NUS National University of Singapore

School of Computing

Map Colouring (for CS1231S)

An applications of Graph Colouring

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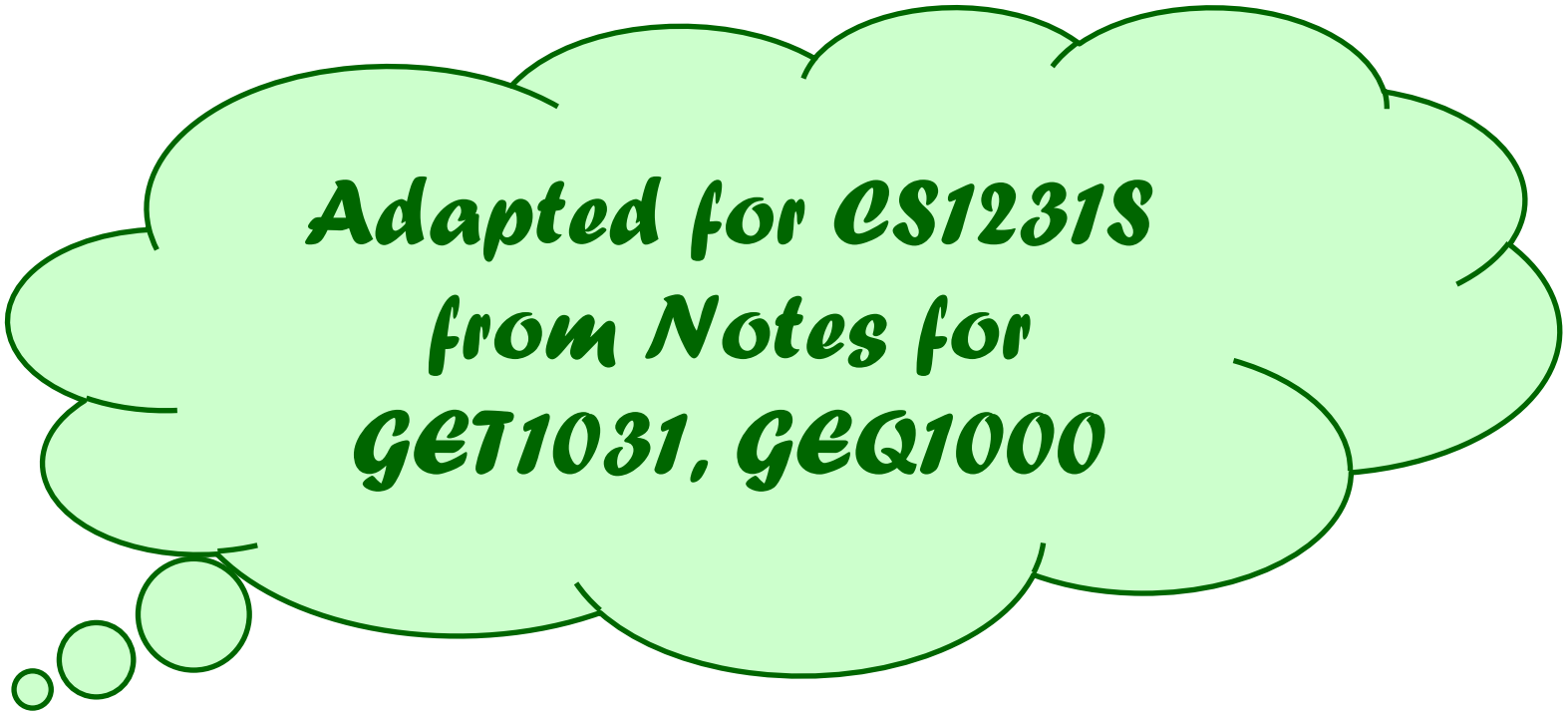

Adapted for CS1231S
from Notes for
GET1031, GEQ1000

The Map Colouring Problem (1)

We want to color countries (oceans, lakes, and islands) on a map so that any two adjacent countries have different colours.

