



# BSM307

## İşaretler ve Sistemler

Dr. Seçkin Arı

Giriş

# Tanıtım

- Seçkin Arı
- Ofis #1161
- [ari@sakarya.edu.tr](mailto:ari@sakarya.edu.tr)
- Kaynak
  - ◆ Ders Notları
  - ◆ A. V. Oppenheim, A.S. Wilsky ve S.H. Nawab, *Signals and Systems*, Prentice Hall (Pearson)
  - ◆ J.G. Proakis ve D.G. Manolakis, *Digital Signal Processing*, Pearson
- Değerlendirme
  - ◆ 3 Kısa sınav
  - ◆ 1 Ara sınav
  - ◆ 1 Dönem sonu sınavı

# Ders İçeriği

- Ayırık Zaman İşaret ve Sistemler
  - ◆ Birim Darbe Cevabı
  - ◆ Fark Denklemleri
  - ◆ Durum Denklemleri
  - ◆  $z$ -Dönüşümü
  - ◆ Ters  $z$ -Dönüşümü
- Sürekli Zaman İşaret ve Sistemler
  - ◆ Fourier Seri Açılımı
  - ◆ Fourier Dönüşümü
  - ◆ Örnekleme

- İşaret (Signal)
- Sistem
- Ayrık (Kesikli) Zaman İşaret ve Sistemler
- Sürekli Zaman İşaret ve Sistemler
- Bağımsız Değişken Dönüşümleri
- Birim Darbe ve Birim Basamak Fonksiyonları

# İşaret

- Matematiksel bir fonksiyon
  - ♦ Fiziksel değişimler
  - ♦ Bilgi taşıyan
  - ♦ Bağımsız değişken: zaman
  - ♦ Bağımlı değişken: voltaj, akım, basınç, sıcaklık, akış hızı, vs....

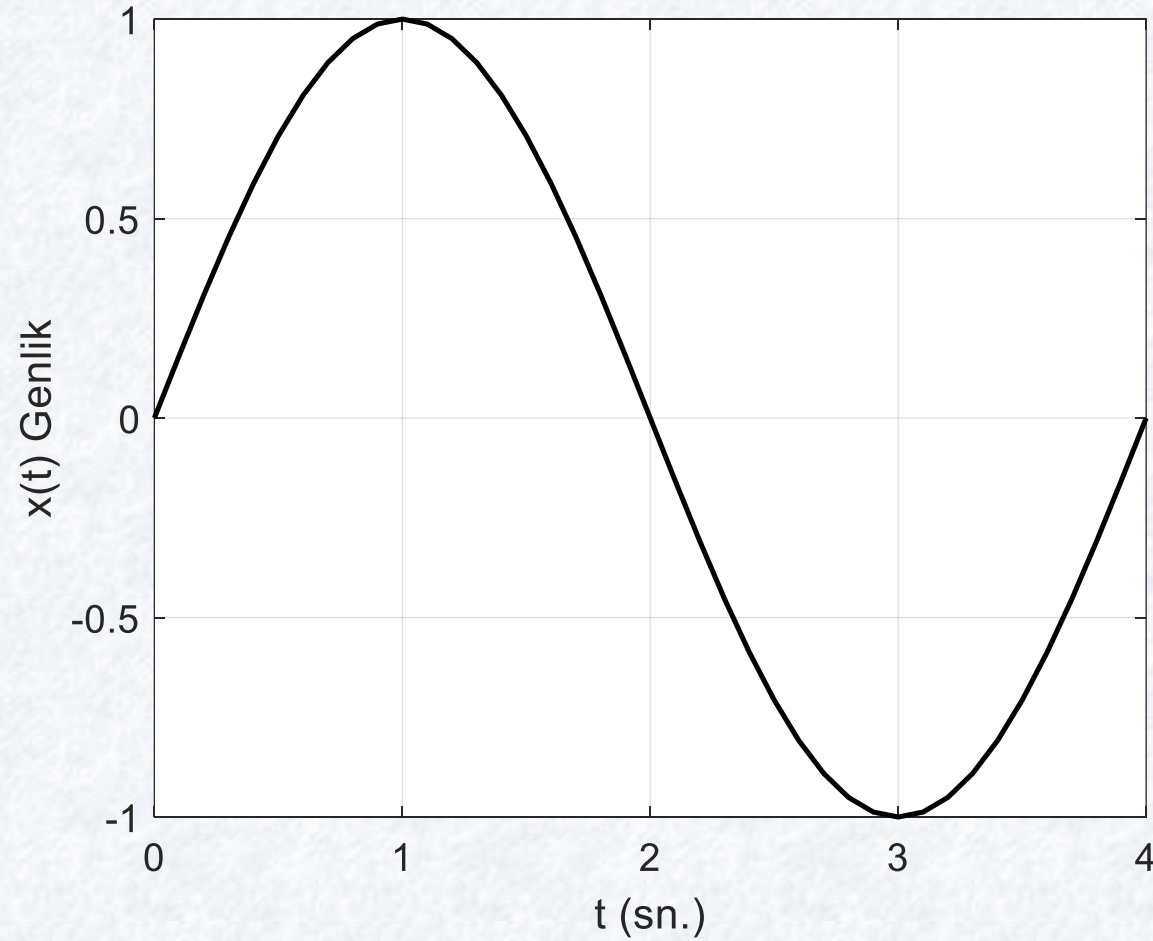


# Örnek İşaretler

- Akıllı telefonlar arasında paylaşılan bilgiler
- Elektromanyetik dalgalar
- Ses
- Görüntü
- Audio
- Video
- Banka faiz oranları
- Borsa indisi
- Döviz oranları
- Tıbbi görüntüler
- EKG
- EEG
- Seçim sonuçları
- Sınav sonuçları

# İşaret Türleri

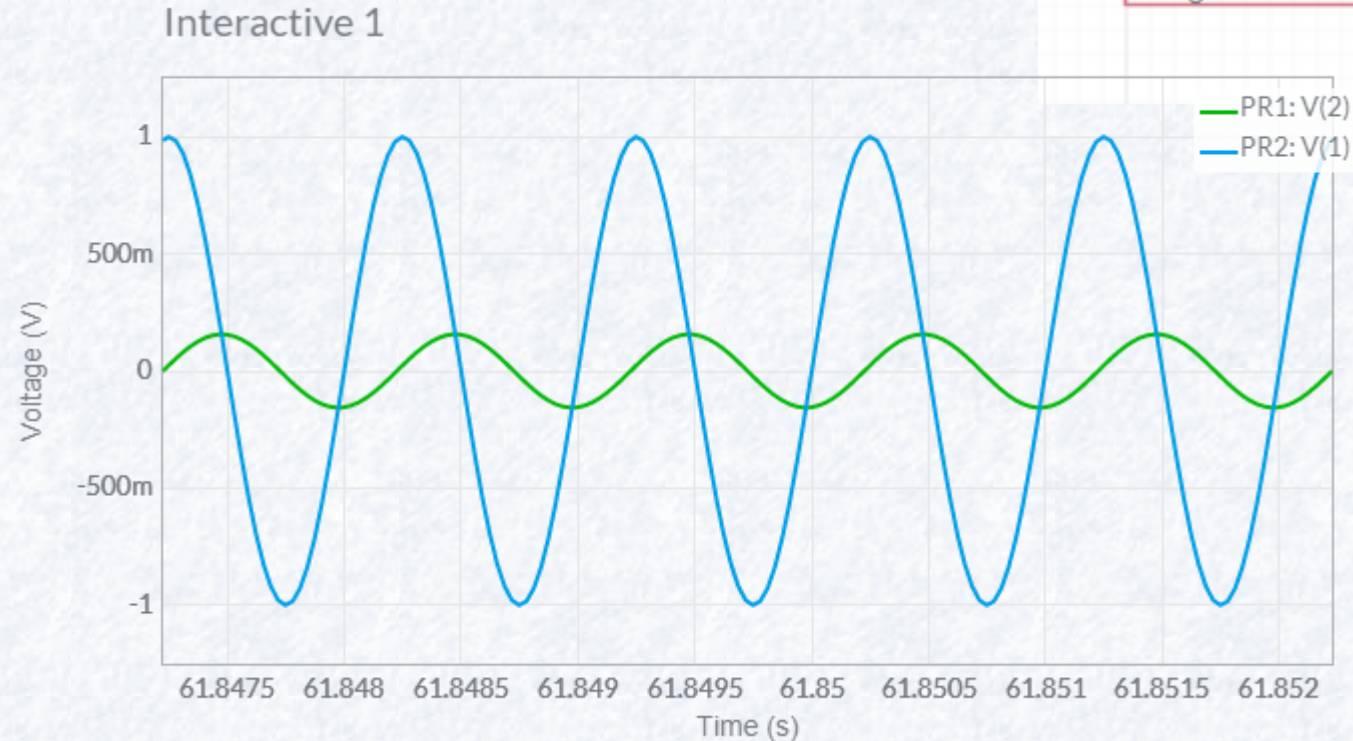
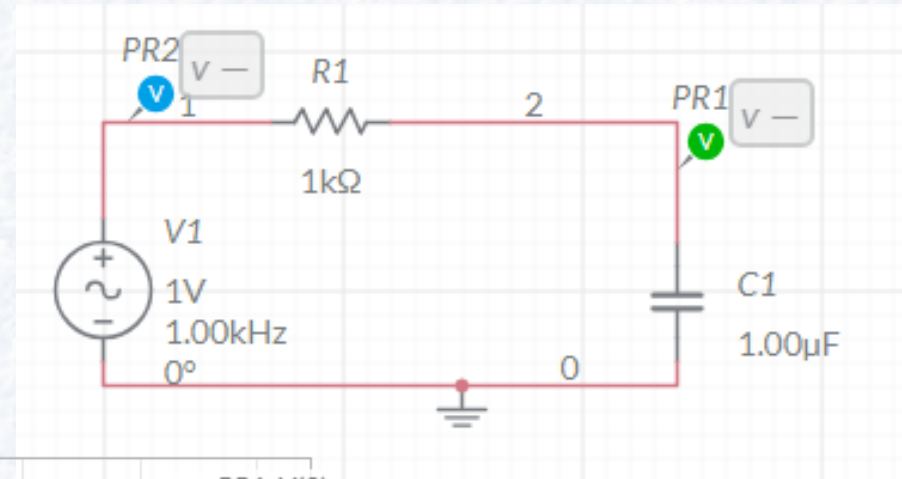
- Sürekli Zaman İşaret
- $x(t)$





# İşaret Türleri

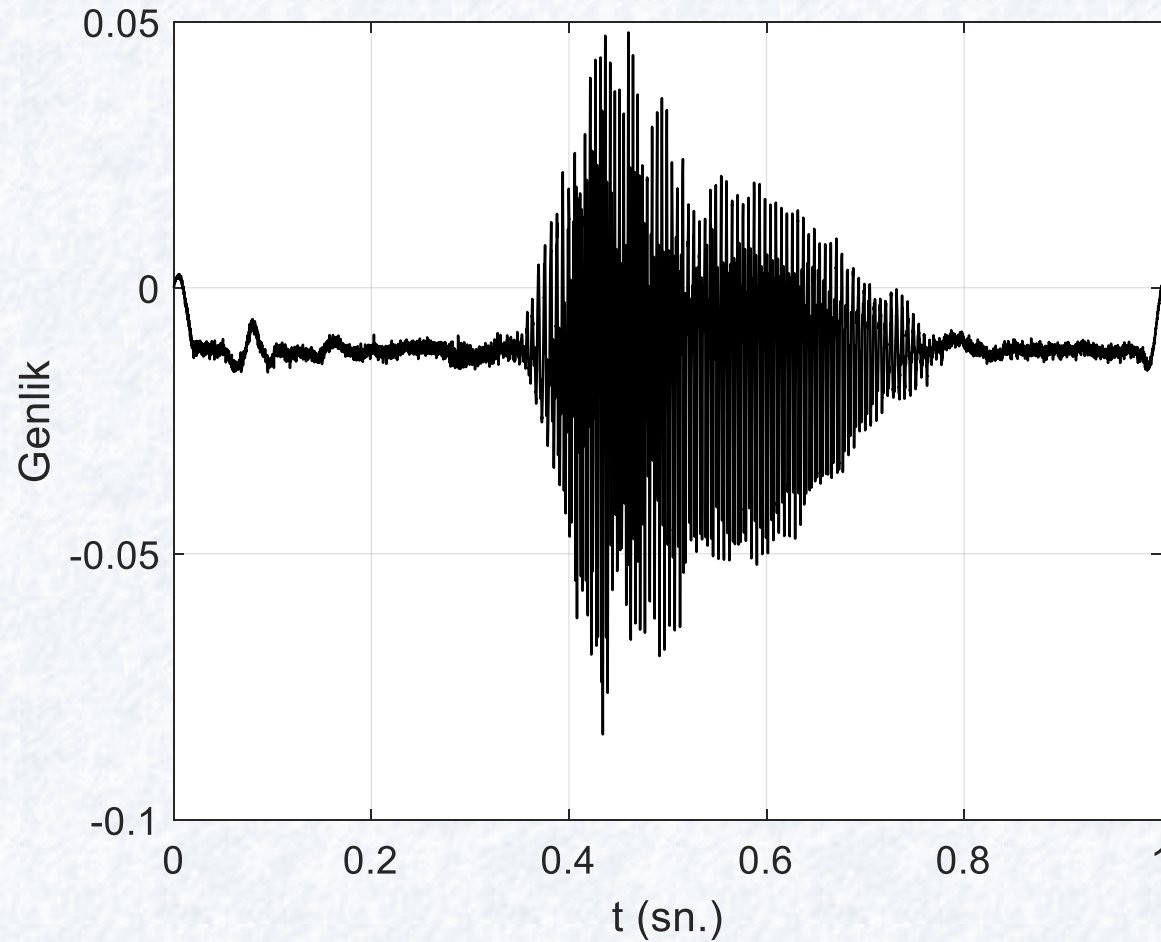
- Sürekli Zaman İşaret -  $x(t)$ 
  - ◆ RC devre çıkışı



