

**NATIONAL UNIVERSITY OF SINGAPORE**  
**SCHOOL OF COMPUTING**  
SOLUTIONS FOR EXAMINATION FOR  
Semester 2, 2006/2007

**CS3243 - Foundations of Artificial Intelligence**

24 April 2007

Time Allowed: 2 Hours

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**INSTRUCTIONS TO CANDIDATES**

1. These solutions contains **SIX (6) questions** questions and comprises **TEN (10) pages**.
2. Weightage of questions in given in square brackets. The maximum attainable score is 100.
3. You are allowed to bring **TWO** double-sided A4 sheets of notes for this exam.
4. You are allowed to use a calculator in this exam, but laptops are not allowed.
5. Write all your answers in the space provided in this

### Question 1 : Do you Understand the Key Concepts? [15 marks]

CS3243 is a survey course of AI techniques. As a warm-up exercise for this exam, the following are short structured questions to test your understanding of some key concepts that were introduced in CS3243.

**A.** In principle, any AI problem can be solved by creating a lookup table and reading off the table. Why then do we need to learn and apply techniques in Artificial Intelligence? [2 marks]

*Because for most practical problems a lookup table is infeasible in terms of computational or storage complexity. Sometimes we may not even know how to build the lookup table, c.f. learning problems.*

**B.** There are different variants of searches, i.e. DFS, BFS, IDS, etc. What is it that gives rise to the differences between these searches? [2 marks]

*The key difference is in the order of expansion for the nodes in the fringe set.*

*Order of traversal of the tree was also accepted.*

*Even if the wrong answer is given here, but order of expansion is mentioned elsewhere in the subsequent parts, credit will still be given since the purpose of this question is to test that students understand that order of expansion is a key concept in search.*

**C.** Suppose you have an agent that executes a search and you find that it runs too slowly. Describe two possible ways to improve the performance of the agent and explain how they are helpful. Note: you may assume that the agent is **not** a strategic (game-playing) agent. [3 marks]

*Any two reasonable search-related answers is okay, e.g. pruning, use a heuristic, or try a different type of search.*

**D.** PSA manages the port operations for Singapore. PSA has a number of berths and there is a fleet of ships that must load and unload cargo at these berths. Suppose you need to come up with a schedule that ensures that all the cargo gets loaded onto the ships within a fixed interval of time (perhaps one day). Which AI technique will you use to solve this problem? Justify your answer. [3 marks]

*Any reasonable answer is okay, e.g. local search, because it is fast for solving constraint satisfaction problems.*

**E.** Consider the cargo loading scheduling problem in Part (D) above. Suppose you need to come up with a schedule that ensures that all the cargo gets loaded onto the ships in the shortest possible time. Which technique will you use to solve this problem? Justify your answer. [2 marks]

*Student needs to demonstrate an understanding of the need for **completeness** in search to mind the optimal schedule.*

**F.** What in your opinion is the most important concept or technique that you learnt in CS3243? Explain. [3 marks]

*Free for all. Student just needs to convince the examiner that he has learnt something in CS3243 and not make an untrue statement here. :-)*

## Question 2 : Prospecting for Gold [17 marks]

You are the owner of a geo-survey company and you have been hired to prospect for gold in Indonesia. Your task is to collect rock samples from three sites (in any order) and bring the samples back to the laboratory for analysis.

Assume that there are five locations of interest: *laboratory*, *town*, *site 1*, *site 2* and *site 3* and that you can travel between any pairs of these locations and you know the time it takes to traverse between them. Each of these locations is associated with an action, i.e. *go-laboratory*, *go-town*, *go-site-1*, *go-site-2* and *go-site-3* respectively. Assume also that you are able to carry as many rock samples as you wish and so there is no need to return to the laboratory to unload between sites.

Your goal is to find a sequence of actions that will complete your task in the shortest amount of time, starting from the laboratory.