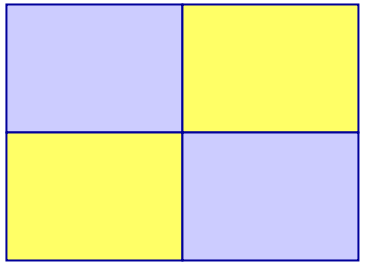
Q1: How many colours do we need?

Problem first posed in 1852
(by Guthrie → deMorgan)

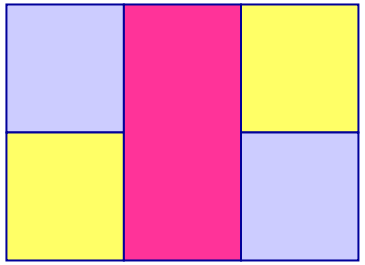
Note:
Two countries are adjacent if they share a common border.
A common point does not count as common border.

Map Colouring Problem: Some small examples

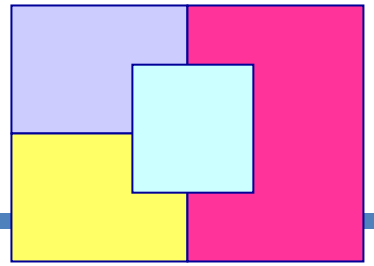
We want to color countries (oceans, lakes, and islands) on a map so that no two adjacent countries have the same colour.



Two colors



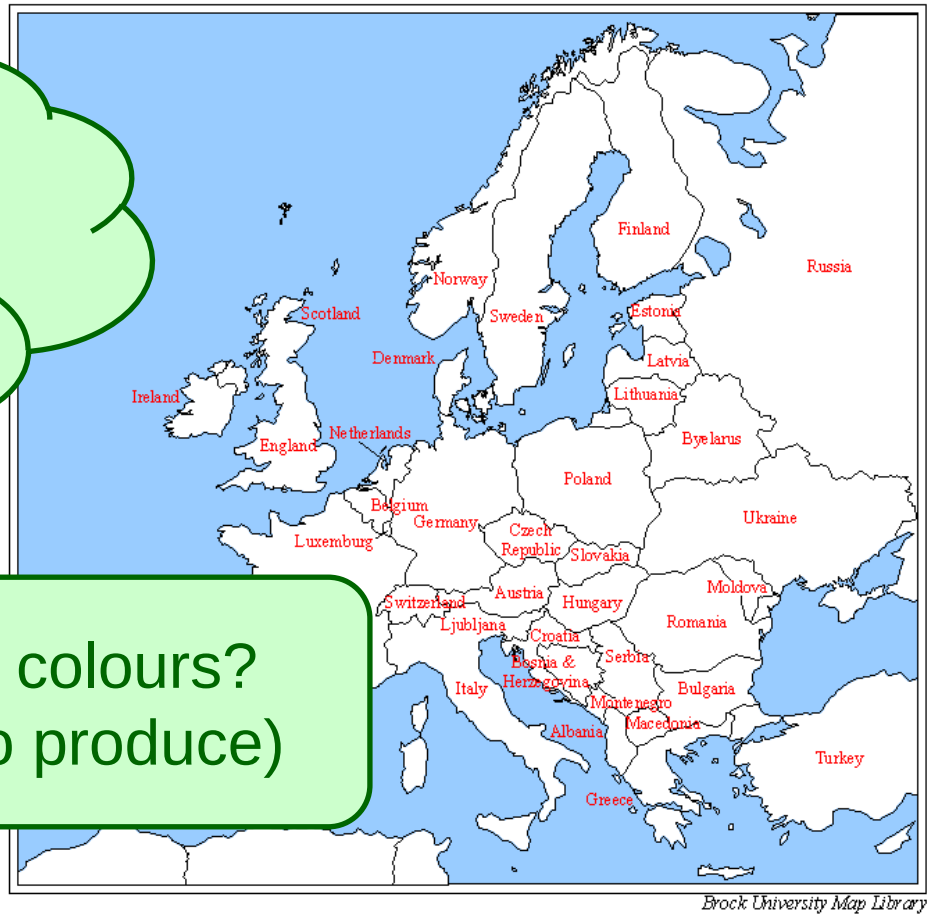
Three colors



Four colors

First, try some small examples

What about maps that need 5 colours?
(This turns out to be difficult to produce)

