Time allowed: 1 hour 15 mins.

The test is open book. You may use online graphic calculator.

Answer all 7 questions. Each question carries 10 marks.

Justify your answers and show your steps clearly.

1. Let a and b be integers. It is known that
$$\lim_{x \to 0} \left(\frac{\sin 2x}{x^3} + \frac{a}{3} + \frac{b}{x^2} \right) = 0.$$
   Determine the value of $a + b$.

2. Let $f(x) = \sqrt{3x + \sqrt{x}}$ for $x > 0$. An equation of the tangent line to the graph of $f(x)$ at $x = 1$ is of the form $ax - by + 9 = 0$, where a and b are integers. Find the value of $a + b$.

3. A robot X moves from left to right along the positive x-axis whose speed at time t is given by $5(1 - \frac{1}{t+1})$ meters per min. Another robot Y moves upward along the positive y-axis whose speed at time t is given by $12(1 - \frac{1}{t+1})$ meters per min. At time $t = 0$ min, they both start moving from rest at the origin O. The distance between the two robots at time $t = 12$ min is increasing at the rate of R meters per min. Determine the value of R.

4. A farmer wishes to employ tomato pickers to harvest 42500 tomatoes. Each picker can harvest 625 tomatoes per hour and is paid $6 per hour. In addition, the farmer must pay a supervisor $10 per hour and pay the union $10 for each picker employed. How many pickers should the farmer employ to minimize the cost of harvesting the tomatoes? Your answer should be a positive integer.

5. The curve $y^4 = 36(y^2 - x^2)$ has a shape like a figure 8. Find the area of the region enclosed by the two loops of the curve.

6. Let $f(x) = \frac{1}{10} \int_{\frac{\pi}{2}}^{x} \sqrt{2 + \sin t + \sin^2 t} \, dt.$

   Show that f^{-1} exists by proving that f is increasing on $\mathbb{R}$. Find also the value of $(f^{-1})'(0)$.

7. It is known that the improper integral $\int_{0}^{1} \frac{1}{x^2} - \frac{1}{(x+1)[\ln(x+1)]^2} \, dx = \frac{p - \ln 8}{\ln 4}.$

   Determine the value of p. Justify your answer.

-**END**-