Quiz 8 (Projection):

Kurosawa is tasked to 'prime' the Island of Eternal Twilight for the 30th Battle Royale on 2 January 2021. 'Prime' is the Ministry's lingo for secretly ridding a place of human presence. Folklore has it that the island was uninhabited until an all-male religious cult migrated there 16 years ago. Kurosawa has never been to the island, nor does he agree with the purpose the government has ascribed to it, but an order is an order.

Hunching over his study desk, his brows furrowed, Kurosawa stares at an unbound tome of facts and figures, a New Year's gift from the Census Bureau. In an impressive demonstration of operational scale and efficacy, the Census Bureau conducts a nationwide population census over the first week of each year. The Island of Eternal Twilight is always one of the few territories to have their part of the census completed on New Year's Day itself. Kurosawa's eyes are fixated on a certain Table 38, which gives some population data of the island:

Year	2009	2010	2011	2012	2013	2014	2015	2016
Population Size	9020	8050	7300	6630	5870	5240	4680	4210

He occasionally reminds himself how remarkable it is that the figure under the year 2016 is a result of census proceedings ending mere hours ago.

[Q1.] *No births, no migration. This place is in a lockdown. In that case, the average number of deaths per year over the years from 2009 to 2015 is _______. If we assume this number of deaths, as well as zero migration, occurs on the island in the upcoming years, the first year its population hits 0 in the census would be _______.*

(i) 601, 2024
(ii) 687, 2022
(iii) 601, 2025
(iv) 687, 2023

[Explanation:

From years 2009 to 2016 inclusive, there are 9020 – 4210 = 4810 deaths. The duration from New Year's Day of 2009 to New Year's Day of 2016 is a period of 2016 – 2009 = 7 years. Hence, the average number of deaths per year is 4810 / 7 = 687 (nearest whole number).
Assuming that there are 687 deaths per year starting from 2016,
the population size censured in 2017 (on New Year's Day) is 4210 – 687 = 3523,

the population size censured in 2018 is 3523 – 687 = 2836,
the population size censured in 2019 is 3523 – 687 = 2149,
the population size censured in 2020 is 3523 – 687 = 1462,
the population size censured in 2021 is 3523 – 687 = 775,
the population size censured in 2022 is 3523 – 687 = 88,
so that the first year that the population hits 0 in the census is 2023.]

Kurosawa heaves a deep sigh. Thinking about people dying is depressing for him. *I really shouldn't be doing this. I am not even that good with numbers.* But the government has no substitute for him, and it is growing more desperate by the day. In last year's population census, Kurosawa is one of only fourteen "exemplary citizens" in the country, who hold university degrees.

Average number of deaths does not seem too useful in population projection, though. From what he has read, **death rate** was what experts normally used, in the past. It is the **number of deaths per 1000 population in a given year**.

[Q2.] *From Table 38, the average death rate in the years 2009 to 2015 is _______ per 1000.* Kurosawa has a knack for calculating averages in his head.

(i) 101
(ii) 102
(iii) 103
(iv) 104

[Explanation:

In 2009, there are 9020 – 8050 = 970 deaths, hence the death rate is (970/9020) * 1000 = 108 (nearest whole number).

In 2010, there are 8050 – 7300 = 750 deaths, hence the death rate is (750/8050) * 1000 = 93.

In 2011, there are 7300 – 6630 = 670 deaths, hence the death rate is (670/7300) * 1000 = 92.

In 2012, there are 6630 – 5870 = 760 deaths, hence the death rate is (760/6630) * 1000 = 115.

In 2013, there are 5870 – 5240 = 630 deaths, hence the death rate is (630/5870) * 1000 = 107.

In 2014, there are 5240 – 4680 = 560 deaths, hence the death rate is (560/5240) * 1000 = 107.

In 2015, there are 4680 – 4210 = 470 deaths, hence the death rate is (470/4680) = 100.

