Tarih: 30/05/2022
Saat : 11:00-12:20

ADI SOYADI:

ÖĞRENCİ NO:

BİLGİSAYAR MÜHENDİSLİĞİ BÖLÜMÜ
DİFERENSİYEL DENKLEMLER DERSİ FİNAL SINAVI

İŞLEM YAPILMADAN VERİLEN CEVAPLAR DİKKATE ALINMAYACAKTIR

1) $x^2y''+3xy'+y=\dfrac{1}{x\ln x}\ (x>1)$ denkleminin genel çözümünü elde ediniz.

$x=e^t$ $\quad y'=\dfrac{dy}{dt}\dfrac{dt}{dx}=\dfrac{1}{x}\dfrac{dy}{dt}$

$y''=\dfrac{1}{x^2}\left(\dfrac{d^2y}{dt^2}-\dfrac{dy}{dt}\right)$ ile

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=\frac{e^{-t}}{t}$$

$r^2+2r+1=0 \quad (r+1)^2=0$

$r_1=r_2=-1$

$$\boxed{y_h=c_1e^{-t}+c_2te^{-t}}$$

$y_p=c_1(t)e^{-t}+c_2(t)te^{-t}$

$$\left.\begin{aligned} c_1'e^{-t}+c_2'te^{-t}&=0\\ -c_1'e^{-t}+c_2'(e^{-t}-te^{-t})&=\frac{1}{t}e^{-t}\end{aligned}\right\} \Rightarrow \begin{aligned} c_1'+c_2't&=0\\ -c_1'+c_2'(1-t)&=\frac{1}{t}\end{aligned}$$

$c_1'=-1 \Rightarrow \boxed{c_1=-t}$

$c_2'=\frac{1}{t} \Rightarrow \boxed{c_2=\ln t}$

$$y_p=-te^{-t}+t\ln t\,e^{-t}$$

$$y_g=c_1e^{-t}+c_2te^{-t}-te^{-t}+t\ln t\,e^{-t}$$

$$\boxed{y_g=c_1x^{-1}+c_2x^{-1}\ln x-\frac{1}{x}\ln x+\frac{\ln x}{x}\ln(\ln x)}$$

2) $y''+y'+xy=0$ denkleminin genel çözümünü $x=0$ noktası civarında kuvvet serileri yardımıyla elde ediniz.

$x=0$ adi nokta. $\quad y=\sum_{n=0}^{\infty} a_n x^n$

$$y'=\sum_{n=1}^{\infty} n a_n x^{n-1} \qquad y''=\sum_{n=2}^{\infty} n(n-1) a_n x^{n-1}$$

$$\sum_{n=2}^{\infty} n(n-1)a_n x^{n-1} + \sum_{n=1}^{\infty} n a_n x^{n-1} + \sum_{n=0}^{\infty} a_n x^{n+1} = 0$$

$$\sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^n + \sum_{n=0}^{\infty}(n+1)a_{n+1}x^n + \sum_{n=1}^{\infty} a_{n-1}x^n = 0$$

$$2a_2+a_1+\sum_{n=1}^{\infty}\left\{(n+1)(n+2)a_{n+2}+(n+1)a_{n+1}+a_{n-1}\right\}x^n=0$$

$2a_2+a_1=0 \qquad \boxed{a_{n+2}=\frac{-(n+1)a_{n+1}-a_{n-1}}{(n+1)(n+2)}} \quad n\geq 1$

