CS4226
Midterm Review

Stevan Gerard Gunawan

Muhammad Reyaaz

School of Computing

National University of Singapore

Priority Queueing

- flows f1 and f2 going through a link, modeled by a single-server queueing system with an infinity queue.
- Poisson arrival with rate λ_1 and λ_2, exponential service rate μ. f1 has a higher priority:

1. if there are packets from flow f1 in the system, served first in a FIFO manner,
2. if no packets from flow f1, then packets from flow f2 are served in a FIFO manner, and
3. if a packet from f1 arrives when a packet from f2 is being served, the server will stop processing f2's packet immediately and process the packets from f1; the server will resume to the unfinished packet of f2 after all packets from f1 are served.

Questions

- If $\lambda_1 = 1, \lambda_2 = 2$ and $\mu = 5$,
  1. what is the average queuing time $\mathbf{E}[Q_1]$ for all the packets from flow f1?
  2. what is the average sojourn time $\mathbf{E}[W]$ for all packets from both flows?
  3. what is the average sojourn time $\mathbf{E}[W_2]$ for all the packets from flow f2?

Question 1

- If $\lambda_1 = 1, \lambda_2 = 2$ and $\mu = 5$,
  - What is the average queuing time $\mathbf{E}[Q_1]$ for all the packets from flow f1?
  - $\mathbf{E}[Q_1] = \mathbf{E}[W_1] - \mathbf{E}[S_1] = \frac{1}{\mu - \lambda_1} - \frac{1}{\mu} = 0.25 - 0.2 = 0.05$

Question 2

- If $\lambda_1 = 1, \lambda_2 = 2$ and $\mu = 5$,
  - What is the average sojourn time $\mathbf{E}[W]$ for all packets from both flows?
  - Think about the aggregate flow $\lambda = \lambda_1 + \lambda_2$
  - $\mathbf{E}[W] = \frac{\mathbf{E}[L]}{\lambda} = \frac{\mathbf{1}}{\lambda} . \frac{\rho}{1-\rho} = \frac{1}{3} . \frac{3/5}{2/5} = \frac{1}{3} . \frac{3}{2} = \frac{1}{2}$
  - Why can we consider aggregate flow?
    - Think about a type of conservation law

Question 3

- If $\lambda_1 = 1, \lambda_2 = 2$ and $\mu = 5$,
  - what is the average sojourn time $\mathbf{E}[W_2]$ for all the packets from flow f2?
  - $\mathbf{E}[W_1] = \frac{1}{\mu - \lambda_1} = \frac{1}{5-1} = \frac{1}{4}$
  - Already know $\mathbf{E}[W] = \frac{1}{2}$
  - $\mathbf{E}[W] = \frac{\lambda_1}{\lambda}\mathbf{E}[W_1] + \frac{\lambda_2}{\lambda}\mathbf{E}[W_2]$
  - $\mathbf{E}[W] = \frac{1}{3} . \frac{1}{4} + \frac{2}{3} . \mathbf{E}[W_2]$
  - $\frac{1}{2} = \frac{1}{3} . \frac{1}{4} + \frac{2}{3} . \mathbf{E}[W_2]$
  - $\mathbf{E}[W_2] = 0.625$

Question 4

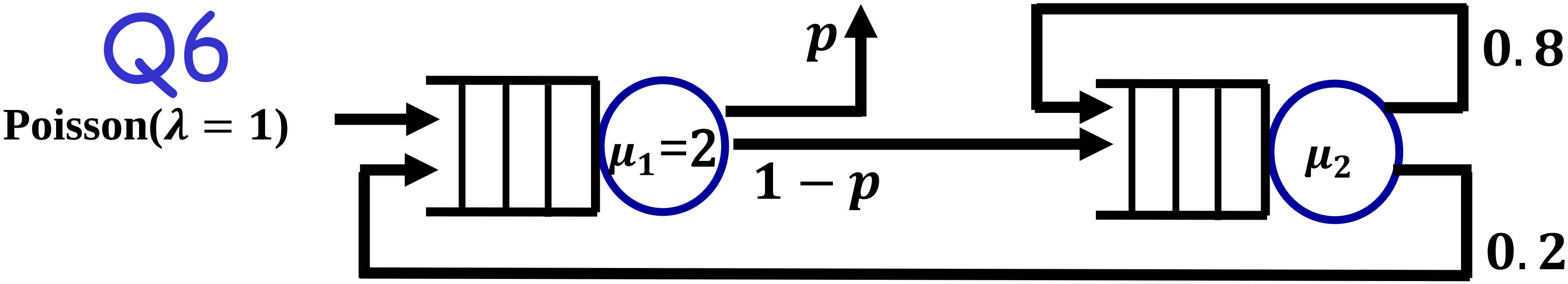
- Consider a variation of the M/M/1 model where there are two servers serving a single infinity-sized queue. The service times of the two servers are IID exponential random variables. The average service times of the two servers are $E[S_1]$ = 1 second and $E[S_2]$ = 4 seconds, respectively. Supppose when you make a random observation at the system and find that both servers are busy. How long (in units of seconds) on average do you need to wait until you see a customer is fully served by a server, i.e., a customer's departure from one of the servers?

