a) $a_1 = 0.5$

$a_n = a_1 + (n-1)d$

$= 3.5$

$S_n = \frac{n}{2}(a_n + a_1)$

$62 = \frac{n}{2}(3.5 + 0.5)$

$n = 31$ //

$a_n = 0.5 + (31-1)d$

$3.5 = 0.5 + 30d$

$d = 0.1$ //

b) $a_1 = a$

$a_2 = ar$

$a_3 = ar^2$

$a + ar + ar^2 = 76$

let $a = 36$,

$36 + 36r + 36r^2 = 76$

$\cancel{9r^2 + 36r - 40 = 0}$

$9r^2 + 9r - 10 = 0$

$(3r+5)(3r-2) = 0$

$r = \frac{2}{3}$ or $-\frac{5}{3}$ (rejected)

$ar^2 \Rightarrow 36\left(\frac{2}{3}\right)^2 = 16$

shortest side 16cm //

a) $\cancel{\left(x-\frac{2}{3}\right)^6}$

$$\left(x-\frac{2}{x}\right)^6 = \binom{6}{0}(x)^6\left(-\frac{2}{3x}\right)^0 + \binom{6}{1}(x)^5\left(-\frac{2}{x}\right)^1 + \binom{6}{2}(x)^4\left(-\frac{2}{3x}\right)^2 + \dots$$

$$= x^6 + 6x^5\left(-\frac{2}{x}\right) + 15x^4\left(\frac{4}{x^2}\right) + \dots$$

$\cancel{= x^6 + -3x^4 +}$

$\cancel{= x^6 +}$

$$= x^6 - 12x^4 + 60x^2 + \dots$$

$$(2+3x^2)\left(x-\frac{2}{x}\right)^6 = (2+3x^2)x^6 - (2+3x^2)12x^4 + (2+3x^2)60x^2 + \dots$$

$$= 2x^6 + 3x^8 - 24x^4 - 36x^6 + 120x^2 + 180x^4 + \dots$$

$$= 3x^8 - 34x^6 + 156x^4 + 120x^2 + \dots$$

coefficient of $x^4 \Rightarrow 156$

b) $$(1-p)^5 = \binom{5}{0}(-p)^0 + \binom{5}{1}(-p)^1 + \binom{5}{2}(-p)^2 + \binom{5}{3}(-p)^3 + \binom{5}{4}(-p)^4 + \binom{5}{5}(-p)^5$$

$$= 1 - 5p + 10p^2 - 10p^3 + 5p^4 - p^5$$

$(1-x-x^2)^5 =$

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i) ~~$\frac{dx}{dt} =$ Let $u = 1+4t^2$~~

~~$\frac{du}{dt} = 8t$~~

~~$du = 8t\,dt$~~

$$\frac{dx}{dt} = \frac{dx}{du} \times \frac{du}{dt}$$

$$\frac{dx}{dt} = -\frac{8t}{(1+4t^2)^2}$$

Let $u = 1 + 4t^2$

$$\frac{du}{dt} = 8t$$

$$\frac{dx}{du} = \frac{d}{du}\left(\frac{1}{u}\right)$$

$$= (-1)\,u^{-2}$$

$$= -\left(\frac{1}{1+4t^2}\right)^2$$

ii) $$\frac{dy}{dt} = \frac{2}{1+4t^2}$$

iii) $$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

$$= \frac{2}{1+4t^2} \times \left(-\frac{(1+4t^2)^2}{8t}\right)$$

$$= -\frac{1+4t^2}{4t}$$

iv) $4y = -5x - 3$

$y = -\frac{5}{4}x - \frac{3}{4}$

$\frac{dy}{dx} = -\frac{5}{4}$

$-\frac{1+4t^2}{4t} = -\frac{5}{4}$

$4 + 16t^2 = 20t$

$16t^2 - 20t + 4 = 0$

$4t^2 - 5t + 1 = 0$

$(4t-1)(t-1) = 0$

$t = \frac{1}{4}$ or 1

when $t = \frac{1}{4}$, $x = \frac{1}{1+4(\frac{1}{4})^2}$

$= \frac{4}{5}$

$y = \tan^{-1}(2(\frac{1}{4}))$

≈ 0.464 (3dp)

when $t = 1$, $x = \frac{1}{1+4(1)^2}$

$= \frac{1}{5}$

$y = \tan^{-1}(2(1))$

$=$ ~~45~~ 0.785 (3dp)

$\left(\frac{4}{5}, 0.464\right)$ or $\left(\frac{1}{5}, 0.785\right)$ //

i) $4y = 100 - 3x$

$y = 25 - \frac{3}{4}x$

ii) $A = \frac{1}{2}(\text{length})(\text{height}) + (\text{length})(\text{length})$
(triangle) (square)

$= \frac{1}{2}(\cancel{4}x)\left(\frac{x}{\sin 90} \cancel{\times} x \sin 60\right) + y^2$

$= \left(\frac{1}{2}\right)\frac{\sqrt{3}}{2}x^2 + \left(\frac{100-3x}{4}\right)^2$

$= \cancel{\frac{\sqrt{3}}{2}} \cdot \frac{\sqrt{3}}{4}x^2 + \frac{(100-3x)^2}{16}$

iii) $\cancel{dt} A = \frac{\sqrt{3}}{4}x^2 + \frac{10000 - 600x + 9x^2}{16}$

$= \frac{4\sqrt{3}}{16}x^2 + \frac{9}{16}x^2 - \frac{600x}{16} + 10000$

$\frac{dA}{dx} \Rightarrow \frac{9+4\sqrt{3}}{8}x - \frac{600}{16} = 0$

$\frac{9+4\sqrt{3}}{8}x = \frac{600}{16}$

$x = \frac{600}{18+8\sqrt{3}}$

$= 18.83$ (2 d.p.)