**A.** Formulate this problem as a search problem by specifying a representation for the search space, initial state, path-cost function, and goal test. You may assume that the world dynamics are deterministic. [6 marks]

Representation: *Tuple (location, has\_sample1, has\_sample2, has\_sample3)*

Initial state: *(laboratory, false, false, false)*

Path-cost function: *Sum of time taken to travel between each pair of locations.*

Goal test: *(laboratory, true, true, true)*

**B.** Which search technique would be most appropriate? Why? If your search technique requires a heuristic, give an appropriate one. [6 marks]

*A\*, because we want to find an optimal solution. Heuristic = (Number of samples remaining) × time taken for shortest leg.*

**C.** What, if any, would be the advantages of treating it as a STRIPS planning problem? What, if any, would be the disadvantages? [4 marks]

*Advantages: You can describe the state and dynamics compactly. For example, you can have a general go operator that takes in a location as a parameter.*

*Disadvantages: STRIPS does not have a way to build optimizations into the goal. So there is no way to require the shortest path. We know the initial state, so STRIPS might not be altogether helpful.*

**D.** Now assume that, in addition, you have to be in town at 12 pm and at 6 pm, for your meals. Any plan that does not satisfy this requirement would be unsatisfactory (since you do not like being starved).

How would you modify the search space, path-cost function, and/or goal-test to handle this additional requirement? With this new requirement, you now have an additional action, called *eat*, which can only be executed while you are in town. Since it turns out that you are very peculiar about your mealtimes, *eat* can only be executed at 6 pm and 12 pm in town. The *eat*

action takes one hour and if you execute it while in town before 6 pm or 12 pm, you will wait until the appropriate time before executing the action. [6 marks]

*We now have to augment the represent the state of the search problem with time and keep track of whether we missed a meal time, i.e. the new state representation is now (location, has\_sample1, has\_sample2, has\_sample3, time, missed\_meal) and the goal state is (laboratory, true, true, true, \*, false).*

### Question 3 : Tortoises and Hares [14 marks]

Write the following sentences in first-order logic, using  $S(x)$  for slow,  $F(x, y)$  to mean that  $x$  is faster than  $y$ ,  $H(x)$  for hare,  $T(x)$  for tortoise,  $B(x)$  for brown.

**A.** All brown hares are not slow. [2 marks]

$$\forall x B(x) \wedge H(x) \Rightarrow \neg S(x)$$

**B.** All hares are either brown or slow, but not both. [3 marks]

$$\forall x H(x) \Rightarrow (B(x) \wedge \neg S(x)) \vee (\neg B(x) \wedge S(x))$$

**C.** Some tortoises are faster than some hare. [3 marks]

$$\exists x, y T(x) \wedge H(y) \wedge F(x, y)$$

**D.** Only one slow tortoise is brown. [3 marks]

$$\forall x, y B(x) \wedge T(x) \wedge S(x) \wedge B(y) \wedge T(y) \wedge S(y) \Rightarrow x = y$$

**E.** The slowest tortoise is not brown. [3 marks]

$$\exists x T(x) \wedge (\forall y T(y) \wedge x \neq y \Rightarrow F(y, x)) \wedge \neg B(x)$$

### Question 4 : How to find a good job [22 marks]

Good programmers either have a good job or are lazy. Lazy people do not work (i.e. have no jobs). Good jobs makes people rich. Rich people are happy. Bad (not good) jobs make people unhappy.

Use the only the following vocabulary when answering this question:

- $Good(x)$  : if  $x$  is good.
- $IsProgrammer(x)$  : if  $x$  is is a programmer.
- $HasJob(x, y)$  : if  $x$  has job  $y$ .
- $Lazy(x)$  : if  $x$  is lazy.
- $Rich(x)$  : if  $x$  is rich.

- $Happy(x)$  : if  $x$  is happy.

Note: Some predicates that you might have expected like  $IsJob$  have been omitted on purpose to simplify this problem. You need only to use the vocabulary stated above for all subsequent parts of this problem.