# İşaret Türleri

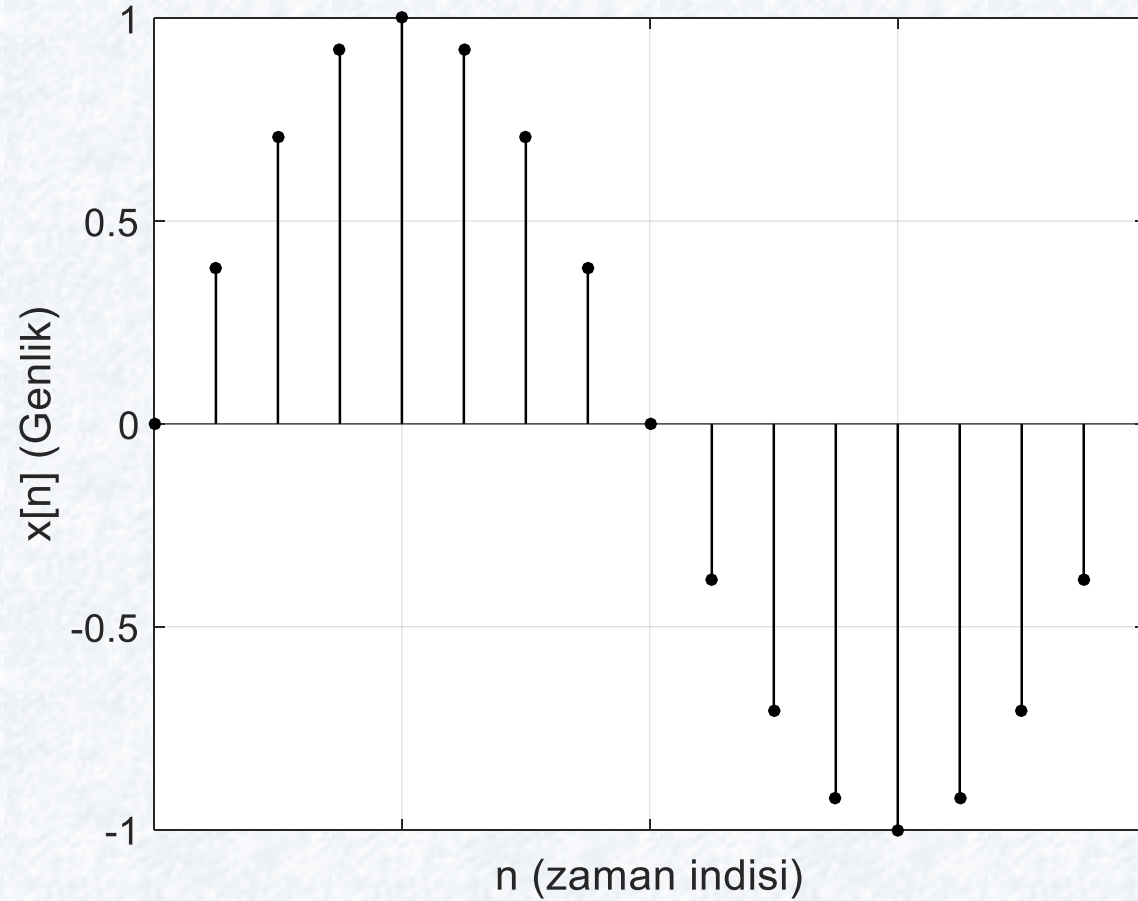
- Sürekli Zaman İşaret -  $x(t)$

- ◆ Ses



# İşaret Türleri

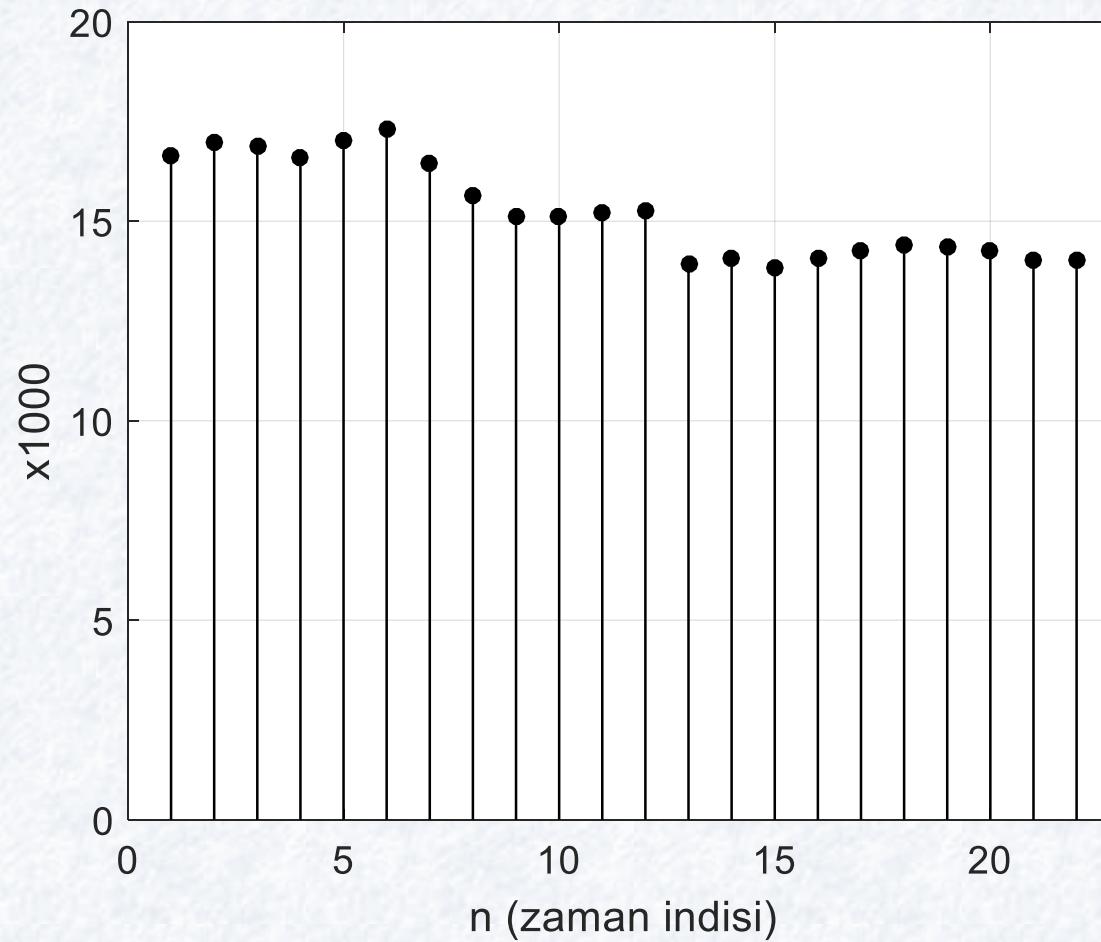
- Ayırık Zaman İşaret
- $x[n]$



# İşaret Türleri

- Ayırık Zaman İşaret -  $x[n]$

- ◆ BIST



# İşaret Türleri

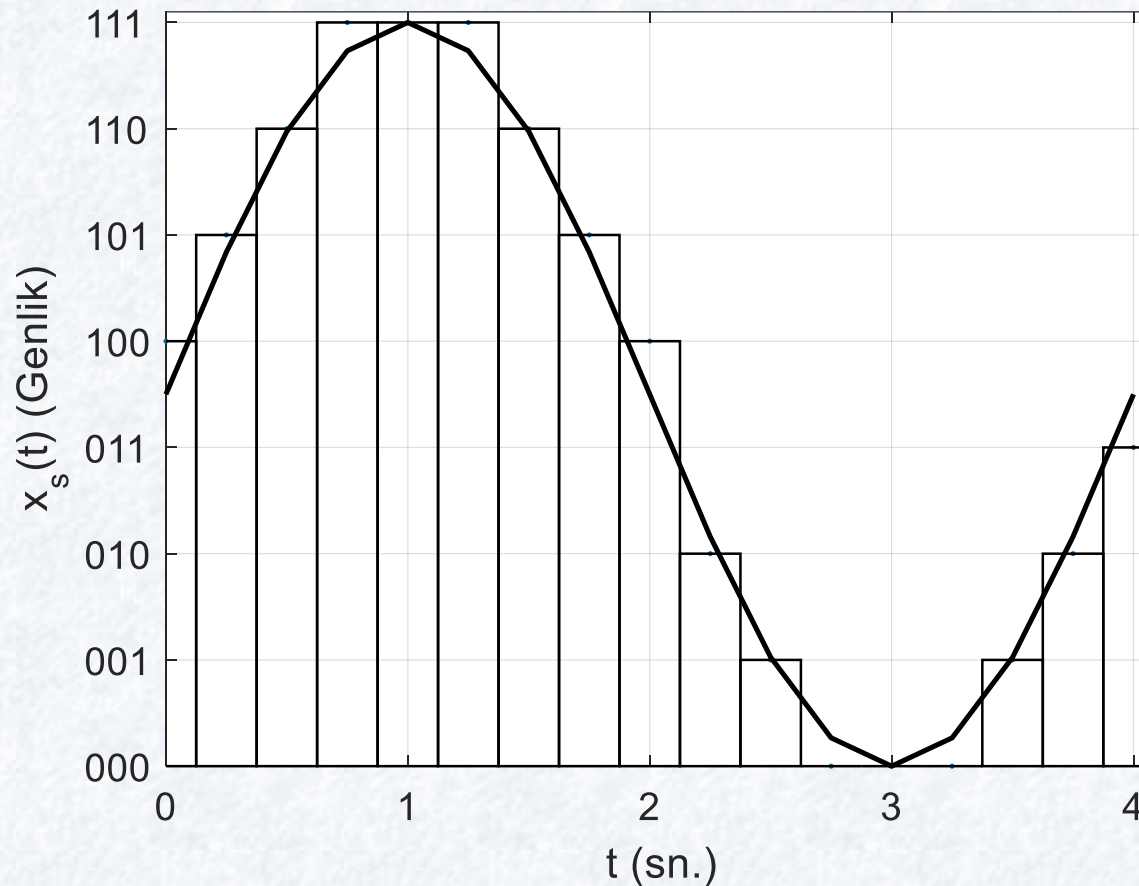
- Ayırık Zaman İşaret -  $x[n]$ 
  - ♦ Görüntü



|     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|
| 29  | 29  | 29  | 29  | 29  | 29  | 29  |
| 29  | 29  | 29  | 29  | 29  | 29  | 29  |
| 173 | 173 | 173 | 173 | 29  | 29  | 29  |
| 7   | 7   | 7   | 173 | 173 | 173 | 173 |
| 7   | 7   | 173 | 173 | 7   | 173 | 173 |
| 173 | 173 | 173 | 173 | 173 | 7   | 7   |
| 173 | 7   | 173 | 7   | 173 | 7   | 7   |
| 173 | 173 | 173 | 173 | 173 | 7   | 7   |
| 173 | 7   | 173 | 7   | 173 | 7   | 7   |
| 173 | 173 | 173 | 173 | 173 | 7   | 7   |
| 173 | 7   | 173 | 7   | 173 | 7   | 7   |
| 173 | 173 | 173 | 173 | 173 | 7   | 7   |
| 173 | 7   | 173 | 7   | 173 | 7   | 7   |
| 173 | 173 | 173 | 173 | 173 | 7   | 7   |
| 173 | 7   | 173 | 7   | 173 | 7   | 7   |

# İşaret Türleri

- Sayısal İşaret, kuantalanmış işaret
- $x_s(t)$

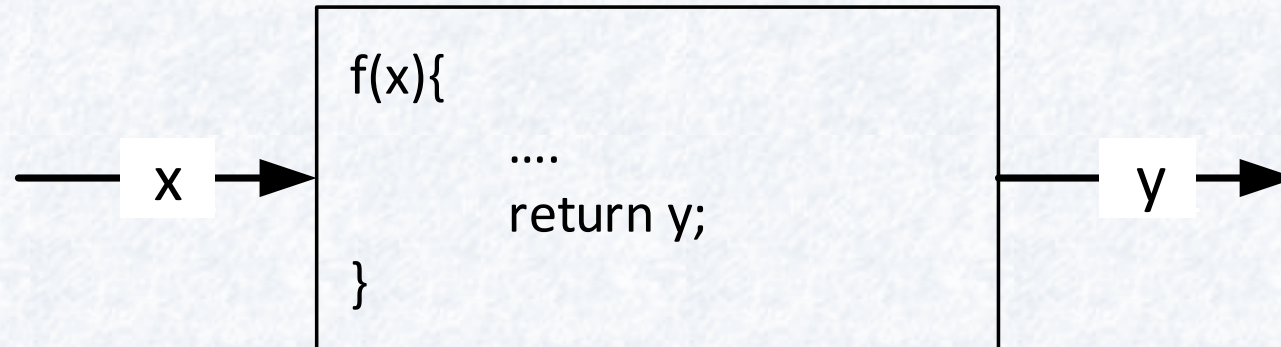




# Sistem

# Sistem

- Giriş işaretini işleyip çıkış işareti oluşturma
  - ◆ Fiziksel
  - ◆ Matematiksel
  - ◆ Bilişimsel



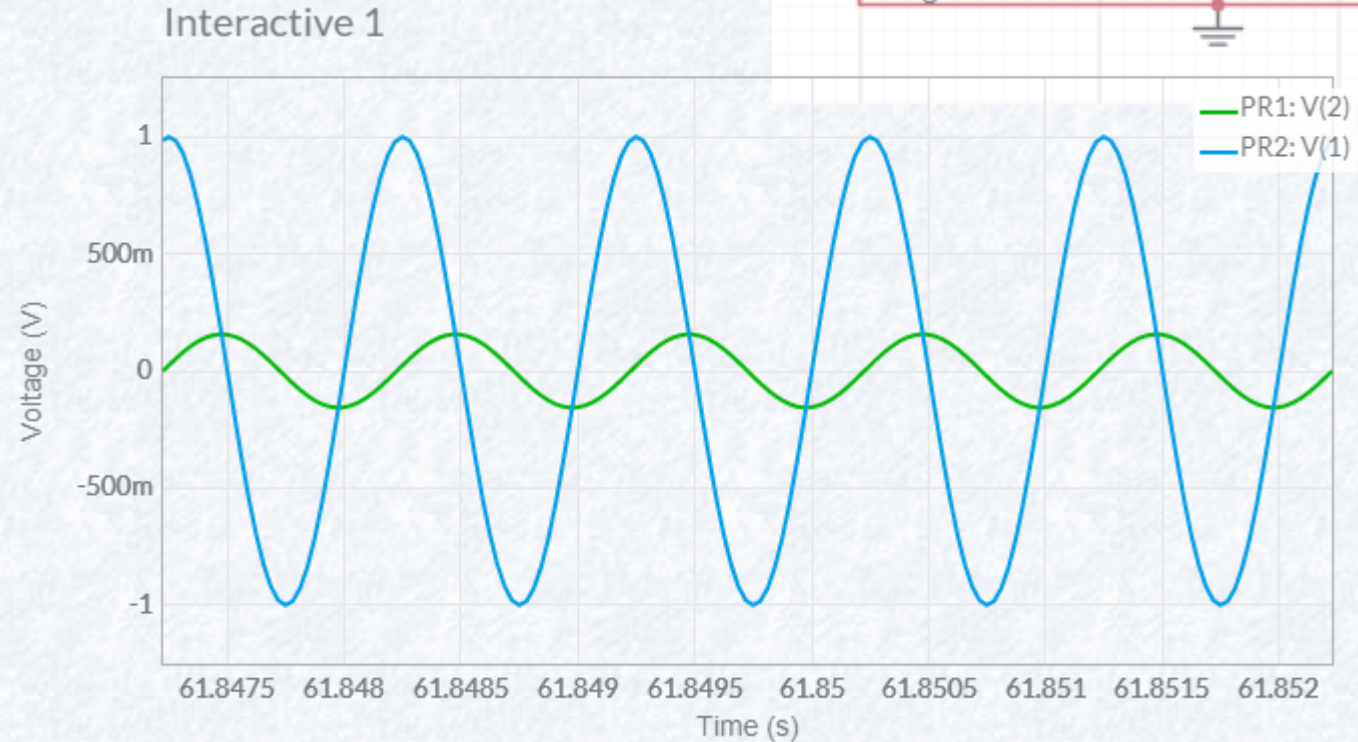
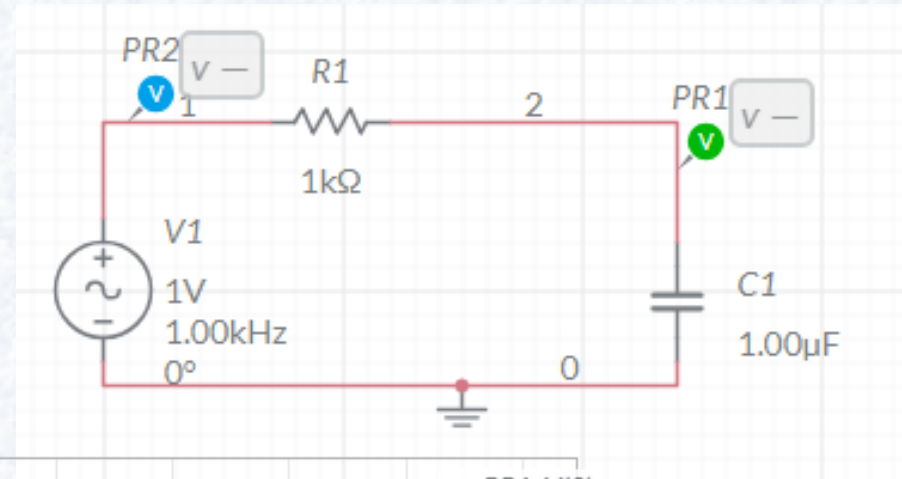
# Sistem Türleri

- Sürekli Zaman Sistemler
  - ♦ RC devreleri (Voltaj, akım...)
  - ♦ Mekanik sistemler
    - Yay
    - Havuz
    - Taşıt

# Sistem

- RC devreleri (Voltaaj, akım...)

