CS3263 MID-TERM TEST
Solution Sketches

Sem 2, AY2022-23

Question #: 1

[1 mark]
Is the following statement True or False?
The size of factors generated during variable elimination is at most twice the size of the largest conditional probability table in the original Bayesian network.

A. True
✓B. False

Question #: 2

[1 mark]
Is the following statement True or False?
During variable elimination, the ordering of elimination does not affect the final answer.

✓A. True **[The algorithm is always sound.]**
B. False

Question #: 3

[1 mark]
Is the following statement True or False?
AI systems that are safe, fair, and accountable can give the most accurate predictions and recommendations in all settings.

A. True
✓B. False [**There is always a trade-off between any responsible AI criterion and technical accuracy.]**

Question #: 4

[1 mark]
Is the following statement True or False?
Prof. Nut wakes up one day with sore throat, fever, and coughing – he immediately thinks that he is down with Covid-19. This could be a cognitive bias emanating from the availability heuristic or representative heuristic.

✓A. True
B. False

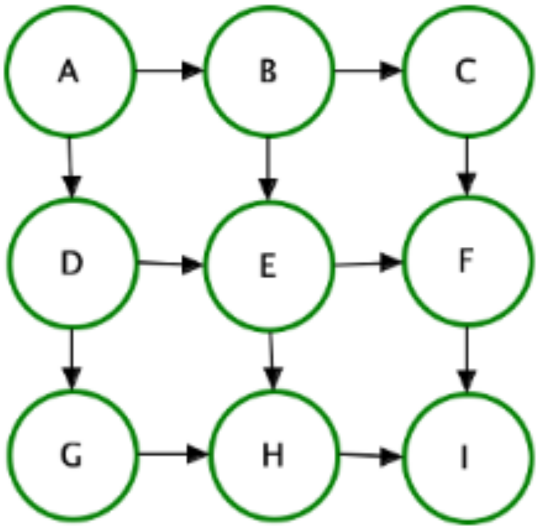
Question #: 5

[1 mark]
Is the following statement True or False?
Both simulated annealing and random restart hill climbing can guarantee global optimality, but the convergence may be slow.

✓A. True
B. False

Question #: 6

[3 marks]
Consider the Bayesian network below with 9 nodes, denoting random variables A to I.



a) Which random variables are independent of G? (Write "NIL" if there is none.)
Ans: __1__

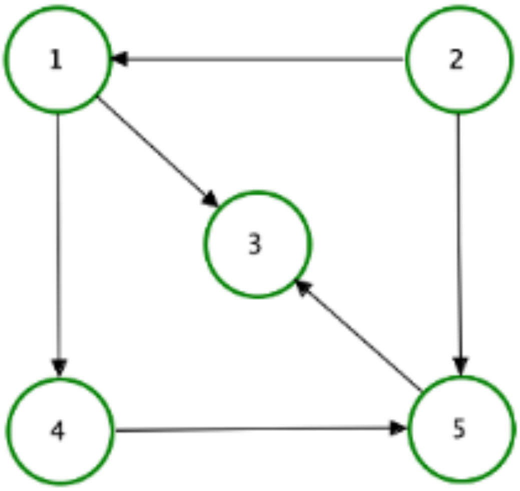
b) Which random variables are conditionally independent of G given A? (Write "NIL" if there is none.)
Ans: __2__

c) Which variables are conditionally independent of G given A and I? (Write "NIL" if there is none.)
Ans: __3__

1. NIL
2. B and C
3. NIL

Question #: 7

[2 marks]
Consider the Bayesian network below with 5 nodes. Label the blank nodes no. N1 to N5 with variables {A, B, C, D, E} such that the following independence conditions are true:
• A is conditionally independent of B given D, E.
• C is conditionally independent of D given A, B.
• D is conditionally independent of E given B.
• E is conditionally independent of C given A, B.



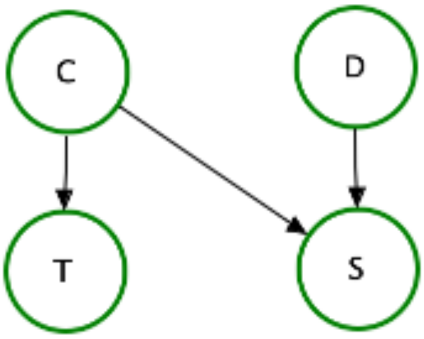
a) What is a possible ordering (of variables) for the nodes N1 to N5?
Ans: __1__

b) Is the ordering given in part a) above unique, i.e., there is no other possible ordering (of variables) for the nodes N1 to N5?
Ans: __2__

1. B, E, C, D, A OR B, D, C, E, A
2. Choice of: YES | NO - Correct Answer:NO

Question #: 8

[6 marks]
The Bayesian network below shows the situation of two different diseases, C and D, that can cause a common symptom S of "blue toes." Disease C is usually rare, and its presence can be tested by a test T.