  - Remember that we can merge Poisson process, similar to our bus example in lecture.
  - $E[S_1] = 1$, $E[S_2] = 4$, $\mu_1 = \frac{1}{1} = 1, \mu_2 = \frac{1}{4}$
  - $\frac{1}{\mu_1+\mu_2} = \frac{1}{1+1/4} = \frac{4}{5} = 0.8$

Question 5

- Consider a variation of the M/M/1 model where there are two servers serving a single infinity-sized queue. The service times of the two servers are IID exponential random variables. The average service times of the two servers are $E[S_1]$ = 1 second and $E[S_2]$] = 4 seconds, respectively. Supppose when you make a random observation at the system and find that both servers are busy. What is the probability that the customer from server 1 complete the service first?
  - Remember that we can merge Poisson process, similar to our bus example in lecture.
  - $\frac{\mu_1}{\mu_1+\mu_2} = \frac{1}{1+1/4} = \frac{4}{5} = 0.8$

Q6

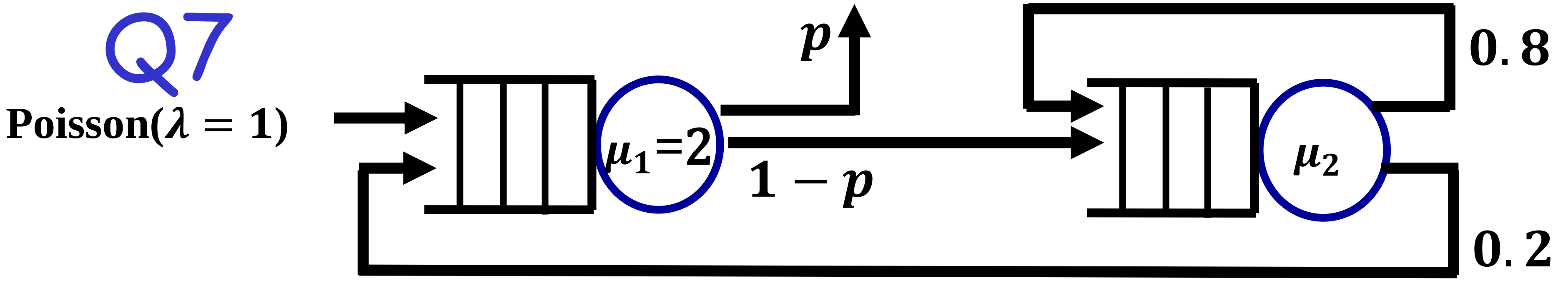


- When p=0.8 and μ 2 is large enough to ensure the system stability, which of the following equals the effective arrival rate λ_1 to the first server?

$$\begin{cases} \lambda_1 = \lambda + 0.2 * \lambda_2 \\ \lambda_2 = (1-p) * \lambda_1 + 0.8 * \lambda_2 \end{cases} \Rightarrow \begin{cases} \lambda_1 = \lambda / p \\ \lambda_2 = \dfrac{1-p}{0.2 * p} \lambda \end{cases}$$

$$\lambda_1 = \frac{\lambda}{p} = \frac{1}{0.8} = 1.25$$

Q7



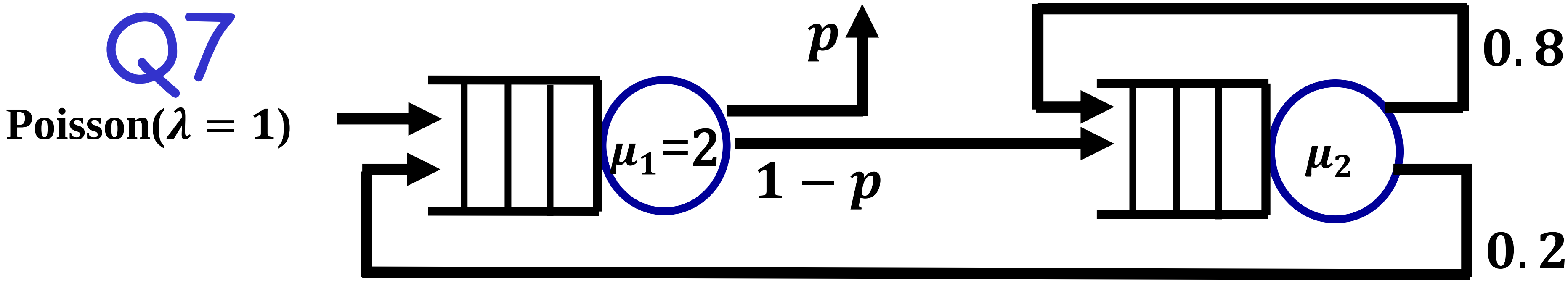
- To guarantee the stability of the Jackson network, what conditions do we need?