**A.** Express the above facts in first-order logic and convert the resulting statements to CNF form. [10 marks]

$$\begin{aligned}
 &\forall x \text{ Good}(x) \wedge \text{IsProgrammer}(x) \Rightarrow (\exists y \text{ HasJob}(x, y) \wedge \text{Good}(y)) \vee \text{Lazy}(x) \\
 &\forall x (\neg(\text{Good}(x) \wedge \text{IsProgrammer}(x)) \vee (\exists y \text{ HasJob}(x, y) \wedge \text{Good}(y))) \vee \text{Lazy}(x) \\
 &\forall x \neg \text{Good}(x) \vee \neg \text{IsProgrammer}(x) \vee (\exists y \text{ HasJob}(x, y) \wedge \text{Good}(y)) \vee \text{Lazy}(x) \\
 &\neg \text{Good}(x) \vee \neg \text{IsProgrammer}(x) \vee \text{Lazy}(x) \vee (\text{HasJob}(x, A(x)) \wedge \text{Good}(A(x))) \\
 &\neg \text{Good}(x) \vee \neg \text{IsProgrammer}(x) \vee \text{Lazy}(x) \vee \text{HasJob}(x, A(x)) \quad (1) \\
 &\neg \text{Good}(x) \vee \neg \text{IsProgrammer}(x) \vee \text{Lazy}(x) \vee \text{Good}(A(x)) \quad (2)
 \end{aligned}$$

$$\begin{aligned}
 &\forall x \text{ Lazy}(x) \Rightarrow \neg \exists y \text{ HasJob}(x, y) \\
 &\forall x \neg \text{Lazy}(x) \vee \neg \exists y \text{ HasJob}(x, y) \\
 &\forall x \neg \text{Lazy}(x) \vee \forall y \neg \text{HasJob}(x, y) \\
 &\neg \text{Lazy}(u) \vee \neg \text{HasJob}(u, v) \quad (3)
 \end{aligned}$$

$$\begin{aligned}
 &\forall x, y \text{ HasJob}(x, y) \wedge \text{Good}(y) \Rightarrow \text{Rich}(x) \\
 &\forall x, y \neg(\text{HasJob}(x, y) \wedge \text{Good}(y)) \vee \text{Rich}(x) \\
 &\forall x, y \neg \text{HasJob}(x, y) \vee \neg \text{Good}(y) \vee \text{Rich}(x) \\
 &\neg \text{HasJob}(r, s) \vee \neg \text{Good}(s) \vee \text{Rich}(r) \quad (4)
 \end{aligned}$$

$$\begin{aligned}
 &\forall x \text{ Rich}(x) \Rightarrow \text{Happy}(x) \\
 &\forall x \neg \text{Rich}(x) \vee \text{Happy}(x) \\
 &\neg \text{Rich}(z) \vee \text{Happy}(z) \quad (5)
 \end{aligned}$$

$$\begin{aligned}
 &\forall x, y \text{ HasJob}(x, y) \wedge \neg \text{Good}(y) \Rightarrow \neg \text{Happy}(x) \\
 &\forall x, y \neg(\text{HasJob}(x, y) \wedge \neg \text{Good}(y)) \vee \neg \text{Happy}(x) \\
 &\forall x, y \neg \text{HasJob}(x, y) \vee \text{Good}(y) \vee \neg \text{Happy}(x) \\
 &\neg \text{HasJob}(p, q) \vee \text{Good}(q) \vee \neg \text{Happy}(p) \quad (6)
 \end{aligned}$$

