NATIONAL UNIVERSITY OF SINGAPORE

CS3241 — Computer Graphics

AY2022/2023 Semester 1

Final Assessment

Time Allowed: 2 Hours

QUESTION PAPER

INSTRUCTIONS

1. This **QUESTION PAPER** contains **22 Questions** in **7 Sections**, and comprises **10** printed pages, including this page.
2. The **ANSWER SHEET** comprises **5** printed pages.
3. Use a pen or pencil to **write** your **Student Number** in the designated space on the front page of the **ANSWER SHEET**, and **shade** the corresponding circle **completely** in the grid for each digit or letter. DO NOT WRITE YOUR NAME!
4. You must **submit only** the **ANSWER SHEET** and no other documents. Do not tear off any pages from the ANSWER SHEET.
5. All questions must be answered in the space provided in the **ANSWER SHEET**; no extra sheets will be accepted as answers.
6. Write legibly with a **pen** or **pencil** (do not use red color). Untidiness will be penalized.
7. For **multiple choice questions (MCQ)**, **shade** in the **circle** of the correct answer **completely**.
8. The full score of this assessment is **90** marks.
9. This is an **Open-Book** assessment.
10. You are allowed to use an approved **calculator**.

Section A [13 marks]

(1) [5 marks]

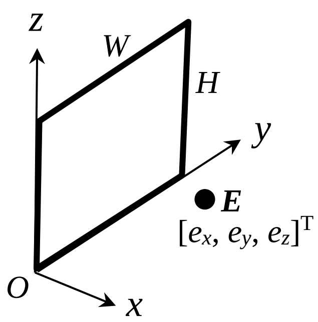
Consider the following view transformation matrix:

$$\mathbf{M}_{\text{view}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & -9 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Write a `gluLookAt()` function call that will produce the above view transformation matrix.

(2) [8 marks]

A rectangle, of size W x H, is located in the **world space** as shown in the diagram. The **center** of the rectangle is located at $[0, W/2, H/2]^T$. A **viewpoint** $\boldsymbol{E}$ is located at world coordinates $[e_x, e_y, e_z]^T$, where $e_x > 0$. You are to complete the following program to set up the view transformation and the perspective view volume so that the rectangle appears to cover the entire viewport *exactly*. The rectangle's corner at the world origin O should appear at the bottom-left corner of the viewport. You should set the near plane exactly on the rectangle, and the far plane exactly 10 units behind the rectangle.




```
...
double W, H;         // rectangle size.
double ex, ey, ez;   // viewpoint world coordinates.

glViewport( 0, 0, vp_width, vp_height );
...
glMatrixMode( GL_PROJECTION );
glLoadIdentity();

glFrustum( ??? );  // set the view volume.

glMatrixMode( GL_MODELVIEW );
glLoadIdentity();

gluLookAt( ??? );  // set the view transformation.
```

Complete the calls to `glFrustum()` and `gluLookAt()`.

Section B [8 marks]

(3) [5 marks] (MCQ)

Consider that the center of each pixel has integer *x*-coordinate and *y*-coordinate. We want to scan convert a **45-degree arc** of a circle of radius 99 and centered at (200, 200). The arc starts at pixel location (200, 299) and extends clockwise to the other end point 45 degrees away. Assuming the curve is drawn as thin as possible and not broken, what is the **number of fragments** (pixels that are turned on) that are produced for this arc? This number includes the two fragments at the two end points of the curve. (Note: $\cos 45° = 1/\sqrt{2} \approx 0.707107$)

- **A.** 25
- **B.** 50
- **C.** 65
- **D.** 71
- **E.** 75
- **F.** 99
- **G.** 100
- **H.** 101

(4) [3 marks] (MCQ)

Now, we want to scan convert the **entire circle**, which has a radius of 99 and is centered at (200, 200). Let N be the correct number of fragments produced for the 45-degree arc in the preceding question, what is the **number of fragments** (pixels that are turned on) that are produced for the full circle?