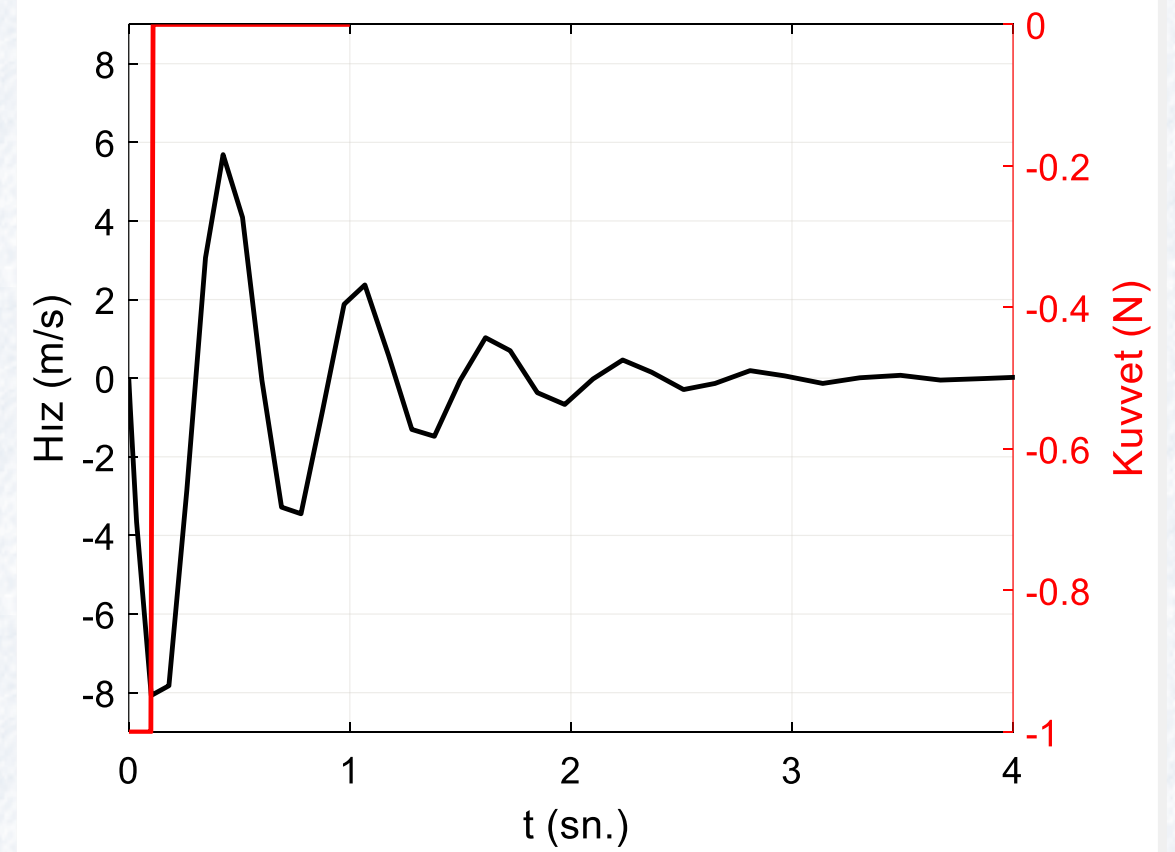
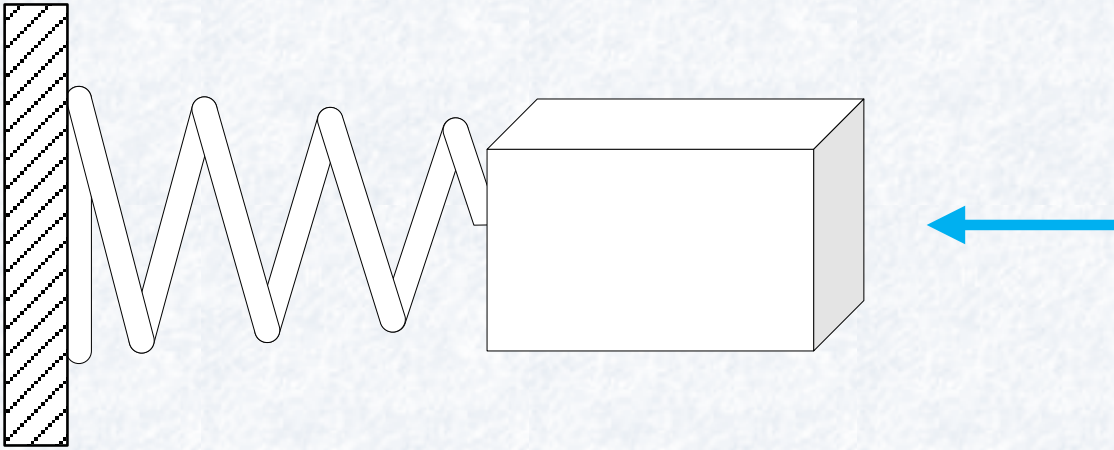
- ◆ Giriş: AC voltaaj kaynağı
- ◆ Çıkış: Kapasitör gerilimi



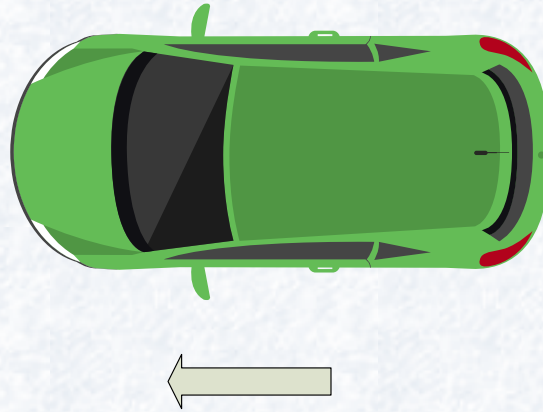
# Sistem

- Kütle yay sistemleri

- ◆ Giriş: Uygulanan kuvvet
- ◆ Çıkış: Hız (Yer değiştirme)



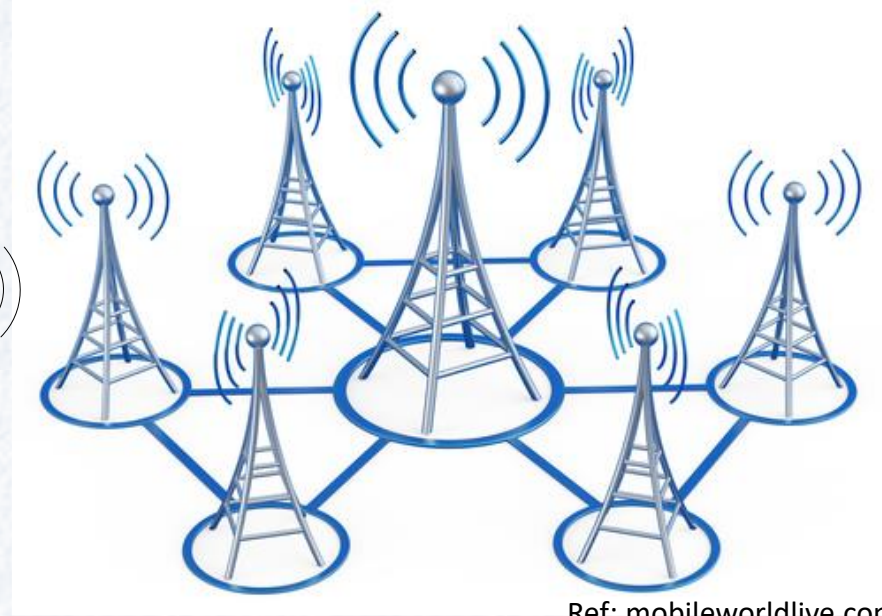
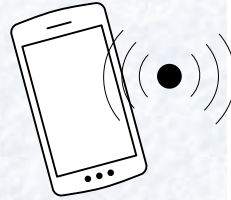
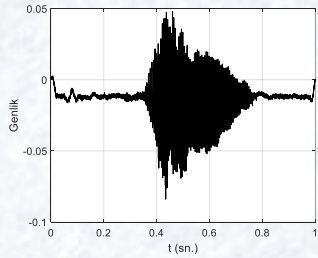
- Taşıt
  - ◆ Giriş: Gaz pedal açısı
  - ◆ Çıkış: Hız



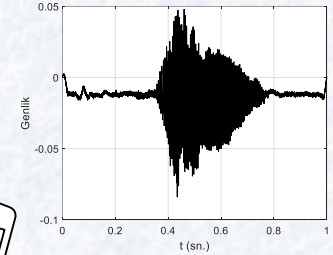
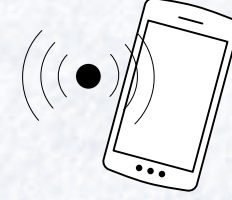


# Sistem

- Cep Telefonu
  - ◆ Giriş: Ses
  - ◆ Çıkış: Ses

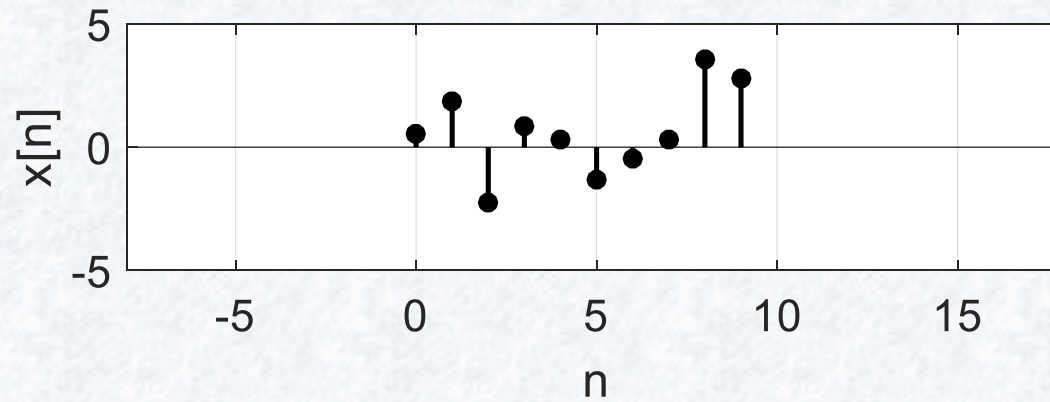


Ref: mobileworldlive.com



# Bağımsız Değişken Dönüşümü

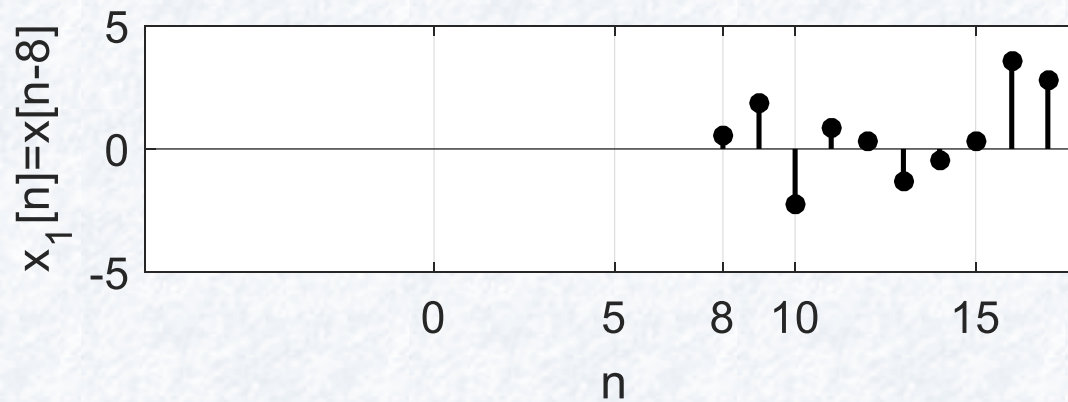
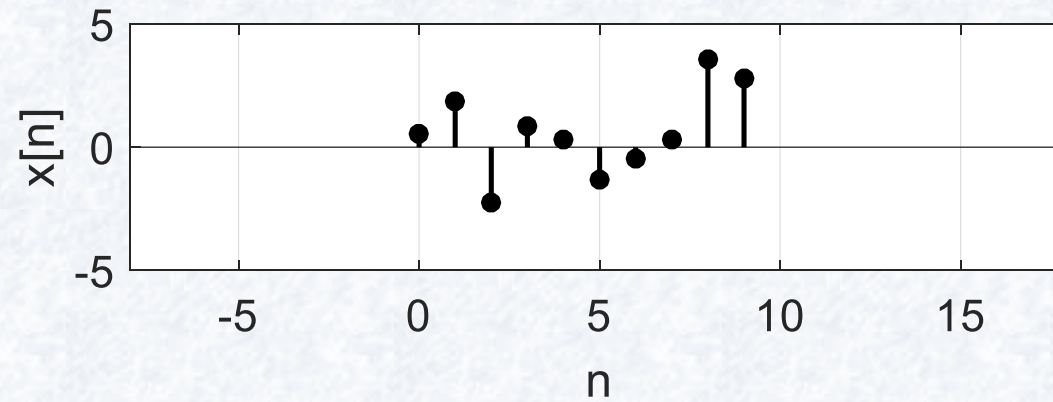
- Zamanda Öteleme
  - ♦  $x_1[n] = x[n - 8]$



# Bağımsız Değişken Dönüşümü

- Zamanda Öteleme

- ♦  $x_1[n] = x[n - 8]$

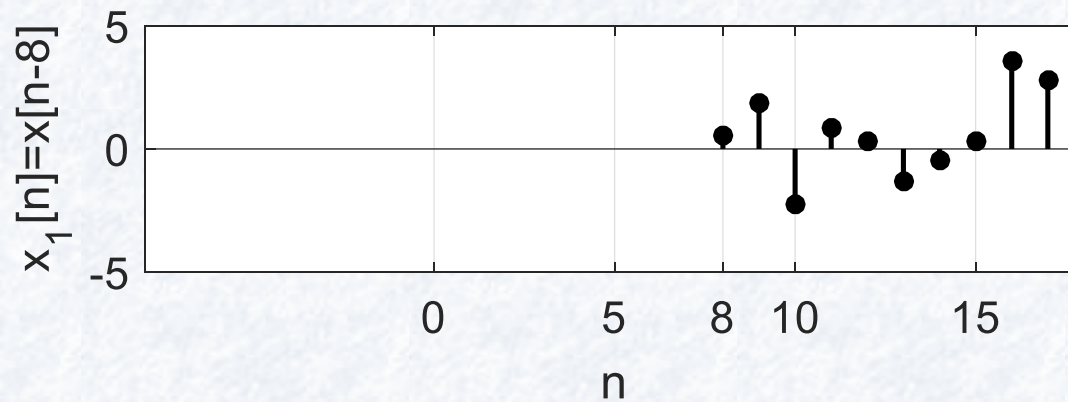
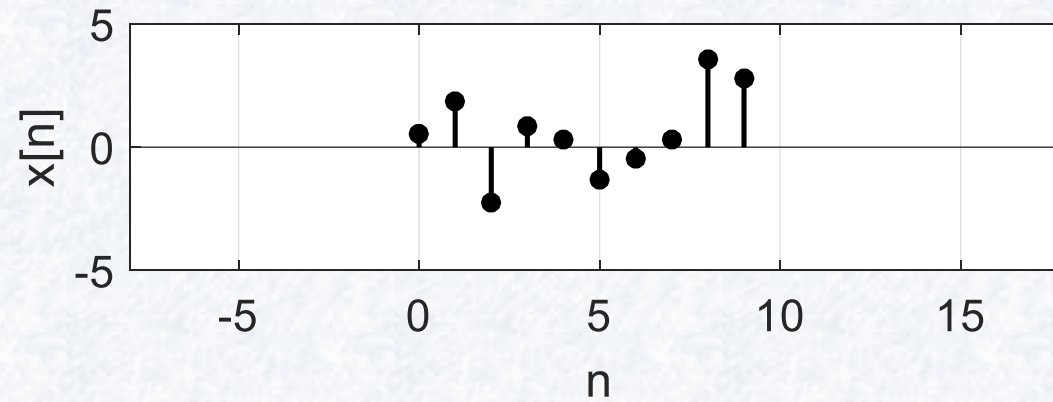