**B.** James is a good and hardworking (not lazy) programmer. Use resolution to show that James is happy. [7 marks]

*New facts:*

$$\text{IsProgrammer}(\text{James}) \quad (7)$$

$$\text{Good}(\text{James}) \quad (8)$$

$$\neg \text{Lazy}(\text{James}) \quad (9)$$

Add negation of statement to KB:

$$\neg \text{Happy}(\text{James}) \quad (10)$$

Resolving (7), (8) and (9) with (1) and (2) yields (11) and (12):

$$\text{HasJob}(\text{James}, \text{A}(\text{James})) \quad (11)$$

$$\text{Good}(\text{A}(\text{James})) \quad (12)$$

$$\neg \text{Good}(\text{A}(\text{James})) \vee \text{Rich}(\text{James}) \quad [\text{From}(11) \& (4), \theta = \{r/\text{James}, s/\text{A}(\text{James})\}] \quad (13)$$

$$\text{Rich}(\text{James}) \quad [\text{From}(13) \& (12)] \quad (14)$$

$$\text{Happy}(\text{James}) \quad [\text{From}(5) \& (14), \theta = \{z/\text{James}\}] \quad (15)$$

$$[] \quad [\text{From}(10) \& (15)] \quad (16)$$

**C.** John is unhappy. Does that mean that John a bad or lazy programmer? Justify your answer. [5 marks]

No, John may just be a lazy and poor non-programmer with no job. Statement (6) is satisfied by  $\neg \text{Happy}(\text{John})$ . The following facts about John will satisfy statements (1) to (5):

$$\neg \text{IsProgrammer}(\text{John}) \quad [\text{satisfies}(1) \& (2)] \quad (17)$$

$$\neg \text{Lazy}(\text{John}) \quad [\text{satisfies}(3)] \quad (18)$$

$$\forall j \neg \text{HasJob}(\text{John}, j) \quad [\text{satisfies}(4)] \quad (19)$$

$$\neg \text{Rich}(\text{John}) \quad (20)$$

Therefore, the statement that “John a bad or lazy programmer” is not entailed by the KB.

### Question 5 : Fishy Business [17 marks]

You have a friend who is a marine biologist. He is currently studying a new species of fish called the *Aigiffismeakea daake* and he is trying to understand the natural habitat for this fish. To this end, he has gone on a number of underwater expeditions to observe this fish. The following are the results of his latest expeditions:

Site	Depth	Plankton?	Corals?	Found?
1	Shallow	Yes	Yes	Present
2	Shallow	Yes	No	Absent
3	Shallow	No	Yes	Absent
4	Medium	No	Yes	Present
5	Medium	Yes	Yes	Present
6	Deep	Yes	No	Absent
7	Medium	Yes	No	Absent
8	Deep	No	Yes	Absent
9	Deep	No	No	Absent
10	Medium	No	No	Absent

**A.** Construct a decision tree to predict whether a given site will be a suitable habitat for the fish based on the **information gain** of each attribute. Is your decision tree *consistent*? Explain.

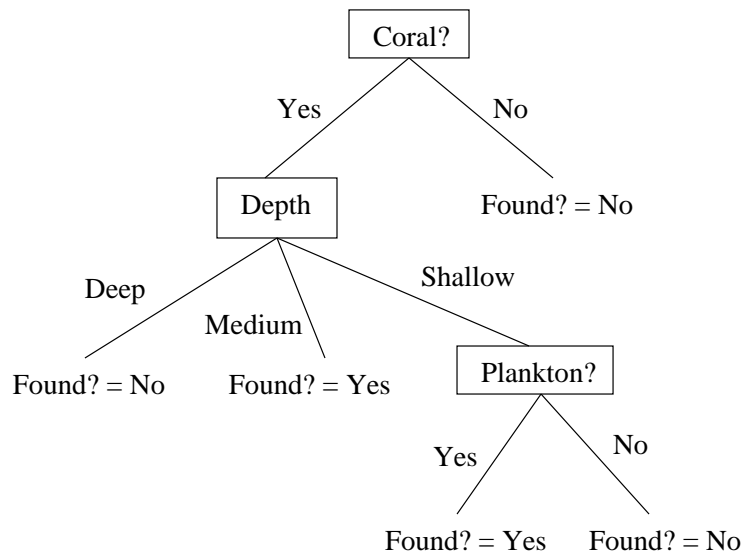
[Note: You should justify your answer by showing your computations for information gain. More space for your answer is available on the next page.] [10 marks]

