

# Balancing informational and social goals in active learning

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## Abstract

Our actions shape what we learn. Because of this dependency, learners are proficient at choosing their actions to maximize their information gain. However, learning often unfolds in social contexts where learners have both informational goals (e.g., to learn how something works) but also social goals (e.g., to appear competent and impress others). How do these goals shape learners' decisions? Here, we present a computational model that integrates the value of social and informational goals to predict the decisions that people will make in a simple active causal learning task. We show that, in a context where the informational and social goals are in conflict, an emphasis on performance or self-presentation goals leads to reduced chances of learning (Exp. 1) and that social context can push learners to pursue performance-oriented actions even when the learning goal is highlighted (Exp. 2). Our formal model of social-active learning successfully captures the empirical results. These findings are first steps towards understanding the role of social reasoning in active learning contexts.

**Keywords:** active learning; social reasoning; information gain; OED; self-presentation; goal tradeoffs

## Introduction

Imagine you are a novice cook and have to decide what meal to prepare for a first date. Should you choose an easy favorite or attempt a new dish? The familiar recipe can ensure a good meal, but the new recipe could be even more delicious, although it has a higher chance of failure. In this type of *explore-exploit* dilemma (Sutton & Barto, 1998), you can either *explore* the new recipe that might result in a more delicious dish (*learning* goal), or *exploit* your previous experience and knowledge to ensure a good meal (*performance* goal). Here, we explore the idea that social factors shape the goals we consider by formalizing the learning-performance goal tradeoff using a simple active learning context.

Active learning occurs when people have control over the sequence of information in a learning context (e.g., press buttons on a toy, one by one, to see their effect). The key assumption of this framework is that learners maximize the utility of their actions by gathering information that is especially helpful for their own learning. Empirical work in education (Grabinger & Dunlap, 1995), machine learning (Settles, 2012), and cognitive psychology (Castro et al., 2009) suggests that active contexts lead to faster learning than passive contexts where people do not have control over the information flow.

Real-world learning, however, usually takes place in rich social contexts with teachers, peers, or others who can directly influence our learning. Indeed, adults and even preschool-aged children modulate their inferences depending on how others (e.g., teachers) select their actions (Shafra, Goodman, & Frank, 2012), and understand that socially communicated information licenses different inferences than in-

formation generated on their own (e.g., Xu & Tenenbaum, 2007). But even when we learn from *our* own actions instead of others', our social environment may affect our self-directed learning process. While previous models have captured how we optimize learning, either from our own actions or from others, they have been agnostic to other social factors that are ubiquitous in a learner's environment. People must integrate the value of social goals (e.g., looking competent or knowledgeable) and informational goals when deciding what to do.

How can active learning models accommodate this richer set of utilities? As a step towards answering this question, we model a learner who considers a mixture of learning and performance goals. A key assumption underlying recent Bayesian models of human social cognition is that people expect others to act approximately optimally given a utility function (e.g., Goodman & Frank, 2016; Jara-Ettinger, Gweon, Schulz, & Tenenbaum, 2016). Our model adopts this utility-theoretic approach, and assumes an agent who reasons about the utility function that represents a weighted combination of multiple goals (Yoon, Tessler, Goodman, & Frank, 2017) in a social active learning context.<sup>1</sup>

We instantiate our model in a simple causal learning task and examine how people choose actions that support learning vs. performance goals in different social contexts. We present a toy with an ambiguous causal mechanism (Fig. 1). For this toy, doing only one of the two possible actions (handle pull or button press) disambiguates its causal mechanism but potentially risks no immediate effect (i.e., neither sound nor light turning on), while doing both actions at the same time is immediately rewarding (both music and light on) but is not informative for learning the toy's causal mechanism.

<sup>1</sup>Such models are commonly used to approximate group-level behavior, without the strong assumption that individuals must be strictly optimal (e.g., Frank, 2013; Goodman et al., 2015).

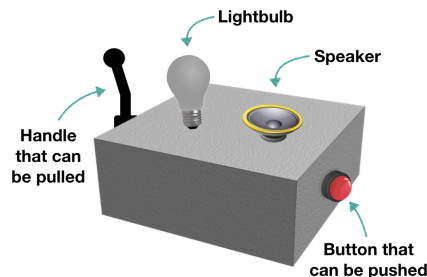


Figure 1: An example of the toy used in our paradigm.