# Bağımsız Değişken Dönüşümü

- Zamanda Öteleme

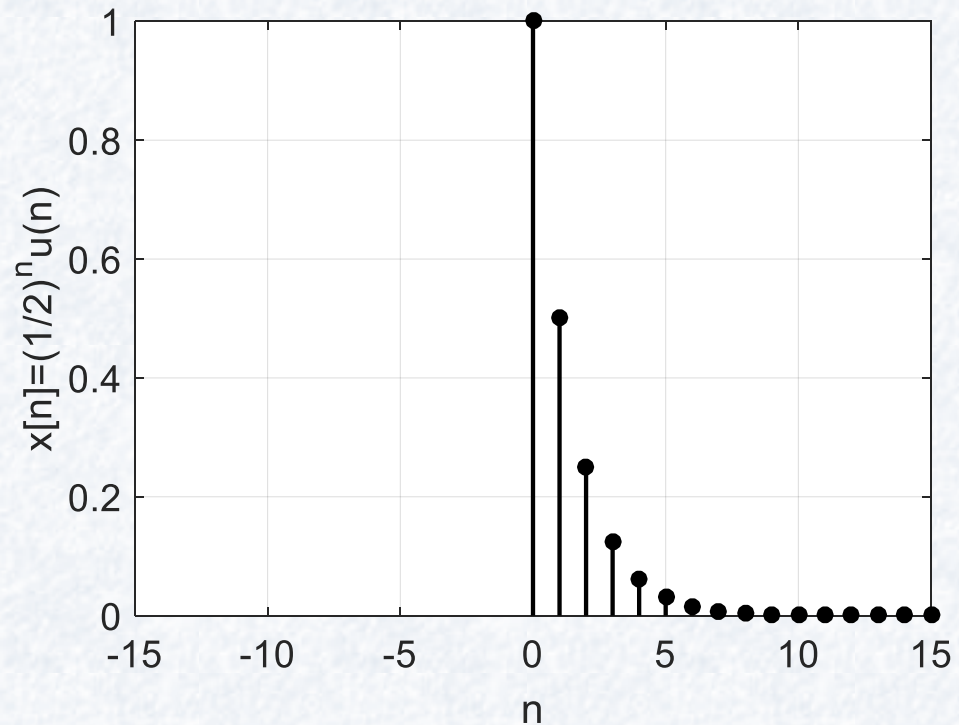
- ◆  $x_1[n] = x[n - 8]$

- Geçmiş



# Örnek 1

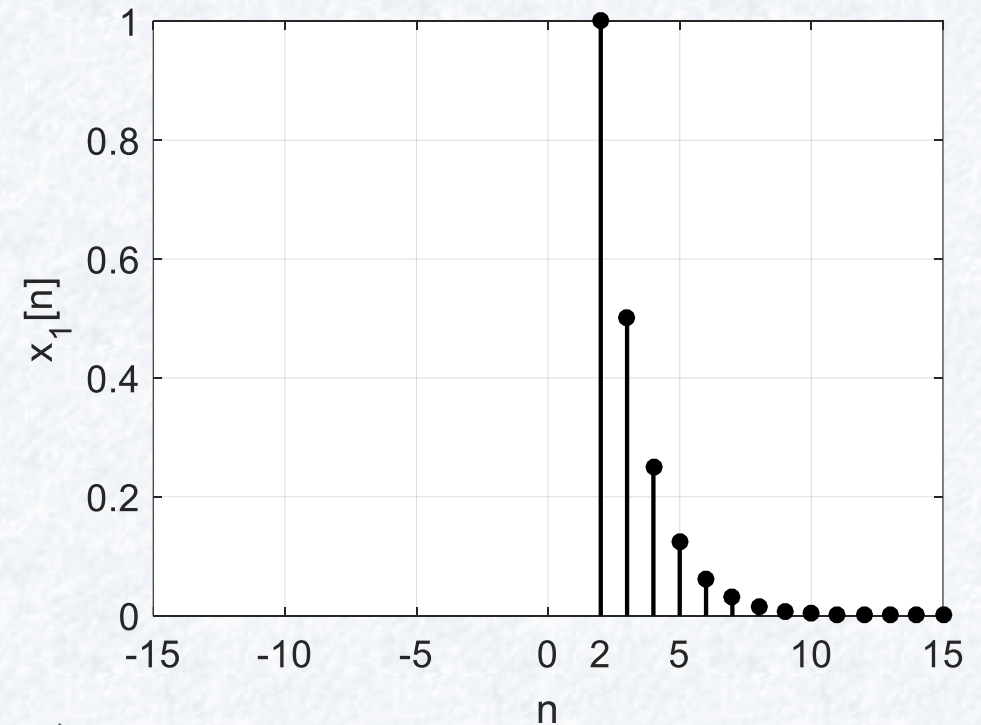
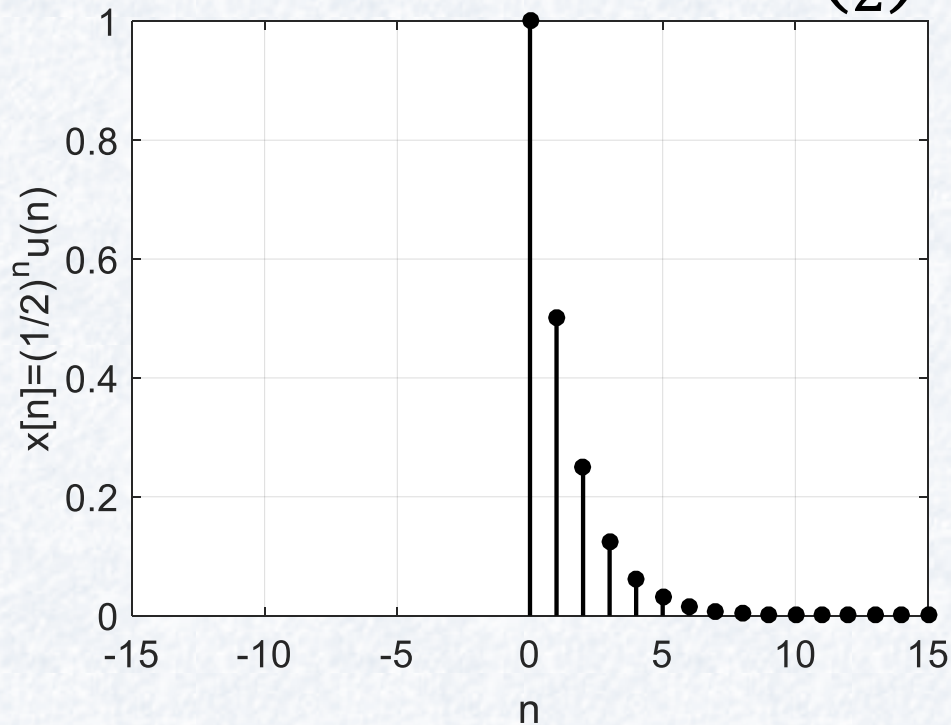
- $x[n] = \left(\frac{1}{2}\right)^n u(n)$
- $x_1[n] = x[n - 2] = ?$



# Örnek

- $x[n] = \left(\frac{1}{2}\right)^n u(n)$

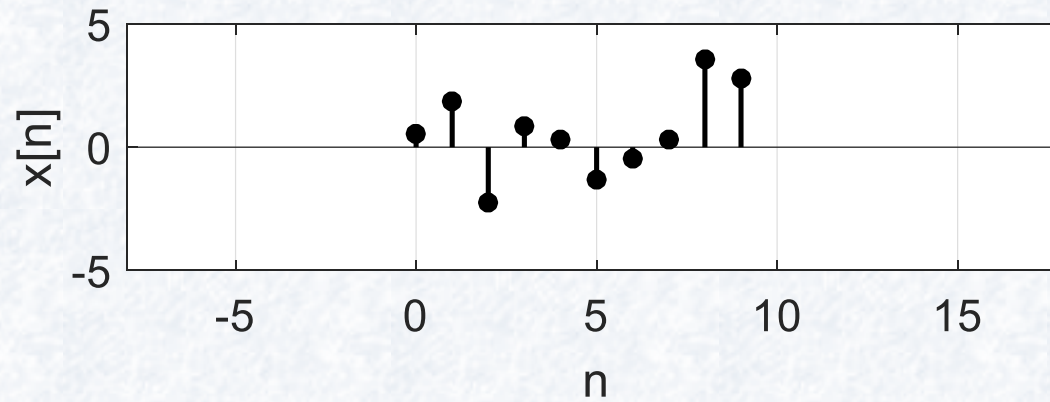
- $x_1[n] = x[n - 2] = \left(\frac{1}{2}\right)^{n-2} u[n - 2]$





# Bağımsız Değişken Dönüşümü

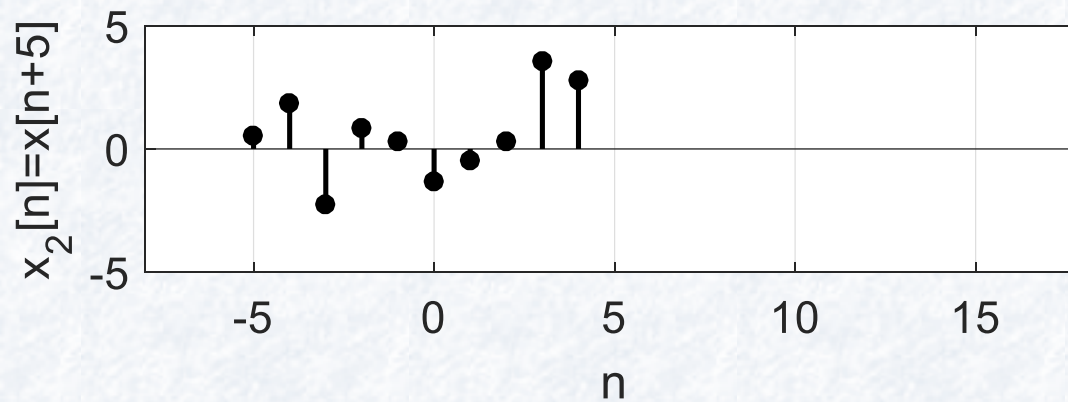
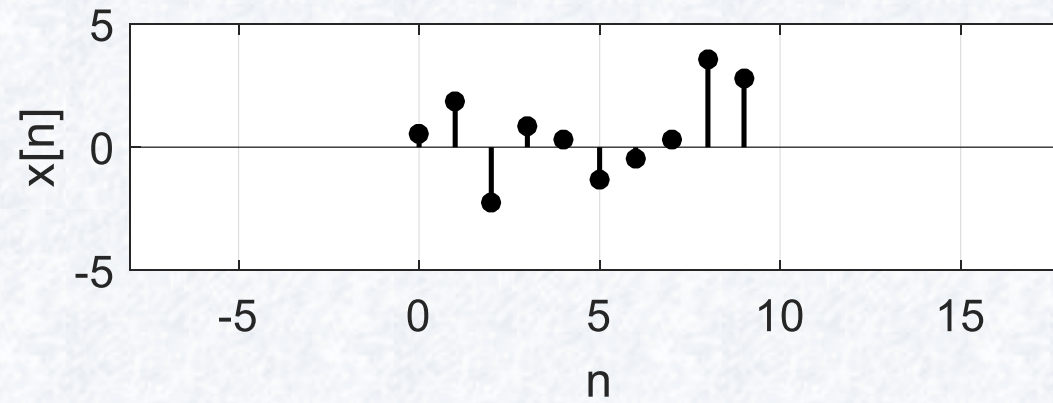
- Zamanda Öteleme
  - ♦  $x_2[n] = x[n + 5]$



# Bağımsız Değişken Dönüşümü

- Zamanda Öteleme

- ◆  $x_2[n] = x[n + 5]$

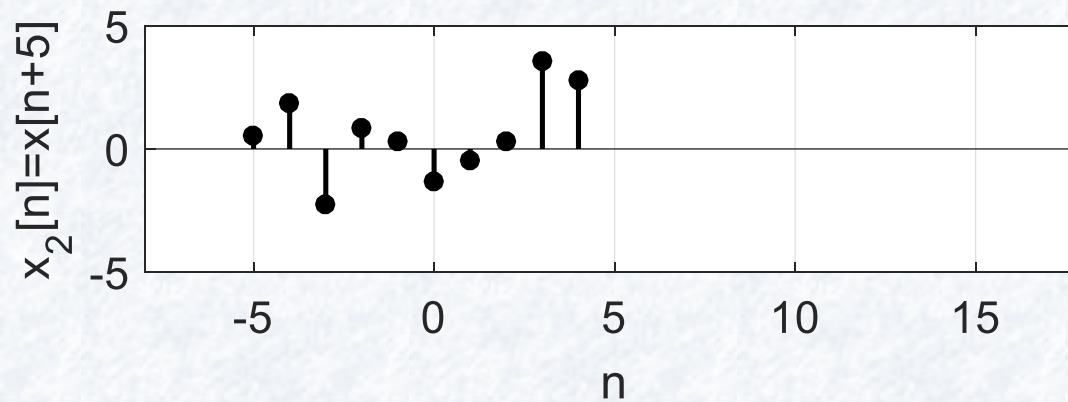
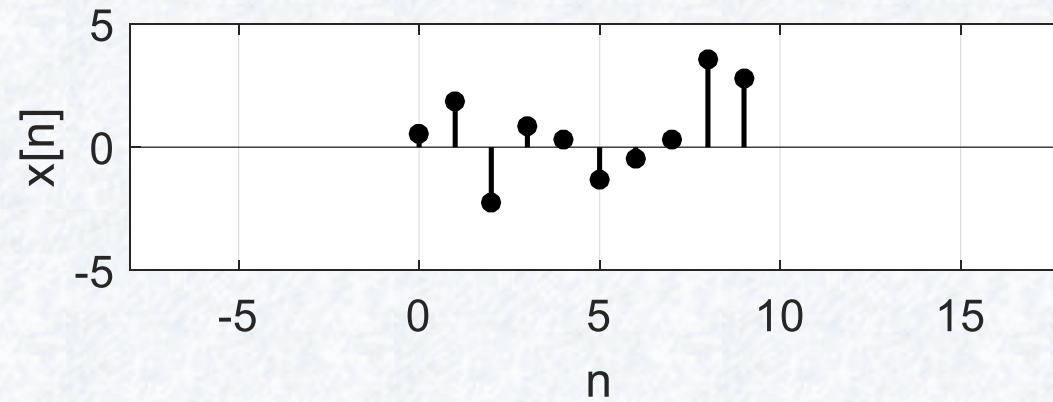


# Bağımsız Değişken Dönüşümü

- Zamanda Öteleme

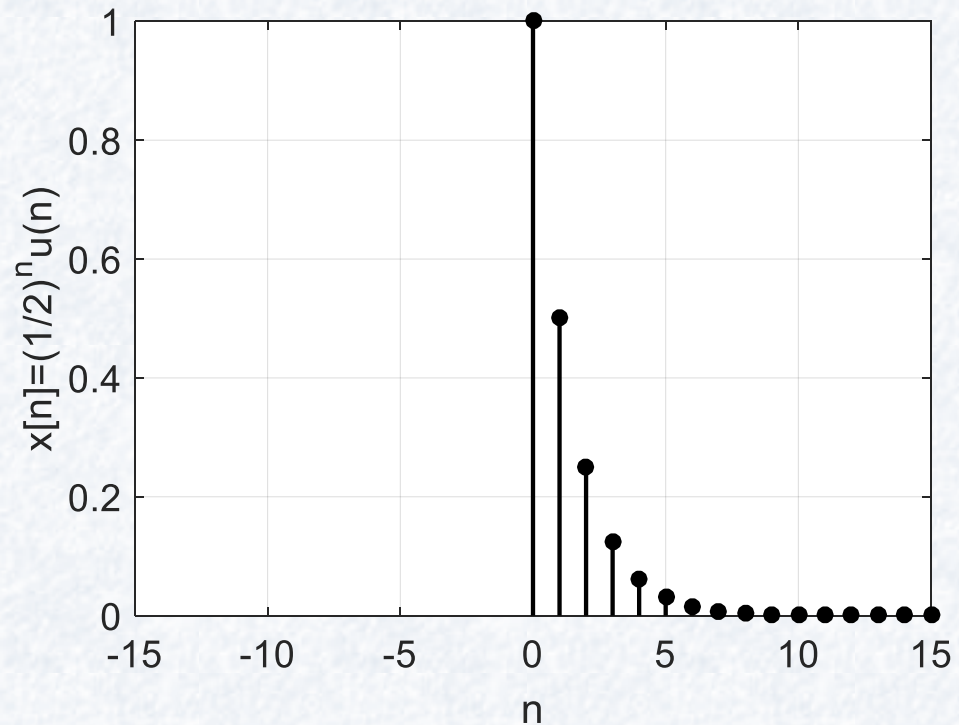
- ◆  $x_2[n] = x[n + 5]$

- Gelecek



## Örnek 2

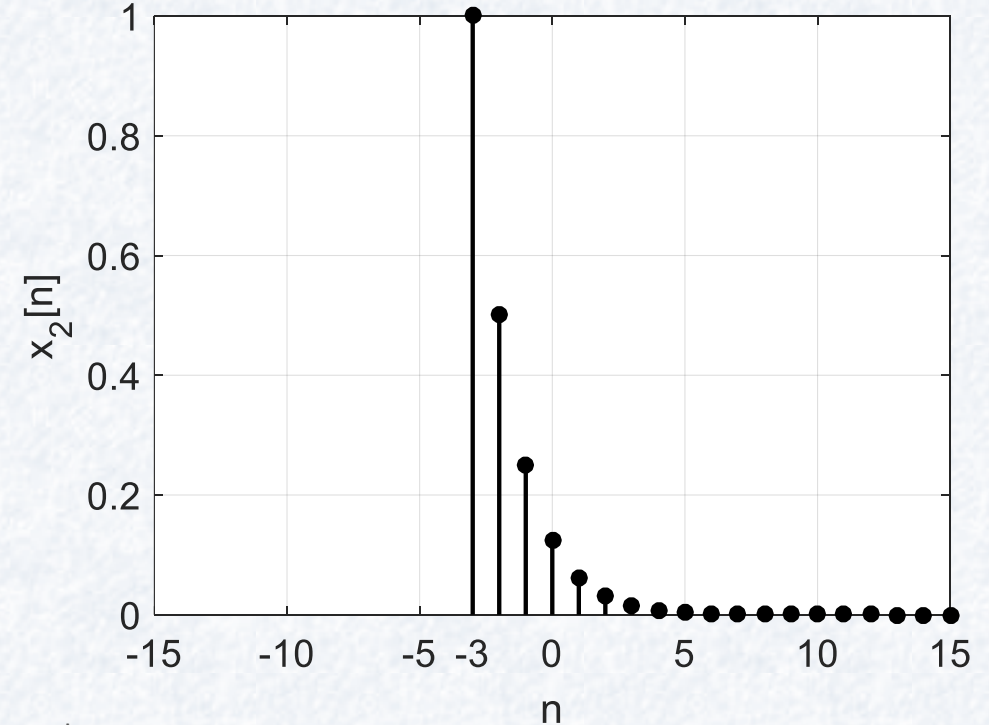
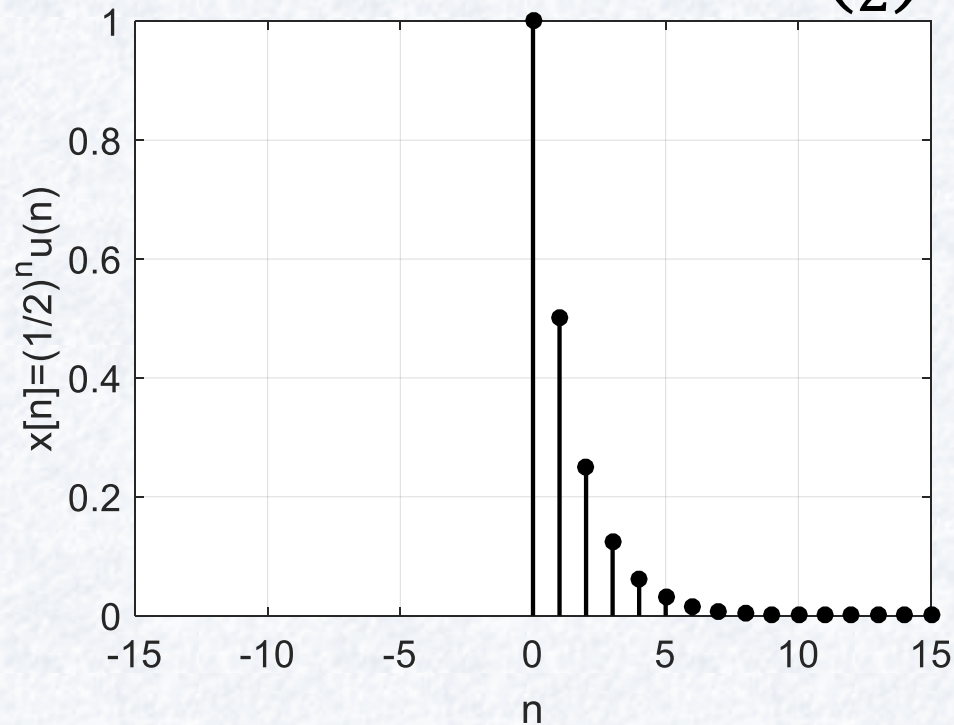
- $x[n] = \left(\frac{1}{2}\right)^n u(n)$
- $x_2[n] = x[n + 3] = ?$



