	TC. SAÜ. MÜH. FAK. METALURJİ VE MALZEME MÜH. BÖL. 2018–2019 ÖĞR. YILI BAHAR DÖNEMİ LİNEER CEBİR DERSİ ARASINAVI	Tarih	08.04.2019	1	2	3	4
		Öğ No					
		Ad					
		Soyad					

Sorular eşit puanlıdır. Sınav Süresi 70 dakikadır.

SORULAR

1) $A=\begin{bmatrix}1 & x & 0\\ 0 & 1 & x\\ 0 & 0 & x\end{bmatrix}$ matrisi için $AA^T=?$ ve $A^TA=?$

2) $\begin{aligned} x_1+x_2-2x_3&=0\\ x_1-2x_2+x_3&=1\\ -2x_1+x_2+x_3&=-1\end{aligned}$ lineer denklem sistemini çözünüz.

3) $\begin{bmatrix}1 & 1 & 1\\ 2 & 1 & 1\\ 1 & 2 & -2\end{bmatrix}$ matrisinin tersini bulunuz.

4) $\begin{vmatrix}1 & 1 & 1 & 1\\ 2 & 2 & 4 & 4\\ 3 & 5 & 5 & 5\\ 4 & 7 & 7 & 8\end{vmatrix}$ determinantının en sade halini hesaplayınız.

CEVAPLAR

ANAHTAR (METALURJİ)

Soru 1. $A=\begin{bmatrix}1 & x & 0\\ 0 & 1 & x\\ 0 & 0 & x\end{bmatrix} \Rightarrow A^T=\begin{bmatrix}1 & 0 & 0\\ x & 1 & 0\\ 0 & x & x\end{bmatrix}$

$$AA^T=\begin{bmatrix}1 & x & 0\\ 0 & 1 & x\\ 0 & 0 & x\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ x & 1 & 0\\ 0 & x & x\end{bmatrix}=\begin{bmatrix}1+x^2 & x & 0\\ x & 1+x^2 & x^2\\ 0 & x^2 & x^2\end{bmatrix}$$

$$A^TA=\begin{bmatrix}1 & 0 & 0\\ x & 1 & 0\\ 0 & x & x\end{bmatrix}\begin{bmatrix}1 & x & 0\\ 0 & 1 & x\\ 0 & 0 & x\end{bmatrix}=\begin{bmatrix}1 & x & 0\\ 0 & x^2+1 & x\\ 0 & x & 2x^2\end{bmatrix}$$

Soru 2. $\left[\begin{array}{ccc|c}1 & 1 & -2 & 0\\ 1 & -2 & 1 & 1\\ -2 & 1 & 1 & -1\end{array}\right] \underset{-R_1+R_2\to R_2}{\overset{2R_1+R_3\to R_3}{\sim}} \left[\begin{array}{ccc|c}1 & 1 & -2 & 0\\ 0 & -3 & 3 & 1\\ 0 & 3 & -3 & -1\end{array}\right] \overset{R_2+R_3\to R_3}{\sim}$

$$\left[\begin{array}{ccc|c}1 & 1 & -2 & 0\\ 0 & -3 & 3 & 1\\ 0 & 0 & 0 & 0\end{array}\right] \overset{-\frac{1}{3}R_2\to R_2}{\sim} \left[\begin{array}{ccc|c}1 & 1 & -2 & 0\\ 0 & 1 & -1 & -1/3\\ 0 & 0 & 0 & 0\end{array}\right]$$

$x_2 - x_3 = -1/3$ $\quad \xRightarrow{x_3=t} \quad x_2 = -\frac{1}{3}+t$

$x_1+x_2-2x_3=0$ $\quad\quad x_1 = 2x_3 - x_2 = 2t - \left(-\frac{1}{3}+t\right) = t+\frac{1}{3}$

Yani $(x_1, x_2, x_3) = \left(t+\frac{1}{3},\ -\frac{1}{3}+t,\ t\right)$; $t\in\mathbb{R}$ şeklinde sonsuz çoklukta çözüm vardır.

Soru 3. $A^{-1} = \frac{1}{\det A} \operatorname{Adj} A$

$$\det A = \det \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{matrix} 1 & 1 & 1 \\ 2 & 1 & 1 \end{matrix} = (-2+4+1) - (1+2-4) = 3-(-1) = 4$$

$$c_{11} = (-1)^{1+1} \cdot \det \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix} = -2-2 = -4; \quad c_{12} = (-1)^{1+2} \cdot \det \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix} = 5$$

$$c_{13} = (-1)^{1+3} \det \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = 3; \quad c_{21} = (-1)^{2+1} \det \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix} = 4$$

$$c_{22} = (-1)^{2+2} \det \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix} = -3; \quad c_{23} = (-1)^{2+3} \det \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = -1$$

$$c_{31} = (-1)^{3+1} \det \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 0; \quad c_{32} = (-1)^{3+2} \cdot \det \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} = -1$$

$$c_{33} = (-1)^{3+3} \det \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} = -1 \qquad \operatorname{Adj}(A) = \begin{bmatrix} -4 & 5 & 3 \\ 4 & -3 & -1 \\ 0 & -1 & -1 \end{bmatrix}^T$$

$$A^{-1} = \frac{1}{4} \cdot \begin{bmatrix} -4 & 4 & 0 \\ 5 & -3 & -1 \\ 3 & -1 & -1 \end{bmatrix}$$

Soru 4.

$$\det\begin{bmatrix}1&1&1&1\\2&2&4&4\\3&5&5&5\\4&7&7&8\end{bmatrix} \overset{-2R_1+R_2\to R_2}{\underset{\substack{-3R_1+R_3\to R_3\\-4R_1+R_4\to R_4}}{=\!=\!=}} \det\begin{bmatrix}1&1&1&1\\0&0&2&2\\0&2&2&2\\0&3&3&4\end{bmatrix}$$

$$\overset{R_2\leftrightarrow R_3}{=\!=\!=} -\det\begin{bmatrix}1&1&1&1\\0&2&2&2\\0&0&2&2\\0&3&3&4\end{bmatrix} \overset{-R_3+R_2\to R_2}{=\!=\!=} -\det\begin{bmatrix}1&1&1&1\\0&2&0&0\\0&0&2&2\\0&3&3&4\end{bmatrix}$$

$$\overset{\frac{1}{2}R_2\to R_2}{\underset{\frac{1}{2}R_3\to R_3}{=\!=\!=}} -2.2.\det\begin{bmatrix}1&1&1&1\\0&1&0&0\\0&0&1&1\\0&3&3&4\end{bmatrix} \overset{-3R_2+R_4\to R_4}{=\!=\!=} -4.\det\begin{bmatrix}1&1&1&1\\0&1&0&0\\0&0&1&1\\0&0&3&4\end{bmatrix}$$

$$\overset{-3R_3+R_4\to R_4}{=\!=\!=} -4.\det\begin{bmatrix}1&1&1&1\\0&1&0&0\\0&0&1&1\\0&0&0&1\end{bmatrix} = -4.\,1.1.1.1 = -4.$$