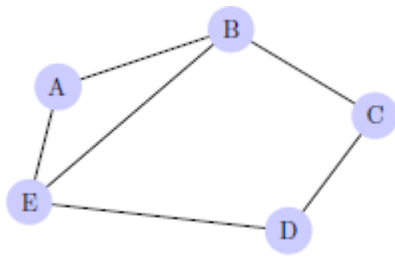


1. Consider the network shown below.



Which of the following statements are true?

- (i) A and B have the same degree centrality.
- (ii) B and E have the same closeness centrality.

(A) (i) and (ii)

(B) (i) only

(C) (ii) only

(D) neither (i) nor (ii)

Explanation:  $D_{\text{cen}}(A) = 2/4$ , and  $D_{\text{cen}}(B) = 3/4$ , so (i) is false.  $C_{\text{cen}}(B) = 5/4$ , and  $C_{\text{cen}}(E) = 5/4$ , hence (ii) is true.

2. The following is the adjacency matrix for a communication network between 5 students. We number the students from 1 to 5.

$$\begin{array}{c}
 \begin{array}{ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
 \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}
 \end{array}
 \end{array}$$

The entry in the  $i$ th row and  $j$ th column of the matrix would be 1 if student  $i$  and student  $j$  can communicate with each other directly and 0 otherwise. Suppose we wish to choose 1 out of the 5 students to be the main point of contact, and our only point of consideration is the closeness centrality measure of the communication network, as represented above. Who is the most appropriate choice?

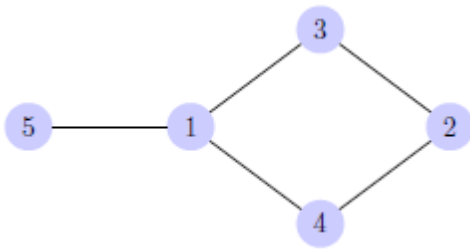
(A) Student 1

(B) Student 2

(C) Student 3

(D) Student 5

Explanation: We have the following network:



$C_{\text{cen}}(\text{student 1}) = 5/4$ ,  $C_{\text{cen}}(\text{student 2}) = 7/4$ ,  $C_{\text{cen}}(\text{student 3}) = 6/4$ , and  $C_{\text{cen}}(\text{student 5}) = 8/4$ . Hence, student 1 has the smallest closeness centrality measure.

3. A class has 5 students. A network is drawn with 5 vertices, each representing a student in the class. Two vertices are joined directly by an edge if the two corresponding students are friends. It is known that student A, student B, and student C are friends with each other. Suppose the other two students in class, student D, and student E, are not friends with each other but both are friends of student A. Which of the following statement is true?

- (A) The degree centrality of student E is 1
- (B) The degree centrality of student A is 1**
- (C) The betweenness centrality of student A is 1
- (D) The betweenness centrality of student D is 1

Explanation: Since student E is not adjacent to student D, its degree centrality is less than 1. Student A is adjacent to all four other students, so its degree centrality is 1. Note that the shortest path between student B and student C does not pass through any other vertices, so the betweenness centrality of student A and student D are both less than 1.

4. The following question is about the Bacon number of an actor, described in Slide 8 of Unit 4 in Chapter 5. Suppose the Bacon number of actor C was computed to be 3 at the end of 2016. We know that the first movie released in 2017 is the comedy-drama “Happiness”, not involving Kevin Bacon. Since this is a low-budget movie, they only hire actors/actresses who are not well known; every one of them has never appeared in a movie with Kevin Bacon. Which of the following statement must be correct after the movie “Happiness” was added to the movie graph?

- (A) The Bacon number of actor C is now 1
- (B) The Bacon number of actor C is now 2
- (C) The Bacon number of actor C is still 3**

Explanation: Bacon number of actor C is 1 only when an edge is added between actor C and Kevin Bacon. However, this is not possible since Kevin Bacon is not in the movie “Happiness.” Bacon number of actor C is 2 only when an edge is added between actor C and another actor/actress with Bacon number 1. This is again not possible since all the actors/actresses who acted in “Happiness” have Bacon numbers of at least 2 at the end of 2016 as they have never appeared in a movie with Kevin Bacon.