$$\begin{aligned}
 I\left(\frac{3}{10}, \frac{7}{10}\right) &= -\frac{3}{10} \log_2 \frac{3}{10} - \frac{7}{10} \log_2 \frac{7}{10} \\
 &= 0.881 \\
 \text{Gain}(\text{Depth}) &= 0.881 - \left[ \frac{3}{10} I\left(\frac{1}{3}, \frac{2}{3}\right) + \frac{4}{10} I\left(\frac{1}{2}, \frac{1}{2}\right) + \frac{3}{10} I(0, 1) \right] \\
 &= 0.881 - \left[ \frac{3}{10} \left( -\frac{1}{3} \log_2 \frac{1}{3} - \frac{2}{3} \log_2 \frac{2}{3} \right) + \frac{4}{10} \left( -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2} \right) \right] \\
 &= 0.881 - \left[ \frac{1}{10} (3 \log_2 3 - 2) + \frac{2}{5} \right] \\
 &= 0.205 \\
 \text{Gain}(\text{Plankton}) &= 0.881 - \left[ \frac{1}{2} I\left(\frac{2}{5}, \frac{3}{5}\right) + \frac{1}{2} I\left(\frac{1}{5}, \frac{4}{5}\right) \right] \\
 &= 0.881 - \left[ \frac{1}{2} \left( -\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5} \right) + \frac{1}{2} \left( -\frac{1}{5} \log_2 \frac{1}{5} - \frac{4}{5} \log_2 \frac{4}{5} \right) \right] \\
 &= 0.035 \\
 \text{Gain}(\text{Coral}) &= 0.881 - \left[ \frac{1}{2} I(0, 1) + \frac{1}{2} I\left(\frac{2}{5}, \frac{3}{5}\right) \right] \\
 &= 0.881 - \left[ \frac{1}{2} \left( -\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5} \right) \right] \\
 &= 0.396
 \end{aligned}$$

Hence, the root node should be “Coral?”. Thereafter, we have the following sample points remaining for “Coral? Yes”:

Site	Depth	Plankton?	Corals?	Found?
1	Shallow	Yes	Yes	Present
3	Shallow	No	Yes	Absent
4	Medium	No	Yes	Present
5	Medium	Yes	Yes	Present
8	Deep	No	Yes	Absent

$$\begin{aligned}
 I\left(\frac{2}{5}, \frac{3}{5}\right) &= -\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5} \\
 &= 0.971 \\
 \text{Gain}(\text{Depth}) &= 0.971 - \left[ \frac{1}{5} I(1, 0) + \frac{2}{5} I(1, 0) + \frac{2}{5} I\left(\frac{1}{2}, \frac{1}{2}\right) \right] \\
 &= 0.971 - \frac{2}{5} \\
 &= 0.571 \\
 \text{Gain}(\text{Plankton}) &= 0.971 - \left[ \frac{2}{5} I(1, 0) + \frac{3}{5} I\left(\frac{1}{3}, \frac{2}{3}\right) \right] \\
 &= 0.971 - \frac{3}{5} \left( -\frac{1}{3} \log_2 \frac{1}{3} - \frac{2}{3} \log_2 \frac{2}{3} \right) \\
 &= 0.971 + \frac{1}{5} (2 - 3 \log_2 3) \\
 &= 0.42
 \end{aligned}$$



Yes, tree is consistent, i.e. fits all the data.

**B.** For a decision tree to be useful, it must have some predictive value. Describe how you would use the available data to test your decision tree. [3 marks]

*Randomly select and withhold a small number of the samples and use the remaining samples for training. Use the withheld samples to test the goodness of the resulting tree.*

**C.** Suppose your friend goes for another two expeditions and returns with the following results:

Site	Depth	Plankton?	Corals?	Found?
11	Shallow	Yes	Yes	Absent
12	Shallow	Yes	No	Present

Does your decision tree yield the correct predictions? Explain.