# Örnek

- $x[n] = \left(\frac{1}{2}\right)^n u(n)$

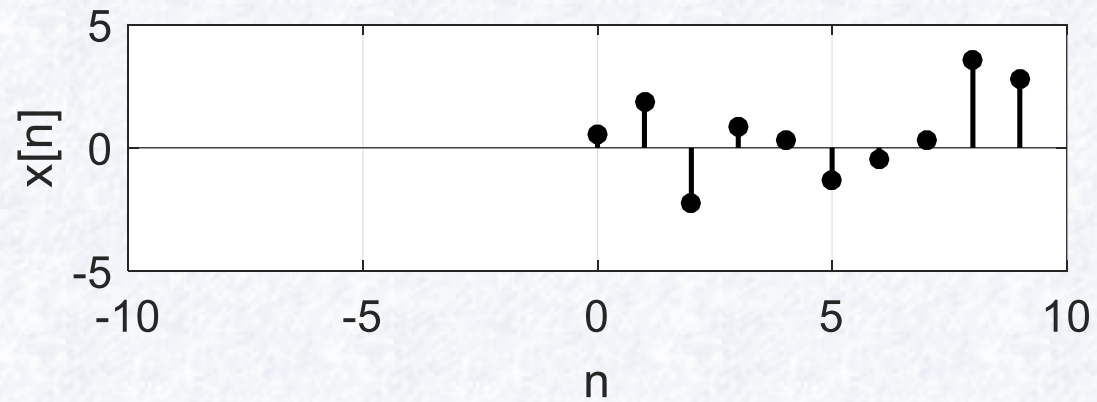
- $x_2[n] = x[n + 3] = \left(\frac{1}{2}\right)^{n+3} u(n + 3)$



# Bağımsız Değişken Dönüşümü

- Zamanda Ters Çevirme

- ◆  $x_3[n] = x[-n]$

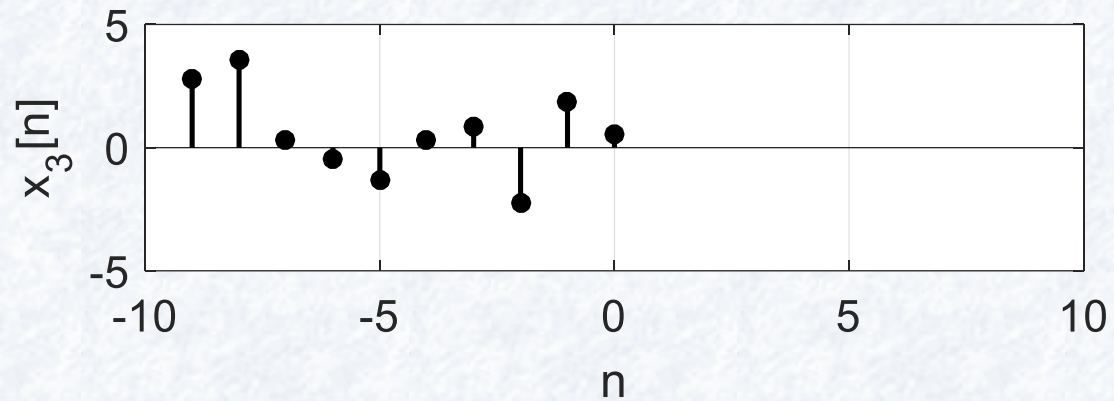
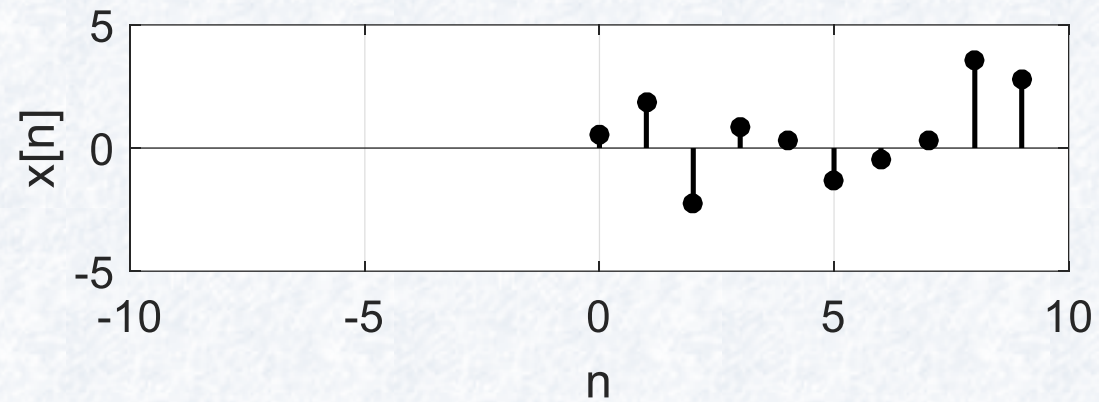




# Bağımsız Değişken Dönüşümü

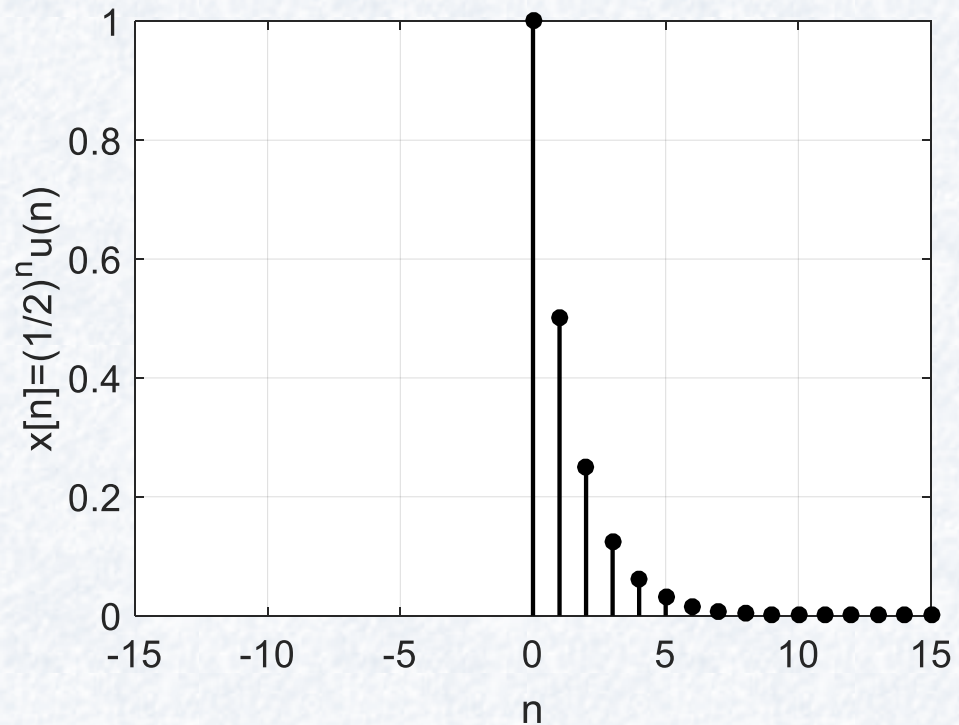
- Zamanda Ters Çevirme

- ◆  $x_3[n] = x[-n]$



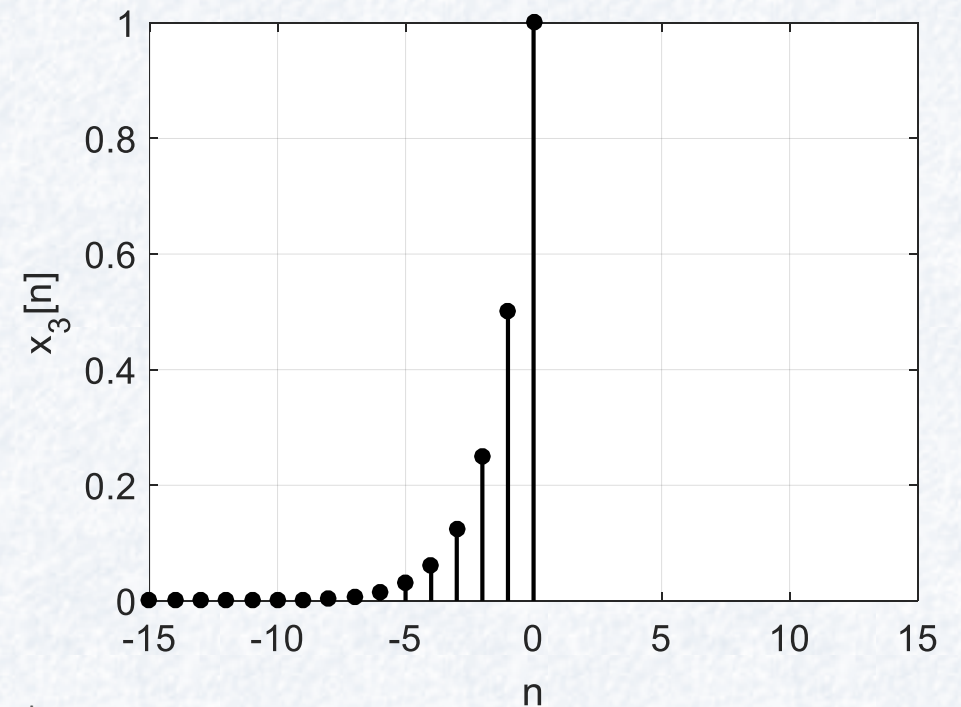
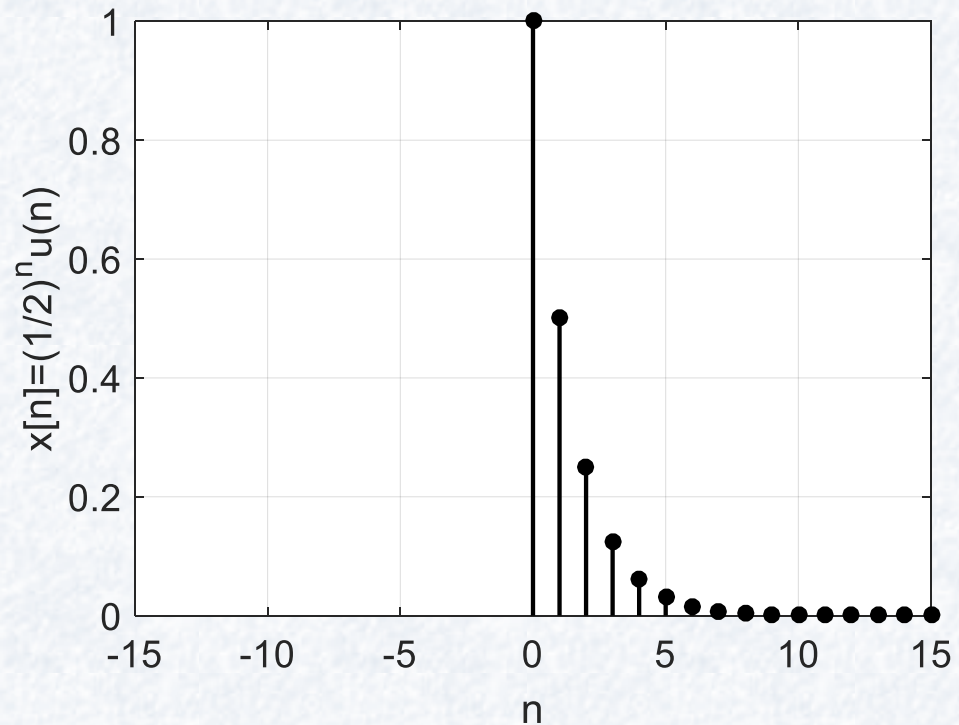
# Örnek 3

- $x[n] = \left(\frac{1}{2}\right)^n u(n)$
- $x_3[n] = x[-n] = ?$



# Örnek 3

- $x[n] = \left(\frac{1}{2}\right)^n u(n)$
- $x_3[n] = x[-n] = 2^n u(-n)$



# Karmaşık sayılar

- $\sigma + j\omega$

- ◆  $r = ? \sqrt{\sigma^2 + \omega^2}$

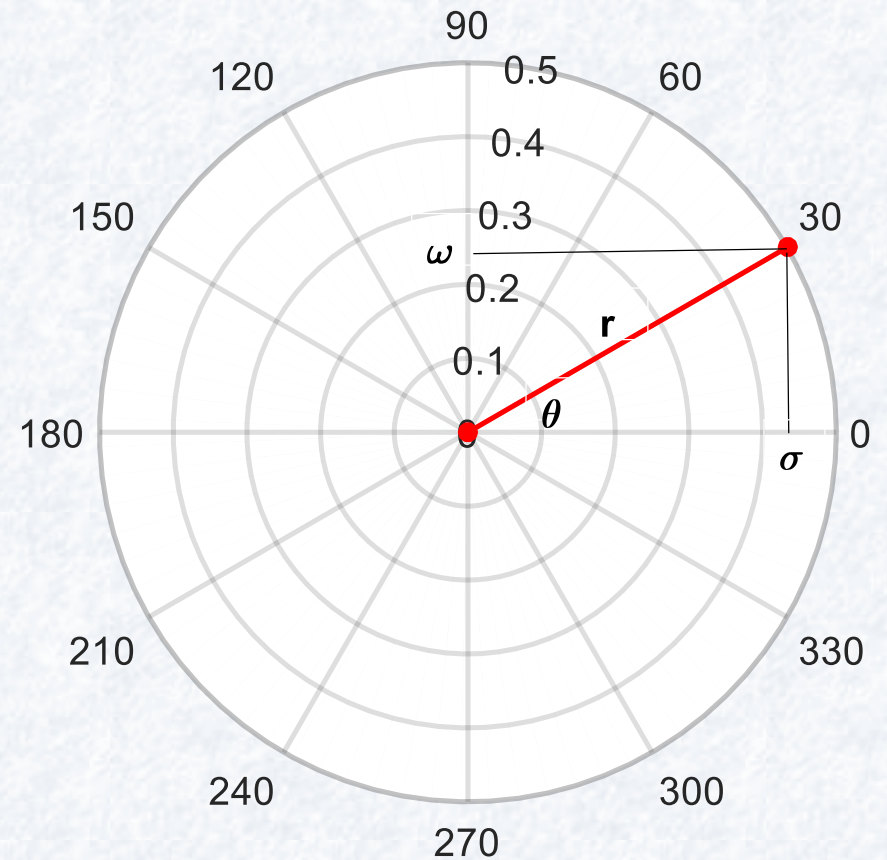
- ◆  $\sigma = ? r \cdot \cos \alpha$

- ◆  $\omega = ? r \cdot \sin \alpha$

$$\text{Sayı} = \underbrace{\sigma}_{\text{Reel}} + j \cdot \underbrace{\omega}_{\text{Fantazya}}$$

