# 📘 Homework # 3

**Due:** November 11 at 11:59 PM **Electronic textbook:** Click here

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📌 Since there are inconsistencies in exercise numbering between Edition 4 and Edition 5, the homework problems are listed explicitly below to avoid confusion.

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In **Questions 1–3**, apply the **Gram–Schmidt process** to the given subset $S$ of the inner product space $V$ to obtain an **orthogonal basis** for $\mathrm{span}(S)$.

Then normalize the vectors in this basis to obtain an **orthonormal basis** $\beta$ for $\mathrm{span}(S)$.

Next, compute the **Fourier coefficients** of the given vector relative to $\beta$, and use **Theorem 6.5** to verify your result.

**Question 1.** $V = \mathbb{R}^3$, $S = \{(1,1,1),(0,1,1),(0,0,1)\}$, and $\mathbf{x} = (1,0,1)$

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**Question 2.** $V = \mathrm{P}_2(\mathbb{R})$ with the inner product $\langle f(x), g(x)\rangle = \displaystyle\int_0^1 f(t)g(t)dt$, $S = \{1, x, x^2\}$, and $h(x) = 1 + x$.

*Hint:* See Section 6.2, page 343, **Example 5** in the textbook for a similar computation.

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**Question 3.** $V = \mathrm{span}(S)$, where $S = \{(1,i,0),(1-i,2,4i)\}$, and $x = (3+i, 4i, -4)$

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**Question 4.** Prove that if $\{w_1, w_2, \ldots, w_n\}$ is an orthogonal set of nonzero vectors, then the vectors $v_1, v_2, \ldots, v_n$ derived from the Gram-Schmidt process satisfy $v_i = w_i$ for $i = 1, 2, \ldots, n$. Hint: Use mathematical induction.

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**Question 5.** Let $S = \{(1,0,i),(1,2,1)\}$ in $\mathbb{C}^3$. Compute $S^\perp$.

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**Question 6.** Let $S_0 = \{x_0\}$, where $x_0$ is a nonzero vector in $\mathbb{R}^3$. Describe $S_0^\perp$ geometrically. Now suppose that $S = \{x_1, x_2\}$ is a linearly independent subset of $\mathbb{R}^3$. Describe $S^\perp$ geometrically.

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**Question 7.** Let $W = \mathrm{span}(\{(i,0,1)\})$ in $\mathbb{C}^3$. Find orthonormal bases for $W$ and $W^\perp$.

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