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2017–2018 BAHAR DÖNEMİ LİNEER CEBİR ARASINAV SORULARI

①
$$\begin{aligned} x_1 - 2x_3 + 3x_4 &= -6 \\ x_1 - x_2 + 8x_3 - 4x_4 &= 5 \\ -x_1 - x_2 + 12x_3 - 10x_4 &= 17 \\ x_1 + x_3 &= -3 \end{aligned}$$

lineer denklem sistemini Gauss-Jordan yoketme yöntemi ile çözünüz.

② $\det\begin{bmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{bmatrix} = x.y.z.(x-y)(y-z)(z-x)$ olduğunu gösteriniz

③ $A = \begin{bmatrix} 3 & -1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & -5 \end{bmatrix}$ matrisinin varsa tersini hesaplayınız.

④ $\begin{bmatrix} 2 & 4 & 0 \\ -1 & 2 & -4 \\ 3 & 1 & 5 \end{bmatrix}$ matrisinin rankını hesaplayınız.

<u>NOT</u>: Sınav süresi 80 dk dır. Cevapları <u>cevap kağıdına</u> yazınız.

Sınav çıkışı bu kağıdı teslim etmeyi unutmayınız.

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(1) $\left[\begin{array}{cccc|c} 1 & 0 & -2 & 3 & -6 \\ 1 & -1 & 8 & -4 & 5 \\ -1 & -1 & 12 & -10 & 17 \\ 1 & 0 & 1 & 0 & -3 \end{array}\right] \underset{\substack{-R_1+R_2\to R_2 \\ -R_1+R_4\to R_4}}{\overset{R_1+R_3\to R_3}{\sim}} \left[\begin{array}{cccc|c} 1 & 0 & -2 & 3 & -6 \\ 0 & -1 & 10 & -7 & 11 \\ 0 & -1 & 10 & -7 & 11 \\ 0 & 0 & 3 & -3 & 3 \end{array}\right] \underset{\frac{1}{3}R_3\to R_3}{\overset{-R_2+R_3\to R_3}{\sim}}$

$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 3 & -6 \\ 0 & -1 & 10 & -7 & 11 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 \end{array}\right] \underset{R_3\leftrightarrow R_4}{\overset{(-1)R_2\to R_2}{\sim}} \left[\begin{array}{cccc|c} 1 & 0 & -2 & 3 & -6 \\ 0 & 1 & -10 & 7 & -11 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \underset{2R_3+R_1\to R_1}{\overset{10R_3+R_2\to R_2}{\sim}}$

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & -4 \\ 0 & 1 & 0 & -3 & -1 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{l} x_3 - x_4 = 1 \\ x_2 - 3x_4 = -1 \\ x_1 + x_4 = -4 \end{array} \Longrightarrow$ $x_4 = t \in \mathbb{R}$ denirse,
$x_3 = 1+t$
$x_2 = 1+3t$
$x_1 = -4-t$ olur.

Böylece denklem sisteminin $(-4-t,\ 1+3t,\ 1+t,\ t)$ şeklinde, t parametresine bağlı sonsuz çözümü vardır.

(2) $\det\begin{bmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{bmatrix} = x.y.z.\det\begin{bmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{bmatrix} \underset{(-1)C_1+C_3\to C_3}{\overset{(-1)C_1+C_2\to C_2}{=}} xyz.\det\begin{bmatrix} 1 & 0 & 0 \\ x & y-x & z-x \\ x^2 & y^2-x^2 & z^2-x^2 \end{bmatrix}$

$= x.y.z\ \underbrace{(-1)^{1+1}.1}_{1}.\det\begin{bmatrix} y-x & z-x \\ y^2-x^2 & z^2-x^2 \end{bmatrix} = x.y.z\left\{(y-x).(z^2-x^2)-(z-x)(y^2-x^2)\right\}$

$= x.y.z\left\{(y-x)(z-x)\left(z-\cancel{x}-y+\cancel{x}\right)\right\}$

$= xyz\,(y-x).(z-x).(z-y)$

$= xyz.(x-y).(y-z)(z-x)$

(3) Önce A matrisinin determinantı hesaplanmalı. Sonra

$A^{-1} = \frac{1}{\det A}.\operatorname{Adj}(A)$ formülü ile A^{-1} hesaplanacaktır.

