NATIONAL UNIVERSITY OF SINGAPORE

MA1301 **Test**

The test carries a total of 60 marks. The marks for each question are as indicated.

1. (a) It is given that $4x^2 - 9xy + 9y^2 = 252$.

   (i) Find $\dfrac{dy}{dx}$ in terms of x and y. [2]

   (ii) Find the equations of the tangents parallel to the x axis. [2]

   (b) A curve is given parametrically by the equations

   $$x = 2t - \frac{1}{2t} \text{ and } y = 2t + \frac{1}{2t},$$

   where $t > 0$.

   (i) Find $\dfrac{dy}{dx}$ in terms of t, giving your answer in the simplest form. [2]

   (ii) Find $\dfrac{d^2y}{dx^2}$, expressing your answer in the form $\dfrac{d^2y}{dx^2} = A\left(\dfrac{t}{Bt^2 + C}\right)^K$, where A, B, C and K are constants. [4]

2. Let $f(x) = 2x^3 - 21x^2 + 60x + 11$, where $x \in \mathbb{R}$.

   (i) Find the intervals on which f is (a) increasing and (b) decreasing. [2]

   (ii) Find the coordinates and nature (local maximum or local minimum or saddle point) of the stationary points of the curve $y = f(x)$. [4]

   (iii) Find the interval on which the graph of f is (a) concave down and (b) concave up. [2]

   (iv) Write down the coordinates of the inflexion point of the curve $y = f(x)$. [2]

3. (a) (i) Show that for any positive integer r,

$$(r+2)! - (r+1)! = (r+1)^2 r!.$$

[3]

(ii) Hence, find an expression in terms of n, for

$$2^2 \times 1! + 3^2 \times 2! + 4^2 \times 3! + \ldots + (n+1)^2 n!.$$

[3]

(b) An arithmetic progression with common difference 3 contains 15 positive terms. Suppose the last term is the square of the first term.

(i) Calculate the first term of the progression. [2]

(ii) Find the sum of the progression. [2]

(c) In a geometric progression, the fourth term exceeds the third term by 2 and exceeds the second by 5.

(i) Find the sum of the first ten terms. [4]

(ii) Find the sum to infinity of this progression. [2]

4. (a) Let p and q be constants. In this question, we consider the binomial expansion of $\left(px + \frac{q}{x}\right)^n$, where n is a positive integer.

(i) Write down the first two terms in the binomial expansion of $\left(px + \frac{q}{x}\right)^n$. [2]

(ii) It is known that the fourth term in the binomial expansion of $\left(px + \frac{q}{x}\right)^n$ is independent of x, find the value of n. [3]

(iii) Suppose p and q are positive integers such that $p - q = 1$ and the value of the fourth term is 160. Using the value of n found in (ii), calculate the values of p and q. [3]

(b) (i) Write down and simplify the expansion of $(2-p)^5$. [2]

(ii) Use the expansion in (i) to find the expansion of $\left(2 - 2x + \frac{x^2}{2}\right)^5$ in ascending powers of x as far as the term in x^2. [4]

5. A piece of wire 360 cm long is used to make the twelve edges of a rectangular box in which the length is twice the breadth. Denote the breadth of the box by x cm and the height by h cm.

   (i) Express h in terms of x. [2]

   (ii) Express V in terms of x, where V cm^3 is the volume of the box. [2]

   (iii) Determine, as x varies, the maximum volume of the box. Show that the volume is a maximum. [6]