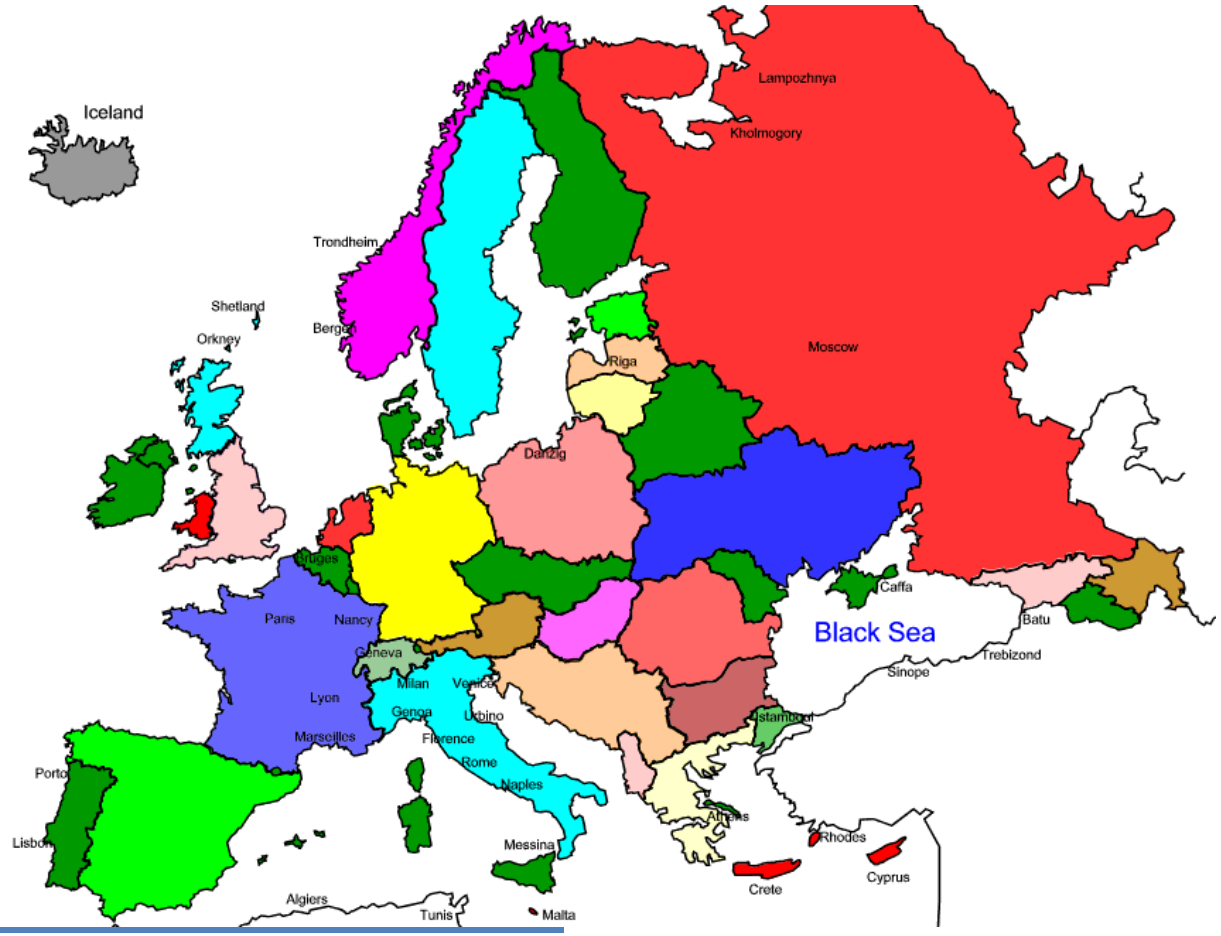


Map Colouring Problem: A Colouring

We want to color countries (oceans, lakes, and islands) on a map so that no two adjacent countries have the same colour.