$P(+c) = 0.1$
$P(+d) = 0.5$
$P(+t \mid +c) = 1$
$P(+t \mid -c) = 0.2$
$P(+s \mid +c, +d) = 1$
$P(+s \mid +c, -d) = 0.8$
$P(+s \mid -c, +d) = 1$
$P(+s \mid -c, -d) = 0$

a) What is the probability that a patient has disease D, (and) does not have disease C, (and) has symptom S, (and) has test T returning negative?

Ans: The probability is: __1__ (up to 2 decimal points)

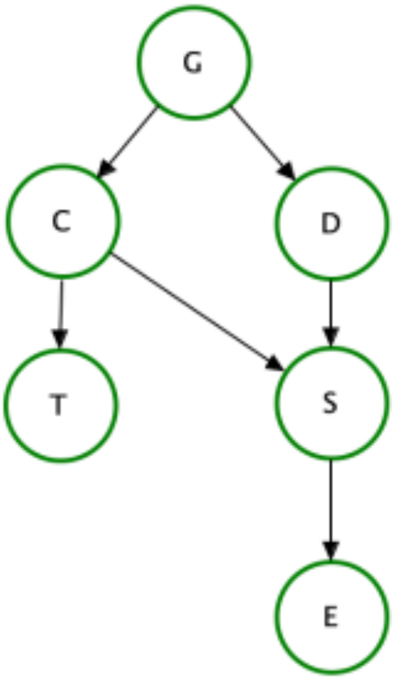
b) What is the probability that a patient has disease C given that he has disease D?

Ans: The probability is: __2__ (up to 2 decimal points)

c) What is the probability that a patient has disease C given that he has symptom S and test T returns positive?

Ans: The probability is: __3__ (up to 2 decimal points)

Suppose that both diseases C and D become more likely if a person has a genetic trait (G) of "green nose", and that the presence of symptom S of "blue toes" can be detected only through the use of a Toe-Color-Detector (E); the detector E will return a positive result if "blue toes" are detected. The revised Bayesian network is shown below. The additional/revised conditional probability tables are **not** given.



For this new situation, indicate if each of the following statement is True, False, or Undetermined (i.e., cannot be determined based on the available information).

d) C is independent of D.

Ans: 4

e) C is conditionally independent of D given G.

Ans: 5

f) C is conditionally independent of D given G and E.

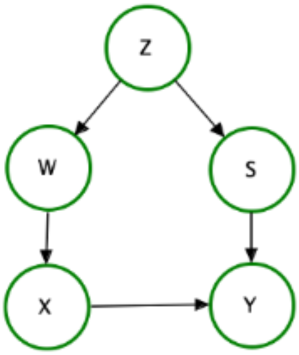
Ans: 6

1. 0.36
2. 0.1
3. 0.5
4. Choice of: True | False | Undetermined - Correct Answer:Undetermined
5. Choice of: True | False | Undetermined - Correct Answer:True
6. Choice of: True | False | Undetermined - Correct Answer:Undetermined

[The graph structure or topology of a Bayesian network cannot assert that a given independence does not hold. Recall: Graph G in a Bayesian network encodes a set of conditional independence assumptions $I(G)$, but $I(G) \subseteq I(P)$ where P is the distribution that factorizes over $G-$ $I(P)$ cannot be determined without the underlying numerical information on conditional probabilities.]

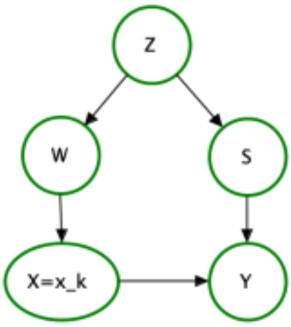
Question #: 9

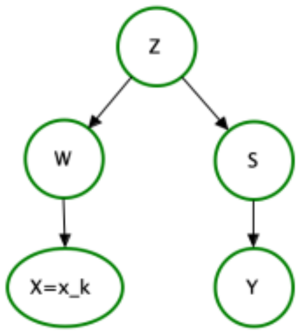
[4 marks]
Consider the following causal network with binary random variables {Z, W, S, X, Y}.



Assume that an intervention do($X=x_k$),where $k \in \{0,1\}$ is performed.
a) Which of the following denote the post-intervention causal network? (Note that the $X = x_k$ is denoted as X =x_k in the diagrams.)

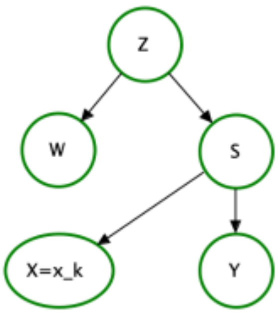
i.

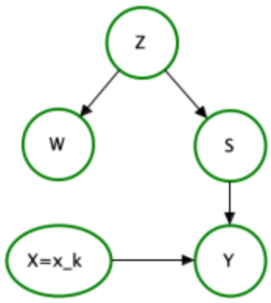




ii.

iii.





iv.

Ans: __1__

b) Write down factorization of the post-interventional joint distribution

$P_{x_k}(x, y, z, w, s)$

in terms of the conditional distributions of the pre-intervention causal network.

