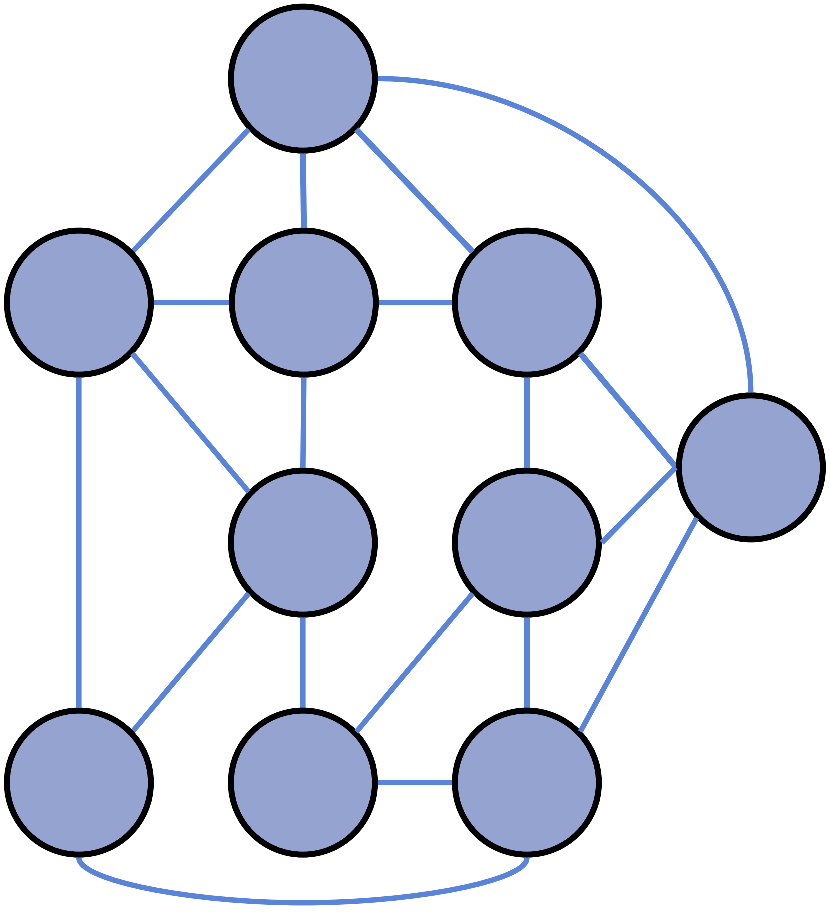
MST Algorithms
(Minimum Spanning Tree)

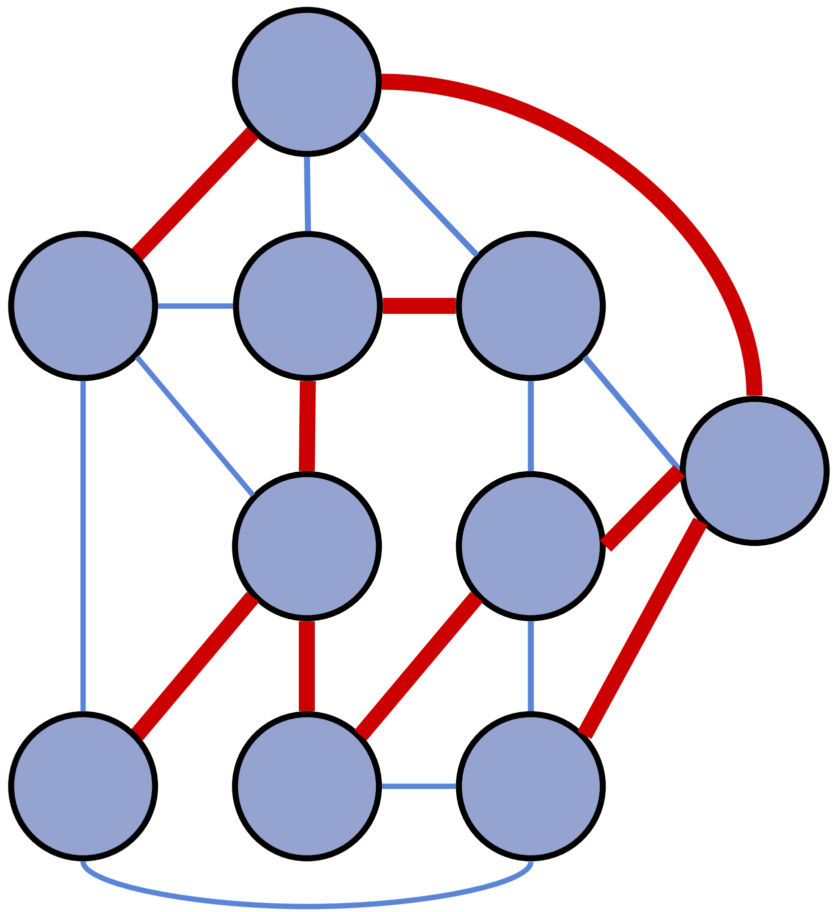
Algorithms by:
Kruskal, Guan, Prim-Dijkstra
Sollin, Yao

Input Graph

Want "minimal" graph that is still *fully connected*



Spanning Tree



Minimum Spanning Tree

Problem:

- Input: A graph G, with costs on the edges.
- ToDo: Find a spanning tree where total cost is minimum.

