Number Systems

Last Week

- Floating Point Numbers
- Arithmetic Operations on Signed Numbers
  - Addition and Subtraction
  - Multiplication and Division

This Week

- Hexadecimal Number System
- Octal Number System
- Conversion Between Hexadecimal/Octal/Decimal Systems

Hexadecimal Number System

There are 16 figures in hexadecimal system. First ten figures are numbers from 0 to 9, and the last 6 figures are the letters from A to F. It gets difficult to write and read large binary numbers. But computers work only with binary numbers. Considering that a computer command consists of 32 bits, it would require a massive effort to write a computer program using binary system. Using hexadecimal system instead makes this process easier. In hexadecimal system, each digit corresponds to a 4 bit binary number.

Decimal	Binary	Hexadecimal
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Hexadecimal Number System

In hexadecimal system F_{16} is followed by 10_{16} and continues as 11_{16},12_{16},...,19_{16},$1A_{16}$, ..., $1F_{16}$.

Two digits in hexadecimal can represent numbers between 0 (00_{16}) and 255 (FF_{16}).

- To convert a binary number to hexadecimal, we divide the binary number into groups of four. We start grouping from the right side of the whole part, and from the left side of the fractional part. Remember that we can add 0’s to the left side of the whole part, and to the right side of the fractional part.

Example

❖ Let's convert 010001111110001_2 to hexadecimal. First, we need to divide this number into groups of four.

$$\underbrace{\mathbf{0}010}_{2}\ \underbrace{0011}_{3}\ \underbrace{1111}_{F}\ \underbrace{0001}_{1}$$ (Notice that, we add an extra **0** to the left)

Then, $(010001111110001)_2 = (23F1)_{16}$

❖ Let's convert 1011001101.110010_2 to hexadecimal.

0010 1100 1101 . 1100 10**00**$_2$ = $2CD.C8_{16}$.

Notice that, we add extra 0's to the left side of the whole part and to the right side of the fractional part.

Conversion

➢ To convert a hexadecimal number to binary, we convert each digit independently, and then concatenate the result.

Example: Let's convert $4A3F_{16}$ to binary.

4	A	3	F
0100	1010	0011	1111

$4A3F_{16} = 0100101000111111_2$

Example: Let's convert $ABC.DE_{16}$ to binary.

$ABC.DE_{16} = 101010111100.11011110_2$

Conversion

- There are two ways to convert hexadecimal numbers to decimal.

1st Way

We convert the number to binary, then convert to decimal.

Example: $2A_{16} = 00101010_2 = 2^5+2^3+2^1 = 42_{10}$

2nd Way

We convert the number to binary using sum of weights method.

Example: Let's convert $(1B.1A)_{16}$ to decimal.

$(1B.1A)_{16} = 1\times16^1+11\times16^0+1\times16^{-1}+10\times16^{-2} = 16+11+1/16+10/256 \cong 27.1$

Conversion

To convert from decimal to hexadecimal, we divide the number to 16 repeatedly, and take the remainders.

Example: Let's convert 2577_{10} to hexadecimal.

$$
\begin{array}{llll}
2577 & \big| \; 16 & & \\
\underline{-16} & \big| \; 161 & \big| \; 16 & \\
97 & \underline{-160} & \big| \; \mathbf{10} \longrightarrow \mathbf{A_{16}} & \\
\underline{-\;96} & \mathbf{1} & & \\
17 & & & \\
\underline{-\;16} & & & \\
\mathbf{1} & & &
\end{array}
$$

Then, $2577_{10} = A11_{16}$

Example

Example: Let's convert 123.256 to hexadecimal.

We need to convert the whole part and the fractional part separately.

Whole Part		
	Quotient	**Remainder**
123/16	7	**11 (B)**
7/16	0	**7**

Fractional Part	
0.256×16	**4**.096
0.096×16	**1**.536
0.536×16	**8**.576

$123.536 \cong 7B.418_{16}$

Addition on Hexadecimal Numbers

In hexadecimal addition, we add up each digits decimal values. If the sum is less than or equal to 15, we write the corresponding hexadecimal digit to the result. If the sum is greater than 15, than we subtract 16 from the sum and write the difference to the sum and carry 1 to the next digit.