$$\det\begin{bmatrix} 3 & -1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & -5 \end{bmatrix} \overset{(-1)R_2+R_3\to R_3}{=\!=} \det\begin{bmatrix} 3 & -1 & 1 \\ 1 & 3 & -1 \\ 0 & -4 & -4 \end{bmatrix} = (-4)\det\begin{bmatrix} 3 & -1 & 1 \\ 1 & 3 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\overset{(-3)R_3+R_2\to R_2}{=\!=} (-4)\det\begin{bmatrix} 3 & -1 & 1 \\ 0 & 0 & -4 \\ 0 & 1 & 1 \end{bmatrix} = (-4)\cdot\underbrace{(-1)^{2+3}\cdot(-4)}_{4}\cdot\underbrace{\det\begin{bmatrix} 3 & -1 \\ 0 & 1 \end{bmatrix}}_{3} = -48 \neq 0$$

Şimdi Adj A nın elemanlarını hesaplayınız

$A_{11} = (-1)^{1+1}\cdot\det\begin{bmatrix} 3 & -1 \\ -1 & -5 \end{bmatrix} = -16$ $\qquad$ $A_{21} = (-1)^{2+1}\cdot\det\begin{bmatrix} -1 & 1 \\ -1 & -5 \end{bmatrix} = -6$

$A_{12} = (-1)^{1+2}\cdot\det\begin{bmatrix} 1 & -1 \\ 1 & -5 \end{bmatrix} = 4$ $\qquad$ $A_{22} = (-1)^{2+2}\cdot\det\begin{bmatrix} 3 & 1 \\ 1 & -5 \end{bmatrix} = -16$

$A_{13} = (-1)^{1+3}\cdot\det\begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix} = -4$ $\qquad$ $A_{23} = (-1)^{2+3}\det\begin{bmatrix} 3 & -1 \\ 1 & -1 \end{bmatrix} = 2$

$A_{31} = (-1)^{3+1}\det\begin{bmatrix} -1 & 1 \\ 3 & -1 \end{bmatrix} = -2$ $\qquad$ $A_{32} = (-1)^{3+2}\cdot\det\begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix} = 4$

$A_{33} = (-1)^{3+3}\cdot\det\begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix} = 10$

$$A^{-1} = \frac{1}{-48}\begin{bmatrix} -16 & 4 & -4 \\ -6 & -16 & 2 \\ -2 & 4 & 10 \end{bmatrix}^T = \frac{-1}{48}\begin{bmatrix} -16 & -6 & -2 \\ 4 & -16 & 4 \\ -4 & 2 & 10 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{8} & \frac{1}{24} \\ \frac{-1}{12} & \frac{1}{3} & \frac{-1}{12} \\ \frac{1}{12} & \frac{-1}{24} & \frac{-5}{24} \end{bmatrix}$$

4) $\begin{bmatrix} 2 & 4 & 0 \\ -1 & 2 & -4 \\ 3 & 1 & 5 \end{bmatrix} \underset{2R_2+R_1\to R_1}{\overset{3R_2+R_3\to R_3}{\sim}} \begin{bmatrix} 0 & 8 & -8 \\ -1 & 2 & -4 \\ 0 & 7 & -7 \end{bmatrix} \underset{\frac{1}{7}R_3\to R_3}{\overset{\frac{1}{8}R_1\to R_1}{\sim}} \begin{bmatrix} 0 & 1 & -1 \\ -1 & 2 & -4 \\ 0 & 1 & -1 \end{bmatrix} \overset{R_1+R_3\to R_3}{\sim}$

$\begin{bmatrix} 0 & 1 & -1 \\ -1 & 2 & -4 \\ 0 & 0 & 0 \end{bmatrix} \overset{(-1)R_2\to R_2}{\sim} \begin{bmatrix} 0 & 1 & -1 \\ 1 & -2 & 4 \\ 0 & 0 & 0 \end{bmatrix} \overset{R_1\leftrightarrow R_2}{\sim} \begin{bmatrix} 1 & -2 & 4 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ rank=2

Rank = hepsi birden 0 olmayan satır
[illegible]