

## 2. The Logic of Compound Statements (aka Propositional Logic) Summary

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# Summary

## 2. The Logic of Compound Statements

### 2.1 Logical Form and Logical Equivalence

- Statements; Compound Statements; Statement Form (Propositional Form)
- Logical Equivalence; Tautologies and Contradictions

### 2.2 Conditional Statements

- Conditional Statements; If-Then as Or
- Negation, Contrapositive, Converse and Inverse
- Only If and the Biconditional; Necessary and Sufficient Conditions

### 2.3 Valid and Invalid Arguments

- Argument; Valid and Invalid Arguments
- Modus Ponens and Modus Tollens
- Rules of Inference
- Fallacies

Reference: Epp's Chapter 2 The Logic of Compound Statements

# Summary

## 2.1 Logical Form and Logical Equivalence

### Definition 2.1.1 (Statement)

A **statement** (or **proposition**) is a sentence that is true or false, but not both.

### Definition 2.1.2 (Negation)

If  $p$  is a statement variable, the **negation** of  $p$  is “not  $p$ ” or “it is not the case that  $p$ ” and is denoted  $\sim p$ .

### Definition 2.1.3 (Conjunction)

If  $p$  and  $q$  are statement variables, the **conjunction** of  $p$  and  $q$  is “ $p$  and  $q$ ”, denoted  $p \wedge q$ .

### Definition 2.1.4 (Disjunction)

If  $p$  and  $q$  are statement variables, the **disjunction** of  $p$  and  $q$  is “ $p$  or  $q$ ”, denoted  $p \vee q$ .

# Summary

## 2.1 Logical Form and Logical Equivalence

### Definition 2.1.5 (Statement Form/Propositional Form)

A **statement form** (or **propositional form**) is an expression made up of **statement variables** and **logical connectives** that becomes a statement when actual statements are substituted for the component statement variables.

### Definition 2.1.6 (Logical Equivalence)

Two statement forms are called **logically equivalent** if, and only if, they have **identical truth values** for each possible substitution of statements for their statement variables. The logical equivalence of statement forms  $P$  and  $Q$  is denoted by  $P \equiv Q$ .

### Definition 2.1.7 (Tautology)

A **tautology** is a statement form that is **always true** regardless of the truth values of the individual statements substituted for its statement variables. A statement whose form is a tautology is a **tautological statement**.

### Definition 2.1.8 (Contradiction)

A **contradiction** is a statement form that is **always false** regardless of the truth values of the individual statements substituted for its statement variables. A statement whose form is a contradiction is a **contradictory statement**.

# Summary

## 2.1 Logical Form and Logical Equivalence

### Theorem 2.1.1 Logical Equivalences

Given any statement variables  $p$ ,  $q$  and  $r$ , a tautology **true** and a contradiction **false**:

1	Commutative laws	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2	Associative laws	$p \wedge q \wedge r$ $\equiv (p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$p \vee q \vee r$ $\equiv (p \vee q) \vee r \equiv p \vee (q \vee r)$
3	Distributive laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4	Identity laws	$p \wedge \text{true} \equiv p$	$p \vee \text{false} \equiv p$
5	Negation laws	$p \vee \sim p \equiv \text{true}$	$p \wedge \sim p \equiv \text{false}$
6	Double negative law	$\sim(\sim p) \equiv p$	
7	Idempotent laws	$p \wedge p \equiv p$	$p \vee p \equiv p$
8	Universal bound laws	$p \vee \text{true} \equiv \text{true}$	$p \wedge \text{false} \equiv \text{false}$
9	De Morgan's laws	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
10	Absorption laws	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11	Negation of <b>true</b> and <b>false</b>	$\sim \text{true} \equiv \text{false}$	$\sim \text{false} \equiv \text{true}$

# Summary

## 2.2 Conditional Statements

### Definition 2.2.1 (Conditional)

If  $p$  and  $q$  are statement variables, the **conditional** of  $q$  by  $p$  is “if  $p$  then  $q$ ” or “ $p$  implies  $q$ ”, denoted  $p \rightarrow q$ .