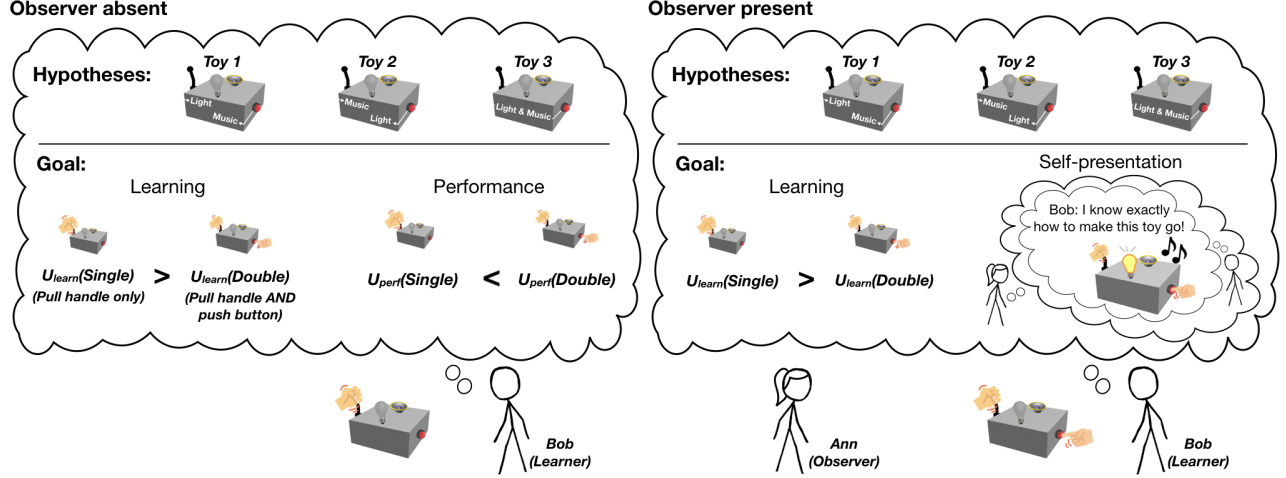


Figure 2: Model schematic for the learner’s inference. The learner considers possible hypotheses: Toy 1 (handle pull turns on the light, button press turns on music, both actions cause both effects); Toy 2 (handle pull turns on music, button press turns on the light, both actions cause both effects); and Toy 3 (both actions cause both effects, but each action on its own does not produce any effect). The learner also considers his goals. When an observer is absent, he considers his learning goal and performance goal and chooses an action. The learning goal favors a “single” action (e.g., pull the handle only) that can fully disambiguate, whereas the performance goal favors the “both” action (pull the handle AND push the button) that guarantees the most salient reward. When an observer is present, the learner considers the learning, performance (not shown), and self-presentation goal.

Thus, the learner can choose between the two actions that will each lead to either a new discovery (exploration; learning) or an immediate reward (exploitation; performance). The learner’s action rests on relative utilities he assigns to learning versus performance, which in turn are determined in part by the social context (e.g., the presence or absence of his boss).<sup>2</sup>

In two experiments, we show that emphasizing performance or self-presentation (social) goals leads to actions that are not informative and thus reduces the chances of learning (Exp. 1). Next, we show that the mere presence of an observer (i.e., a boss) pushes learners to consider performance/presentation-oriented actions even when the learning goal is highlighted (Exp. 2). Finally, we show that the empirical results are consistent with predictions of our cognitive model of social-active learning.

### Computational model

We model a learner  $L$  who chooses his action  $a$  approximately optimally (as per optimality parameter  $\lambda$ ) with respect to his goal based on the expected total utility  $U_t$  given his action and presence of an observer  $o$ :

$$P_L(a|o) \propto \exp(\lambda \cdot \mathbb{E}[U_t(a, o)]).$$

The total utility is defined as:

$$U_t(a, o) = \phi_{\text{learn}} \cdot U_{\text{learn}}(a) + \phi_{\text{perf}} \cdot U_{\text{perf}}(a) + \delta^o \cdot \phi_{\text{pres}} \cdot U_{\text{pres}}(a),$$

where  $\phi$ s are weights that are inferred for each utility from data and  $\delta^o$  is a Dirac delta function that is 1 if there is an

<sup>2</sup>From here on, we use a male pronoun for Bob, the learner, and female pronoun for Ann, the boss and observer.

observer, and 0 if there is no observer. Below we describe the structure of each utility (see Fig. 2 for the model schematic).