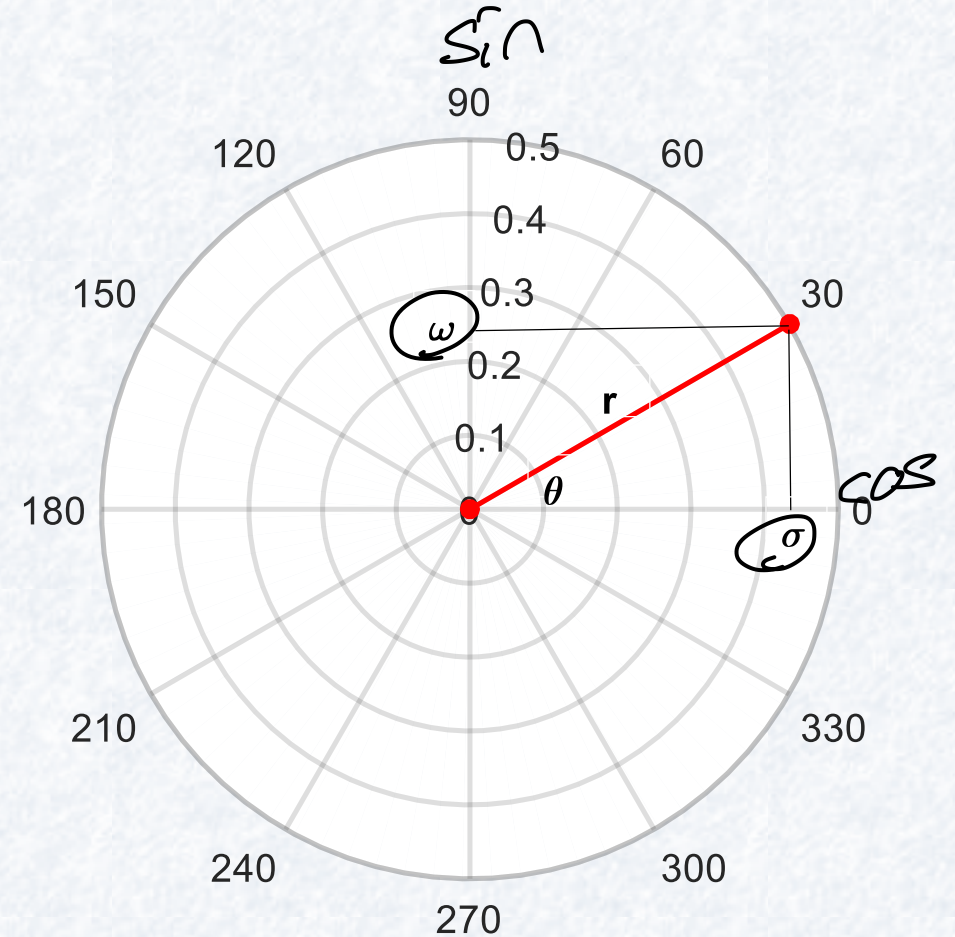
$$\sigma + j\omega = r \cos \alpha + j r \sin \alpha$$
$$\sigma - j\omega = r \cos \alpha - j r \sin \alpha$$

$$r = \sqrt{\sigma^2 + \omega^2} = \sqrt{r^2 \cos^2 \alpha + r^2 \sin^2 \alpha} = \sqrt{r^2 (\cos^2 \alpha + \sin^2 \alpha)} = r$$



# Karmaşık sayılar

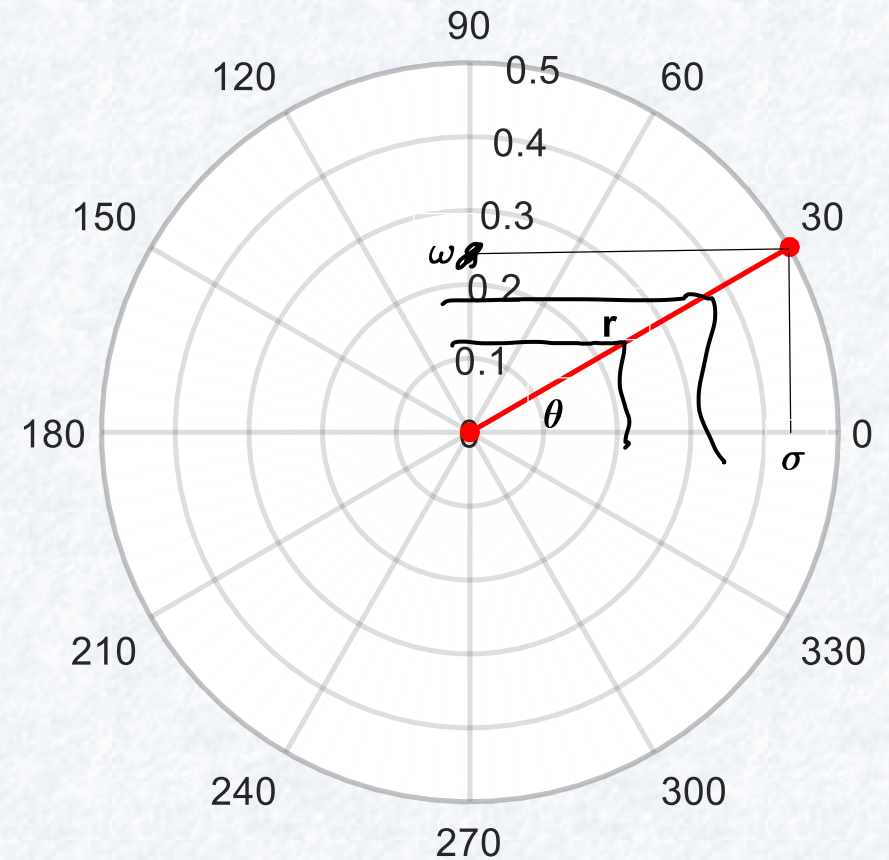
- $\sigma + j\omega$ 
  - ♦  $r = \sqrt{\sigma^2 + \omega^2}$
  - ♦  $\sigma = r \cos(\theta)$
  - ♦  $\omega = r \sin(\theta)$
- $\sigma + j\omega = r \cos(\theta) + jr \sin(\theta)$
- $\sigma + j\omega = r(\cos(\theta) + j \sin(\theta))$
- $\sigma - j\omega = r(\cos(-\theta) + j \sin(-\theta))$
- $\sigma - j\omega = r(\cos(\theta) - j \sin(\theta))$





# Karmaşık sayılar

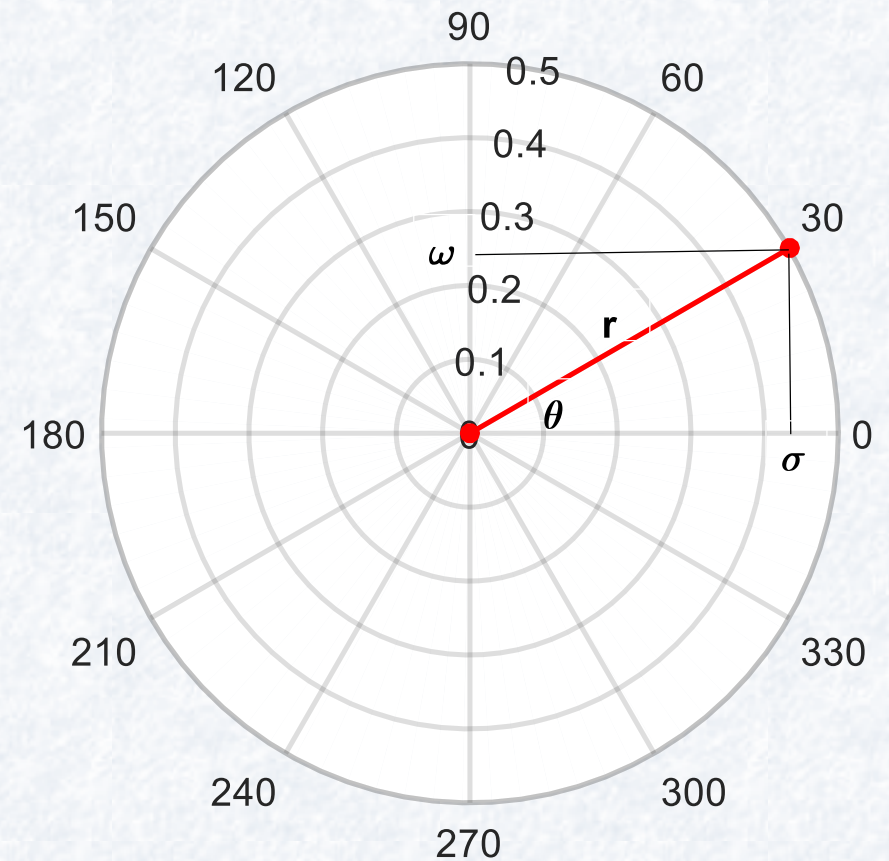
- $f(\theta) = \cos(\theta) + j \sin(\theta)$
- $f'(\theta) = ?$





# Karmaşık sayılar

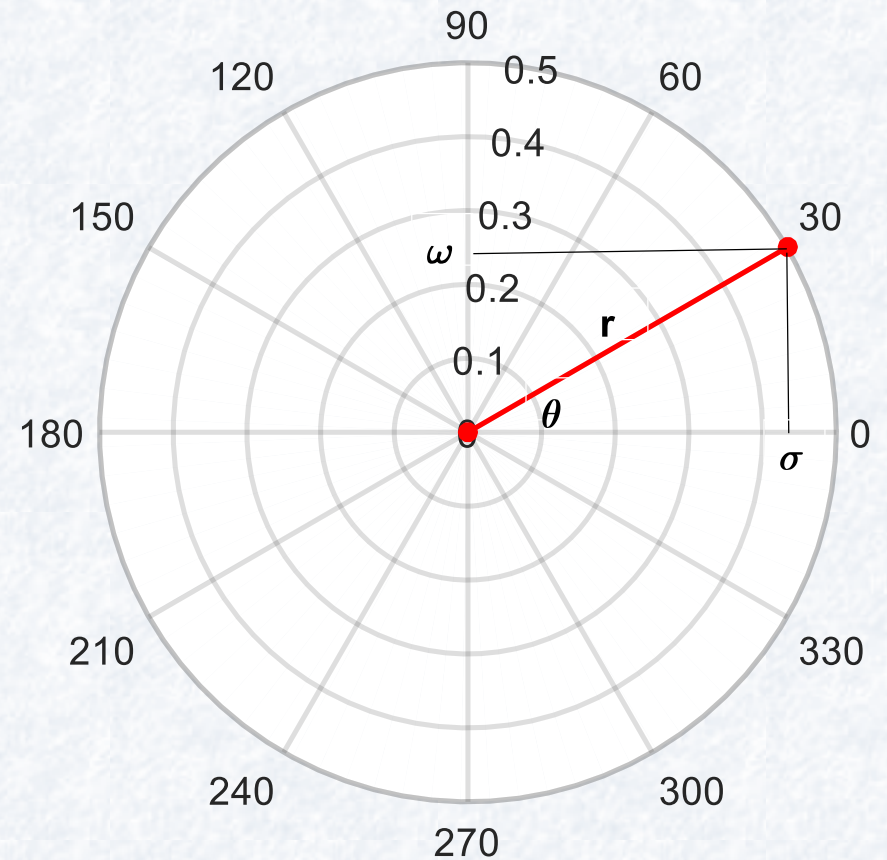
- $f(\theta) = \cos(\theta) + j \sin(\theta)$
- $f'(\theta) = -\sin(\theta) + j \cos(\theta)$
- $f'(\theta) = ?$



# Karmaşık sayılar

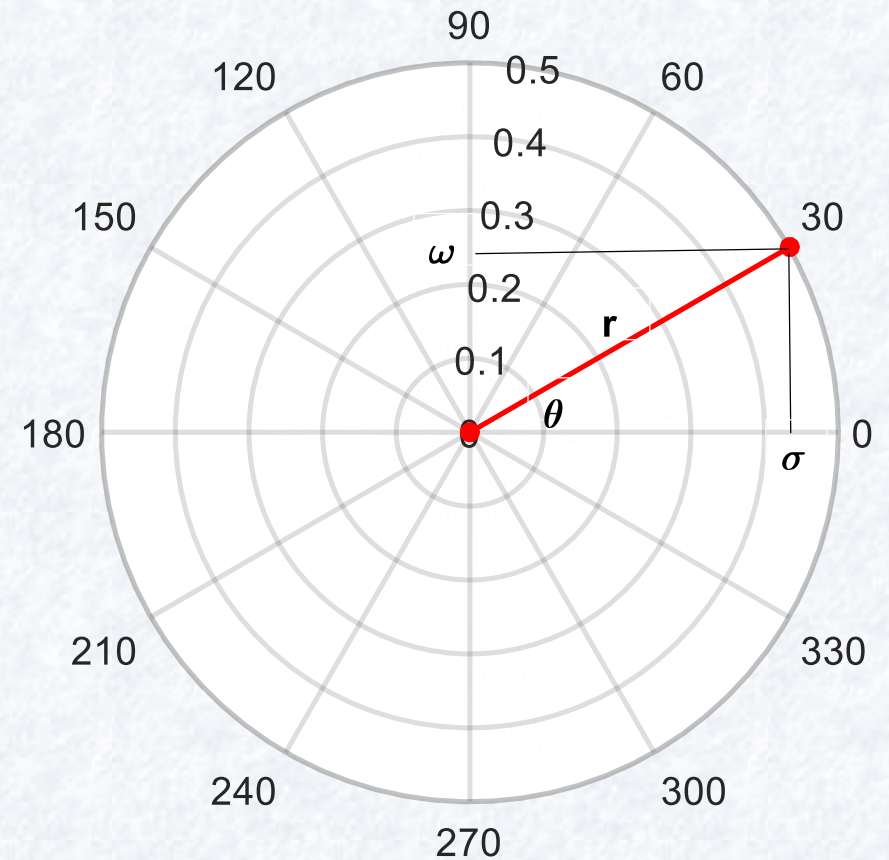
- $f(\theta) = \cos(\theta) + j \sin(\theta)$
- $f'(\theta) = -\sin(\theta) + j \cos(\theta)$   
 $= j \cos(\theta) - \sin(\theta)$
- $f'(\theta) = j f(\theta)$
- $f(\theta) = ?$

$$j = \sqrt{-1}$$
$$j^2 = -1$$



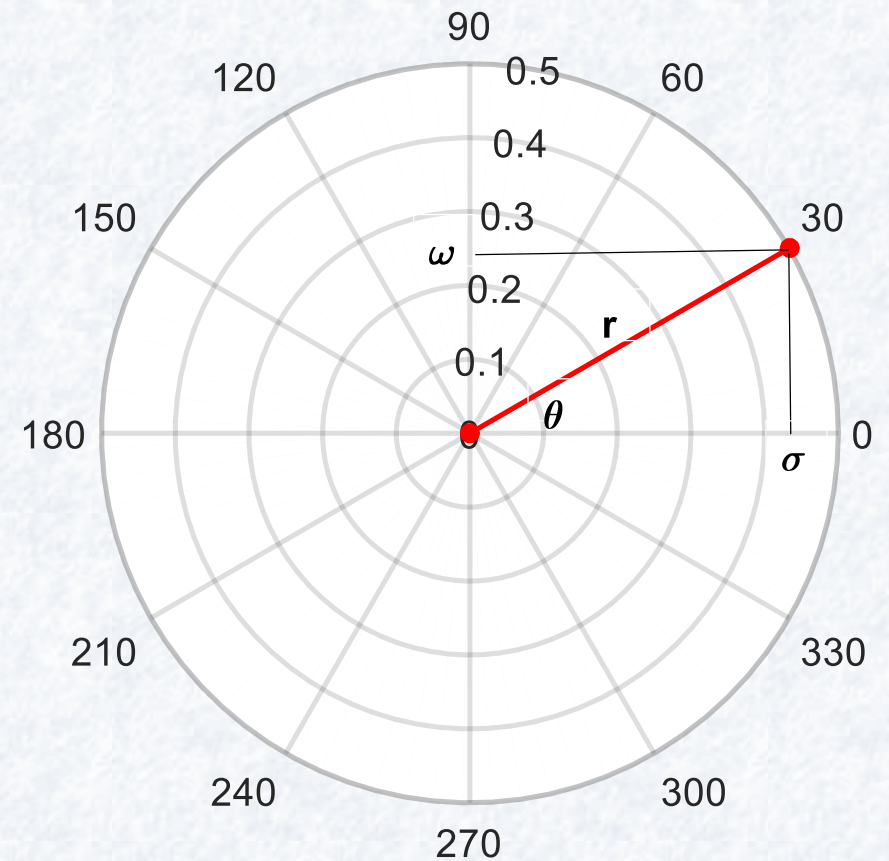
# Karmaşık sayılar

- $f(\theta) = \cos(\theta) + j \sin(\theta)$
- $f'(\theta) = -\sin(\theta) + j \cos(\theta)$
- $f'(\theta) = jf(\theta)$
- $f(\theta) = e^{j\theta}$
- $\sigma + j\omega = re^{j\theta}$
- $\sigma - j\omega = re^{-j\theta}$



