GER1000 AY2018/19 Semester 1

Quiz 7 and solutions (Uncertainty part 1)

1. This question is based on the game called Decisions for the Decade. You can check it out online if you are curious, but there is no need for other information to answer this question. In the game, a fair die is rolled every year, independently. The outcome represents the amount of rain in the year. “1” results in a drought, “6” results in a flood, and every other outcome is fine. A water supply crisis will be declared if drought occurs three years in a row. What is the probability that in three years, there is no water supply crisis?
   (a) 1/216
   (b) 3/216
   (c) 15/216
   (d) 125/216
   (e) 215/216

   Explanation: The complement of the event is there is a crisis, meaning three years of drought. This event has probability 1/6 x 1/6 x 1/6 = 1/216, by independence. So the answer is 1 – 1/216 = 215/216.

2. In a population, 52% are women and 48% are men. 40% of the women wear turtleneck sweaters to bed, and 40% of the men wear turtleneck sweaters to bed. A person is picked at random from the population. What is the probability that this person is a man and does not wear turtleneck sweaters to bed?
   (a) 0.192
   (b) 0.208
   (c) 0.288
   (d) 0.312

   Explanation:

   1. It is given in the question that the proportion of men in this population is 48%.

   2. Since 40% of men wear turtleneck sweaters to bed, this means 60% of men do not wear turtleneck sweaters to bed.

   3. Let A denote the event that the person is a man, and B denote the event that the person does not wear a turtleneck sweater to bed. To calculate P(A and B), we can use the multiplication rule. We know that P(A) = 0.48, and P(B | A) = 0.6, so the answer is 0.48x0.6 = 0.288.

3. In a given population, 40% of men wear turtleneck sweaters to bed, and 40% of women also wear turtleneck sweaters to bed. Suppose in this population 52% are women and 48% are men, and a person is picked at random from the population. Are the events “person picked is a woman” and “person picked wears turtleneck sweaters to bed” independent of each other?
   (a) Yes, the events are independent.
   (b) No, the events are not independent.

Explanation:

1. It is given in the question that the proportion of men in this population is 48%.

2. Since 40% of men wear turtleneck sweaters to bed, and 40% of women also wear turtleneck sweaters to bed, it follows that the proportion of people who wear turtleneck sweaters is also 40% in this population. This means 60% of people do not wear turtleneck sweaters to bed.

3. The proportion of men who don't wear turtleneck sweaters to bed is 0.48*0.6=28.8%.

4. Let A denote the event that the person is a man, and B denote the event that the person does not wear a turtleneck sweater to bed. Since this person is picked at random from the population, we have P(A) = 0.48, P(B) = 0.6, P(A and B) = 0.288. Hence P(A and B)=P(A)*P(B).

In general, if for two events A and B, we have P(A|B)=P(A|not B) then the events A and B must be independent.

4. You are invited to play the following game: a fair coin is flipped and a head wins $15 while a tail wins $3. It costs $10 to play the game. What is the average profit of playing this game?
   (a) $15
   (b) $9
   (c) $(-1)
   (d) $(-2)

Explanation: The profit earned if the coin come up heads is $15-$10=$5, while the profit earned if the coin came up tails is $3-$10=$(-7). Since each outcome has probability 0.5, we have the average value equals 0.5*$5+0.5*$(-7)=$(-1).

5. A fair coin is flipped three times. What is the probability of obtaining 2 or 3 heads?
   (a) 0.25
   (b) 0.375
   (c) 0.5
   (d) 0.66

Explanation: The events "2 heads" and "3 heads" denote the event of obtaining exactly 2 heads and the event of obtaining exactly 3 heads respectively. There are 3 ways of obtaining exactly 2 heads in three tosses: HHT where we obtain heads for the first two coins and tail for the third coin, HTH where we obtain tail for the second coin and heads for the other two, or THH where we obtain tail for the first coin and heads for the other two. These outcomes each have probability 0.5^3 = 0.125, since the tosses are independent. Hence P(2 heads) = P(HHT or HTH or THH) = 3*0.5^3 = 0.375 by the addition rule. Similarly, P(3 heads) = P(HHH) = 0.5^3 = 0.125. By the addition rule, we have P(2 heads or 3 heads) = P(2 heads) + P(3 heads) = 0.375 + 0.125 = 0.5.