$$\begin{cases} \lambda_1 = \lambda + 0.2 * \lambda_2 \\ \lambda_2 = (1-p) * \lambda_1 + 0.8 * \lambda_2 \end{cases} \Rightarrow \begin{cases} \lambda_1 = \lambda / p \\ \lambda_2 = \dfrac{1-p}{0.2 * p} \lambda \end{cases}$$

$$\lambda_1 = \lambda / p < \mu_1 = 2 \Rightarrow p > 1/2$$

$$\lambda_2 = \frac{1-p}{0.2 * p} \lambda < \mu_2 \Rightarrow \mu_2 > \frac{5 * (1-p)}{p}$$

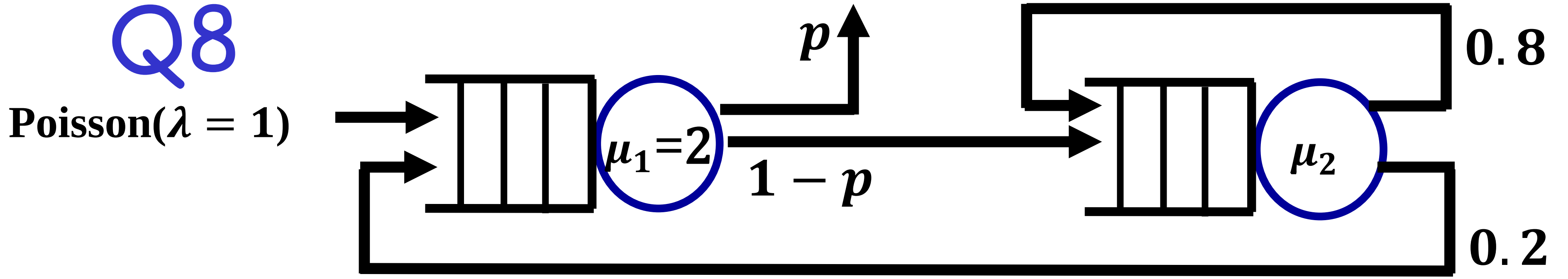
Q7



- When p=0.8, which of the following equals the maximum service rate of the second server μ_2 such that the system is still unstable?

$$\mu_2 = \frac{5*(1-p)}{p} = \frac{5*(1-0.8)}{0.8} = 1.25$$

Q8



- When μ_2 =3, which of the following equals the maximum value of p which will still make the system unstable?

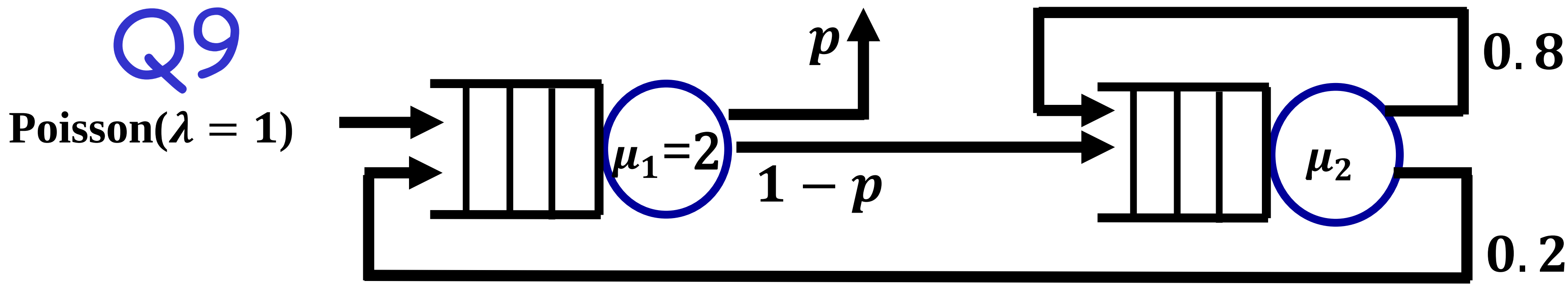
$$\lambda_1 = \lambda/p < \mu_1 = 2 \Rightarrow p > 1/2$$

$$\lambda_2 = \frac{1-p}{0.2*p}\lambda < \mu_2 \Rightarrow \mu_2 > \frac{5*(1-p)}{p}$$

$$3 > \frac{5*(1-p)}{p}, p > \frac{5}{8}$$

$$p > \frac{5}{8} \cap \mathrm{p} > \frac{1}{2} = \mathrm{p} > \frac{5}{8}$$

Q9



- When μ_2 =15, which of the following equals the maximum value of p which will still make the system unstable?

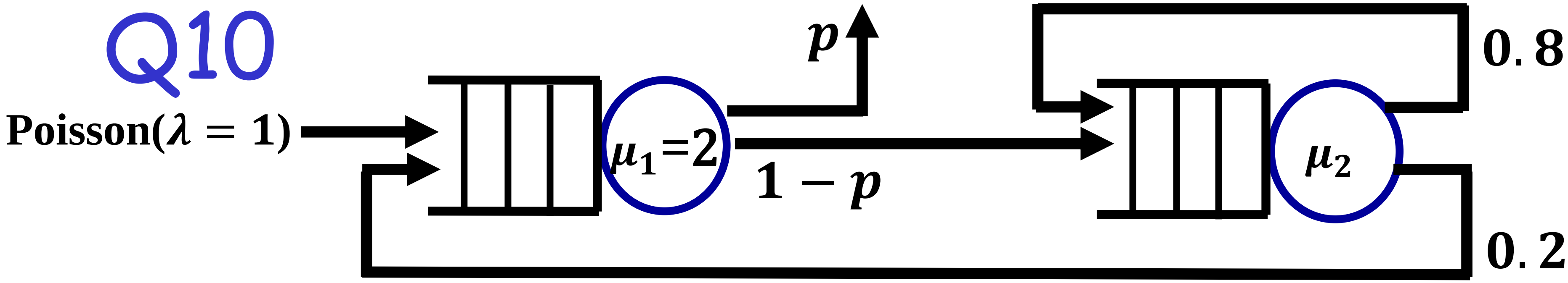
$$\lambda_1 = \lambda/p < \mu_1 = 2 \Rightarrow p > 1/2$$