# Karmaşık sayılar

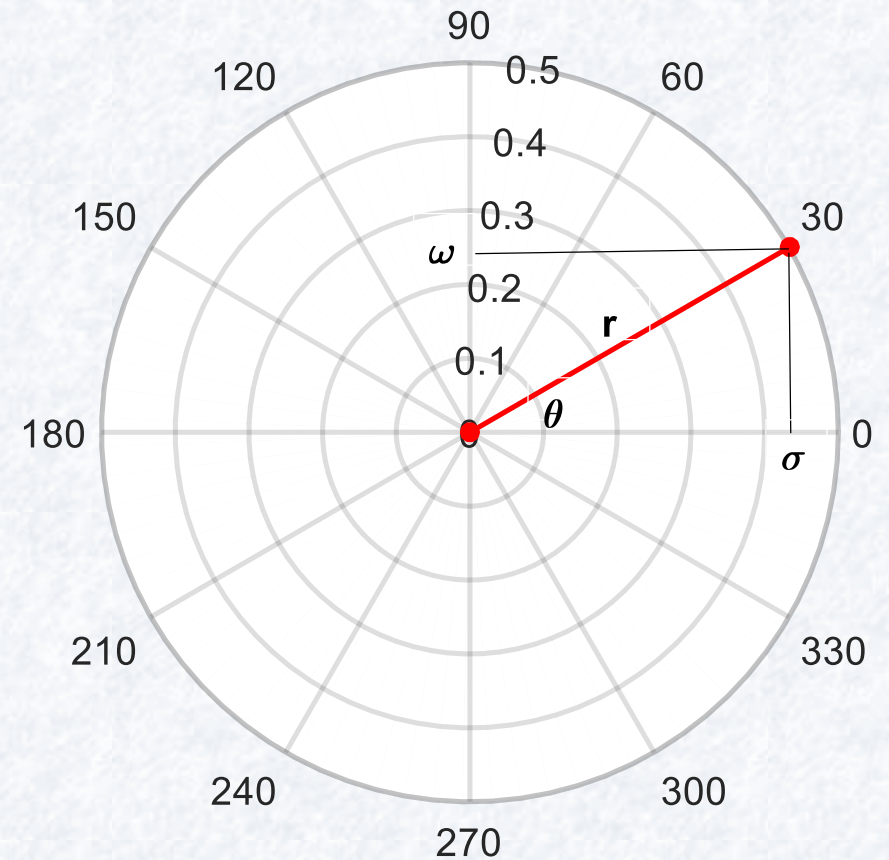
- $e^{j\theta} = \cos(\theta) + j \sin(\theta)$
- $e^{-j\theta} = \cos(\theta) - j \sin(\theta)$
- $\cos(\theta) = ?$





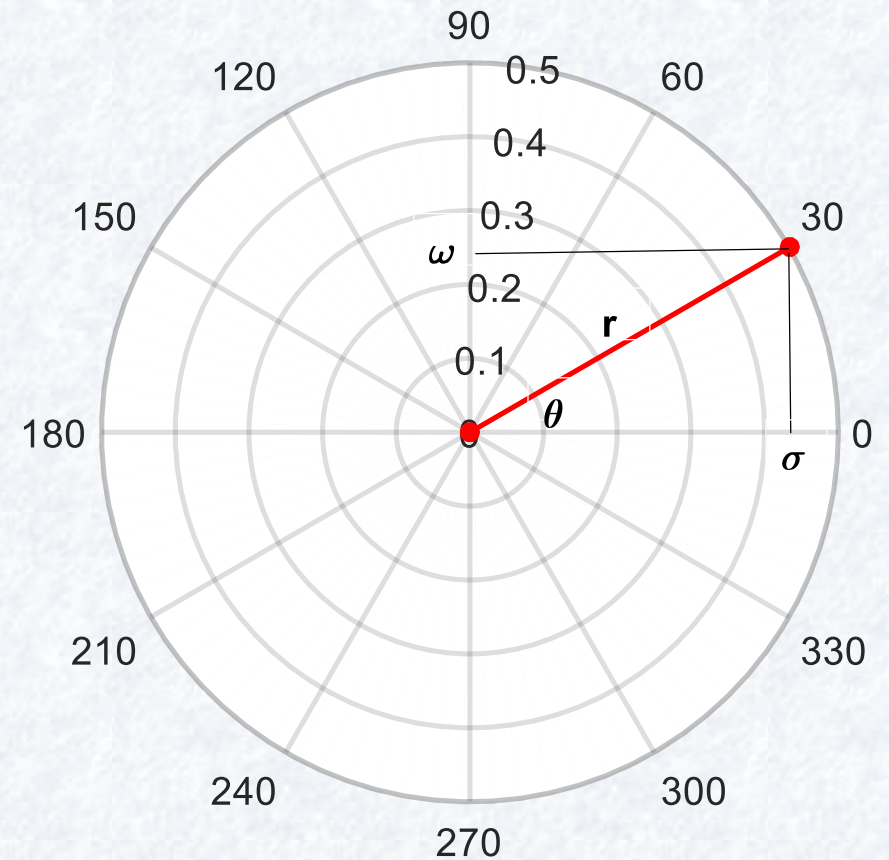
# Karmaşık sayılar

- $e^{j\theta} = \cos(\theta) + j \sin(\theta)$
- $e^{-j\theta} = \cos(\theta) - j \sin(\theta)$
- $e^{j\theta} + e^{-j\theta} = ?$



# Karmaşık sayılar

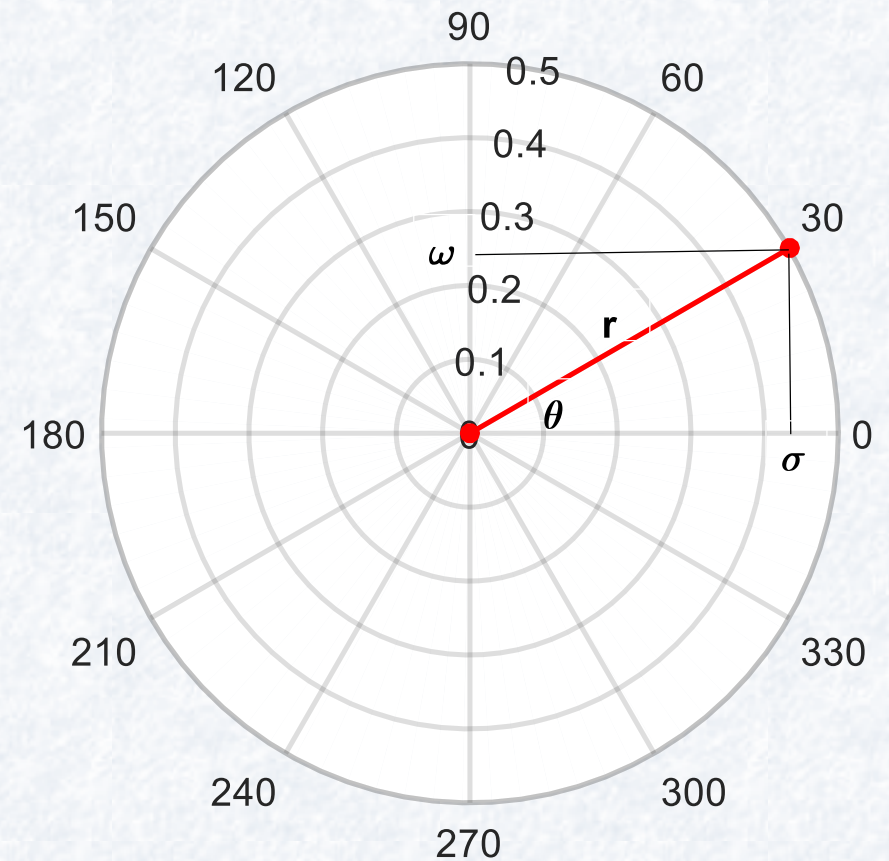
- $e^{j\theta} = \cos(\theta) + j \sin(\theta)$
- $e^{-j\theta} = \cos(\theta) - j \sin(\theta)$
- $e^{j\theta} + e^{-j\theta} = 2\cos(\theta)$
- $\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$





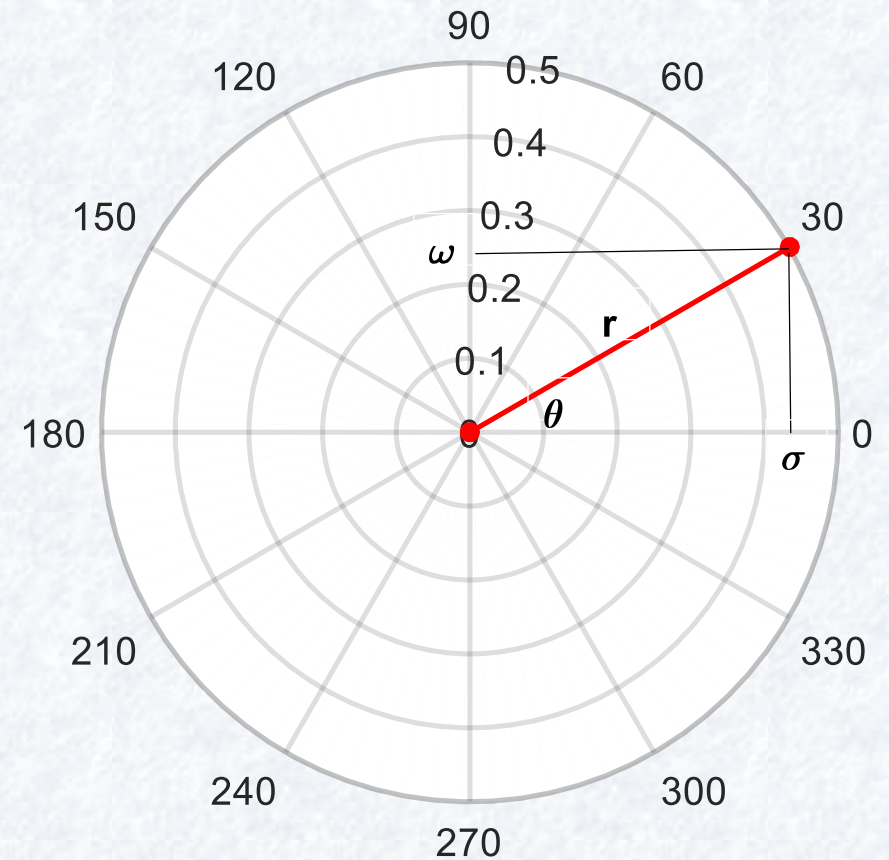
# Karmaşık sayılar

- $e^{j\theta} = \cos(\theta) + j \sin(\theta)$
- $e^{-j\theta} = \cos(\theta) - j \sin(\theta)$
- $\sin(\theta) = ?$



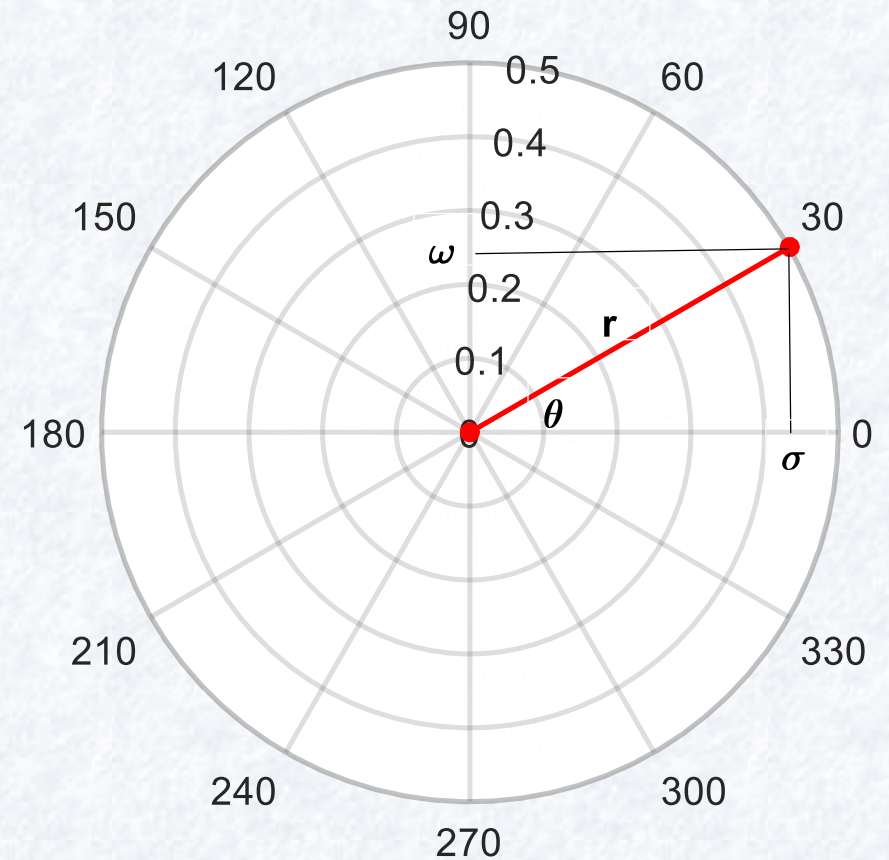
# Karmaşık sayılar

- $e^{j\theta} = \cos(\theta) + j \sin(\theta)$
- $e^{-j\theta} = \cos(\theta) - j \sin(\theta)$
- $e^{j\theta} - e^{-j\theta} = ?$



# Karmaşık sayılar

- $e^{j\theta} = \cos(\theta) + j \sin(\theta)$
- $e^{-j\theta} = \cos(\theta) - j \sin(\theta)$
- $e^{j\theta} - e^{-j\theta} = 2j \sin(\theta)$
- $\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$



# Periyodik İşaretler

- $x[n] = x[n + N] = x[n + kN] \rightarrow \text{Periyot}$ 
  - ♦ Tam sayı bir  $N > 0$  değeri var ise  $x[n]$  periyodiktir.
  - ♦  $N$  örnekte bir genlik tekrar eder.



# Periyodik İşaretler

- $x[n] = e^{j\omega_0 n}$
- $x[n] = x[n + N]$
- $e^{j\omega_0 n} =$



# Periyodik İşaretler

- $x[n] = e^{j\omega_0 n}$
- $x[n] = x[n + N]$
- $e^{j\omega_0 n} = e^{j\omega_0(n+N)}$
- $e^{j\omega_0 n} =$



# Periyodik İşaretler

- $x[n] = e^{j\omega_0 n}$
- $x[n] = x[n + N]$
- $e^{j\omega_0 n} = e^{j\omega_0(n+N)}$
- $e^{j\omega_0 n} = e^{j\omega_0 n} e^{j\omega_0 N}$
- $1 = e^{j\omega_0 N}$
- $1 + j0 = ?$

# Periyodik İşaretler

- $x[n] = e^{j\omega_0 n}$
- $x[n] = x[n + N]$
- $e^{j\omega_0 n} = e^{j\omega_0(n+N)}$
- $e^{j\omega_0 n} = e^{j\omega_0 n} e^{j\omega_0 N}$
- $1 = e^{j\omega_0 N}$
- $1 + j0 = ?$ 
  - ♦  $r = 1$
  - ♦  $\theta = 0 = \dots$

# Periyodik İşaretler

- $x[n] = e^{j\omega_0 n}$
- $x[n] = x[n + N]$
- $e^{j\omega_0 n} = e^{j\omega_0(n+N)}$
- $e^{j\omega_0 n} = e^{j\omega_0 n} e^{j\omega_0 N}$
- $1 = e^{j\omega_0 N}$
- $1 + j0 = ?$ 
  - ♦  $r = 1$
  - ♦  $\theta = 0 = 2\pi = 4\pi = \dots = 2\pi k$

# Periyodik İşaretler

- $x[n] = e^{j\omega_0 n}$
- $1 = e^{j\omega_0 N}$
- $1 = 1e^{j2\pi k}$
- $1e^{j2\pi k} = e^{j\omega_0 N}$
- $N = ?$

# Periyodik İşaretler

- $x[n] = e^{j\omega_0 n}$
- $1 = e^{j\omega_0 N}$
- $1 = 1e^{j2\pi k}$
- $1e^{j2\pi k} = e^{j\omega_0 N}$
- $\omega_0 N = 2\pi k$
- $N = \frac{2\pi}{\omega_0} k$ 
  - ♦  $k > 0$ , olabilecek en küçük tam sayı

## Örnek 4

- $x[n] = \cos\left(\frac{2\pi}{12}n\right)$  periyodik midir?

$$x(n) = x(n+N) = x(n+kN) \Rightarrow \cos\left(\frac{\pi}{6}n\right) = \cos\left(\frac{\pi}{6}(n+N)\right) = \cos\left(\frac{\pi}{6}(n+kN)\right)$$

$$\frac{\pi n}{6} = \frac{\pi n}{6} + \frac{\pi kN}{6} \Rightarrow \frac{\pi kN}{6} = 0$$

$$\Rightarrow \pi N = 0$$

$$k \in \mathbb{Z}^+ \\ N \in \mathbb{R}^+$$



## Örnek 4

- $x[n] = \cos\left(\frac{2\pi}{12}n\right)$  periyodik midir?
- $N = \frac{2\pi}{2\pi/12}k$
- $N = 12k$
- $N = 12$

$$\cos \alpha = \frac{e^{i\alpha+b} - e^{-i\alpha+b}}{2}$$

## Örnek 4

- $x[n] = \cos\left(\frac{2\pi}{12}n\right)$  periyodik midir?
- $N = \frac{2\pi}{2\pi/12}k$
- $N = 12k$
- $N = 12$
- $x[0] = x[12] = x[24] = \dots$

## Örnek 5

- $x[n] = \sin\left(\frac{8\pi}{25}n\right)$  periyodik midir?
- $N = ?$

## Örnek 5

- $x[n] = \sin\left(\frac{8\pi}{25}n\right)$  periyodik midir?
- $N = \frac{2\pi}{8\pi/25}k$
- $N = \frac{25}{4}k$
- $N =$

## Örnek 5

- $x[n] = \sin\left(\frac{8\pi}{25}n\right)$  periyodik midir?
- $N = \frac{2\pi}{8\pi/25}k$
- $N = \frac{25}{4}k$
- $N = 25$
- $x[0] = x[25] = x[50] = \dots$



## Örnek 6

- $x[n] = \cos\left(\frac{n}{6}\right)$  periyodik midir?
- $N = ?$

## Örnek 6

- $x[n] = \cos\left(\frac{n}{6}\right)$  periyodik midir?
- $N = \frac{2\pi}{1/6} k$
- $N = 12\pi k$
- $N =$

## Örnek 6

- $x[n] = \cos\left(\frac{n}{6}\right)$  periyodik midir?
- $N = \frac{2\pi}{1/6} k$
- $N = 12\pi k$
- Geçerli bir  $N$  değeri yok. Periyodik değil.