$\frac{d^2A}{dx^2} \Rightarrow \frac{9+4\sqrt{3}}{8} > 0$

This value makes A a minimum

a) let $u = 7 - x^2$ $\quad \frac{du}{dx} = -2x$

$\cancel{\frac{1}{3}du =}$ $du = -2x\,dx$

$-3du = 6x\,dx$

$$\int_1^2 \frac{6x}{\sqrt{7-x^2}}\,dx = \int_1^2 -3u^{-\frac{1}{2}}\,du$$

$$= -3\left[\frac{1}{2}u^{\frac{1}{2}}\right]_1^2$$

$$= \left[-\frac{3}{2}(7-x^2)\right]_1^2$$

$$= \left(-\frac{3}{2}(7-(2)^2)\right) - \left(-\frac{3}{2}(7-(1)^2)\right)$$

$$= \frac{9}{2} \text{ //}$$

b) let $u = 4 - 3\ln x$, $\quad \frac{du}{dx} = -\frac{3}{x}$

$du = -\frac{3}{x}dx$

$-\frac{1}{3}du = \frac{1}{x}dx$

$$\int_1^e \frac{\sqrt{4-3\ln x}}{x}\,dx = \int_1^e u^{\frac{1}{2}}\left(-\frac{1}{3}du\right)$$

$$= \left[-\frac{1}{2}u^{\frac{3}{2}}\right]_1^e$$

$$= \left[-\frac{1}{2}(4-3\ln x)^{\frac{3}{2}}\right]_1^e$$

$$= -\frac{1}{2}(4-3\ln e)^{\frac{3}{2}} + \frac{1}{2}(4-3\ln 1)^{\frac{3}{2}}$$

$$= \frac{7}{2} \text{ //}$$

i) $3x^2+1 = 2x^2+5$

$x^2 - 4 = 0$

$x^2 = 4$

$x = 2$ or -2

when $x = 2$, $y = 3(2)^2 + 1$

$= 13$

when $x = -2$, $y = 3(-2)^2+1$

$= 13$

$A(-2, 13)$ $B(2, 13)$

$p \Rightarrow -2$
$q \Rightarrow 13$
$r \Rightarrow 2$
$s \Rightarrow 13$

iii) $A = \cancel{\int_{-2}^{2}(2x^2-5) - (3x^2+1)\, dx}$ $\int_{-2}^{2}(3x^2+1) - (2x^2-5)\, dx$

$= \int_{-2}^{2} x^2 + 6\, dx$

$= \left[\frac{1}{3}x^3 + 6x\right]_{-2}^{2}$

$= \left(\frac{1}{3}(2)^3 + 6(2)\right) - \left(\frac{1}{3}(-2)^3 + 6(-2)\right)$

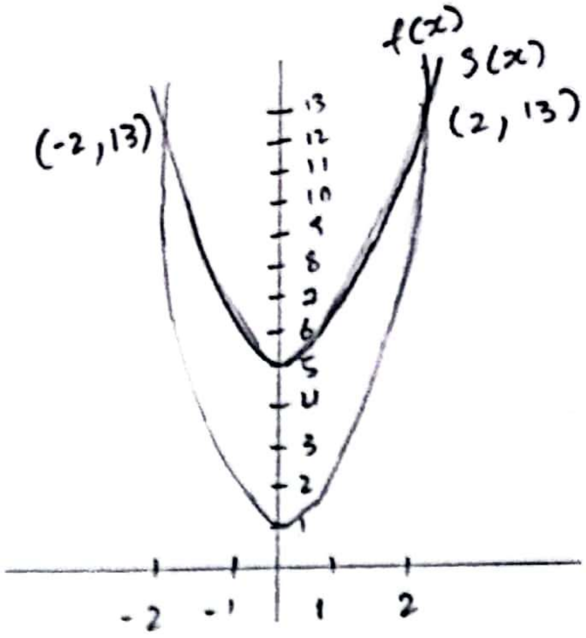
$= \frac{8}{3} + 12 + \frac{8}{3} + 12$

$= \frac{88}{3}$

$\cancel{A = \int_{-2}^{2}(3x^2+1) - (2x^2-5)} \, dx$

$\cancel{=}$

ii) when $x = 0$, $f(x) = 1$ $g(x) = 5$



iv) $V = \int_{-2}^{2} \pi(3x^2+1)^2 - \pi(2x^2-5)^2 \, dx$

$= \int_{-2}^{2} \pi(9x^4 + 6x^2 + 1 - 4x^4 + 20x^2 - 25) \, dx$

$= \cancel{459x} \left[(25x^5 + 78x^3 - 24x)\pi \right]_{-2}^{2}$

$=$

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$$y^2 \csc x \frac{dy}{dx} = 4x\sqrt{1+y^3}$$

$$\frac{dy}{dx} = \frac{4x\sqrt{1+y^3}}{y^2 \csc x}$$

i) $\begin{pmatrix} -3 \\ 2 \\ 7 \end{pmatrix}$ point A

$\underset{\sim}{r} = \underset{\sim}{a} + \underset{\sim}{\lambda} \underset{\sim}{u}$

$\underset{\sim}{r} \cdot \underset{\sim}{n} = d = \underset{\sim}{a} \cdot \underset{\sim}{n}$