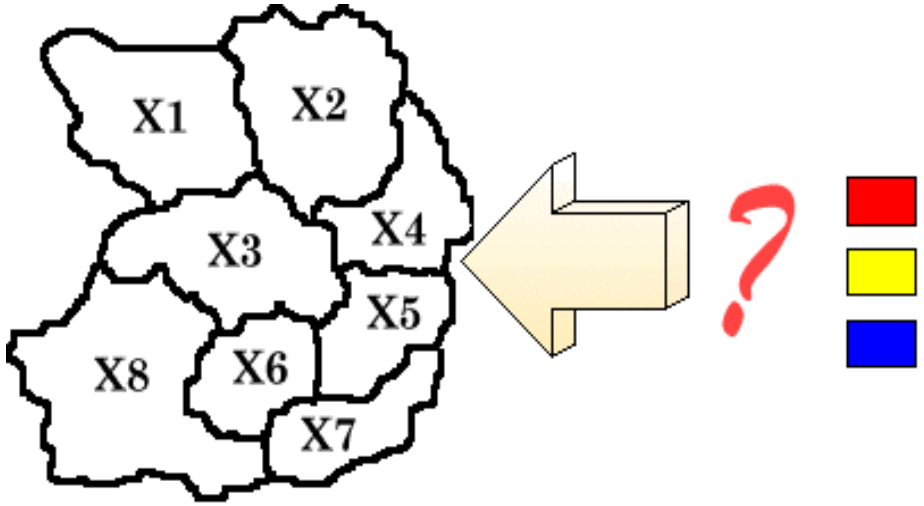
A legal colouring.
But uses >10 colours!

Four Color Conjecture:
Does 4 colours suffice?



Map Colouring (an example)

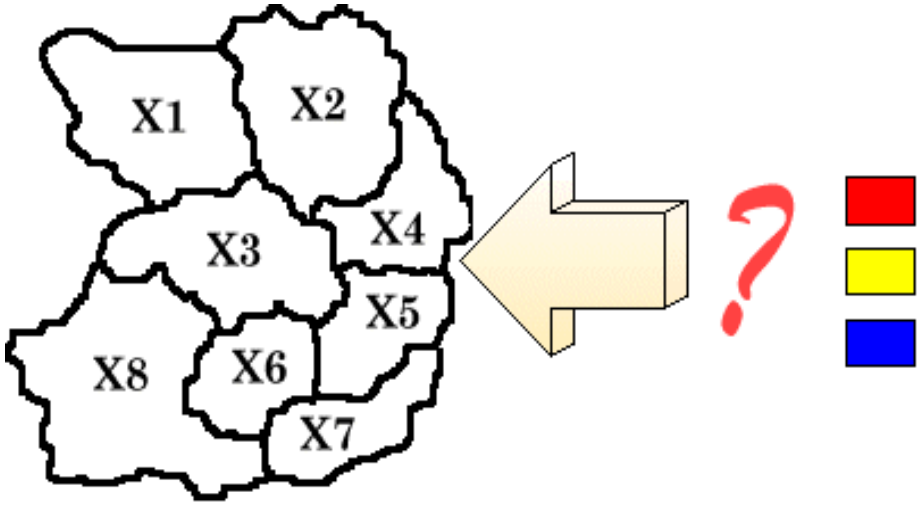
Given this map (with 8 countries), can we colour with 3 colours? (red, yellow, and blue)