**Learning utility** The *learning utility* captures the goal to learn new information, which in our paradigm is associated with figuring out how a given toy works. The learning utility ( $U_{\text{learn}}$ ) in our model is derived from Optimal Experiment Design models (OED; Nelson, 2005), which quantify the expected utility of different information seeking actions. The learner is uncertain about the mechanism of toy  $t$  and wants to decrease his uncertainty by taking an action. This decrease is captured by *information gain*, the change in the learner’s entropy (uncertainty) before and after seeing an outcome of the action. To maximize information gain, the learner sums the information gain due to each outcome  $m$  in the set of possible outcomes  $M$  (e.g., music playing), weighted by the probability of that outcome given the action. Thus,

$$U_{\text{learn}}(a) \propto \sum_{m \in M} P(m|a)[H(t) - H(t|m, a)],$$

where  $H(t)$  is the Shannon entropy of the learner’s guess about the toy (MacKay, 2003). Once the learner chooses an action and observes an outcome, he updates his beliefs about each hypothesis via standard Bayesian updating. Finally, we scale the utility by  $\log_2 n$ , where  $n$  is the number of possible actions, to convert the utility to a value between 0 and 1.

**Performance utility** The *performance utility* is the utility of achieving an immediate rewarding outcome. Within our paradigm, the learner gains utility from an immediate effect of music or light turning on. The expected performance utility ( $U_{\text{perf}}$ ) before the learner chooses an action is the likelihood

of an effect  $m$  given the action  $a$ :

$$U_{perf}(a) = P_L(m|a).$$

**Presentation utility** When there is another person present to observe the learner’s action, the observer  $O$  is expected to reason about the learner  $L$ ’s competence, equal to whether the learner was able to make the toy produce an effect. The learner thinks about the observer’s inferential process, and the expected *presentation utility* ( $U_{pres}$ ) is based on maximizing the apparent competence inferred by the observer:

$$U_{pres}(a) = P_O(m|a),$$

where  $P_O(m|a)$  is the observer’s own estimate of the likelihood of an effect given the learner’s action.<sup>3</sup>

## Experiment 1

In Exp. 1, we first wanted to confirm that participants would choose different actions depending on what goal was highlighted. We were also interested in how people would act when no explicit goal was specified within the task. Participants were asked to act on a toy with an ambiguous causal structure, and were assigned to different goal conditions: (1) Learning (i.e., learn how the toy works), (2) Performance (e.g., make the toy play music), (3) Presentation (i.e., impress their boss), and (4) No goal specified. We hypothesized that participants would choose an informative action more often in the following order of goal conditions (decreasing): Learning, No-goal, Performance, and Presentation.<sup>4</sup>

## Method

**Participants** We recruited 196 participants (45-51 per condition) on Amazon’s Mechanical Turk, with IP addresses in the US and a task approval rate above 85%. We excluded 7 participants who failed to answer at least two out of three manipulation check questions correctly (see Procedure section for details on the manipulation check), and thus the remaining 189 participants were included in our final analysis.

**Stimuli and Design** We presented instructions for three different toys that looked very similar but worked in different ways in a deterministic world (see captions for Fig. 2). The instructions conveyed that pressing the button and pulling the handle at the same time was immediately rewarding but uninformative (fails to disambiguate the causal mechanism). In contrast, either of the single actions was completely disambiguating, but was uncertain to produce an immediate outcome. Each toy had a label at the front, indicating the correct action(s)–outcome link.

<sup>3</sup>We assume that the observer is naive about the toy’s causal structure; if the observer is knowledgeable,  $U_{perf}$  and  $U_{pres}$  will diverge, which is an important consideration for future work.

<sup>4</sup>Our hypothesis, method, model and data analysis were pre-registered prior to data collection at <https://osf.io/kcjjau>.