$$\lambda_2 = \frac{1-p}{0.2 * p}\lambda < \mu_2 \Rightarrow \mu_2 > \frac{5*(1-p)}{p}$$

$$15 > \frac{5*(1-p)}{p}, p > \frac{1}{4}$$

$$p > \frac{1}{4} \cap \mathrm{p} > \frac{1}{2} = \mathrm{p} > \frac{1}{2}$$

Q10



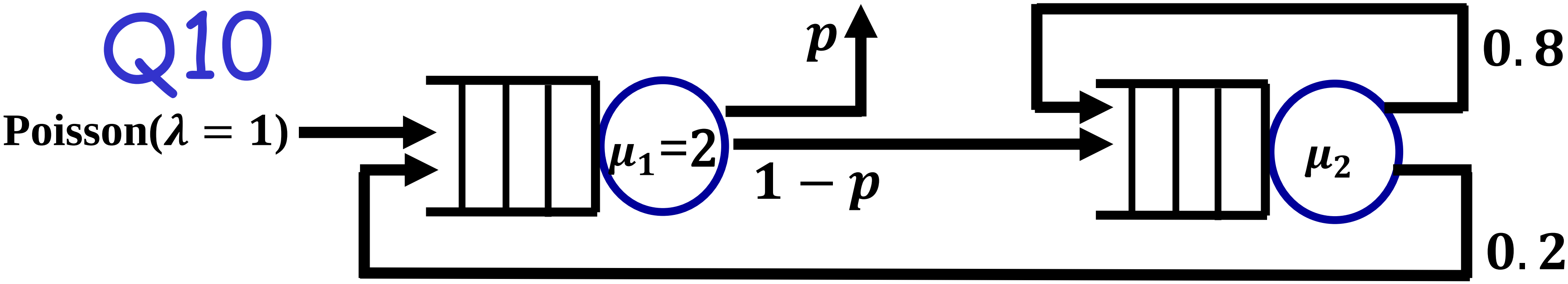
- Determine the mean sojourn time of the jobs $\mathbf{E}[W]$ as a function of μ_2 and p.

$$\mathbf{E}[W] = \frac{\mathbf{E}[L]}{\lambda}, \mathbf{E}[L] = \mathbf{E}[L_1] + \mathbf{E}[L_2]$$

$$\mathbf{E}[L_1] = \frac{\rho_1}{1-\rho_1} = \frac{\lambda_1}{\mu_1 - \lambda_1} = \frac{1/p}{2 - 1/p} = \frac{1}{2p-1}$$

$$\mathbf{E}[L_2] = \frac{\rho_2}{1-\rho_2} = \frac{\lambda_2}{\mu_2 - \lambda_2} = \frac{\frac{1-p}{0.2*p}}{\mu_2 - \frac{1-p}{0.2*p}}$$

Q10

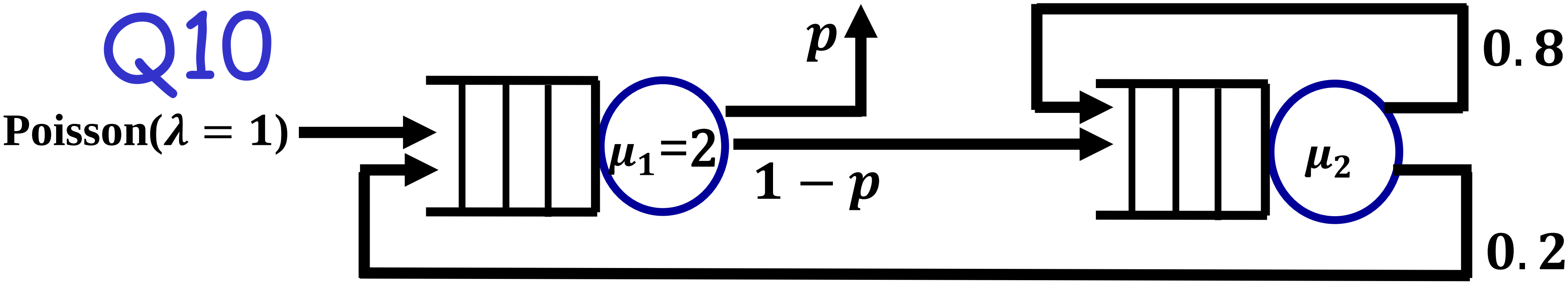


- When p=0.6 and μ_2 =10, which of the following equals the average sojourn time E[W] of the packets?

