1. B.

No. of undergraduates in the treatment group $= 300 \times 0.7 = 210$;
No. of postgraduate students in the control group $= 700 \times 0.3 = 210$.

2. A.
Let

No. of men drinking wine = a;
No. of men not drinking wine = b;
No. of women drinking wine = c;
No. of women not drinking wine = d.

So we have

$$\text{odds of drinking wine among men} = \frac{\text{no. of men drinking wine}}{\text{no. men not drinking wine}} = \frac{a}{b},$$

$$\text{odds of drinking wine among women} = \frac{\text{no. of women drinking wine}}{\text{no. women not drinking wine}} = \frac{c}{d},$$

and

$$\frac{a}{b} = 36 \times \frac{c}{d}.$$

This implies

$$\frac{ad}{bc} = 36 > 1.$$

Since $ad > bc$, we have

$$\begin{aligned}\text{risk ratio of drinking wine (women as baseline)} &= \frac{\text{risk of drinking wine among men}}{\text{risk of drinking wine among women}} \\ &= \frac{\frac{a}{a+b}}{\frac{c}{c+d}} \\ &= \frac{ac+ad}{ac+bc} > 1.\end{aligned}$$

Thus, the risk of drinking wine among men is higher than the risk of drinking wine among women.

3. D.
The target population is all undergraduate students in Singapore but the sampling frame is all undergraduate students in National University of Singapore. Since sampling frame is smaller than target population, sample is not representative of the target population.

4. C.
The sampling frame is all readers of the newspaper who have participated in this particular poll. So results can be generalised to the sampling frame.

5. D.
The sampling scheme used is not systematic sampling. The first student is not selected randomly and time interval between selected students is likely not consistent since students may enter at different times after each 5 minute mark. Moreover, interviewing students at entrances involves the judgement of interviewers (when more than one student enter at the same time).

6. B.
Statement (ii) tells us the response rate is likely to be around 20%. Sample is biased when the response rate is low.

7. D.
We are assuming ethnicity does not affect result of the study. Since this is stratified sampling, the sample is representative of Singapore population. As there are more Chinese than Malay in the population and only 200 respondents are selected in each ethnic group, the chance a Chinese individual is selected is smaller than that a Malay individual is selected.

8. D.
P-value is the probability of obtaining an outcome that is equivalent to or more extreme than the observed outcome assuming the null hypothesis is true. A large p-value does not provide any evidence to reject or accept the null hypothesis.

9. A.
To receive an A grade, one has to answer $60 \times 0.95 = 57$ or more questions correctly. Since 54 questions have been answered correctly, one only has to answer 3 or more questions correctly to receive an A. We have

$$\begin{aligned} \text{P(not receiving an A grade)} &= \text{P(0 out of 6 questions answered correctly)} \\ &\quad + \text{P(1 out of 6 questions answered correctly)} \\ &\quad + \text{P(2 out of 6 questions answered correctly)} \\ &= \left(\frac{3}{4}\right)^6 + \binom{6}{1}\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^5 + \binom{6}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^4 \\ &= 0.8306. \end{aligned}$$

Thus,

$$\begin{aligned} \text{P(receiving an A grade)} &= 1 - \text{P(not receiving an A grade)} \\ &= 1 - 0.8306 \\ &= 0.169. \end{aligned}$$

10. B.
The biased coin has probability of $1-p$ of getting tails. We have

$$\begin{aligned}
a &= \binom{2}{1} p(1-p) = 2\left[p(1-p)\right]; \\
b &= \binom{3}{1} p^2(1-p) = 3p\left[p(1-p)\right]; \\
c &= \binom{3}{2} p(1-p)^2 = 3(1-p)\left[p(1-p)\right].
\end{aligned}$$

Note that it suffices to compare 2, $3p$ and $3(1-p)$. Since $0.4 < p < 0.5$, we have

$$1.2 < 3p < 1.5 < 3(1-p) < 1.8 < 2$$

Thus, $b < c < a$.

11. D.
Method 1
From the question, we have

$$\begin{aligned}
&\mathrm{P}\left(\text{brown eyes}\right) = 0.3; \\
&\mathrm{P}\left(\text{blue eyes}\right) = 0.2; \\
&\mathrm{P}\left(\text{other color eyes}\right) = 0.5; \\
&\mathrm{P}\left(\text{brown eyes} \mid \text{in the game}\right) = 0.45; \\
&\mathrm{P}\left(\text{blue eyes} \mid \text{in the game}\right) = 0.20; \\
&\mathrm{P}\left(\text{other color eyes} \mid \text{in the game}\right) = 0.35.
\end{aligned}$$