Therefore the average death rate in the years 2009 to 2015 is (108 + 93 + 92 + 115 + 107 + 107 + 100)/7 = 103.]

For the purpose of population projection, we can assume this to be the constant death rate of the island for the upcoming years. Kurosawa paused. *Or, we can use another number in its place.* He proceeds with a series of computations: this time, with pen and paper. *Assuming that the death rate is a constant **d** for the 7 years from 2009 to 2015, then the population size in 2016 can be expressed as **9020 * (1 - d / 1000)^7**. This expression equals 4210 according to Table 38, so we obtain **d = 103**.* After comparing the two candidates for future death rate, he scribbles on his notebook, "ave. DR = 103".

If this average annual death rate of 103 persists until 2020, would it be high enough to rid the island of people by 2021? Kurosawa suspects not. In that case, he may have to start evicting people. He was hoping to avoid that. *The public backlash could be ugly, and I am only given a 6-hour window in the early morning of 1 January each year, **right before the census is taken**, to get people out of the island.*

For a while, Kurosawa paces back and forth the length of his room. *Labels maketh plan,* he chants silently. Then the notion of migration rate strikes him. Specifically, **emigration rate: the number of emigrants per 1000 existing residents in a given year**. *Projected emigration is the term. Calling it emigration, instead of eviction, seems to suggest it is a matter of choice.* He rummages through the heap of paper on his desk, and retrieves from the mess a sheet containing migration data of the offshore islands. A quick calculation gives 121 as the average annual migration rate from all offshore islands to the mainland.

[Q3.] *If we assume the death rate of the island is kept constant at 103 per 1000 from 2016, the emigration rate is 0 in 2016 and a constant 121 per 1000 from 2017 onwards, and there is no immigration, there would still be _______ people left on the island at the end of 2020.*

(i) 1283
(ii) 1459
(iii) 1627
(iv) 1851

[Explanation:
There are 4 years worth of non-zero migration and 5 years worth of non-zero deaths. Evaluate 4210 * (1 - 121 / 1000)^4 * (1 - 103 / 1000)^5 to get 1459.]
This is bad, Kurosawa frowned. The year 2021 is a landmark anniversary of the BR Act. The hectic schedule leading up to the year’s Battle Royale is bound to drain resources, and Kurosawa imagines being able to move no more than 200 people on 1 January 2021.

[Q4.] *Again, let's assume the death rate is constant at 103 per 1000 from 2016, and there is no migration other than the 'emigration' I shall set in motion on New Year's Day of each year. Now we want* ***at most*** *200 living men on the island at the end of 2020, so that all of them can be shipped out in the wee hours of 1 January 2021. If we also want the emigration rate to be* ***constant*** *from 2017 to 2020, then _____ per 1000 is the* ***minimum*** *emigration rate we should aim for.*

(i) 394
(ii) 466
(iii) 535
(iv) 606

[Rephrasing the problem, we want to have at most 200 people on the island on New Year's Day, 2021. Let m = emigration rate, d = death rate = 103, x = 1 - m / 1000, y = 1 - d / 1000 = 0.897.

We have the equation 4210 * (x^4) * (y^5) = 4210 * (0.897^5) * (x^4) = 200. Solving for x gives 0.534 (rounded down), which means m is 466.]

How should I account for the sudden wave of emigration from the island? Kurosawa muses on the question to no avail, before deciding to call it a night. With a slight recoil at the ankle, he springs onto his bed. He wishes, as he does every other night, for the good fortune of meeting his high school classmates. More than anything though, he wishes for comeuppance. And he knows, only the latter has a chance of being fulfilled.

"Which would you choose, a world with pyramids or a world without?" This is a quote from an old film about flying and dreaming, but mostly flying. Kurosawa watched it when he was younger, but only felt the significance of this line much later. Not everyone can afford to keep his hands clean in life.

"But still, I choose a world with pyramids in it."