$$\mathbf{E}[L_1] = \frac{\rho_1}{1-\rho_1} = \frac{\lambda_1}{\mu_1 - \lambda_1} = \frac{1/p}{2 - 1/p} = \frac{1}{2p-1}$$

$$\mathbf{E}[L_1] = \frac{1}{2p-1} = \frac{1}{2 * 0.6 - 1} = 5$$

Q10

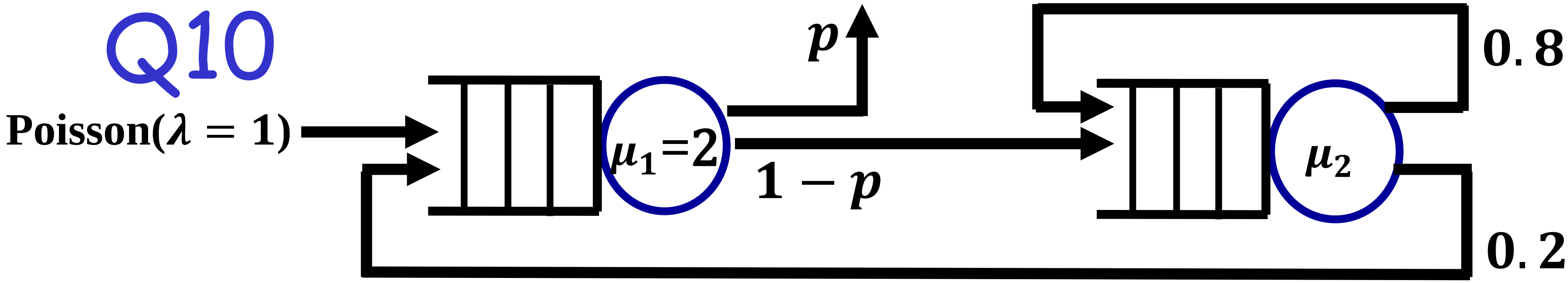


- When p=0.6 and μ_2 =10, which of the following equals the average sojourn time E[W] of the packets?

$$\mathbf{E}[L_2] = \frac{\rho_2}{1-\rho_2} = \frac{\lambda_2}{\mu_2 - \lambda_2} = \frac{\frac{1-p}{0.2*p}}{\mu_2 - \frac{1-p}{0.2*p}}$$

$$\mathbf{E}[L_2] = \frac{1}{2}$$

Q10

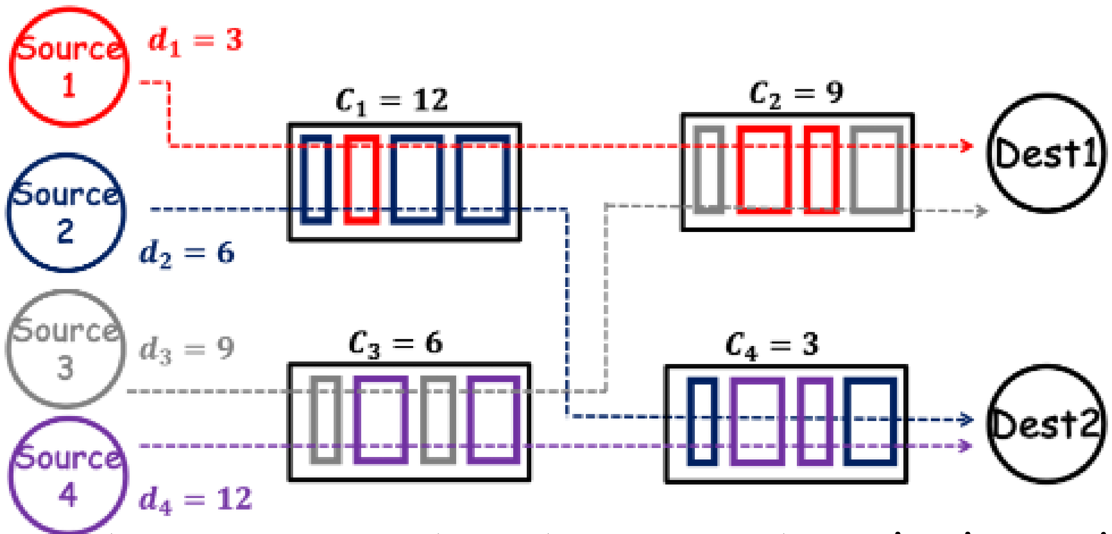


- When p=0.6 and μ_2 =10, which of the following equals the average sojourn time E[W] of the packets?

$$\mathbf{E}[L] = \mathbf{E}[L_1] + \mathbf{E}[L_2] = \frac{1}{2} + 5 = \frac{11}{2}$$

$$\mathbf{E}[W] = \frac{\mathbf{E}[L]}{\lambda} = \mathbf{E}[L] = \frac{11}{2} = 5.5$$

Q11



- When $\varphi = (\varphi 1, \varphi 2, \varphi 3, \varphi 4) = (1,\ 2,\ 3,\ 4)$, calculate the weighted max-min fair allocation for the 4 flows.