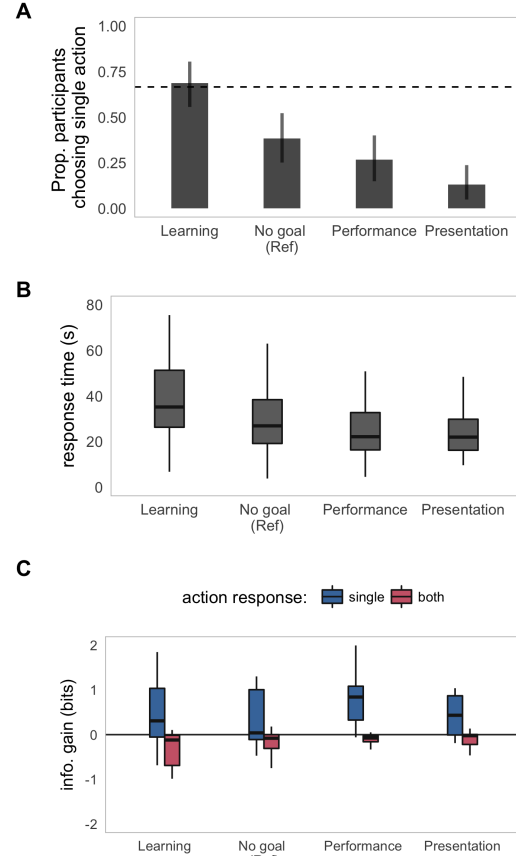


Figure 3: Behavioral results for Exp. 1. A: Proportion of action decisions for each goal condition. Error bars are 95% binomial CIs based on a Bayesian beta-binomial model. Dashed line represents chance level at 66.7% (two single actions out of three action choices). B: Distribution of response times on the action decisions. C: Distribution of participants’ belief change (information gain in bits) as a function of condition. Higher values represent more information gained from the action selection.

We asked participants to act on one of these toys; importantly, the given toy was missing its label, leading to uncertainty about its causal structure. We randomly assigned participants into four goal conditions. In the *No-goal* condition we did not specify any goal for participants. In the *Learning*, *Performance*, and *Presentation* conditions, we asked participants to imagine they were toy developers and one day their boss approached them. We instructed participants to: figure out the correct label for the toy (*Learning*); make the toy play music (or turn the light on; *Performance*); or impress their boss and show that they are competent (*Presentation*). We asked participants to select an action out of the following set: “press the button”, “pull the handle”, or “press the button and pull the handle.” The order of actions was randomized.

**Procedure** In the *exposure phase*, we showed participants an example toy and gave instructions for three toy types. We

first presented the instructions for the single action toys (Toy 1 and Toy 2) in a randomized order, and then presented the instructions for the both action toy (Toy 3). After instructions, participants indicated what action would make each toy operate (e.g., “How would you make [this] toy play music?”) to show that they understood how the different toys worked.

In the *test phase*, participants read a scenario for one of the four goal conditions, followed by the question: “If you only had one chance to try a SINGLE action [to pursue the specified goal], which action would you want to take? You will get a 10 cent bonus ... if you [achieve the given goal]”.

Both before and after the critical action decision trial, we asked participants to rate the likelihood that the unknown toy was Toy 1, 2, or 3, which indexed participants’ prior beliefs about how the toys were likely to function and their *belief change* after selecting an action and observing its effect.

## Results and discussion

**Action decisions:** We modeled action decisions (informative “single” vs. rewarding “both” action) using a logistic regression  $action \sim goal\_condition$  with the No-goal condition as the reference category.<sup>5</sup> Participants’ tendency to select a “single” action varied across conditions as predicted (Fig. 3A), with the highest proportion in the Learning condition<sup>6</sup>, followed by No-goal, Performance, and Presentation.

Compared to the No-goal condition, participants selected the single action at a greater rate in the Learning condition ( $\beta = 1.18$ , [0.82, 1.55]) and at lower rate in the Presentation context ( $\beta = -1.53$ , [-2, -1.06]), with the null value of zero difference condition falling well outside the 95% HDI, and at similar rate in the Performance condition ( $\beta = -0.65$ , [-1.04, -0.27]) with the 95% HDI including the null.