Ans:

$If\ x = x_k, \quad P_{x_k}(x, y, z, w, s) =$ __2__

$If\ x \neq x_k, \quad P_{x_k}(x, y, z, w, s) =$ __3__

c) Which of the following denote all the sets of variable(s) that satisfy the backdoor criterion to estimate causal effect of X on Y.

i.{Z, W, S}, {Z, W}, {Z, S}, {W, S}, {Z}, {W}, {S}
ii.{Z, W}, {Z, S}, {W, S}, {Z}, {W}, {S}
iii.{Z, W, S}, {Z}, {W}, {S}
iv.{Z}, {W}, {S}
v.{W, S}, {Z}, {W}, {S}

Ans: __4__

1. Choice of: i. | ii. | iii. | iv. - Correct Answer:iv.
2. P(z)P(w|z)P(s|z)P(y|x,s)
3. 0
4. Choice of: i. | ii. | iii. | iv. | v. - Correct Answer:i.

Question #: 10

[5 marks]
Persy the Happy Rover is on its first mission to Mars! Persy is ready to explore the surroundings of its landing location in Jezero Crater to find a good place to set up rock-analysis experiments.
At any location, assume that Persy can only explore the k adjacent locations (or neighbors) from where it is. Persy can measure the "goodness" of a location and that of its neighbors for collecting rock deposits with a special equipment.

Assume that at any one time only one of the neighbors has a strictly better score than any one location. Assume also that Percy has unlimited battery power B (with re-charging) but limited time T that it can spend on exploration.

Consider the various search algorithms that Percy can use:

Algorithm A: Randomly choose one of the neighbors with equal probability. If the chosen new location has strictly better "rock goodness score" than its current location, move there; otherwise stay at the current location.

a) If Persy uses Algorithm A, what is the probability that Persy will move out of a location in N iterations or fewer?

i.

$$1-(1-\frac{1}{k})^{N}$$

ii.

$$1-(1-\frac{1}{k})\times N$$

iii.

$$(1-\frac{1}{k})^{N}$$

iv.

$$(\frac{1}{k})^{N}$$

Ans: __1__

b) If Persy uses Algorithm A, will it always find the best location in the Jazero Crater to set up rock-analysis experiments?

Ans: __2__

Algorithm B: Randomly choose one of the neighbors, but with a probability that varies directly according to the neighbor's rock goodness score.

c) If Persy uses Algorithm B, will it always find the best location in the Jazero Crater to set up rock-analysis experiments?

Ans: __3__

Algorithm C: Run Algorithm A to find a good location. Then randomly start from a different location and run Algorithm A again to find another good location. Repeat R times to find the best location among the different searches. Assume that each random start search takes time T_i, $i \in \{1, 2, 3, \ldots, R\}$.
d) If Persy uses Algorithm C, and assuming that

$\sum_{i=1}^{R} T_i \leq T$, will it always find the best location in the Jazero Crater to set up rock-analysis experiments?

Ans: __4__

Algorithm D: Randomly choose one of the neighbors with the best rock goodness score. After moving to the new location, record the score of the previous ("from") location in a table. Continue with the search and recording of the locations and the scores. Stop if all the neighbors have been visited before, and the current (final) location has the best score in the table.

e) If Persy uses Algorithm D, and assuming that the search terminates within time T, will it always find the best location in the Jazero Crater to set up rock-analysis experiments?

Ans: __5__

1. Choice of: i. | ii. | iii. | iv - Correct Answer:i.
2. Choice of: YES. The best location will always be found within a finite time. | NO. The best location may not be found within a finite time. - Correct Answer:NO. The best location may not be found within a finite time.
3. Choice of: YES. The best location will always be found within a finite time. | NO. The best location may not be found within a finite time. - Correct Answer:NO. The best location may not be found within a finite time.
4. Choice of: YES. The best location will be always be found within a finite time. | NO. The best location may not be found within a finite time. - Correct Answer:NO. The best location may not be found within a finite time.
5. Choice of: YES. The best location will be always be found within a finite time. | NO. The best location may not be found within a finite time. - Correct Answer:NO. The best location may not be found within a finite time.

[In this hypothetical and highly uncertain scenario, based on the assumptions, all the algorithms will likely not be able to find the global optimum – and exploration has to stop within a finite time T. Hence, the best location may not be found in a finite time in all the following cases.

Note: Implications on assumptions: "better positions" are arranged in centric ovals/shapes "outwards toward the borders of the Jazero Crator from any location – with better/best positions located at or beyond the borders of the Jazero Crator.

In general: in any environment, within a finite time:

Algorithm A is a version of hill climbing search - it is likely to find only a local optimum.

Algorithm B is a version of Stochastic hill-climbing search – it is not guaranteed to find the global optimum within a finite time period.

Algorithm C is a version of random restart hill climbing – it may find the global optimum only if $\sum_{i=1}^{R} T_i \leq T$. But random restart is not practical in this case, as Persy cannot be easily transported to random locations to start the search.

Algorithm D is a version of LRTA* - it may find the global optimal the search time allowed is long enough – but "long enough" is not easily defined]