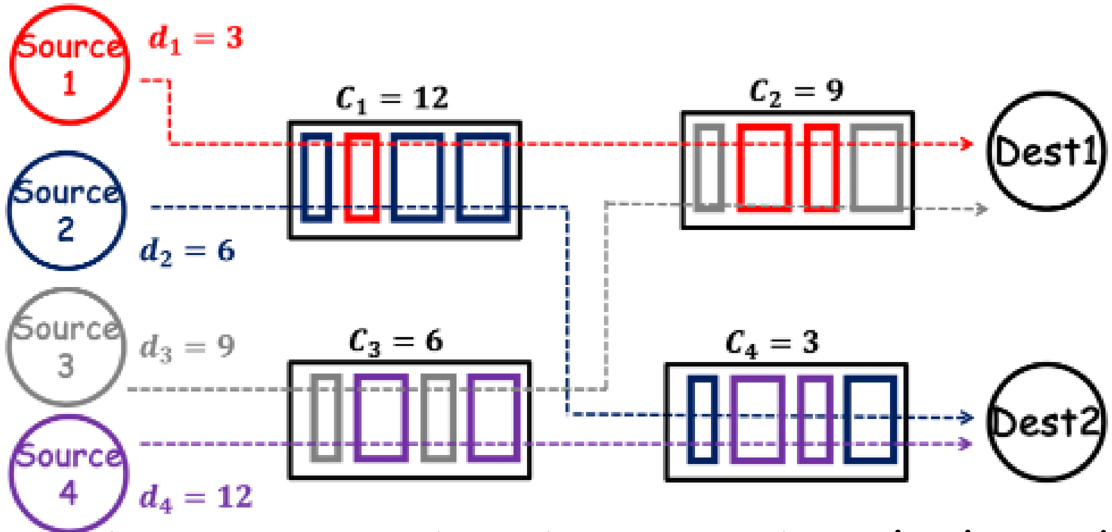
C1		C2		C3		C4	
f1	f2	f1	f3	f3	f4	f2	f4
3	6	9/4	27/4	18/7	24/7	1	2

We can see that C4 will be the most bottlenecking link by comparing the weighted allocations.

f2 = 3*2/(2+4) = 1

f4 = 3*4/(2+4) = 2

Q11



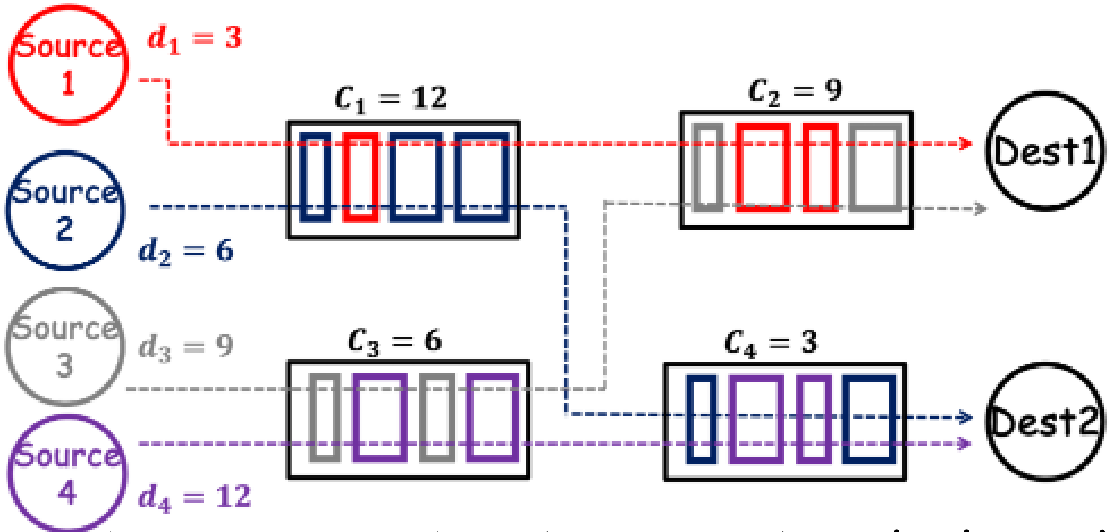
- When $\varphi = (\varphi1, \varphi2, \varphi3, \varphi4) = (1,\ 2,\ 3,\ 4)$, calculate the weighted max-min fair allocation for the 4 flows. Fixing f2 = 1 and f4 = 2

