Practice Exam

MA1521 Calculus for Computing

Time allowed: 2 hours

Answer all 10 questions. Each question carries 10 marks.

Show your steps clearly.

1. The plane that passes through the point $(1,-1,1)$ and contains the line with parametric equations $x = t, y = t/2, z = t/3$ has an equation of the form $9z = ax + by$. Determine the value of $a + b$.

2. Determine whether the following series converges or diverges. Justify your answers.

   (a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n}{2n+1}\right)^n$

   (b) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n \ln n}{n - \ln n}$

3. Find the number of saddle points of the function $f(x,y) = 3x^2y + x^2 - 6x - 3y - 2$.

4. Let $f(x,y,z) = \sqrt{x^2+y^2+z^2}$. It is known that $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} + \dfrac{\partial^2 f}{\partial z^2} = \dfrac{k}{f}$, where k is a positive integer. Determine the value of k.

5. Given $\displaystyle\int_{-2}^{0}\int_{0}^{x^2} e^{y-\frac{1}{3}y^{\frac{3}{2}}}\,dydx = ae^{\frac{4}{3}} + b$, where a and b are integers, determine the value of $a + b$.

6. Let R be the circular region bounded by the circle $x^2 + (y-1)^2 = 1$. It is known that

$$\iint_R \frac{dA}{(1+2x^2+2y^2)^2} = \frac{\pi}{a},$$

   where a is a positive integer. Determine the value of a.

   [Hint: Use polar coordinates and evaluate the resulting integral by means of the substitution $t = \tan\theta$].

7. Let a be a positive number. Let R be the smaller cap region of the circular disk $x^2 + y^2 \leq a^2$ cut off by the line $x + y = a$. Suppose $\iint_R xy^2\, dA = 5000$. Determine the value of a.

8. Let $y(x)$ be the solution of the differential equation

$$\frac{dy}{dx} + \frac{2}{x}y = \frac{y^3}{x^2}, \text{ with } x > 0, y > 0 \text{ and } y(1) = \sqrt{\frac{5}{7}}.$$

Find the value of $y(\frac{1}{10})$. Give your answer correct to two decimal places.

9. You started an experiment with 100 mg of a radioactive substance X which has a half life of 30 minutes. After 0.86 hour, you had m mg of X left. Find the value of m. Give your answer correct to the nearest integer.

10. The growth of the sandhill crane population follows a logistic model (the modified Malthus model) with a birth rate per capita of 10% per year. Initially at time $t = 0$ there were 1521 sandhill cranes. It is known that at time $t = 10$ years there were 2019 sandhill cranes. How many sandhill cranes will there be after a very long time? Give your answer correct to the nearest integer.