$a_2=-\frac{a_1}{2} \qquad a_3=\frac{-2a_2-a_0}{6}=-\frac{1}{3}a_2-\frac{1}{6}a_0$

$$\boxed{a_3=\frac{1}{6}a_1-\frac{1}{6}a_0}$$

$$a_4=\frac{-3a_3-a_1}{12}=-\frac{1}{4}a_3-\frac{1}{12}a_1=-\frac{1}{24}a_1+\frac{1}{24}a_0-\frac{2}{24}a_1$$

$$=\frac{1}{24}a_0-\frac{3}{24}a_1$$

$$y=a_0+a_1x-\frac{1}{2}a_1x^2+\left(\frac{1}{6}a_1-\frac{1}{6}a_0\right)x^3+\left(\frac{1}{24}a_0-\frac{3}{24}a_1\right)x^4+\cdots$$

$$=a_0\left(1-\frac{1}{6}x^3+\frac{1}{24}x^4+\cdots\right)+a_1\left(x-\frac{x^2}{2}+\frac{1}{6}x^3-\frac{1}{8}x^4+\cdots\right)$$

3) $\left(2y^2-xy-2xy^2\right)dx+\left(x+4xy+1\right)dy=0$ denklemi için önce $\lambda=\lambda(x)$e bağlı bir integrasyon çarpanı bulunuz. Daha sonra bu çarpan yardımıyla denklemin genel çözümünü elde ediniz.

$P(\cancel{x}y) = 2y^2 - xy - 2xy^2 \qquad Q(x,y) = 1 + x + 4xy$

$$\frac{Q_x - P_y}{-Q} = -1 \Rightarrow \boxed{\lambda = e^{-x}}$$

$$e^{-x}(2y^2 - xy - 2xy^2)\,dx + e^{-x}(x + 4xy + 1)\,dy = 0$$

$$\frac{\partial F}{\partial x} = e^{-x}(2y^2 - xy - 2xy^2)$$

$$\frac{\partial F}{\partial y} = e^{-x}(x + 4xy + 1) \Rightarrow F(x,y) = e^{-x}(xy + 2xy^2 + y) + h(x)$$

$$e^{-x}(2y^2 - xy - 2xy^2) = -e^{-x}(xy + 2xy^2 + y) + e^{-x}(y + 2y^2) + h'(x)$$

$$= e^{-x}(y + 2y^2 - xy - 2xy^2 - y) + h'(x)$$

$$= e^{-x}(2y^2 - xy - 2xy^2) + h'(x)$$

$$\Rightarrow h'(x) = 0 \Rightarrow h(x) = C$$

$$\Rightarrow \boxed{e^{-x}(xy + 2xy^2 + y) = C}$$

4) $\begin{cases} y'+z=x \\ z'+4y=0 \end{cases}$ $y(0)=z(0)=0$ sisteminin genel çözümünü Laplace dönüşümü yardımıyla elde ediniz. $L\{y^{(n)}\}=s^nY(s)-s^{n-1}y(0)-s^{n-2}y'(0)-...-y^{(n-1)}(0)$

$$\begin{aligned} L\{y'+z\} &= L\{x\} \\ L\{z'+4y\} &= L\{0\} \end{aligned} \Rightarrow \begin{aligned} sY(s)+Z(s) &= \frac{1}{s^2} \\ sZ(s)+4Y(s) &= 0 \end{aligned}$$

$$4Z(s)-s^2Z(s)=\frac{4}{s^2} \Rightarrow \boxed{Z(s)=\frac{4}{s^2(4-s^2)}}$$

$$Z(s)=\frac{4}{s^2(2-s)(2+s)}=\frac{A}{s}+\frac{B}{s^2}+\frac{C}{2-s}+\frac{D}{2+s}$$

$A=0 \quad B=1 \quad C=\frac{1}{4} \quad D=\frac{1}{4}$

$$Z(s)=\frac{1}{s^2}-\frac{1}{4}\frac{1}{s-2}+\frac{1}{4}\frac{1}{s+2}$$

$$\boxed{z(x)=x-\frac{1}{4}e^{2x}+\frac{1}{4}e^{-2x}}$$

$y(x)=-\frac{1}{4}z'(x)$ ise

$$y(x)=-\frac{1}{4}\left[1-\frac{2}{4}e^{2x}-\frac{2}{4}e^{-2x}\right)$$

$$\boxed{y(x)=-\frac{1}{4}+\frac{1}{8}e^{2x}+\frac{1}{8}e^{-2x}.}$$