C1		C2		C3		C4	
f1	f2	f1	f3	f3	f4	f2	f4
3	1	9/4	27/4	4	2	1	2

Now our allocation is bottlenecked by C3

f4 = C3 – f4 = 6 – 2 = 4

Q11



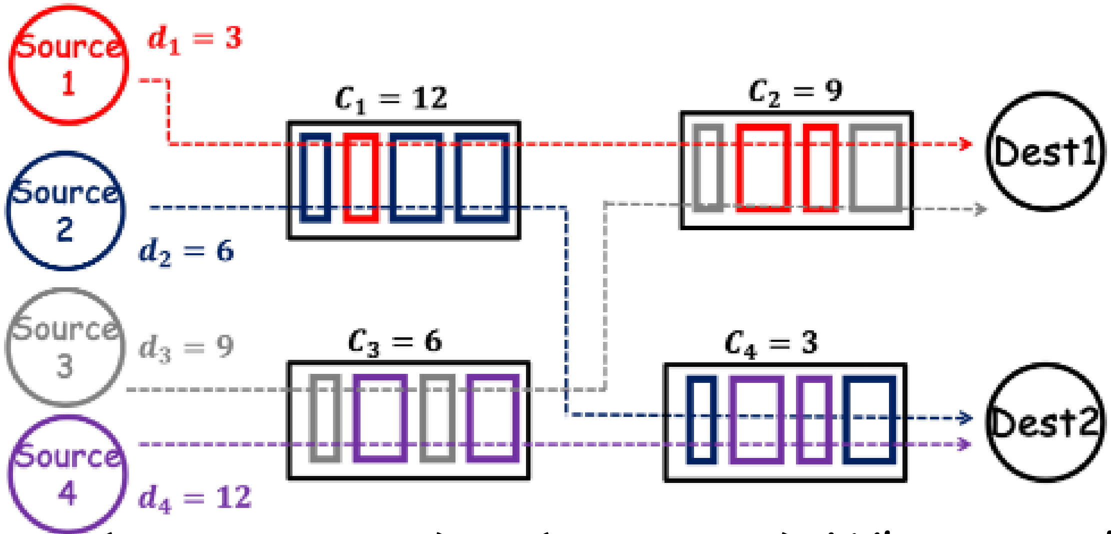
- When $\varphi = (\varphi 1, \varphi 2, \varphi 3, \varphi 4) = (1,\ 2,\ 3,\ 4)$, calculate the weighted max-min fair allocation for the 4 flows. Fixing f2 = 1; f4 = 2; f3 = 4

C1		C2		C3		C4	
f1	f2	f1	f3	f3	f4	f2	f4
3	1	3	4	4	2	1	2

Now our allocation is bottlenecked by C3

f1 = C2 - f3 = 9 - 4 = 5 -> 3 (restricted by its own demand)

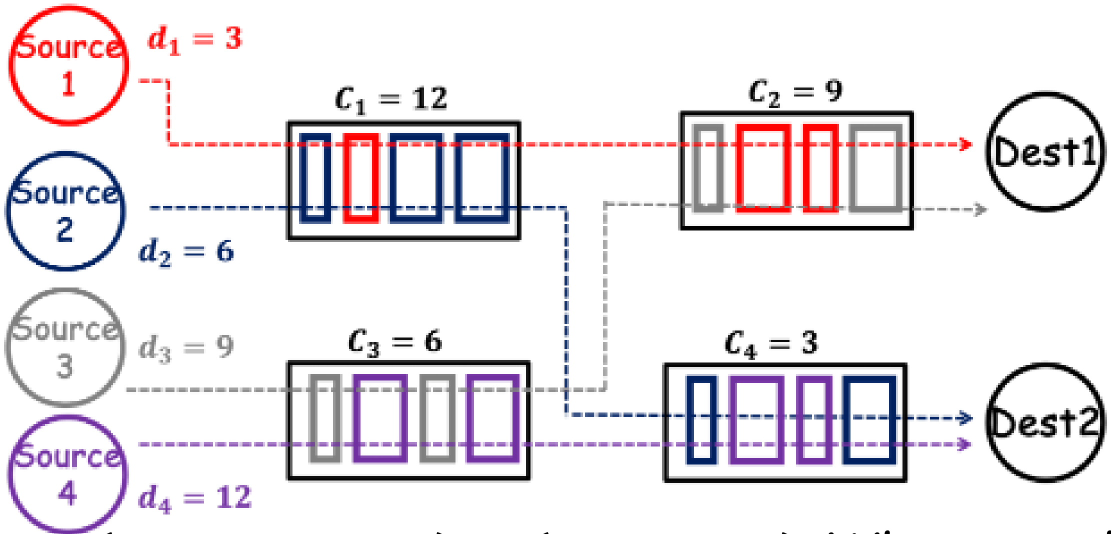
Q12



- When $\varphi = (\varphi 1, \varphi 2, \varphi 3, \varphi 4) = (1,\ 2,\ 3,\ 4)$, *What are the bottleneck links for f1? f = (3, 1, 4, 2)*

f1 use links C1 and C2. Both links not saturated (f1 + f2 = 4 < C1; f1 + f3 =7 < C2). So the answer is None.

