



MA1301 INTRODUCTORY MATHEMATICS

TEST REVISION

September/October 2021 – Time allowed: 1 hour

Question 1 [10 marks]

(a) Given that $100x^2 - 90xy + 36y^2 = 1575$,

(i) find $\frac{dy}{dx}$ in terms of x and y , [2]

(ii) find the equations of the tangents parallel to the x – axis. [3]

Question 1 [continued]

- (b)** A curve is defined by the parametric equations

$$x = \frac{6k}{t^2}, \quad y = 12kt^2,$$

where $t > 0$ and k is a non-zero constant.

- (i)** Find $\frac{dy}{dx}$. [2]

- (ii)** Find the value of k for which $\frac{d^2y}{dx^2} = 2017$ when $t = 1$. [3]

Question 2 [10 marks]

Let $f(x) = 8x^3 - 36x^2 + 48x - 7$, where $x \in \mathbf{R}$.

(i) Find the intervals on which f is

(a) increasing (b) decreasing .

[2]

(ii) Find the coordinates and nature (local maximum or local minimum or saddle point) of the stationary points of the curve $y = f(x)$.

[4]

Question 2 [continued]

(iii) Find the interval on which the graph of f is

(a) concave down (b) concave up.

[2]

(iv) Write down the coordinates of the inflexion point of the curve $y = f(x)$.

[2]

Question 3 [16 marks]

(a)(i) Show that for any positive integer n ,

$$n(n+1)(n+2) - (n-1)(n)(n+1) = 3n(n+1). \quad [2]$$

(a)(ii) Use the result in (a)(i) to find the sum:

$$1017 \times 1018 + 1018 \times 1019 + 1019 \times 1020 + \dots + 2016 \times 2017. \quad [4]$$

Question 3 [continued]

(b) The first three terms of a geometric progression are $x + 5$, $x + 1$ and x . Calculate

(i) the value of x , [2]

(ii) the common ratio, [1]

(iii) the sum to infinity of the geometric progression. [2]

Question 3 [continued]

- (c) An arithmetic progression has 12 terms. The sum of the last six terms is three times the sum of the first 5 terms. Find the ratio of the sixth term to the fourth term. [5]

Question 4 [14 marks]

- (a) Given that the term which is independent of x in the binomial expansion of $\left(x^2 + \frac{k}{x}\right)^6$ is 240, calculate the possible values of k .

[6]

Question 4 [continued]

(b) Find the values of a, b, c and d if the binomial expansion, in ascending powers of x , up to

$$x^4 \text{ term, of } \sqrt{\frac{1+x}{1-x}} \text{ is } 1 + ax + bx^2 + cx^3 + dx^4. \quad [8]$$

Question 5 [10 marks]

An event organizer needs to build a fence to enclose a rectangular region of area 2400 square meters. As one side of the region is facing a main road, the organizer decides to make that side more attractive by using higher quality fencing that costs \$6 per meter. For the other three sides, he intends to use fencing that costs \$3 per meter. What dimensions of the rectangular region will minimize the cost of the fence?

END OF PAPER