[4 marks]

*Decision tree does not yield the same predictions. Could be due to errors in some of the observations or we may have a missing attribute.*

## Question 6 : How Not to Fail CS3243 [15 marks]

It is known that whether a student  $x$  fails CS3243, i.e.  $Fail(x)$ , is influenced by a number of factors. Some of the known factors are:

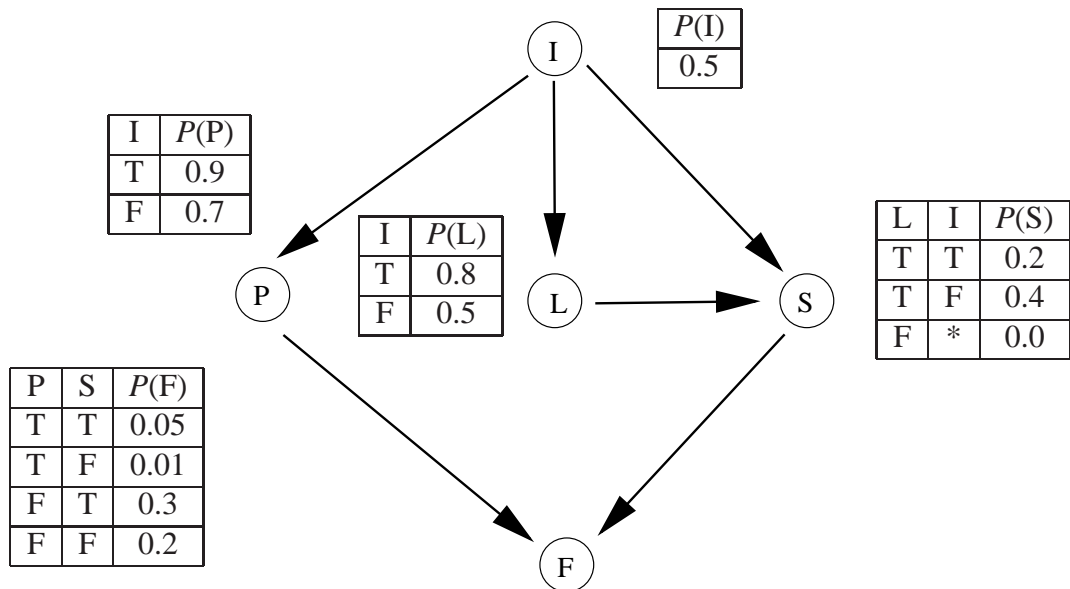
1.  $Interested(x)$  : whether student  $x$  is interested in Artificial Intelligence.
2.  $Lecture(x)$  : whether student  $x$  attends lectures.
3.  $Sleep(x)$  : whether student  $x$  sleeps during lectures (Note: student  $x$  can only sleep during lectures if he actually attends lectures).
4.  $Project(x)$  : whether student  $x$  puts effort into doing the Final Project.

For this question, you may abbreviate  $Fail$ ,  $Interested$ ,  $Lecture$ ,  $Sleep$  and  $Project$  with  $F$ ,  $I$ ,  $L$ ,  $S$  and  $P$  respectively. Assume that the following conditional probabilities are available:



$P(\text{Interested})$	0.5
$P(\text{Lecture} \mid \text{Interested})$	0.8
$P(\text{Lecture} \mid \neg \text{Interested})$	0.5
$P(\text{Project} \mid \text{Interested})$	0.9
$P(\text{Project} \mid \neg \text{Interested})$	0.7
$P(\text{Sleep} \mid \text{Interested} \wedge \text{Lecture})$	0.2
$P(\text{Sleep} \mid \neg \text{Interested} \wedge \text{Lecture})$	0.4
$P(\text{Fail} \mid \text{Project} \wedge \neg \text{Sleep})$	0.01
$P(\text{Fail} \mid \text{Project} \wedge \text{Sleep})$	0.05
$P(\text{Fail} \mid \neg \text{Project} \wedge \neg \text{Sleep})$	0.2
$P(\text{Fail} \mid \neg \text{Project} \wedge \text{Sleep})$	0.3