It is false when  $p$  is true and  $q$  is false; otherwise it is true.

We called  $p$  the **hypothesis** (or **antecedent**) and  $q$  the **conclusion** (or **consequent**).

### Definition 2.2.2 (Contrapositive)

The **contrapositive** of a conditional statement “if  $p$  then  $q$ ” is “if  $\sim q$  then  $\sim p$ ”.

Symbolically, the contrapositive of  $p \rightarrow q$  is  $\sim q \rightarrow \sim p$ .

### Definition 2.2.3 (Converse)

The **converse** of a conditional statement “if  $p$  then  $q$ ” is “if  $q$  then  $p$ ”.

Symbolically, the converse of  $p \rightarrow q$  is  $q \rightarrow p$ .

### Definition 2.2.4 (Inverse)

The **inverse** of a conditional statement “if  $p$  then  $q$ ” is “if  $\sim p$  then  $\sim q$ ”.

Symbolically, the inverse of  $p \rightarrow q$  is  $\sim p \rightarrow \sim q$ .

# Summary

## 2.2 Conditional Statements

$$p \rightarrow q \equiv \sim p \vee q \quad \text{Implication law}$$

$$p \rightarrow q \equiv \sim q \rightarrow \sim p$$

*conditional statement* *contrapositive*

$$q \rightarrow p \equiv \sim p \rightarrow \sim q$$

*converse* *inverse*

Note that:

$$p \rightarrow q \not\equiv q \rightarrow p$$

# Summary

## 2.2 Conditional Statements

### Definition 2.2.5 (Only If)

If  $p$  and  $q$  are statements,

“ $p$  only if  $q$ ” means “if not  $q$  then not  $p$ ” or “ $\sim q \rightarrow \sim p$ ”

Or, equivalently,

“if  $p$  then  $q$ ” or “ $p \rightarrow q$ ”

### Definition 2.2.6 (Biconditional)

Given statement variables  $p$  and  $q$ , the **biconditional** of  $p$  and  $q$  is “ $p$  if, and only if,  $q$ ” and is denoted  $p \leftrightarrow q$ .

It is true if both  $p$  and  $q$  have the same truth values and is false if  $p$  and  $q$  have opposite truth values.

The words *if and only if* are sometimes abbreviated *iff*.

### Definition 2.2.7 (Necessary and Sufficient Conditions)

If  $r$  and  $s$  are statements,

“ $r$  is a sufficient condition for  $s$ ” means “if  $r$  then  $s$ ” or “ $r \rightarrow s$ ”

