Bob attends a party in which him and 99 other guests are evenly distributed around a round table, so that there is someone sitting directly opposite every guest at the table. At first, a naive system of discreet message passing is implemented: every message is to be passed around the table, instead of across it. We can model this system with a network *G*, defined as follows:

- The vertices are precisely the guests at the table.
- There is an edge between two vertices if they are sitting next to each other.

Abstracting to *G*, we can say that every message has to be passed from one vertex to another, along the edges of *G*.

Q1. What is the degree of Bob in *G*?

a) 2
b) 3
c) 4
d) There is insufficient information to determine the answer.

Explanation: Since the guests are seated in a circle, there are exactly two guests seated next to Bob, one to his left and one to his right.

Q2. How many vertices in *G*, excluding Bob himself, are strictly less than a distance of 30 away from Bob?

a) 29
b) 30
c) 58
d) 60
e) There is insufficient information to determine the answer.

Explanation: There are the first 29 guests to the left of Bob have distances strictly less than 30 from Bob. And so do the first 29 guests to the right of Bob. Note that there is no overlap in these two groups of guests, and that anyone outside these two groups will have a distance 30 or more from Bob. So the answer is just 29 + 29 = 58.

Q3. What is the maximum attainable distance between any two vertices in *G*?

a) 49
b) 50
c) 99
d) 100

Explanation: Since the entire network *G* is one big cycle, the furthest two vertices can be from each other is when they are directly opposite each other. The distance between them, in this case, is 50.

To shorten the duration of message passing, there is a unanimous decision to utilise the miniature cable transportation network that links each guest to the guest sitting directly opposite him/her, in conjunction with the original system. We can model the new and improved system with a network *H*, defined by adding new edges to the network *G*, according to the following criterion:

- There is an edge between two vertices if they are directly opposite each other.

Again, in *H*, every message has to be passed from one vertex to another along the edges.

Q4. What is the degree of Bob in *H*?

a) 2
b) 3
c) 4
d) There is insufficient information to determine the answer.

Explanation: The vertices adjacent to Bob are the guests to his left and right, and the guest directly opposite him. Note that these three vertices have to be distinct, because of how big the circular table is.

Q5. How many vertices in *H*, excluding Bob himself, are strictly less than a distance of 30 away from Bob?

a) 58
b) 98
c) 99
d) 100
e) There is insufficient information to determine the answer.

Explanation: There are 99 vertices in *H* excluding Bob. The answer to Q6 tells us all these vertices are of distances no more than 25 (hence strictly less than 30) away from Bob.

Q6. What is the maximum attainable distance between any two vertices in *H*?

a) 24
b) 25
c) 49
d) 50

Explanation: The following argument works for any vertex in *H*, so just choose an arbitrary vertex X. The first 25 vertices to the left and right of X are of distances up to 25 from X, considering only the paths that use edges already in *G*. As for the other vertices in the network, there are paths from X to them via the vertex directly opposite X, and these paths are of length no more than 25. So the furthest vertices from X are of distance 25 from X.