GER1000 AY2018/19 Semester 1
Quiz 10 and solutions

1. John flips a fair coin and observes the outcome of H-H-H-H-T for the first 5 tosses. On his sixth toss, what is the probability of getting an H? (H is head, T is tail.)
   a. P(H) is 0.5.
   b. P(H) is less than 0.5.
   c. P(H) is more than 0.5.
   d. There is insufficient information to answer the question.

   Ans: a. "Fair" means P(H) = 0.5, at every toss.

2. John is having a bet with Elaine in game where he has a 20% chance of winning. Which of the following terms maximises his expected profit, i.e., the average value of the profit?
   a. Receives $1000 on winning, pays $200 on losing.
   b. Receives $100 on winning, pays nothing on losing.
   c. Receives $20 on winning, receives $10 on losing.
   d. Refuse to bet.

   Ans: a. The average profits are:
   a. 0.2x1000 + 0.8x(-200) = 40
   b. 0.2x100 + 0.8x0 = 20
   c. 0.2x20 + 0.8x10 = 12
   d. No bet, no profit: 0

3. An observational study on the association between having a pet/pets and having insomnia was conducted on 200 randomly selected NUS students. Below is the contingency table. It is found that 10% of the students suffered from insomnia.

	Insomnia	No insomnia
Have a pet/pets	4	40
No pet		

   What is the association observed in this study?
   a. Having a pet/pets and having insomnia are positively associated.
   b. Having a pet/pets and having insomnia are negatively associated.
   c. There is no association between having a pet/pets and having insomnia.
   d. The association cannot be determined with the given information.

   Ans: b. The number of people who suffer from insomnia is 10% of the sample of 200, which is 20. 4 of them has a pet/pets, hence the remaining 16 of them should be those without a pet/pets. Then we can fill the contingency table. The number of people who do not have a pet/pets and who do not suffer from insomnia is 200-60=140. Then you can see that rate(insomnia|Pet) = 4/44 = 1/11 < rate(insomnia|No pet) = 16/156 = 4/39. Hence, having a pet/pets and having insomnia is negatively associated.

4. The correlation coefficient for 100 pairs of (X,Y) values is -0.4. What would be the correlation coefficient for 100 pairs of (Y,3X) values?
   a. 0.4.
   b. -0.4.
   c. Cannot be determined with the given information.

   Ans: b. The correlation coefficient of (Y,X) is the same as the correlation coefficient of (X,Y). And the correlation coefficient of (Y,3X) is the same as the correlation coefficient of (Y,X). See lecture notes Chapter 2 slide 52. Hence, the answer is -0.4.

5. Tommy was passing by a supermarket and found that there was a big sale on coke going on. He decided to buy a large number of bottles of coke to share with his numerous friends. When he came back home with the cokes, however, he found that some of the cokes he purchased had already expired. Suppose the odds of expiry was ½. What is the fraction of coke bottles that have expired, among those bought by Tommy?
   a. 1
   b. $\frac{1}{2}$
   c. $\frac{1}{3}$
   d. $\frac{1}{4}$

   Ans (c). By definition, $\frac{\text{number of cokes expired}}{\text{number of cokes not expired}} = \frac{1}{2}$: the number of cokes not expired is double the number of cokes expired. Let's denote the number of cokes expired as x, then the number of cokes not expired is $2x$. And the total number bought is $x + 2x = 3x$. So the rate of expired cokes is $\frac{\text{number of cokes expired}}{\text{total number bought}} = \frac{x}{3x} = \frac{1}{3}$. Students can also refer to lecture notes Chapter 5 slide 17 and use the relationship between odds and risks to solve this problem.