.
5. The curve $\pi^2 x^2 = y(2-y)^3$ has a shape like a teardrop . Find the area of the region enclosed by the curve.
6. Let $f(x) = \int_e^x \frac{1}{\ln t} dt$. Show that f^{-1} exists by proving that f is increasing on $(1, \infty)$. Find also the value of $(f^{-1})'(0)$.
7. Let $p > 1$ be an integer. Suppose $\int_e^\infty \frac{dx}{x(\ln x)^p} = \frac{1}{6}$. Determine p .

Answers: 1. 8, 2. 11, 3. 41, 4. 25, 5. 1, 6. 1, 7. 7.