**Action decision times:** As an exploratory measure, we analyzed decision times, or the latency to make an action selection as measured from the start of the action decision trial (all RTs were analyzed in log space), using the same model specification as action decisions. Fig. 3A shows the full RT data distribution. Compared to the No-goal condition, participants took longer to generate a decision in the Learning condition. In contrast, participants in the Performance and Presentation conditions produced similar decision times. These findings suggest that reaching a decision in the Learning condition was cognitively more effortful than the other conditions.

<sup>5</sup>In all of the analyses for Exp. 1 and Exp. 2, we used the `rstanarm` package to fit Bayesian regression models estimating the differences across conditions. We report the uncertainty in our point estimates using 95% Highest Density Intervals (HDI). The HDI provides a range of credible values given the data and model. Also, we combined the two action choices – button press or handle pull, each on its own – into one “single” action in our analysis, as both of those action choices are fully informative and do not differ in their salience as suggested by judgments in a separate prior elicitation task.

<sup>6</sup>Note that the “single” action choices did not go above the chance level of 66.7% in any of the conditions, including the Learning condition. One possible reason is the relative salience of the “both” action choice, given the single-attempt decision-making context, where participants may have drawn to observing the immediate rewarding effect.

**Belief change:** We quantified participants’ change in beliefs about the toy using information gain. We computed the Kullback-Leibler (KL) divergence both before and after participants’ action selections. The KL divergence gives a measure of the distance between the correct<sup>7</sup> probability distribution and the participant’s beliefs about the identity of the unlabeled toy. We notate participants’ belief distributions as  $B_{prior}$  and  $B_{prior+a}$  and the correct distribution as  $B_{correct}$ . The difference between these KL divergences provides the number of bits of information gained due to the action:  $IG(a) = D_{KL}(B_{correct}||B_{prior}) - D_{KL}(B_{correct}||B_{prior+a})$ .

We modeled information gain as a function of goal condition and action choices:  $IG \sim goal\_condition + action\_response$  (Fig. 3C). Across all conditions, people who selected the single action showed a greater gain in bits of information ( $\beta_{single} = 0.91$ , [0.77, 1.05], i.e., learned more from their action. We did not see evidence of an interaction between goal and action selection. However, a larger proportion of participants selected a single action in the Learning context, so learning was more likely in this condition.

## Experiment 2

In Exp. 1, we confirmed that participants selected different actions depending on the type of goal emphasized. In Exp. 2, our goals were three-fold: (1) to replicate the results from Exp. 1; (2) to manipulate goals *and* the presence/absence of another person (social/no-social) independently, allowing us to measure the interaction between goals and social context; and (3) to compare empirical data with predictions of our computational model. Our key behavioral prediction was an interaction: that participants would be less likely to select a single (more informative) action in the Learning goal and No-goal conditions when their boss was present. We also predicted a null result: that the presence of the boss should not affect action decisions in the Performance condition.

## Method

**Participants** Using the same recruitment and exclusion criteria as Exp. 1, we recruited 347 participants (42-51 per condition), and included 325 participants in our final analysis.

**Stimuli and Design** The stimuli and design were identical to Exp. 1, except we had 7 different goal  $\times$  social conditions. Goals were identical to Exp. 1; social context varied depending on whether the boss was present (*Social*) or absent (*No-social*) in the story. The conditions were: *Social-learning*, *Social-performance*, *Social-presentation*, *No-social-no-goal*, *No-social-learning*, *No-social-performance*, and *Social-no-goal*. Note that we did not have *No-social-presentation* condition, because the presentation goal was defined by presenting oneself as competent to another person.

**Procedure** The procedure was identical to Exp. 1.

<sup>7</sup>Note that since the action-effect link was deterministic, the correct belief distribution is a function of participant’s action decision. For example, if a participant selected the button action, then  $B_{correct}$  placed 100% of the probability mass on the button hypothesis.

## Results and discussion

**Action decisions:** We modeled action decisions using a logistic regression specified as  $action \sim goal\_condition * social\_context$  with the No-social-no-goal condition as the reference category. We replicated the key finding from Exp. 1: participants selected a “single” action more often in the Learning goal condition, followed by the No-goal, Performance, and Presentation conditions (Fig. 4A). There was a main effect of social context, with participants being less likely to select the single action when their boss was present ( $\beta = -0.521, [-1.005, -0.053]$ ). Finally, there was evidence for a reliable interaction between goal condition and social context such that the effect of social context was present in the Learning and No-goal conditions, but not in the Performance condition ( $\beta_{perf-social-int} = 1.148, [0.049, 2.296]$ ).