- **A.** $4N$
- **B.** $4(N-2)$
- **C.** $4(N-1)$
- **D.** $4(N+1)$
- **E.** $8N$
- **F.** $8(N-1)$
- **G.** $8(N-1)+4$
- **H.** $8(N+1)$

Section C [12 marks]

(5) [5 marks]

Consider the following **model-view matrix**:

$$\begin{bmatrix} s_x & 0 & 0 & t_x \\ 0 & s_y & 0 & t_y \\ 0 & 0 & s_z & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

where s_x, s_y, and s_z are non-zero scaling factors. What is the **matrix** used for **transforming normal vectors**?

(6) [4 marks] (MCQ)

Suppose the lighting computation in OpenGL uses the basic Phong Illumination Equation $I_{phong} = I_a k_a + I_p k_d (N \bullet L) + I_p k_s (R \bullet V)^n$, where $R = 2(N \bullet L)N - L$. Assuming that normal vectors are not automatically renormalized to unit vectors after they have been transformed. Suppose the light reflection from a 3D object looks normal when no scaling is applied to the geometry of the 3D object, which of the followings most accurately describes the change in the reflection from the 3D object when it is **scaled smaller**?

A. There is no change to the reflection.
B. The **diffuse** and **specular** reflection appear **brighter**.
C. The **diffuse** and **specular** reflection appear **darker**.
D. The **diffuse** reflection appears **brighter**, and **specular** reflection remains the **same**.
E. The **diffuse** reflection appears **darker**, and **specular** reflection remains the **same**.
F. The **specular** reflection appears **brighter**, and **diffuse** reflection remains the **same**.
G. The **specular** reflection appears **darker**, and **diffuse** reflection remains the **same**.
H. None of the other options is the correct answer.

(7) [3 marks] (MCQ)

Suppose polygons are rendered using **phong shading**. The basic Phong Illumination Equation (PIE) is used for all the lighting computations. Suppose one of the polygons has 10 vertices and is covering 100 fragments, how many times is the PIE evaluated to render this polygon?

A. 0
B. 1
C. 10
D. 90
E. 100
F. 110
G. 1000
H. The number of pixels in the viewport.

Section D [10 marks]

(8) [3 marks] (MCQ)

Suppose in a mipmap, each texel in a mipmap level is the average of the corresponding 2x2 texels in the previous level, and all these 2x2-texel regions do not overlap. Given a texture image of size 4096 x 4096 texels, we want to create a **mipmap** from it. What would be the **number of levels** (including the original texture image) in the mipmap?

A. 11
B. 12
C. 13
D. 121
E. 144
F. 169
G. 1024
H. 2048

(9) [5 marks]

Consider the mipmap from the preceding question, whose Level 0 has a size of 4096 x 4096 texels. The fragment *F* at window coordinates (20, 20) has texture coordinates (0.3, 0.6). Given that the fragment at window coordinates (21, 20) has texture coordinates (0.4, 0.6), and the fragment at window coordinates (20, 21) has texture coordinates (0.3, 0.7), what would be the **exact ideal mipmap level** to texture map fragment *F*? This mipmap level can be **non-integer**, and you must round your answer to **2 decimal places**. You may use the formula $\log_a(x) = \log_b(x) / \log_b(a)$.

(10) [2 marks]

Consider the fragment *F* from the preceding question. Suppose the integer mipmap level closest to the ideal mipmap level is actually used to texture map fragment *F*, what texture coordinates should be used to read from that mipmap level for fragment *F*?

Section E [15 marks]

Suppose there is an **opaque sphere** in an **enclosed** environment. All the other surfaces of the environment are **opaque** and have materials that have only **diffuse** component (i.e. $k_d > 0$, $k_r = 0$ and $k_{rg} = 0$). However, the sphere's material has both **diffuse and specular** components (i.e. k_d, k_r, and k_{rg} are all greater than 0). There are **three point light sources** in the scene.

