NATIONAL UNIVERSITY OF SINGAPORE

MA1521 - CALCULUS FOR COMPUTING

(Semester 2 : AY2021-2022)

Time allowed: 2 hours

<u>INSTRUCTIONS TO CANDIDATES</u>

1. Please be reminded to submit a signed copy of the examination declaration form.
2. Use A4 size white colour paper and black (or blue) colour pen to write your answers. Do not write your name. Write down your student number clearly on the top left corner of every answer sheets.
3. This examination paper consists of **TEN (10)** questions and comprises **THREE (3)** printed pages.
4. Answer **ALL** questions. Each question carries 10 marks. The total mark of the paper is 100.
5. This is an OPEN BOOK examination. You are allowed to use any computer/calculator/online calculator. However, you should lay out systematically the various steps in the calculations.
6. At the end of the examination,

   • scan or take photos of your work (make sure the images can be read clearly);

   • merge all the images into one pdf file (arrange them in the order: Q1, Q2, ... in their page sequence);

   • upload your pdf file to the exam submission folder in LumiNUS. Late submission will not be accepted, unless there is a valid reason.

Answer **all** *the questions. Total is 100 marks. You need to show detail work of your solution.*

Question 1. [10 marks]

An equation of the plane passing through the three points $(3,-1,2)$, $(8,2,4)$ and $(-1,-2,-3)$ has the form $ax+by+cz=42$. Find the value of $a+b+c$.

Question 2. [10 marks]

For each of the following series, determine whether it converges or diverges. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty}\frac{n!(2n)!}{(3n)!}$,

(b) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^3}$.

Question 3. [10 marks]

Let a be a constant and let $f(x)=\dfrac{ax+a+1}{x^2-x-2}$. It is known that $f^{(7)}(0)=0$, where $f^{(7)}(0)$ denotes the seventh derivative of f evaluated at the point $x=0$. By finding the Maclaurin series of f, determine the value of a.

Question 4. [10 marks]

Let $f(x,y)$ be a differentiable function defined on $\mathbb{R}^2$. It is known that the directional derivative of f at $(1,2)$ along the direction $3\mathbf{i}+4\mathbf{j}$ is 15 and along the direction of $-3\mathbf{i}+4\mathbf{j}$ is 9. Find the directional derivative of f in the direction of $24\mathbf{i}+7\mathbf{j}$.

Question 5. [10 marks]

Let $f(x,y)=x^3+3xy^2-3x^2-3y^2+4$. Find all critical points of f. For each critical point, determine whether f has a local maximum, a local minimum or a saddle point.

Question 6. [10 marks]

Let a denote a positive constant. It is known that the graph of

$$y^2 = x^2a^3 - 3x^3a^2 + 3x^4a - x^5$$

has a loop which bounds a region R in the xy-plane and the area of R is $\frac{8}{35}a^{\frac{7}{2}}$. Suppose

$$\iint_R (x-4)\,dA = 0.$$

Determine the value of a.

Question 7. [10 marks]

Find the surface area of the portion of the surface $z = 6y^2 + \sqrt{323}x$ lying above the triangular region R in the xy-plane with vertices at $(0,0), (0,2)$ and $(2,2)$.

Question 8. [10 marks]

Find the volume of the solid bounded by the paraboloid $z = 150 + (2x-1)^2 + 4y^2$, the cylinder $x^2 + y^2 = 9$ and the xy-plane. Express your answer in terms of π.

Question 9. [10 marks]

Let $y(x)$ be the solution of the differential equation

$$x\frac{dy}{dx} + y = xy^3, \ \ 0 < x < \frac{8}{7},$$

satisfying $y(\frac{1}{2}) = \frac{4}{3}$ and $y(x) > 0$ for $0 < x < \frac{8}{7}$. Find the exact value of $y(1)$.

Question 10. [10 marks]

At time $t = 0$, a tank contains 400 grams of salt dissolved in 4 litres of water. Assume that water containing 40 grams of salt per litre is entering the tank at a rate of $\ln(4)$ litres per minute and that the well-mixed solution is draining from the tank at the same rate. Find the amount of salt in grams in the tank at time $t = 4$ minutes. Give your answer correct to two decimal places.

END OF PAPER