	TC. SAÜ. MÜH. FAK. ENDÜSTRİ MÜH. BÖL. 2018–2019 ÖĞR. YILI BAHAR DÖNEMİ LİNEER CEBİR DERSİ ARASINAVI	Tarih	08.04.2019	1	2	3	4
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Sorular eşit puanlıdır. Sınav Süresi 70 dakikadır.

SORULAR

1) $A=\begin{bmatrix}1 & x & 0\\ x & 0 & 1\\ 1 & 0 & x\end{bmatrix}$ matrisi için $AA^T=?$ ve $A^TA=?$

2) $\begin{matrix} x_1+x_2-2x_3=0\\ x_1-2x_2+x_3=1\\ -2x_1+x_2+x_3=-1\end{matrix}$ lineer denklem sistemini çözünüz.

3) $\begin{bmatrix}1 & 3 & -2\\ 2 & 5 & -3\\ -3 & 2 & -4\end{bmatrix}$ matrisinin tersini bulunuz.

4) $\begin{vmatrix}1 & 1 & 0 & 1\\ x & 1 & 1 & 1\\ x & 0 & x & x\\ x & x & x & 1\end{vmatrix}$ determinantının en sade halini hesaplayınız.

CEVAPLAR

ANAHTAR (ENDÜSTRİ)

① $A=\begin{bmatrix}1 & x & 0\\ x & 0 & 1\\ 1 & 0 & x\end{bmatrix} \Rightarrow A^T=\begin{bmatrix}1 & x & 1\\ x & 0 & 0\\ 0 & 1 & x\end{bmatrix}$

$$AA^T=\begin{bmatrix}1 & x & 0\\ x & 0 & 1\\ 1 & 0 & x\end{bmatrix}\begin{bmatrix}1 & x & 1\\ x & 0 & 0\\ 0 & 1 & x\end{bmatrix}=\begin{bmatrix}1+x^2 & x & 1\\ x & x^2+1 & 2x\\ 1 & 2x & 1+x^2\end{bmatrix}$$

$$A^TA=\begin{bmatrix}1 & x & 1\\ x & 0 & 0\\ 0 & 1 & x\end{bmatrix}\begin{bmatrix}1 & x & 0\\ x & 0 & 1\\ 1 & 0 & x\end{bmatrix}=\begin{bmatrix}2+x^2 & x & 2x\\ x & 0 & 0\\ 2x & 0 & 1+x^2\end{bmatrix}$$

② $$\left[\begin{array}{ccc|c}1 & 1 & -2 & 0\\ 1 & -2 & 1 & 1\\ -2 & 1 & 1 & -1\end{array}\right] \underset{-R_1+R_2\to R_2}{\overset{2R_1+R_3\to R_3}{\sim}} \left[\begin{array}{ccc|c}1 & 1 & -2 & 0\\ 0 & -3 & 3 & 1\\ 0 & 3 & -3 & -1\end{array}\right] \overset{R_2+R_3\to R_3}{\sim}$$

$$\left[\begin{array}{ccc|c}1 & 1 & -2 & 0\\ 0 & -3 & 3 & 1\\ 0 & 0 & 0 & 0\end{array}\right] \overset{-\frac{1}{3}R_2\to R_2}{\sim} \left[\begin{array}{ccc|c}1 & 1 & -2 & 0\\ 0 & 1 & -1 & -1/3\\ 0 & 0 & 0 & 0\end{array}\right]$$

$x_2 - x_3 = -1/3 \quad \overset{x_3=t}{\Longrightarrow} \quad x_2 = -\frac{1}{3}+t$

$x_1 + x_2 - 2x_3 = 0 \qquad x_1 = 2x_3 - x_2 = 2t-\left(-\frac{1}{3}+t\right) = t+\frac{1}{3}$

Yani $(x_1, x_2, x_3) = \left(t+\frac{1}{3},\ -\frac{1}{3}+t,\ t\right);\ t\in\mathbb{R}$ şeklinde sonsuz