**Action decision times:** We replicated the key decision time finding from Exp. 1: Participants made slower decisions in the Learning context than in Performance/Presentation. On average, participants took 39.32 seconds to make a response in the No-goal condition and 40.72 sec in the Learning condition. In contrast, decisions were faster in the Performance ( $\beta = -7.67$  sec,  $[-14.01, -1.25]$ ) and Presentation ( $-10.66$  sec,  $[-18.37, -3.36]$ ) conditions, which were similar to one another (Fig. 4B). There was no evidence of a main effect of social context or an interaction between goal condition and social context. Note that here we did not see a difference in decision times between the Learning and No-goal conditions, which is different from the pattern in Exp. 1.

**Belief change:** We replicated the information gain effect from Exp. 1: Participants who selected a single action showed greater information gain across all conditions ( $\beta_{single} = 0.63, [0.4, 0.86]$ ). There was no evidence of a main effect of social context or two-/three-way interactions between social context, goal, and action choice. As in Exp. 1, more participants selected the single action in the Learning condition, especially in the No-social context, meaning information gain was most likely in this learning context.

**BDA model-data fit:** In our paradigm, participants chose an action based on a certain goal.<sup>8</sup> We assumed that the goal descriptions (e.g., “impress your boss”) conveyed to the participants a particular set of goal weights  $\{\phi_{learn}, \phi_{perf}, \phi_{pres}\}$  used to generate action choices. We put uninformative priors on these weights ( $\phi \sim Unif(0, 1)$ ) and inferred their credible values for each social-goal condition, using Bayesian data analytic techniques (Lee & Wagenmakers, 2014).

The inferred goal weights were consistent with our predictions (Fig. 4D).  $\phi_{learn}$  was highest for the No-social learning condition, whereas  $\phi_{perf}$  and  $\phi_{pres}$  together made up the highest portion in the Presentation condition, with high social pressure to appear competent.

<sup>8</sup>For action priors, we used a separate task in which people indicated the likelihood for selecting an action without any information about possible hypotheses or goals. We used the mean likelihood for each action choice as baseline priors in our model.

