NATIONAL UNIVERSITY OF SINGAPORE

MA1301 — INTRODUCTORY MATHEMATICS

2021 – 2022   SEMESTER I

29 November 2021,   17:00 − 19:00

INSTRUCTIONS TO CANDIDATES

1. Use A4 size paper and pen (blue or black ink) to write your answers.

2. Write down your student number clearly on the top left of every page of the answers.

3. Write on one side of the paper only. Start a new question in a new page. Write the question number and page number on the top right corner of each page (e.g., Q1Page1, Q1Page2, ..., Q2Page1, Q2Page2 ...).

4. This exam paper contains **EIGHT (8)** questions. Answer **ALL** questions.

5. The total mark for this paper is **ONE HUNDRED (100)**.

6. This is an **OPEN BOOK** examination.

7. You should lay out systematically the various steps in your calculations.

8. At the end of the test,

   (i) Scan or take pictures of your work (make sure the images can be read clearly).

   (ii) Merge all your images into one PDF file in correct order.

   (iii) Name the PDF file by "Student Number MA1301 Exam" (e.g., A1234567X MA1301 Exam.pdf).

   (iv) Upload your PDF into the LumiNUS Folder "Exam Submission Folder".

   (v) Review your submission to ensure that it is successful.

   (vi) The Exam Submission folder will close on 29 November 2021, 19:15. After the folder is closed, exam answers that are not submitted will not be accepted.

Question 1 [12 marks]

(a) A 62 m length of rope is cut into n pieces whose lengths are in arithmetic progression with a common difference of d m. Given that the lengths of the shortest and longest pieces are 0.5 m and 3.5 m respectively, find the values of n and d.

(b) The length of the sides of a triangle are in geometric progression and the longest side has a length of 36 cm. Given that the perimeter of the triangle is 76 cm, find the length of the shortest side.

Question 2 [12 marks]

(a) Find the first three terms in the expansion of $\left(x - \dfrac{2}{x} \right)^6$. Hence, find the coefficient of x^4 in the expansion of $\left(2 + 3x^2 \right) \left(x - \dfrac{2}{x} \right)^6$.

(b) Write down and simplify the expansion of $(1 - p)^5$. Use this result to find the expansion of $\left(1 - x - x^2 \right)^5$ in ascending powers of x as far as the term in x^3. Find the value of x which would enable you to estimate $(0.9899)^5$ from this expansion.

Question 3 [12 marks]

A curve has parametric equations $x = \dfrac{1}{1 + 4t^2}$ and $y = \tan^{-1}(2t)$, where $t \in \mathbb{R}$ is the parameter.

  (i) Find $\dfrac{dx}{dt}$.

  (ii) Find $\dfrac{dy}{dt}$.

  (iii) Show that for $t \neq 0$, $\dfrac{dy}{dx} = -\dfrac{1 + 4t^2}{4t}$.

  (iv) Find the coordinates of the two points on the curve at which the tangent is parallel to the line $5x + 4y + 3 = 0$.

Question 4 [12 marks]

A piece of wire 100 cm in length is divided into two parts. One part is bent to form an equilateral triangle of side x cm and the other part is bent to form a square of side y cm.

  (i) Express y in terms of x.

 (ii) Show that the total area enclosed by the two shapes, A cm^2, is given by $A = \dfrac{\sqrt{3}}{4}x^2 + \dfrac{(100 - 3x)^2}{16}$.

(iii) Calculate the value of x for which A has a stationary value, giving your answer to two decimal places. Determine whether this value of x makes A a maximum or a minimum.

Question 5 [12 marks]

Find the exact values of the following integrals.

(a) $\displaystyle\int_{1}^{2} \dfrac{6x}{\sqrt{7 - x^2}}\,dx$.

(b) $\displaystyle\int_{1}^{e} \dfrac{\sqrt{4 - 3\ln x}}{x}\,dx$.

Question 6 [16 marks]

Let $f(x) = 3x^2 + 1$ and $g(x) = 2x^2 + 5$.

  (i) The curve of $y = f(x)$ intersects the curve of $y = g(x)$ at the points $A(p, q)$ and $B(r, s)$, where $p < r$. Find the values of p, q, r and s.

 (ii) Sketch on a **single** diagram the graphs of $f(x)$ and $g(x)$. The region R is bounded by the curve of $y = f(x)$ and the curve of $y = g(x)$.

(iii) Find the area R.

(iv) Find the volume generated when R is rotated through $360°$ about the x-axis.

 (v) Find the volume generated when R is rotated through $360°$ about the y-axis.

Question 7 [12 marks]

Find the general solution of the differential equation

$$y^2 \csc x \frac{dy}{dx} = 4x\sqrt{1+y^3}.$$

Question 8 [12 marks]

A straight line L has vector equation
$$\vec{r} = -\vec{i} + \vec{j} + 4\vec{k} + \lambda(2\vec{i} - \vec{j} - 3\vec{k}),$$
where $\lambda \in \mathbb{R}$ is the parameter. The plane Π contains the point $(7, -2, 0)$ and is perpendicular to the vector $-3\vec{i} + 2\vec{j} + 7\vec{k}$.

  (i) Find the Cartesian equation of the plane Π.

 (ii) Determine the position vector of the point at which L intersects Π.

(iii) Calculate, to the nearest degree, the acute angle between L and Π.

END OF PAPER