Quiz 5

1. Jack, a 1.75m tall, bearded male, stands accused of armed robbery. Witness reports indicate the man who robbed the bank sported a beard, was about 1.7-1.8m tall, and was wearing a yellow coloured cap and a pair of hammer pants. Jack fits this description, and hence is a suspect. During his trial, the prosecutor states the following probabilities:

In the general population,

fraction(men with beards) = 0.294

fraction(own a yellow cap) = 0.01

fraction(own hammer pants) = 0.001

fraction(taller than 1.7m) = 0.4.

He then claims that the probability of someone chosen at random from the general population fitting that description is

0.294*0.01*0.001*0.4=1.176*10^-6, ------- (*)

about a one in a million. Assuming the probabilities quoted are accurate, should you be convinced by the computation in (*)?

(a) Yes, we conclude there is a one in a million chance of someone fitting that description being at the crime scene.
(b) No, the computation in (*) is invalid because the events considered **may not be** mutually exclusive.
(c) No, the computation in (*) is invalid because the events considered **may not be** independent.
(d) No, the probabilities in (*) should have been summed, not multiplied.

Explanation:

C is the answer. The argument in (*) ignores possible dependencies between the events. For example, the events "Man with beard" and "Taller than 1.7m" are unlikely to be independent, as are the events "Owning a yellow cap" and "Owning hammer pants". For a similar real life example, see the case of People vs. Collins: https://en.wikipedia.org/wiki/People_v._Collins

2. You visit your grandparents and notice an old die and notebook. The die is handmade and the edges are worn. Grandpa explains that he rolled this die 10,000 times, independently and under the same conditions, when he was a prisoner of war and recorded the results of the die rolls in his notebook:

Side of the die	1	2	3	4	5	6
Number of outcomes	2607	1633	1148	1839	2552	221

He adds that he used to run an illegal gambling ring and that he fashioned this die out of bone himself. Which of the following follow from these observations?

(I) The data suggests the die has a much lower probability of producing a ‘6’ compared to other outcomes.
(II) It is possible, though very unlikely, that the die is fair.

(a) (I) only.
(b) (II) only.
(c) Both (I) and (II).
(d) Neither (I) nor (II).

Answer: C

From the appearance of the die, its lack of symmetry suggests that it would be better to estimate the probability of obtaining a ‘6’ from the empirical data than simple assume that all outcomes for this die are equally likely. However, strictly speaking, the true probability of obtaining a ‘6’ with this die is unknowable. Hence, despite the small p-value of observing such an outcome under the null hypothesis that the die is fair, we cannot rule out this possibility with certainty.

3. Based on the data in the previous question, what is the average value of a single roll of this die?

(a) 1
(b) 3.5
(c) 3.076
(d) 5

Answer: C

The average value of a single roll of this die is given by

1*(2607/10,000) + 2*(1633/10,000) + 3*(1148/10,000) + 4*(1839/10,000) + 5*(2552/10,000) + 6(221/10,000) = 3.076

4. In a game of Monopoly, 2 (fair) six-sided dice are rolled. Every face of each die is labelled 1 to 6.

A *double* occurs if both dice land up on the same number.

Instead of rolling both dice together, Brandon rolls one die at a time. The first die lands on a ‘6’. What is the probability that he rolls a double?

If the first die lands on either ‘1’, ‘2’, ‘3’, ‘4’, ‘5’ or ‘6’, what is the probability that he rolls a double?

(A) 1/36, 2/36
(B) 1/6, 2/36
(C) 1/6, 2/6
(D) 1/6, 1/6

Answer: D

For the first case where Brandon first rolls a ‘6’, since the die rolls are independent, the probability of rolling a double is just the probability of rolling a ‘6’: 1/6. For the second case, this is simply the probability of rolling *any* one of the six doubles, which is 1/36 + 1/36 + 1/36 + 1/36 + 1/36 + 1/36 = 1/6.

5. In another unrelated game called Oligopoly, Brandon has 3 chances to roll a single die. He rolls 2 '6's, and a '5' on his turn. Assuming the dice are fair, what is the probability of (i) '6's on the first two rolls, and a non-six in the third roll, and (ii) getting exactly two '6's?

(A) 0.0278, 0.005

(B) 0.0231, 0.005

(C) 0.0231, 0.0278

(D) 0.0231, 0.0694

(E) 0.0694, 0.0231

(D) 0.0278, 0.0231

(E) 0.005, 0.0231

(F) 0.005, 0.0278

Answer: D

Since the die rolls are independent, (i) the probability rolling sixes on the first two rolls and a non-six on the third roll is given by (1/6)^2*(5/6) and (ii) the probability of rolling exactly two sixes is given by 3*(1/6)^2*(5/6).