1 Sigmoid functions

The sigmoid can be specified using the logistic function

$$g(x) = \frac{1}{1 + \exp(\varsigma(v_0 - x))} \tag{1}$$

which has the derivative

$$g'(x) = \varsigma g(x)(1 - g(x)) \tag{2}$$

Alternatively, using the error function

$$g(x) = \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{x - v_0}{\xi\sqrt{2}}\right)\right) \tag{3}$$

which has the derivative

$$g'(x) = \frac{1}{\xi\sqrt{2\pi}} \exp\left(-\left(\frac{x - v_0}{\xi\sqrt{2}}\right)^2\right) \tag{4}$$

2 Relationship between widths

Through numerical simulation we can deduce that setting the error function width to

$$\xi = \frac{1.699}{\varsigma} \tag{5}$$

gives a sigmoid with approximately the same shape as a logistic function with width ς.