**A.** Construct a Bayesian network corresponding to these conditional probabilities. [5 marks]



**B.** What is the probability of failing CS3243? [5 marks]

$$\begin{aligned}
 P(S|I) &= P(S|L, I)P(L|I) + P(S|\neg L, I)P(\neg L|I) \\
 &= 0.2 \times 0.8 \\
 &= 0.16 \\
 P(S|\neg I) &= P(S|L, \neg I)P(L|\neg I) + P(S|\neg L, \neg I)P(\neg L|\neg I) \\
 &= 0.4 \times 0.5 \\
 &= 0.2 \\
 P(P, S) &= P(P, S|I)P(I) + P(P, S|\neg I)P(\neg I) \\
 &= P(P|I)P(S|I)P(I) + P(P|\neg I)P(S|\neg I)P(\neg I) \\
 &= 0.9 \times 0.16 \times 0.5 + 0.7 \times 0.2 \times 0.5 \\
 &= 0.142 \\
 P(P, \neg S) &= P(P, \neg S|I)P(I) + P(P, \neg S|\neg I)P(\neg I)
 \end{aligned}$$

$$\begin{aligned}
 &= P(P|I)P(\neg S|I)P(I) + P(P|\neg I)P(\neg S|\neg I)P(\neg I) \\
 &= 0.9 \times 0.84 \times 0.5 + 0.7 \times 0.8 \times 0.5 \\
 &= 0.658 \\
 P(\neg P, S) &= P(\neg P, S|I)P(I) + P(\neg P, S|\neg I)P(\neg I) \\
 &= P(\neg P|I)P(S|I)P(I) + P(\neg P|\neg I)P(S|\neg I)P(\neg I) \\
 &= 0.1 \times 0.16 \times 0.5 + 0.3 \times 0.2 \times 0.5 \\
 &= 0.038 \\
 P(\neg P, \neg S) &= P(\neg P, \neg S|I)P(I) + P(\neg P, \neg S|\neg I)P(\neg I) \\
 &= P(\neg P|I)P(\neg S|I)P(I) + P(\neg P|\neg I)P(\neg S|\neg I)P(\neg I) \\
 &= 0.1 \times 0.84 \times 0.5 + 0.3 \times 0.8 \times 0.5 \\
 &= 0.162 \\
 P(F) &= P(F|P, S)P(P, S) + P(F|P, \neg S)P(P, \neg S) \\
 &\quad + P(F|\neg P, S)P(\neg P, S) + P(F|\neg P, \neg S)P(\neg P, \neg S) \\
 &= 0.05 \times 0.142 + 0.01 \times 0.658 + 0.3 \times 0.038 + 0.2 \times 0.162 \\
 &= 0.05748
 \end{aligned}$$

**C.** If a student fails CS3243, what is the probability that he was interested in AI to begin with?  
[5 marks]

$$\begin{aligned}
 P(I|F) &= \frac{P(F|I)P(I)}{P(F)} \\
 &= \frac{P(I)}{P(F)} (P(F|P, S, I)P(P|I)P(S|I) + P(F|P, \neg S, I)P(P|I)P(\neg S|I) \\
 &\quad + P(F|\neg P, S, I)P(\neg P|I)P(S|I) + P(F|\neg P, \neg S, I)P(\neg P|I)P(\neg S|I)) \\
 &= \frac{0.5}{0.05748} (0.05 \times 0.9 \times 0.16 + 0.01 \times 0.9 \times 0.84 + 0.3 \times 0.1 \times 0.16 + 0.2 \times 0.1 \times 0.84) \\
 &= 0.316
 \end{aligned}$$