Q13



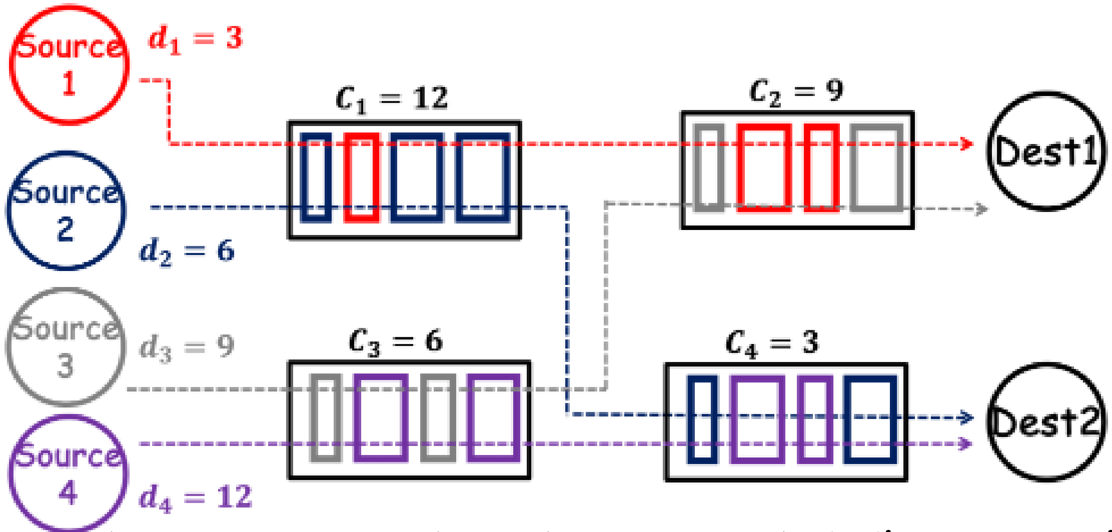
- When $\varphi = (\varphi1, \varphi2, \varphi3, \varphi4) = (1,\ 2,\ 3,\ 4)$, *What are the bottleneck links for f2? f = (3, 1, 4, 2)*

f2 use links C1 and C4. C1 not saturated (f1 + f2 = 4 < C4). C4 is saturated (f2 + f4 = 3 = C4), calculate the normalized allocation. N2 = 1/2, N4 = 2/4 = 1/2. So f2 has maximum normalized rate for C4.

The bottleneck link is C4 only

Ni is the normalized allocation for ith flow.

Q14



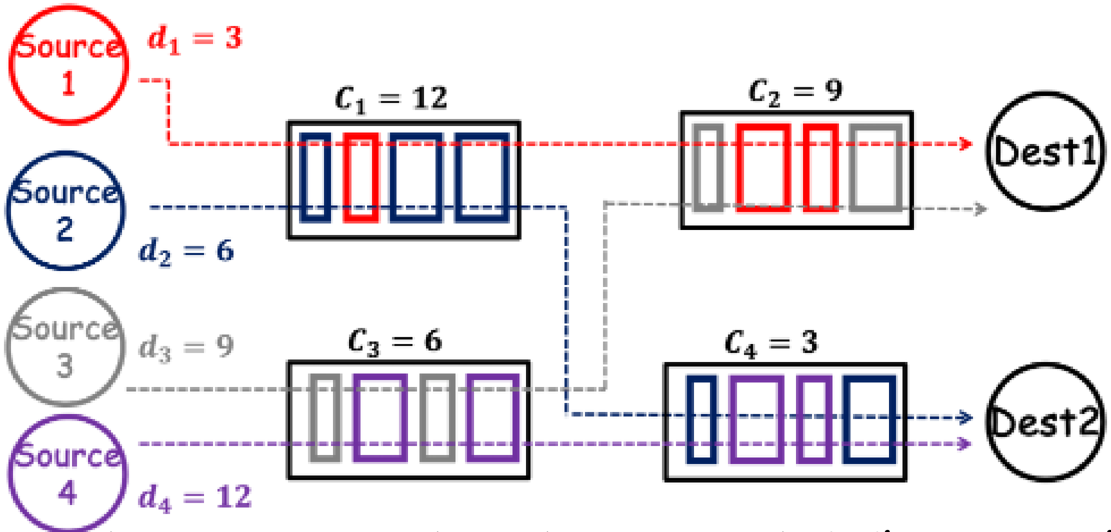
- When $\varphi = (\varphi 1, \varphi 2, \varphi 3, \varphi 4) = (1,\ 2,\ 3,\ 4)$, *What are the bottleneck links for f2? f = (3, 1, 4, 2)*

f3 use links C2 and C3. C2 not saturated (f1 + f3 = 7 < C3). C3 is saturated (f3 + f4 = 6 = C3), calculate the normalized allocation. N3 = 4/3, N4 = 2/4 = 1/2. So f3 has maximum normalized rate for C3.

The bottleneck link is C3 only

Ni is the normalized allocation for ith flow.

Q15



- When $\varphi = (\varphi 1, \varphi 2, \varphi 3, \varphi 4) = (1,\ 2,\ 3,\ 4)$, *What are the bottleneck links for f2? f = (3, 1, 4, 2)*

f4 use links C3 and C4. C3 is saturated (f3 + f4 = 6 = C3). C4 is saturated (f2 + f4 = 3 = C4). Calculate the normalized allocation for both links. N2 = 1/2, N3 = 4/3, N4 = 2/4 = 1/2. So f4 has maximum normalized rate for C4, not for C3 (N4 < N3).

The bottleneck link is C4 only

Ni is the normalized allocation for ith flow.