We want to render a **200x200 pixels image** of the scene using **Whitted Ray Tracing**, with **two levels of recursion**.

Assume the camera is within the environment but outside the sphere, and the **sphere occupies 5,000 pixels** (i.e. number of primary rays that hit the sphere directly) in the rendered image.

In addition, at any surface point where lighting computation is to be performed, a **shadow ray** is always shot towards each light source even when $N \bullet L \leq 0$ (where N is the normal vector at the surface point and L is the vector towards the light source).

(11) [2 marks] (MCQ)

What is the total number of **primary rays** shot?

- **A.** 5,000
- **B.** 40,000
- **C.** 35,000
- **D.** 80,000
- **E.** 120,000
- **F.** 70,000
- **G.** 105,000
- **H.** 10,000

(12) [3 marks] (MCQ)

What is the total number of **shadow rays** shot from those points intersected by the primary rays?

- **A.** 105,000
- **B.** 15,000
- **C.** 40,000
- **D.** 5,000
- **E.** 35,000
- **F.** 80,000
- **G.** 70,000
- **H.** 120,000

(13) [3 marks] (MCQ)

How many **first-level** secondary **reflection rays** are spawned? First-level secondary reflection rays are spawned from surface points hit by the primary rays.

- **A.** 0
- **B.** 70,000
- **C.** 40,000
- **D.** 35,000
- **E.** 10,000
- **F.** 80,000
- **G.** 5,000
- **H.** 15,000

(14) [3 marks] (MCQ)

How many **second-level** secondary **reflection rays** are spawned? Second-level secondary reflection rays are spawned from surface points hit by the first-level secondary reflection rays.

- **A.** 0
- **B.** 5,000
- **C.** 40,000
- **D.** 35,000
- **E.** 10,000
- **F.** 80,000
- **G.** 70,000
- **H.** 15,000

(15) [4 marks] (MCQ)

How many **rays** are shot **altogether**? This includes all the primary rays, reflection rays and shadow rays.

- **A.** 80,000
- **B.** 120,000
- **C.** 300,000
- **D.** 180,000
- **E.** 480,000
- **F.** 160,000
- **G.** 200,000
- **H.** 440,000

Section F [10 marks]

(16) [5 marks] (MCQ)

Consider the **ray** (in parametric representation)

$$\boldsymbol{P}(t) = \begin{bmatrix} 3 \\ 5 \\ 4 \end{bmatrix} + t \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}$$

and the **sphere *S*** (in implicit representation)

$$(x - 14)^2 + (y - 8)^2 + (z - 21)^2 - 5^2 = 0$$

Suppose we are only concerned with finding the value of t at the first intersection between the ray $\boldsymbol{P}(t)$ and the sphere $\boldsymbol{S}$. This is equivalent to finding the value of t at the first intersection between the ray $\boldsymbol{R}(t)$ and the sphere

$$x^2 + y^2 + z^2 - 5^2 = 0$$

Which of the followings is the correct $\boldsymbol{R}(t)$?

- **A.** $\boldsymbol{R}(t) = [0\ \ 0\ \ 0]^T + t\,[4\ \ 2\ \ 6]^T$
- **B.** $\boldsymbol{R}(t) = [-3\ \ -5\ \ -4]^T + t\,[4\ \ 2\ \ 6]^T$
- **C.** $\boldsymbol{R}(t) = [14\ \ 8\ \ 21]^T + t\,[4\ \ 2\ \ 6]^T$
- **D.** $\boldsymbol{R}(t) = [-14\ \ -8\ \ -21]^T + t\,[4\ \ 2\ \ 6]^T$
- **E.** $\boldsymbol{R}(t) = [11\ \ 3\ \ 17]^T + t\,[4\ \ 2\ \ 6]^T$
- **F.** $\boldsymbol{R}(t) = [-11\ \ -3\ \ -17]^T + t\,[4\ \ 2\ \ 6]^T$
- **G.** $\boldsymbol{R}(t) = [17\ \ 13\ \ 25]^T + t\,[4\ \ 2\ \ 6]^T$
- **H.** $\boldsymbol{R}(t) = [-17\ \ -13\ \ -25]^T + t\,[4\ \ 2\ \ 6]^T$