çoklukta çözüm vardır

③ $A^{-1} = \dfrac{1}{\det A} \operatorname{Adj}(A)$

$$\det A = \det \begin{bmatrix} 1 & 3 & -2 \\ 2 & 5 & -3 \\ -3 & 2 & -4 \end{bmatrix} = (-20-8+27)-(30-6-24)$$

$$\begin{matrix} 1 & 3 & -2 \\ 2 & 5 & -3 \end{matrix}$$

$$= (-1) - 0 = -1$$

$$C_{11} = (-1)^{1+1} \det \begin{bmatrix} 5 & -3 \\ 2 & -4 \end{bmatrix} = -20+6 = -14$$

$$C_{12} = (-1)^{1+2} \det \begin{bmatrix} 2 & -3 \\ -3 & -4 \end{bmatrix} = (-1)\cdot(-8-9) = 17$$

$$C_{13} = (-1)^{1+3} \det \begin{bmatrix} 2 & 5 \\ -3 & 2 \end{bmatrix} = 4+15 = 19$$

$$C_{21} = (-1)^{2+1} \det \begin{bmatrix} 3 & -2 \\ 2 & -4 \end{bmatrix} = (-1)\cdot(-12+4) = 8$$

$$C_{22} = (-1)^{2+2} \det \begin{bmatrix} 1 & -2 \\ -3 & -4 \end{bmatrix} = -4-6 = -10$$

$$C_{23} = (-1)^{2+3} \det \begin{bmatrix} 1 & 3 \\ -3 & 2 \end{bmatrix} = (-1)(2+9) = -11$$

$$C_{31} = (-1)^{3+1} \det \begin{bmatrix} 3 & -2 \\ 5 & -3 \end{bmatrix} = -9+10 = 1$$

$$C_{32} = (-1)^{3+2} \det \begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix} = (-1)(-3+4) = -1$$

$$\operatorname{Adj} A = \begin{bmatrix} -14 & 17 & 19 \\ 8 & -10 & -11 \\ 1 & -1 & -1 \end{bmatrix}^T$$

$$C_{33} = (-1)^{3+3} \det \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} = 5-6 = -1$$

$$A^{-1} = \frac{1}{-1} \begin{bmatrix} -14 & 8 & 1 \\ 17 & -10 & -1 \\ 19 & -11 & -1 \end{bmatrix} \checkmark$$

$$④\ \det\begin{bmatrix} 1 & 1 & 0 & 1 \\ x & 1 & 1 & 1 \\ x & 0 & x & x \\ x & x & x & 1 \end{bmatrix} \overset{\substack{(-x)R_1+R_2\to R_2 \\ (-x)R_1+R_3\to R_3 \\ (-x)R_1+R_4\to R_4}}{=\!=\!=\!=} \det\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1-x & 1 & 1-x \\ 0 & -x & x & 0 \\ 0 & 0 & x & 1-x \end{bmatrix}$$

$$= (-1)^{1+1}\cdot 1\cdot \det\begin{bmatrix} 1-x & 1 & 1-x \\ -x & x & 0 \\ 0 & x & 1-x \end{bmatrix} = (-1)^{3+2}\cdot x\cdot \det\begin{bmatrix} 1-x & 1-x \\ -x & 0 \end{bmatrix} +$$

$$(-1)^{3+3}\cdot(1-x)\cdot\det\begin{bmatrix} 1-x & 1 \\ -x & x \end{bmatrix} = (-x)\cdot x(1-x) + (1-x)\cdot\big(\underbrace{x(1-x)+x}_{x-x^2+x}\big)$$

$$= (1-x)\left(-x^2+2x-x^2\right)$$

$$= (1-x)\cdot(2x-x^2) = (1-x)\cdot x(2-x)$$

$$= x(1-x)(2-x)$$