Example:

$$\begin{array}{r} 29_{16} \\ +\ \underline{1A_{16}} \\ 43_{16} \end{array}$$

$9_{16}+A_{16} = 9_{10}+10_{10} = 19_{10}$

$19-16 = 3$ carry 1

$2+1+1(\text{carry}) = 4$

Example:

$$\begin{array}{r} EF_{16} \\ +\ \underline{9B_{16}} \\ 18A_{16} \end{array}$$

$F_{16}+B_{16} = 15_{10}+11_{10} = 26_{10}$

$26-16 = 10 = A_{16}$ carry 1

$E_{16}+9_{16}+1(\text{carry}) = 14_{10}+9_{10}+1=24_{10}$

$24-16 = 8$ carry 1

Subtraction on Hexadecimal Numbers

As you remember, we do binary subtraction by converting the second number to two's complement and then adding up these two numbers.

We can use the same method to do hexadecimal subtraction. To do this, we first convert the second number to binary, calculate two's complement, and convert back to hexadecimal. Then add the numbers.

Example: Let's calculate 25_{16} - $1B_{16}$

$1B_{16}$ = 00011011_2. Two's complement of 00011011 is 11100101_2 which is $E5_{16}$.

$$\begin{array}{r} 25_{16} \\ \underline{+\ E5_{16}} \\ \mathbf{1}0A_{16} \end{array}$$

The result is 0A (we ignore the carry bit)

Octal Number System

Octal system is not as widely used as hexadecimal system. In octal system, numbers from 0 to 7 are used. 7 is followed by 10.

0,1,…,6,7,10,11,…,16,17,20,21,…

We convert octal numbers to decimal using sum of weights method.

Example: Let's convert 1234_8 to decimal.

$1234_8 = 1\times8^3 + 2\times8^2 + 3\times8^1 + 4\times8^0 = 668_{10}$

Example: Let's convert 12.34_8 to decimal.

$12.34_8 = 1\times8^1 + 2\times8^0 + 3\times8^{-1} + 4\times8^{-2} = 8+2+3/8+4/64 \cong 10.44$

Conversion

➢ To convert a decimal number to octal, we divide the number by 8 repeatedly. Then, we concatenate the remainders and form the octal number.

Example: Let’s convert 123_{10} to octal.

$$
\begin{array}{l|l}
123 & 8 \\ \cline{2-2}
\underline{-\ 8} & 15 \ | \ \underline{8} \\
\ \ 43 & \underline{-\ 8} \ | \ \mathbf{1} \\
\underline{-\ 40} & \ \ \mathbf{7} \\
\ \ \ \ \mathbf{3} &
\end{array}
$$

Then, $123_{10} = 173_8$

Conversion

Example: Let's convert 34.45_{10} to octal.

Whole Part		
	Quotient	**Remainder**
34/8	4	**2**
4/8	0	**4**

Fractional Part	
0.45×8	**3**.6
0.6×8	**4**.8
0.8×8	**6**.4

$34.45 \cong 42.346_8$

➢Conversion from octal to binary is just like hexadecimal system. We convert each digit to 3-bit binary numbers and concatenate these numbers.

Example: Let's convert 456_8 to binary.

$456_8 = \underline{100}\,\underline{\mathbf{101}}\,\underline{110}_2$

Example: Let's convert 56.34_8 to binary.

$56.34_8 = \mathbf{101}110.\mathbf{011}100_2$

Conversion

➢ Conversion of binary numbers to octal is just like hexadecimal numbers. The only difference is that we divide the binary number into groups of four. We start grouping of the whole part from right to left, and the fractional part from left to right. Remember that we can add extra 0's to the left of the whole part, and to the right of the fractional part.

Example: Let's convert 101110001_2 to octal.

$\underline{101}\,\underline{110}\,\underline{001}_2 = 561_8$

Example: Let's convert 10011.11010_2 to octal.

010 011 . 110 10**0** (Divided into groups of three and added extra 0's.)

Then, $10011.11010_2 = 23.64_8$

Conversion

- To convert from octal to hexadecimal, we first convert from octal to binary, then to hexadecimal. Likewise, to convert from hexadecimal to octal, we first convert from hexadecimal to binary, then to octal.

Example: Let’s convert 45.63_8 to hexadecimal.

$\mathbf{4}5.\mathbf{6}3_8 = \mathbf{100}101.\mathbf{110}011_2$

0010 0101 . 1100 11**00** (Divide the binary number to groups of four)

Then, $45.63_8 = 25.CC_{16}$

Example: Let’s convert $A2.B3_{16}$ to octal.

$\mathbf{A}2.\mathbf{B}3_{16} = \mathbf{1010}0010.\mathbf{1011}0011_2$

010 100 010 . 101 100 11**0** (Divide the binary number to groups of three)

Then, $A2.B3_{16} = 242.546_8$