(17) [5 marks] (MCQ)

Using Whitted Ray Tracing. Consider a **shadow ray** $\boldsymbol{P}(t) = \boldsymbol{O} + t\,\boldsymbol{D}$ that is shot from a surface point at $\boldsymbol{O}$ towards a point light source at $\boldsymbol{S}$. The **magnitude** of vector $\boldsymbol{D}$ is 2, and the **distance** between points $\boldsymbol{O}$ and $\boldsymbol{S}$ is 100. Assuming that all surfaces are opaque, what is the range of values of t in which an intersection point $\boldsymbol{P}(t)$ is considered blocking the light from reaching the surface point at $\boldsymbol{O}$? (Note: ε is a very small positive number.)

- **A.** $\varepsilon < t$
- **B.** $\varepsilon < t < 1 - \varepsilon$
- **C.** $\varepsilon < t < 50 - \varepsilon$
- **D.** $\varepsilon < t < 100 - \varepsilon$
- **E.** $2 + \varepsilon < t < 100 - \varepsilon$
- **F.** $t > 1 + \varepsilon$
- **G.** $t > 50 + \varepsilon$
- **H.** $t > 100 + \varepsilon$

Section G [22 marks]

(18) [4 marks]

Consider the following **parametric 2D curve segment** $\mathbf{q}(w)$, for $-5 \le w \le 5$:

$$\mathbf{q}(w) = \begin{bmatrix} w \\ 4w^2 \end{bmatrix}$$

Let $\mathbf{p}(u) = [x(u), y(u)]^T$, for $0 \le u \le 1$, represent the same parametric 2D curve segment as $\mathbf{q}(w)$. Write the expressions $x(u)$ and $y(u)$. Your answer must be in the following form:

$x(u)$ = *expression in terms of u*
$y(u)$ = *expression in terms of u*

(19) [4 marks]

Consider the following **parametric cubic polynomial 2D curve segment** for $0 \le u \le 1$:

$$\mathbf{p}(u) = \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 9 \\ 3 \end{bmatrix} u + \begin{bmatrix} 18 \\ 9 \end{bmatrix} u^2 + \begin{bmatrix} 27 \\ 54 \end{bmatrix} u^3$$

Write the coordinates of the **four control points**, $\mathbf{p}_0$, $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$, for the **cubic interpolating curve segment** that corresponds to $\mathbf{p}(u)$. The control points must be given in the right order such that $\mathbf{p}_0$ corresponds to $\mathbf{p}(0)$, and so on. You must write your answer in the following form:

$\mathbf{p}_0$ = (*x-coordinate*, *y-coordinate*)
$\mathbf{p}_1$ = (*x-coordinate*, *y-coordinate*)
$\mathbf{p}_2$ = (*x-coordinate*, *y-coordinate*)
$\mathbf{p}_3$ = (*x-coordinate*, *y-coordinate*)

(20) [5 marks]

Consider the following **parametric cubic polynomial 2D curve segment** for $0 \le u \le 1$:

$$\mathbf{p}(u) = \begin{bmatrix} 7 \\ 4 \end{bmatrix} + \begin{bmatrix} 9 \\ 12 \end{bmatrix} u + \begin{bmatrix} 6 \\ 9 \end{bmatrix} u^2 + \begin{bmatrix} 9 \\ 6 \end{bmatrix} u^3$$

Write the coordinates of the **four control points**, $\mathbf{p}_0$, $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$, for the **cubic Bézier curve segment** that corresponds to $\mathbf{p}(u)$. The control points must be given in the right order such that $\mathbf{p}_0$ corresponds to $\mathbf{p}(0)$, and so on. You must write your answer in the following form:

