

1 What kind of roots exist for a polynomial function besides rational root?

2 How many roots exist for a polynomial equation  $P(X) = 0$

3 When you find a root using the theorem above, you find a root.  
How can you find other roots?

4 Does the theorem above guarantee that we can find all roots using the method described?

irrational root, and Imaginary root.

The number of roots is  $n$  when  $P(X) = a_n X^n + a_{n-1} X^{n-1} + \dots + a_0$

If the first root is  $s_1$ , then  $P(X) = a_n X^n + a_{n-1} X^{n-1} + \dots + a_0 = P(X) = P^1(X) * (X - s_1)$   
where  $P^1(X) = (a_{n-1} X^{n-1} + a_{n-2} X^{n-2} + \dots + a_0)$

Repeat the same process for  $P^1(X)$  to find a new root.

No, it does not guarantee it. The theorem finds a root only if there is rational root.