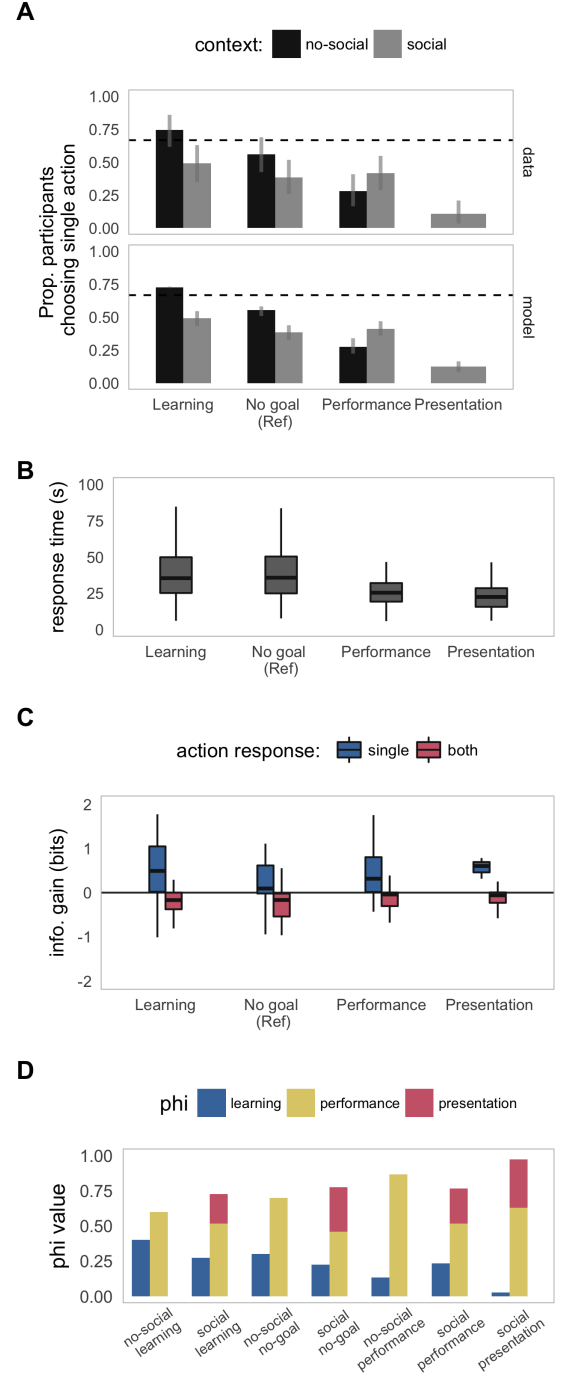


Figure 4: Behavioral and model results for Exp. 2. A: Action decisions from human data (top) and fitted model predictions (bottom). Color represents social context. B and C: Decision times and belief change respectively, collapsing across social contexts. D: Inferred phi values for each condition. All other plotting conventions are the same as Fig. 3.

We also inferred another parameter of the cognitive model, the optimality parameter  $\lambda$ , which represents how optimally the agent acts with respect to the total utility. We put uninfor-



mative prior on the parameter ( $\lambda \sim \text{Unif}(0, 10)$ ) and inferred its posterior credible value from the data. We ran 4 MCMC chains for 100,000 iterations, discarding the first 50,000 for burnin. The Maximum A-Posteriori estimate and 95% HDI Interval for  $\lambda$  was 4.79 [3.96, 6.2].

The fitted model predictions of action choices are shown in Fig. 4A (bottom). The model's expected posteriors over action choices capture key differences between conditions: the single action was more likely for No-social than Social conditions overall, but not when the performance goal was highlighted. The model was able to predict the distribution of action responses with high accuracy  $r^2(21) = 0.9$ .

## General Discussion

How do social contexts shape active learning? We proposed that people integrate informational vs. social goals when deciding what to do. In two experiments, we showed that people chose more informative actions when learning goals were highlighted and in the absence of a relevant social context (no boss present), while they chose more immediately rewarding actions when performance/presentational goals were highlighted, especially when a boss was present. When no goal was specified, people's behavior seemed to reflect a mixture of goals. Our model of social-active learning successfully captured key patterns in the people's action decisions.

This work begins to bring active learning accounts into contact with social learning theories. We used ideas from Optimal Experiment Design, which models active learning as a process of rational choice to maximize information gain, and Rational Speech Act models, which formalize recursive social reasoning within a Bayesian framework. We included social information within a formal utility-theoretic framework, building a richer utility function that represented a weighted combination of multiple goals – informational and social.

There are limitations to this work that present opportunities for future work. First, we did not differentiate between performance and presentation goals, since the choice of doing both actions satisfies both of these goals in our paradigm. Enriching the space of possible actions could tease apart actions driven by self-presentation, especially when optimal actions for demonstrating your competence and knowledge may be different from actions to produce immediately rewarding outcomes. Second, we used a particular social context (the presence of a boss) to emphasize presentational goals. Our model can be extended to explain a richer set of social considerations, such as other kinds of observers (e.g., a teacher who wants the learner to select actions that help her learn). Third, we limited people to a single action choice. While this allowed a clean measurement of our condition manipulations, real-world learning often involves sequential decision-making (Rafferty, Brunskill, Griffiths, & Shafto, 2011, Ho, Littman, MacGlashan, Cushman, & Austerweil (2016)) that could cause learners to prioritize different goals depending on their prior actions or the probability of interacting with an observer in the future.

Another interesting open question is how our model could be used to understand active learning over development. Our framework could allow us to measure changes in children's goal preferences as they develop better social reasoning and meta-cognitive abilities. One prediction is that young children focus on learning goals earlier on when they are surrounded by familiar caregivers who scaffold learning-relevant actions. But as their social reasoning abilities mature and their social environments become more complex, children may start to emphasize performance or presentation goals.

Overall, this work represents a first step to answering these rich questions that ultimately seek to unify theories on active learning and social reasoning.

All experiments, data, model, and analysis codes are available in the public repository for the project: <https://github.com/kemacdonald/soc-info>

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