$\mathbf{p}_0$ = (*x-coordinate*, *y-coordinate*)
$\mathbf{p}_1$ = (*x-coordinate*, *y-coordinate*)
$\mathbf{p}_2$ = (*x-coordinate*, *y-coordinate*)
$\mathbf{p}_3$ = (*x-coordinate*, *y-coordinate*)

(21) [5 marks]

Consider the following **two parametric cubic polynomial 2D curve segments** for $0 \le u \le 1$:

$$\mathbf{p}(u) = (1-u)^3 \begin{bmatrix}1\\2\end{bmatrix} + 3u(1-u)^2 \begin{bmatrix}2\\5\end{bmatrix} + 3u^2(1-u) \begin{bmatrix}6\\7\end{bmatrix} + u^3 \begin{bmatrix}7\\4\end{bmatrix}$$

$$\mathbf{q}(u) = (1-u)^3 \begin{bmatrix}15\\6\end{bmatrix} + 3u(1-u)^2 \begin{bmatrix}17\\9\end{bmatrix} + 3u^2(1-u) \begin{bmatrix}12\\12\end{bmatrix} + u^3 \begin{bmatrix}17\\18\end{bmatrix}$$

Suppose we want to use a **cubic Bézier curve segment** $\mathbf{s}(u)$ to join to curve $\mathbf{p}(u)$ and $\mathbf{q}(u)$, such that $\mathbf{p}(1) = \mathbf{s}(0)$ and $\mathbf{s}(1) = \mathbf{q}(0)$. We also require G^1 continuity at all joint points, such that $\mathbf{s}'(0) = 2\mathbf{p}'(1)$ and $\mathbf{s}'(1) = 2\mathbf{q}'(0)$.

Write the coordinates of the **four control points**, $\mathbf{s}_0$, $\mathbf{s}_1$, $\mathbf{s}_2$ and $\mathbf{s}_3$, for the **cubic Bézier curve segment** that corresponds to $\mathbf{s}(u)$. The control points must be given in the right order such that $\mathbf{s}_0$ corresponds to $\mathbf{s}(0)$, and so on. You must write your answer in the following form:

$\mathbf{s}_0$ = (*x-coordinate*, *y-coordinate*)
$\mathbf{s}_1$ = (*x-coordinate*, *y-coordinate*)
$\mathbf{s}_2$ = (*x-coordinate*, *y-coordinate*)
$\mathbf{s}_3$ = (*x-coordinate*, *y-coordinate*)

(22) [4 marks]

Consider the **cubic Bézier 2D curve segment** $\mathbf{p}(u)$, for $0 \le u \le 1$, whose control points are

$\mathbf{p}_0 = (0, 0)$
$\mathbf{p}_1 = (0, 32)$
$\mathbf{p}_2 = (32, 32)$
$\mathbf{p}_3 = (32, 0)$

Suppose $\mathbf{p}(u)$ is split into two segments at $u = 0.5$, where the sub-segment $\mathbf{s}(t)$, for $0 \le t \le 1$, is the same as $\mathbf{p}(u)$ for $0.5 \le u \le 1$.

Write the coordinates of the **four control points**, $\mathbf{s}_0$, $\mathbf{s}_1$, $\mathbf{s}_2$ and $\mathbf{s}_3$, for the **cubic Bézier curve segment** that corresponds to $\mathbf{s}(t)$. The control points must be given in the right order such that $\mathbf{s}_0$ corresponds to $\mathbf{s}(0)$, and so on. You must write your answer in the following form:

$\mathbf{s}_0$ = (*x-coordinate*, *y-coordinate*)
$\mathbf{s}_1$ = (*x-coordinate*, *y-coordinate*)
$\mathbf{s}_2$ = (*x-coordinate*, *y-coordinate*)
$\mathbf{s}_3$ = (*x-coordinate*, *y-coordinate*)

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