Hence,

$$\begin{aligned}
\mathrm{P}\left(\text{blue eyes} \mid \text{not in the game}\right) &= \frac{\mathrm{P}(\text{blue eyes and not in the game})}{\mathrm{P}(\text{not in the game})} \\
&= \frac{\mathrm{P}(\text{blue eyes}) - \mathrm{P}(\text{blue eyes and in the game})}{1 - \mathrm{P}(\text{in the game})} \\
&= \frac{\mathrm{P}(\text{blue eyes}) - \mathrm{P}(\text{blue eyes} \mid \text{in the game})\,\mathrm{P}(\text{in the game})}{1 - \mathrm{P}(\text{in the game})} \\
&= \frac{0.2 - 0.2\,\mathrm{P}(\text{in the game})}{1 - \mathrm{P}(\text{in the game})} \\
&= 0.2
\end{aligned}$$

Method 2
We know that

$$\mathrm{P}(\text{blue eyes}) = a\,\mathrm{P}(\text{blue eyes} \mid \text{in the game}) + b\,\mathrm{P}(\text{blue eyes} \mid \text{not in the game}) \tag{1}$$

where $0 < a < 1$ and $0 < b < 1$ such that $a+b = 1$. Since $\mathrm{P}(\text{blue eyes}) = \mathrm{P}(\text{blue eyes}|\text{in the game}) = 0.2$, equation (1) holds if and only if $\mathrm{P}(\text{blue eyes} \mid \text{not in the game}) = 0.2$.

12. C.
Let $\Omega =$ average amount of time for 1 call.

$$\begin{aligned}\Omega &= (0.2 \times 0.4)(150) + (0.2 \times 0.6)(60) + (0.4 \times 0.8)(60) + (0.8 \times 0.6)(30) \\ &= 12 + 7.2 + 19.2 + 14.4 \\ &= 52.8.\end{aligned}$$

So we have

$$\text{average amount of time for 45 calls} = 45\Omega = 45 \times 52.8 = 2376.$$

13. C.
We assume population size is 100000. Using information from question, we have the following contingency table

	Positive	Negative	Row total
Disease	980	20	1000
No disease	4950	94050	99000
Column total	5930	94070	100000

Thus,

$$\mathrm{P}(\text{disease} \mid \text{positive}) = \frac{980}{5930} = 0.165.$$

14. C.
From question 13, we have

$$\begin{aligned}&\mathrm{P}(\text{disease}) = 0.01; \\ &\mathrm{P}(\text{no disease}) = 1 - \mathrm{P}(\text{disease}) = 0.99; \\ &\mathrm{P}(\text{positive} \mid \text{disease}) = 0.98; \\ &\mathrm{P}(\text{negative} \mid \text{no disease}) = 0.95; \\ &\mathrm{P}(\text{positive} \mid \text{no disease}) = 1 - \mathrm{P}(\text{negative} \mid \text{no disease}) = 0.05;\end{aligned}$$

Since outcomes of each test are independent regardless of Patrick's disease status,

$$\begin{aligned}&\mathrm{P}(\text{two positives} \mid \text{disease}) = \mathrm{P}(\text{positive} \mid \text{disease}) \times \mathrm{P}(\text{positive} \mid \text{disease}), \\ &\mathrm{P}(\text{two positives} \mid \text{no disease}) = \mathrm{P}(\text{positive} \mid \text{no disease}) \times \mathrm{P}(\text{positive} \mid \text{no disease}).\end{aligned}$$

Hence,

$$\begin{aligned}
&\mathrm{P}\left(\text{disease} \mid \text{two positives}\right)\\
&= \frac{\mathrm{P}\left(\text{two positives and disease}\right)}{\mathrm{P}\left(\text{two positives}\right)}\\
&= \frac{\mathrm{P}\left(\text{two positives} \mid \text{disease}\right) \times \mathrm{P}\left(\text{disease}\right)}{\mathrm{P}\left(\text{two positives and disease}\right) + \mathrm{P}\left(\text{two positives and no disease}\right)}\\
&= \frac{\mathrm{P}\left(\text{two positives} \mid \text{disease}\right) \times \mathrm{P}\left(\text{disease}\right)}{\left[\mathrm{P}\left(\text{two positives} \mid \text{disease}\right) \times \mathrm{P}\left(\text{disease}\right)\right] + \left[\mathrm{P}\left(\text{two positives} \mid \text{no disease}\right) \times \mathrm{P}\left(\text{no disease}\right)\right]}\\
&= \frac{(0.98)^2 \times 0.01}{\left[(0.98)^2 \times 0.01\right] + \left[(0.05)^2 \times 0.99\right]}\\
&= 0.795
\end{aligned}$$