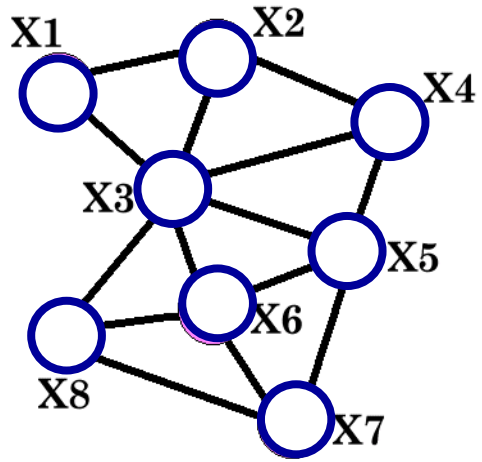


Direct Method:
Start colouring map,
Erase colours if we make mistake…

Indirect Method?
Model with Graph Colouring

Model with a Graph





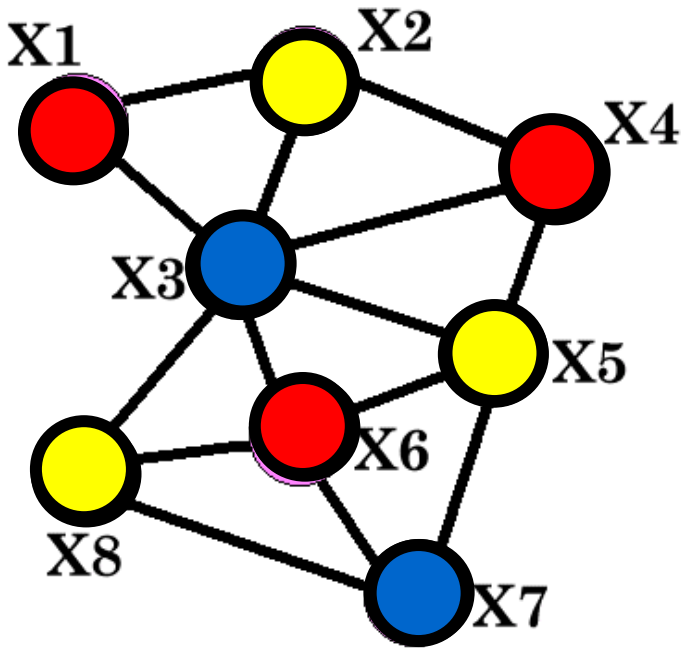
Graph model

How to use a Graph Model:
what are the vertices & edges?

Define
* every vertex "is a" country,
* edge {x,y} iff countries x and y share a common border

Note: A common point does not count as common border.

Colouring the Graph (1)



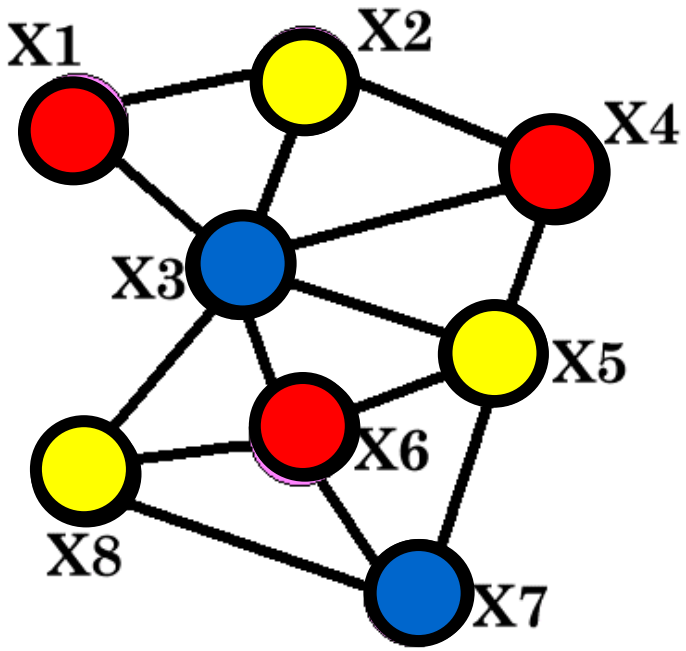
Graph coloured with 3 colours!