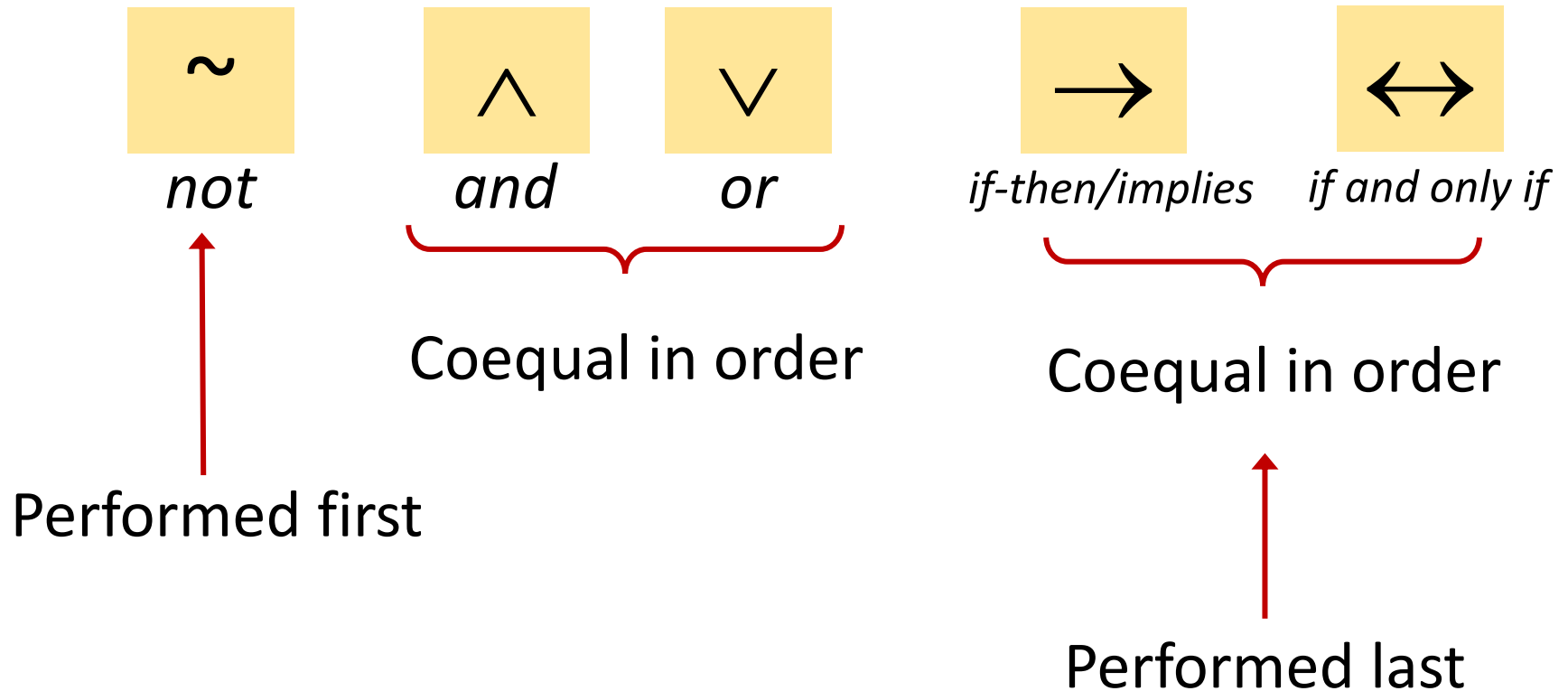
“ $r$  is a necessary condition for  $s$ ” means “if not  $r$  then not  $s$ ”  
or “if  $s$  then  $r$ ” or “ $s \rightarrow r$ ”



# Summary

## Order of Operations

Order of operations:



# Summary

## 2.3 Valid and Invalid Arguments

### Definition 2.3.1 (Argument)

An **argument** (**argument form**) is a sequence of statements (statement forms). All statements in an argument (argument form), except for the final one, are called **premises** (or **assumptions** or **hypothesis**). The final statement (statement form) is called the **conclusion**. The symbol  $\bullet$ , which is read “therefore”, is normally placed just before the conclusion.

To say that an argument form is **valid** means that no matter what particular statements are substituted for the statement variables in its premises, if the resulting premises are all true, then the conclusion is also true.

### Definition 2.3.2 (Sound and Unsound Arguments)

An argument is called **sound** if, and only if, it is valid and all its premises are true.

An argument that is not sound is called **unsound**.

# Summary

## 2.3 Valid and Invalid Arguments

Table 2.3.1 Rules of Inference

Rule of inference			Rule of inference		
Modus Ponens	$p \rightarrow q$ $p$ $\bullet \quad q$		Elimination	$p \vee q$ $\sim q$ $\bullet \quad p$	$p \vee q$ $\sim p$ $\bullet \quad q$
Modus Tollens	$p \rightarrow q$ $\sim q$ $\bullet \quad \sim p$		Transitivity	$p \rightarrow q$ $q \rightarrow r$ $\bullet \quad p \rightarrow r$	
Generalization	$p$ $\bullet \quad p \vee q$	$q$ $\bullet \quad p \vee q$	Proof by Division Into Cases	$p \vee q$ $p \rightarrow r$ $q \rightarrow r$ $\bullet \quad r$	
Specialization	$p \wedge q$ $\bullet \quad p$	$p \wedge q$ $\bullet \quad q$	Contradiction Rule	$\sim p \rightarrow \mathbf{false}$ $\bullet \quad p$	
Conjunction	$p$ $q$ $\bullet \quad p \wedge q$				

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