P $(3,-1,2)$
Q $(8,2,4)$
R $(-1,-2,-3)$

$\vec{PQ} = \langle 8-3,\ 2-(-1),\ 4-2 \rangle = \langle 5, 3, 2 \rangle$

$\vec{PR} = \langle -1-3,\ -2-(-1),\ -3-2 \rangle = \langle -4, -1, -5 \rangle$

$$n = \vec{PQ} \times \vec{PR}$$

$$= \begin{vmatrix} i & j & k \\ 5 & 3 & 2 \\ -4 & -1 & -5 \end{vmatrix}$$

$$= \big((3)(-5) - (2)(-1)\big)i - \big((5)(-5) - (2)(-4)\big)j + \big((5)(-1) - (3)(-4)\big)k$$

$$= (-15+2)i - (-25+8)j + (-5+12)k$$

$$= -13i + 17j + 7k$$

$$-13(x-1) + 17(y+1) + 7(z-2) = 0$$

$$-13x + 13 + 17y + 17 + 7z - 14 = 0$$

$$-13x + 17y + 7z = -16$$

Scanned with CamScanner

a)

$$L = \lim_{n\to\infty} \left| \frac{\frac{(n+1)!\,\cancel{(2n)}\,(2(n+1))!}{(3(n+1))!}}{\frac{n!\,(2n)!}{(3n)!}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(n+1)!\,(2n+2)!\,(3n)!}{n!\,(2n)!\,(3n+3)!} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(2n+2)(2n+1)}{3(3n+1)(3n+2)} \right|$$

$$= \frac{2}{3} \lim_{n\to\infty} \left| \frac{(n+1)(2n+1)}{(3n+1)(3n+2)} \right|$$

$$= \frac{2}{3} \lim_{n\to\infty} \left| \frac{2n^2+3n+1}{9n^2+9n+2} \right|$$

$$= \frac{2}{3} \frac{\lim_{n\to\infty}\left(\cancel{2n}\,2 + \frac{3}{n} + \frac{1}{n^2}\right)}{\lim_{n\to\infty}\left(9 + \frac{9}{n} + \frac{2}{n^2}\right)}$$

$$= \frac{2 \times 2}{3 \times 9}$$

$$= \frac{4}{27}$$

Since ~~4~~ $0 \leq L < 1$, the series ~~is~~ convergent by ratio test

b) $\int_2^{\infty} \frac{1}{x(\ln(x))^3}\, dx = \cancel{\int \frac{1}{u^3}\, du}\ \left[-\frac{1}{2(\ln x)^2}\right]_2^{\infty}$

let $u = \ln x$, $\frac{du}{dx} = \frac{1}{x}$

$x\, du = dx$

$$\int \frac{1}{u^3}\, du = -\frac{1}{2u^2} + C$$

$$= -\frac{1}{2(\ln x)^2} + C$$

$$\left[-\frac{1}{2(\ln x)^2}\right]_2^{\infty} = -\frac{1}{2(\ln \infty)^2} + \frac{1}{2(\ln 2)^2}$$

$$= \frac{1}{2(\ln 2)^2} \quad \text{(convergent)}$$

Thus the series ~~is~~ ~~ex~~ converges by integral test

$$f(x) = \frac{ax + a + 1}{x^2 - x - 2}$$

$$= \frac{ax + a + 1}{(x-2)(x+1)}$$

~~$\langle f_x, f_y \rangle \cdot \langle 3i, 4j \rangle = 15$~~

$\sqrt{3^2 + 4^2} = 5$

~~$3i, 4j = \frac{1}{5}\langle \frac{1}{5} \rangle i \quad \langle \frac{3}{5} i, \frac{4}{5} j \rangle$~~

$3i + 4j \Rightarrow \frac{1}{5} \langle 3i, 4j \rangle$

$\langle f_x, f_y \rangle \cdot \langle \frac{3}{5} i, \frac{4}{5} j \rangle = 15$

$\sqrt{(-3)^2 + (4)^2} = 5$

$-3i + 4j \Rightarrow \langle -\frac{3}{5} i, \frac{4}{5} j \rangle$ ~~$= 7$~~

$\langle f_x, f_y \rangle \cdot \langle -\frac{3}{5} i, \frac{4}{5} j \rangle = 9$

$\frac{3}{5} f_x + \frac{4}{5} f_y = 15$

$-\frac{3}{5} f_x + \frac{4}{5} f_y = 9$

$\frac{6}{5} f_x = 6$

$f_x = 5$

$f_y = 15$

$\sqrt{24^2 + 7^2} = 25$

$24i + 7j \Rightarrow \langle \frac{24}{26} i, \frac{7}{25} j \rangle$

directional derivative $\Rightarrow \langle 5, 15 \rangle \langle \frac{24}{25} i, \frac{7}{25} j \rangle$

$= 5 \times \frac{24}{26} + 15 \times \frac{7}{25}$

$= \frac{24}{5} + \frac{21}{5}$

$= 9$

$f_x(x,y) = 3x^2 + 3y^2 - 6x$

~~$= 3x^2 + 6$~~

$f_y(x,y) = 6xy - 6y$

when f_x and $f_y = 0$,

$3x^2 - 6x + 3y^2 = 6xy - 6y$

$3x^2 - 6x - 6xy + 6y + 3y^2 = 0$

$x^2 - 2x - 2xy + 2y + y^2 = 0$

critical points $= (0,0), (2,0), (0,2)$ (using graph)

$f_{xx} = 6x - 6$

$f_{yy} = 6x - 6$

$f_{xy} = 6y$

$D = (6x-6)^2 - (6y)^2$

$(0,0) \Rightarrow D = 36 > 0$, $f_{xx} = -6 < 0$ (maximum ~~minimum~~)

$(2,0) \Rightarrow D = (12-6)^2 = 36 > 0$, $f_{xx} = 6 > 0$, (minimum)

$(0,2) \Rightarrow D = 36 - 144 = -108 < 0$, (saddle point)

$(0,0)$ maximum

$(2,0)$ minimum

$(0,2)$ saddle point

Scanned with CamScanner

$$y^2 = x^2a^3 - 3x^3a^2 + 3x^4a - x^5$$
$$= x^2(a^3 - 3xa^2 + 3x^2a - x^3)$$

when $y = 0$, $x = 0$ or a

$$\int_0^a \cancel{x^2(a^3} \; x^2a^3 - 3x^3a^2 + 3x^4a - x^5 \, dx$$
$$= \left[\frac{x^3a^3}{3} - \frac{3x^4a^2}{4} + \frac{3x^5a}{5} - \frac{x^6}{6}\right]_0^a$$
$$= \frac{a^6}{3} - \frac{3a^6}{4} + \frac{3a^6}{5} - \frac{a^6}{6}$$
$$= \frac{1}{60}a^6$$

let $y = vx$, $y' = v'x + v$

$$x(v'x + v) + vx = x(vx)^3$$

$$\Rightarrow v'x^2 + vx + vx = v^3x^4$$

$$\frac{v'x^2 + 2vx}{v^3x^4} = 1$$

$$\frac{v'x + 2v}{v^3x^3} = 1$$

Let Q = amount of salt

dQ = salt input - salt output

$$\frac{dQ}{dt} = 40\ln(4) - \frac{Q\ln(4)}{4}$$

$$= 80\ln 2 - \frac{Q}{2}\cancel{\ln(4)}\ln 2$$

$Q =$