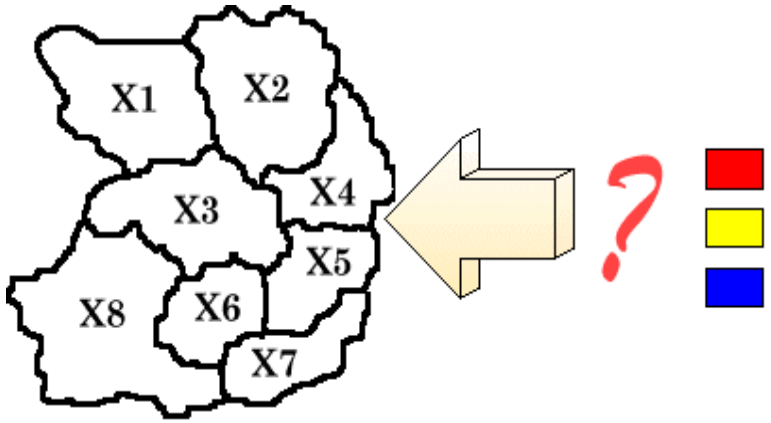
Use an informal algorithm (for now…)

Algorithm: (vertex-by-vertex)

1. First colour {X1,X2,X3}
2. Then colour X4,
3. Then X5,
4. Then X6, X7, X8…

Colouring the Graph (2)





Use an informal algorithm (for now…)

Algorithm: (vertex-by-vertex)

1. First colour {X1,X2,X3}
2. Then colour X4,
3. Then X5,
4. Then X6, X7, X8…

Colour the map accordingly.

The Four Colour Conjecture.

- Question: Does four colours suffice to colour all maps?

HISTORY:

1852 Conjecture (*Guthrie → DeMorgan*)

1878 Publication (*Cayley*)

1879-1891 …Many incorrect “proof”, but “5 suffices”

1913 …Reducibility, connexity (*Birkhoff*)

1969 Discharging (*Heesch*)

1976 Four Color Theorem (*Appel & Haken*) @UIUC

1995 Streamlining (*Robertson et al*)

2005 COQ proof (*Gonthier*)

Early controversies over 1976 Proof

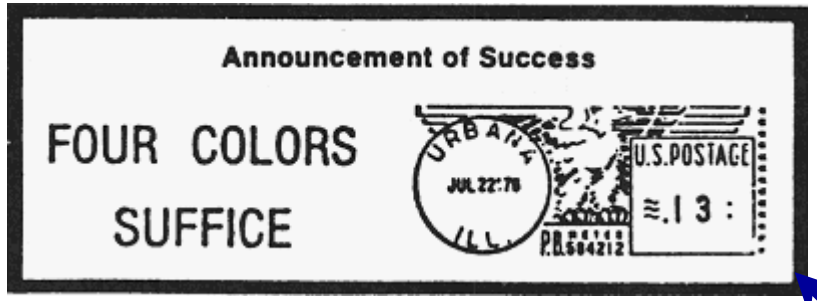
At the time, many mathematicians could-not
and would-not accept the proof of the
"Four Colour Theorem" by Appel and Haken, UIUC.