Kruskal's MST Algorithm

- Joseph Kruskal, 1956

2. ^ Kruskal, J. B. (1956). "On the shortest spanning subtree of a graph and the traveling salesman problem". *Proceedings of the American Mathematical Society* **7**: 48–50. doi:10.1090/S0002-9939-1956-0078686-7. JSTOR 2033241.

Idea:

"Repeatedly,

add shortest edge whenever possible"

Kruskal’s Algorithm

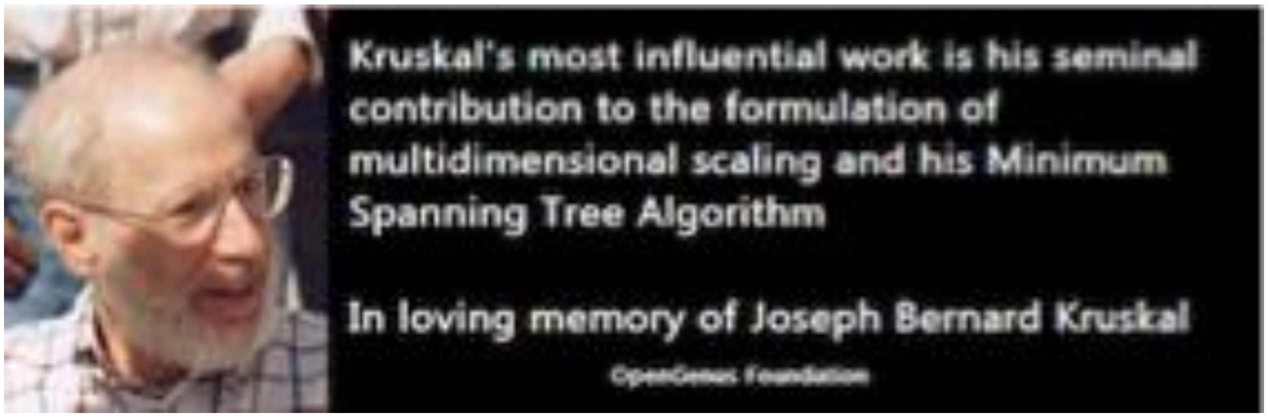
while there are unprocessed edges left
    pick an edge e with *minimum* cost
    if adding e to MST does not form a cycle
        add e to MST
    else
        throw e away

Data Structures

- How to pick edge with minimum cost?
  - Use a Priority Queue
- How to check if adding an edge can form a cycle?
  - Use a Disjoint Set

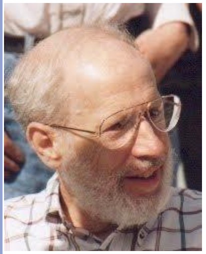
Data Structures needed

What about that *Cool* Kruskal's Algorithm?



Joseph B. Kruskal (1928 – 2010)
https://iq.opengenus.org/kruskal-minimum-spanning-tree-algorithm/

Joe, Clyde, Encounter @UIUC



I (LeongHW) don't know
Joseph Kruskal. Never met him.

But, I **do** know his nephew, Clyde.

Clyde Kruskal (now a professor in U. Maryland, College Park).
He was doing his post-doc at UIUC around 1980, when I was PhD student at UIUC.

Kruskal on Kruskal's Algorithm

https://www.youtube.com/watch?v=qOv8K-AJ7o0

Guan's MST Algorithm

- Guan Meigu (管梅谷), 1975

  Shandong Normal University

[Gua75] Guan Mei-Gu. The method of eliminating cycles for finding minimum spanning trees (in Chinese). *Shuxue de Shijian yu Renshi* 4 (1975).

管梅谷

Idea:

"In any cycle in the graph,

remove the longest edge."

LeongHW met Prof Guan Meigu in 1979 in a mathematics conference in Nantah (Nanyang University, 南洋大学)

Pic with Guan Mei-ko (管梅谷), 1979 @SG

Picture with Guan Meigu (管梅谷) at the **Franco-Southeast Asia Mathematics Conference**, @Nanyang University, May 1979.
(I was tutor with MU)

Prim-Dijkstra's MST Algorithm

- R. C. Prim, "Shortest Connection Networks and some Generalizations, Bell System Tech. J, 36, (1957), pp. 1389-1401.
- E. W. Dijkstra, "A note on two problems in connections with graphs," Numerical Math, 1, (1959), pp. 269-271.

Idea:

"Repeatedly, add shortest edge connecting a red vertex (in A) with a yellow vertices (in (V-A))"

Prim-Dijkstra's MST Algorithm

color all vertices yellow

color the root red

while there are yellow vertices

    pick an edge (u,v) such that

        u is red, v is yellow & cost(u,v) is min

    color v red

Included for your fun reading

Sollin's Algorithm,
Yao's algorithm,
(General idea only)

Sollin's MST Algorithm

- G. Sollin, "Probleme de l'arbre minimum",(unpublished manuscript prepared for C. Berge Paris' Seminar), 1961.
- G. Sollin, "Problemes de recherche operationelle," Report C.41, Meeting of Technical Directors, S.E.G. Paris, (1962), pp. 15-23.