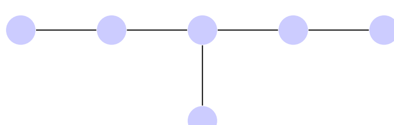
15. C.
Note that

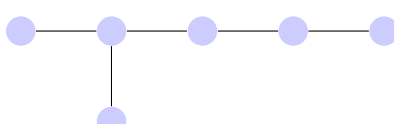
$$\begin{aligned}
\deg\left(\text{Freddie}\right) &= \mathrm{Dcen}\left(\text{Freddie}\right) \times (n-1) = 0.537 \times 54 = 29\\
\deg\left(\text{Gavin}\right) &= \mathrm{Dcen}\left(\text{Gavin}\right) \times (n-1) = 0.611 \times 54 = 33
\end{aligned}$$

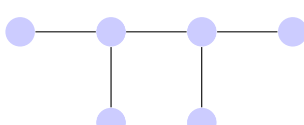
There are 53 vertices in total excluding Freddie and Gavin. Suppose Freddie and Gavin are not adjacent, then Freddie is adjacent to 29 vertices and Gavin is adjacent to 33 vertices. So smallest N is 33 - (53 - 29) = 9. Suppose Freddie and Gavin are adjacent, then Freddie is adjacent to 28 other vertices excluding Gavin and Gavin is adjacent to 32 other vertices excluding Freddie. Hence, smallest N is 32 - (53 - 28) = 7. Minimum value N can take is 7.

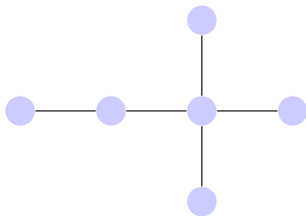
16. A.
The smallest possible size of a connected graph of order 6 is 5. There are 6 possible graphs of order 6 and size 5:

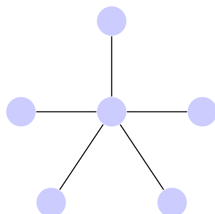
1)

2) 

3) 

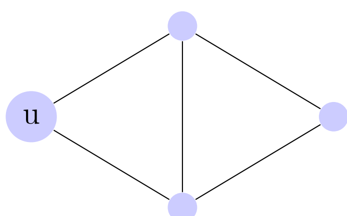
4) 

5) 

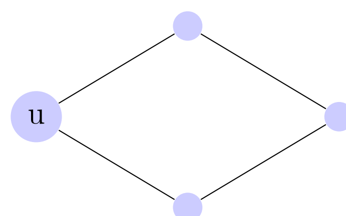
6) 

Hence, the smallest closeness centrality measure is 1.

17. B.
There are two possible networks:



or



The smallest closeness centrality measure for the two networks above are different.

18. B.
Since student D is not adjacent to student E and both students are adjacent to student A, one of the shortest paths between student D and student E passes through student A. So Bcen (A) > 0. Since student B is adjacent to student C, no shortest paths between student B and student C passes through student A. Hence, Bcen (A) < 1.

19. C.
Let S and T be any two distinct students in class excluding H. Since d(S,T) = d(S,H) + d(H,T), one of the shortest paths between S and T passes through H. Thus, Bcen (H) > 0. Next, we show that for any student S, d(S,H) = 1. Suppose d(S,H) > 1, then there exists a student, M, in the shortest path between S and H such that d(S,M) + d(M,H) = d(S,H). This leads to d(S,M) = d(S,H) − d(M,H) $<$ d(S,H) + d(H,M) which contradicts the condition given in question. So for any two students, S and T, d(S,H) and d(H,T) is always 1, and d(S,T) is always 2. Thus, all 42 students (excluding H) are only adjacent to H, and the network is a star graph.

20. A.
Since actor A and actor C acted in only one movie "Frank" and actor B is also in this movie, bacon number of B is at most 2 and degree of actor B is larger or equal to degree of actor

C. Thus, (i) and (ii) are false. All vertices (other than actor B) adjacent to actor A are also adjacent to actor B, so closeness centrality measure of actor B may be smaller than the closeness centrality measure of actor A which means (iii) is not necessarily true. Actor A and actor C are interchangeable (and symmetrical) in the movie graph since they both only acted in one movie "Frank", hence betweenness centrality measure of actor A is the same as the betweenness centrality measure of actor C. Only (iv) is true.

21. C.
Bacon number of actor C is now 1 only when an edge is added between actor C and Kevin Bacon. However, this is not possible since Kevin Bacon is not in the movie Happiness. Bacon number of actor C is now 2 only when an edge is added between actor C and another actor/actress with Bacon number 1. This is again not possible since all the actors/actresses who acted in Happiness have initial Bacon numbers of at least 2.

22. C.
23. C.
24. C.
25. D.
26. A.
27. B.
28. D.