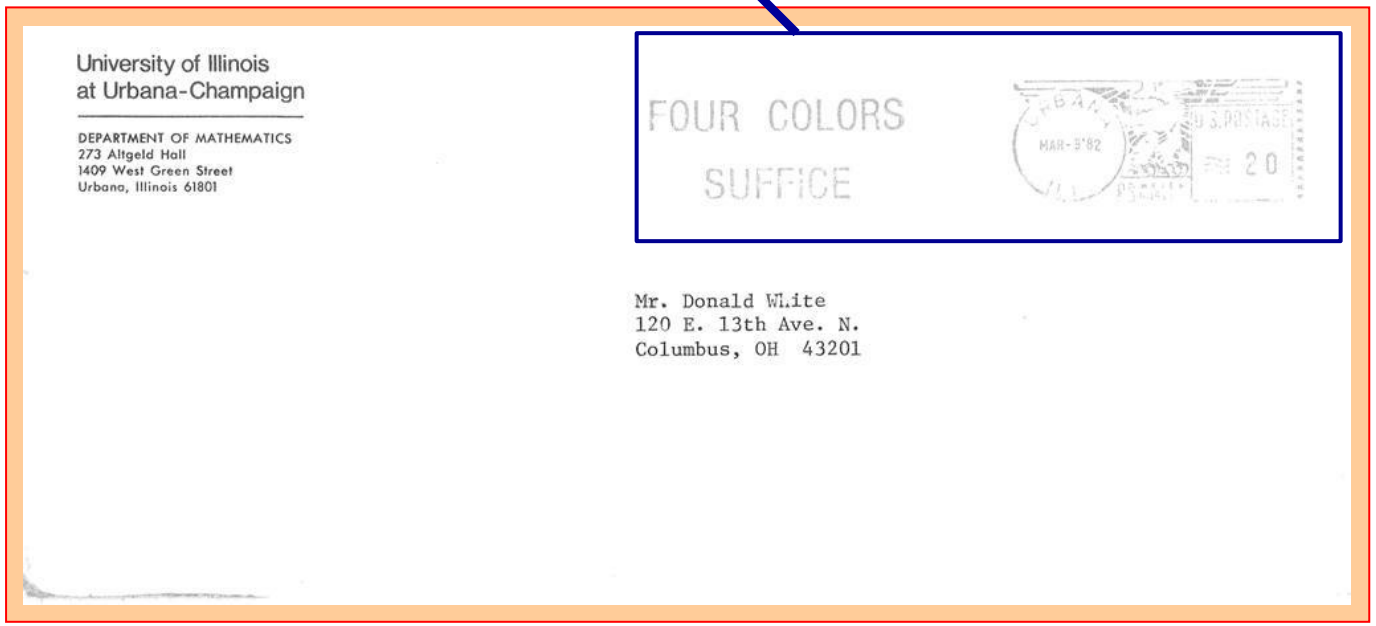
because…
it requires the help of computer programs
to check some tedious and very time-consuming parts
(taking thousand+ hours on the computer).

…they say that
those parts cannot be
mathematically verified.

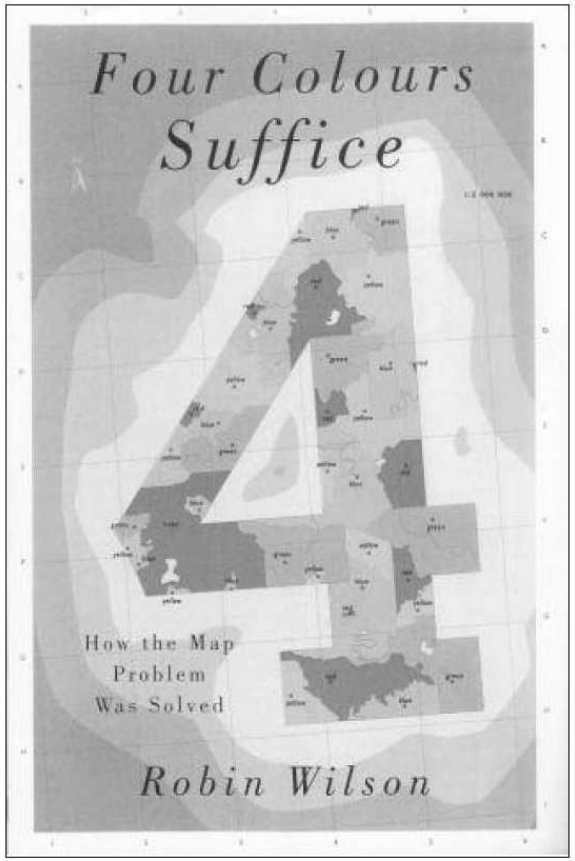
Note: Appel and Haken had the "luxury" of using the ILLIAC "supercomputers" built at the Digital Computer Lab of UIUC

“Four Colors Suffices” postage stamp @UIUC





Learning “Four Colour Theorem” @UIUC



Kenneth Appel and Wofgang Haken in the 1970s

In Fall 1979,
I took a course
MA313 *Combinatorics*
taught by Ken Appel

he spent 2 weeks on
Four Colour Theorem

Ken Appel & Wolfgang Haken @UIUC
(University of Illinois at Urbana-Champaign

Oct 2015, at UIUC Quad

A chance meeting with
Carole Appel
(wife of Ken Appel)
at UIUC in Oct 2015.

End of

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