Idea:

"Repeatedly,

add shortest edges to
each "component" in parallel; "

Sollin's Algorithm

T ← empty tree;

foreach vertex v in G,

    choose min edge e adjacent to v;
    Add e to tree T

Collapse/Merge connected components formed to get reduced graph G′

Repeat process on reduced graph G′

Andy Yao's MST Algorithm

- A. C.-C. Yao, "An O(*e* log log *v*) algorithm for finding minimum spanning trees", Information Processing Letters, 4 (1975), pp. 21-23.

Key Ideas:

"Improve Sollin's algorithm,

Use smarter priority queues."

Improve from O(e log v) to O(e log log v)

The O(e log log v) paper…

AN O(|E| log log |V|) ALGORITHM FOR FINDING MINIMUM SPANNING TREES *

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Received 30 December 1975, revised version received 9 June 1975

Minimum spanning tree, linear median finding algorithm

(2000)

1. Introduction

Given a connected, undirected graph $G = (V, E)$ and a function c which assigns a cost $c(e)$ to every edge $e \in E$, it is desired to find a spanning tree T for G such that $\Sigma_{e \in T}\, c(e)$ is minimal. In this note we describe an algorithm which finds a minimum spanning tree (MST) in $O(|E| \log\log |V|)$ time. Previously the best MST algorithms known have running time $O(|E| \times \log |V|)$ for sparse graphs [1], and more recently Tarjan [2] has an algorithm that requires $O(|E| \times \sqrt{\log |V|})$ time.

Our algorithm is a modification of an algorithm by Sollin [3]. His method works by successively enlarging components of the MST. In the first stage the minimum-cost edge incident upon each node of G is found.

plying the linear median-finding algorithm [4]. Having accomplished this, we follow basically Sollin's algorithm as outlined above. Note that the number of operations needed in this phase is now reduced to

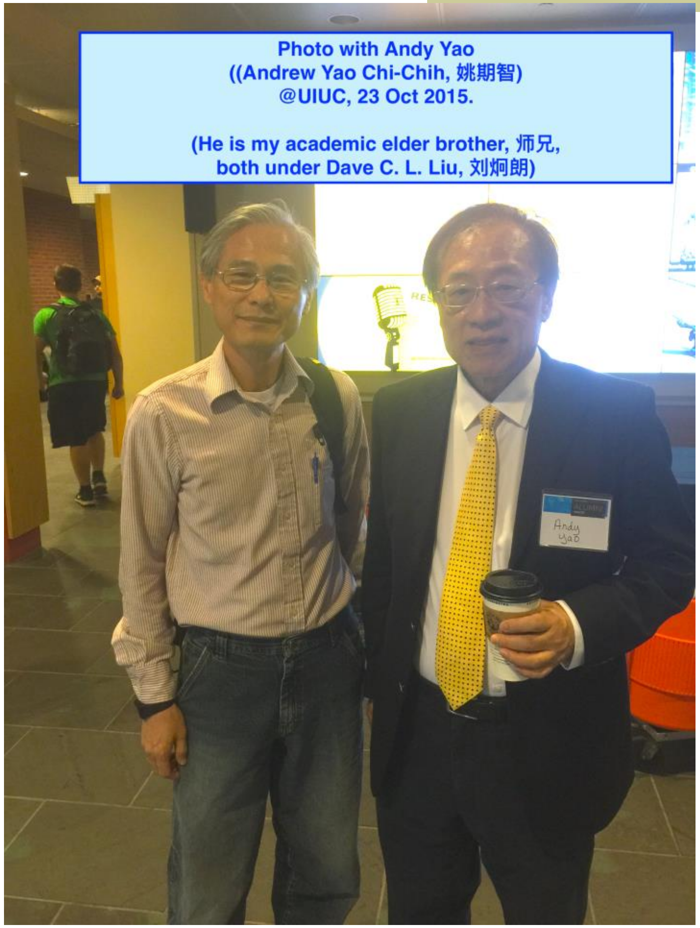
$$O\left(\frac{|E|}{k} \log |V|\right)$$

since only approximately $|E|/k$ edges have to be examined at each stage to find the minimum-cost edges incident with all the nodes. Therefore, the total number of operations required by our algorithm is

$$O\left(|E| \log k + \frac{|E|}{k} \log |V|\right),$$

which is $O(|E| \log\log |V|)$ if we choose k to be $\log |V|$.

Yao @UIUC (Oct-29, 2015)



https://cs.illinois.edu/news/alumnus-andrew-yao-sees-quantum-computing-next-great-science

Andy Yao @Tsinghua

Started “Yao Class” 姚班 @ 清华 Tsinghua

-- emulate US style undergraduate program in CS.

-- invited many visiting professors to Yao Class