# Örnek 7

- $x_1[n]$ , periyot:  $N_1$
- $x_2[n]$ , periyot:  $N_2$
- $x[n] = x_1[n] + x_2[n]$ , periyodik midir?

# Örnek 7

- $x_1[n]$ , periyot:  $N_1$
- $x_2[n]$ , periyot:  $N_2$
- $x[n] = x_1[n] + x_2[n]$ , periyodik midir?
- $x[n] = x[n + N]$
- $x_1[n] = ?$
- $x_2[n] = ?$



# Örnek 7

- $x_1[n]$ , periyot:  $N_1$
- $x_2[n]$ , periyot:  $N_2$
- $x[n] = x_1[n] + x_2[n]$ , periyodik midir?
- $x_1[n] = x_1[n + N_1] = x_1[n + kN_1]$
- $x_2[n] = x_2[n + N_2] = x_2[n + mN_2]$
- $x[n] = x[n + N]$

# Örnek 7

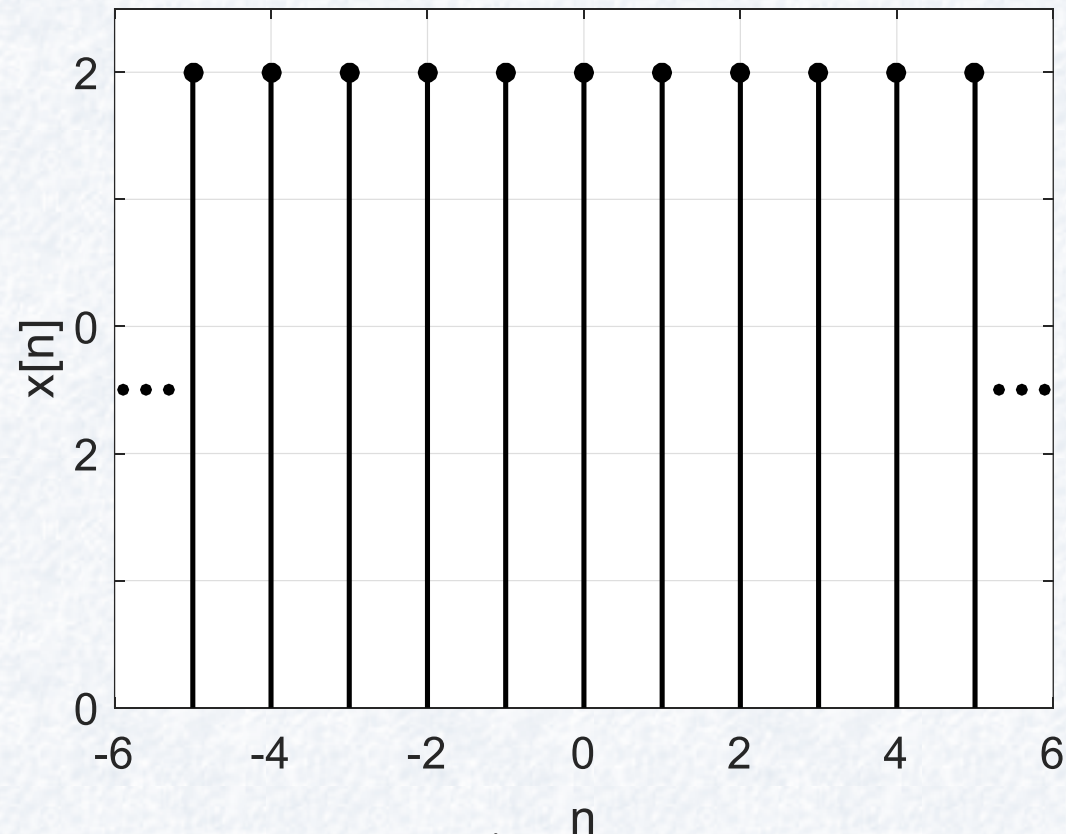
- $x_1[n]$ , periyot:  $N_1$
- $x_2[n]$ , periyot:  $N_2$
- $x[n] = x_1[n] + x_2[n]$ , periyodik midir?
- $x_1[n] = x_1[n + N_1] = x_1[n + kN_1]$
- $x_2[n] = x_2[n + N_2] = x_2[n + mN_2]$
- $x[n] = x[n + N]$
- $x_1[n + kN_1] + x_2[n + mN_2] = x_1[n + N] + x_2[n + N]$

# Örnek 7

- $x_1[n + kN_1] + x_2[n + mN_2] = x_1[n + N] + x_2[n + N]$
- $N = kN_1 = mN_2$ 
  - ♦ EKOK

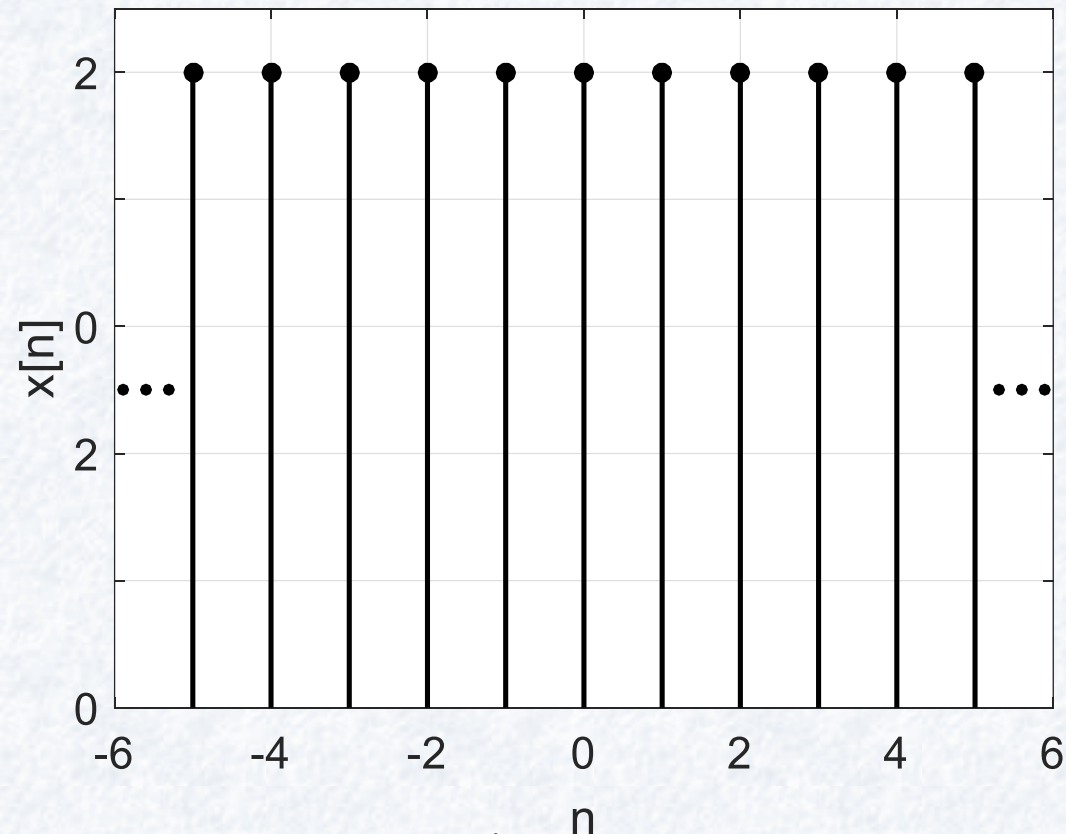
# Örnek 8

- $x[n] = 2$ , periyodik midir?



# Örnek 8

- $x[n] = 2$ , periyodik midir?
- $N = 1$





## Örnek 8

- $x[n] = \cos^2\left(\frac{\pi}{8}n\right)$ , periyodik midir?
- $x[n] =$

## Örnek 8

- $x[n] = \cos^2\left(\frac{\pi}{8}n\right)$ , periyodik midir?
- $x[n] = \frac{1 + \cos\left(\frac{\pi}{4}n\right)}{2} = \frac{1}{2} + \frac{1}{2}\cos\left(\frac{\pi}{4}n\right)$

## Örnek 8

- $x[n] = \cos^2\left(\frac{\pi}{8}n\right)$ , periyodik midir?
- $x[n] = \frac{1 + \cos\left(\frac{\pi}{4}n\right)}{2} = \underbrace{\frac{1}{2}}_{x_1[n]} + \underbrace{\frac{1}{2}\cos\left(\frac{\pi}{4}n\right)}_{x_2[n]}$
- $x_1[n]$ , periyodik midir?

## Örnek 8

- $x[n] = \cos^2\left(\frac{\pi}{8}n\right)$ , periyodik midir?
- $x[n] = \frac{1 + \cos\left(\frac{\pi}{4}n\right)}{2} = \underbrace{\frac{1}{2}}_{x_1[n]} + \underbrace{\frac{1}{2}\cos\left(\frac{\pi}{4}n\right)}_{x_2[n]}$
- $x_1[n], N_1 = ?$

## Örnek 8

- $x[n] = \cos^2\left(\frac{\pi}{8}n\right)$ , periyodik midir?
- $x[n] = \frac{1 + \cos\left(\frac{\pi}{4}n\right)}{2} = \underbrace{\frac{1}{2}}_{x_1[n]} + \underbrace{\frac{1}{2}\cos\left(\frac{\pi}{4}n\right)}_{x_2[n]}$
- $x_1[n], N_1 = 1$
- $x_2[n], N_2 = ?$



## Örnek 8

- $x[n] = \cos^2\left(\frac{\pi}{8}n\right)$ , periyodik midir?
- $x[n] = \frac{1 + \cos\left(\frac{\pi}{4}n\right)}{2} = \underbrace{\frac{1}{2}}_{x_1[n]} + \underbrace{\frac{1}{2}\cos\left(\frac{\pi}{4}n\right)}_{x_2[n]}$
- $x_1[n], N_1 = 1$
- $x_2[n], N_2 = 8$

## Örnek 8

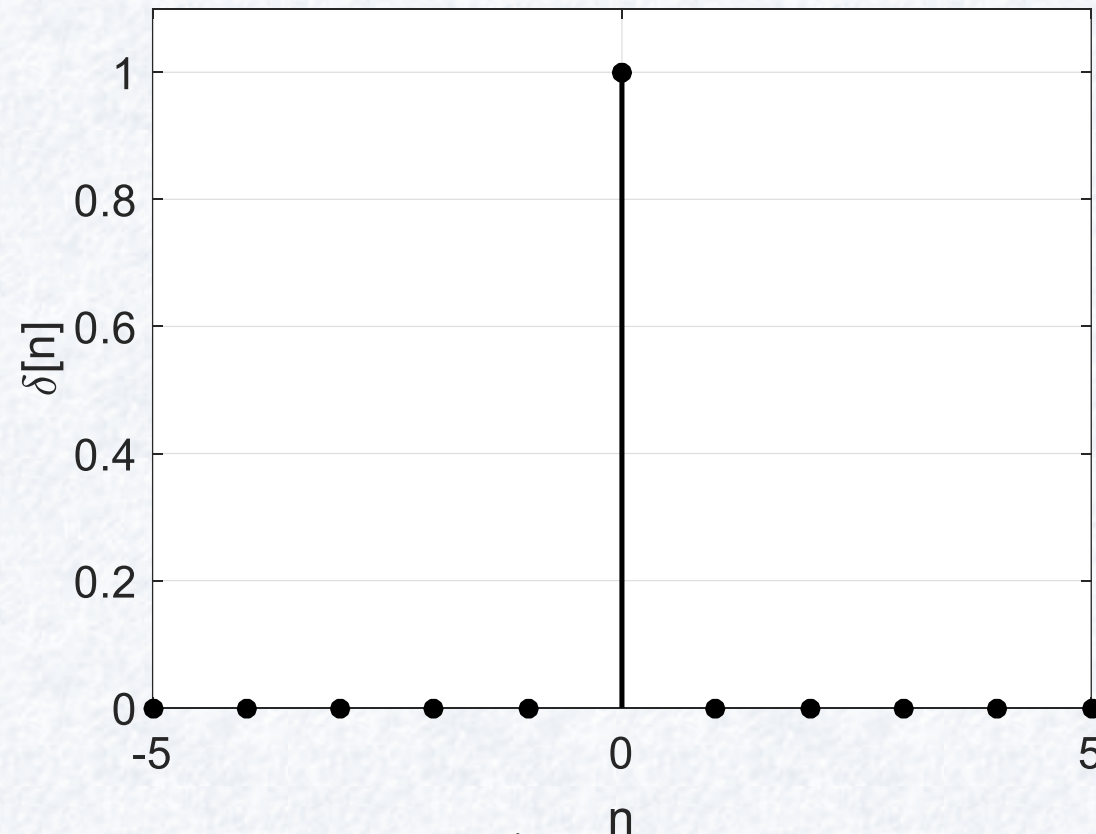
- $x[n] = \cos^2\left(\frac{\pi}{8}n\right)$ , periyodik midir?
- $x[n] = \frac{1 + \cos\left(\frac{\pi}{4}n\right)}{2} = \underbrace{\frac{1}{2}}_{x_1[n]} + \underbrace{\frac{1}{2}\cos\left(\frac{\pi}{4}n\right)}_{x_2[n]}$
- $x_1[n], N_1 = 1$
- $x_2[n], N_2 = 8$
- $N = k1 = m8 = 8$

## Örnek 8

- $x[n] = \cos\left(\frac{\pi}{8}n^2\right)$ , periyodik midir?

# Birim Darbe İşareti

- $\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$

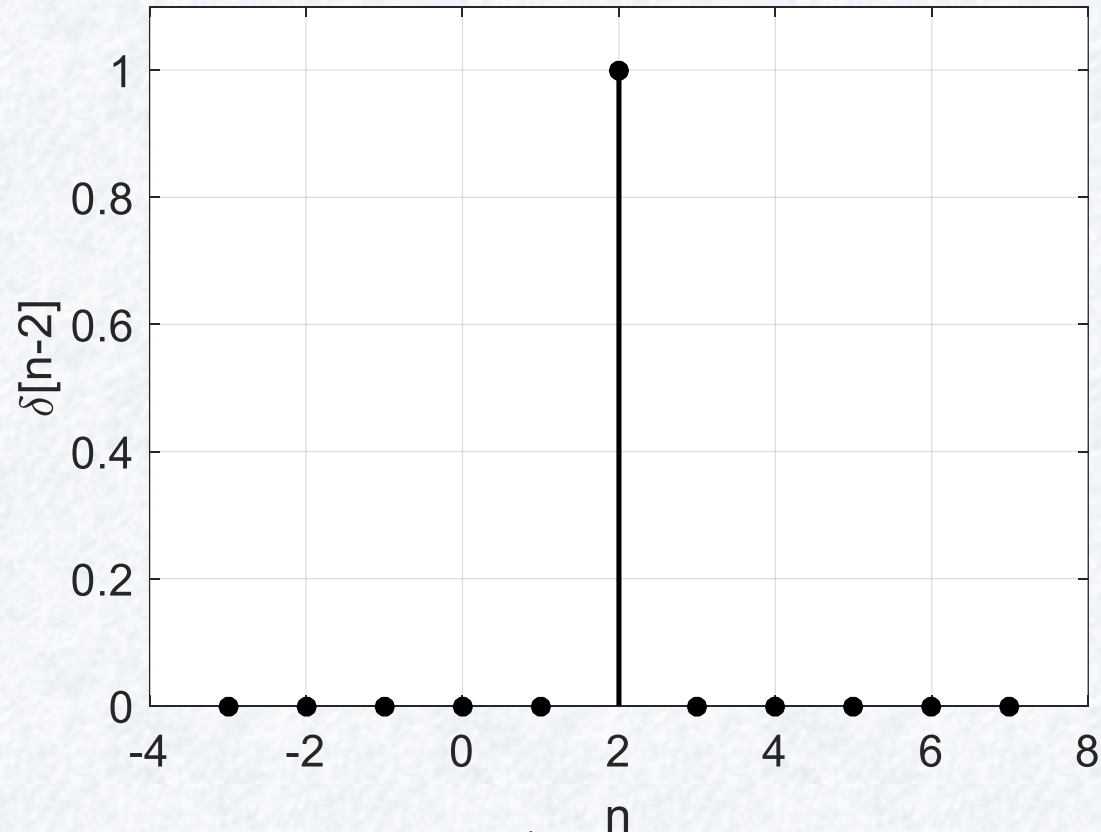


# Birim Darbe İşareti

- $\delta[n - 2] = ?$

# Birim Darbe İşareti

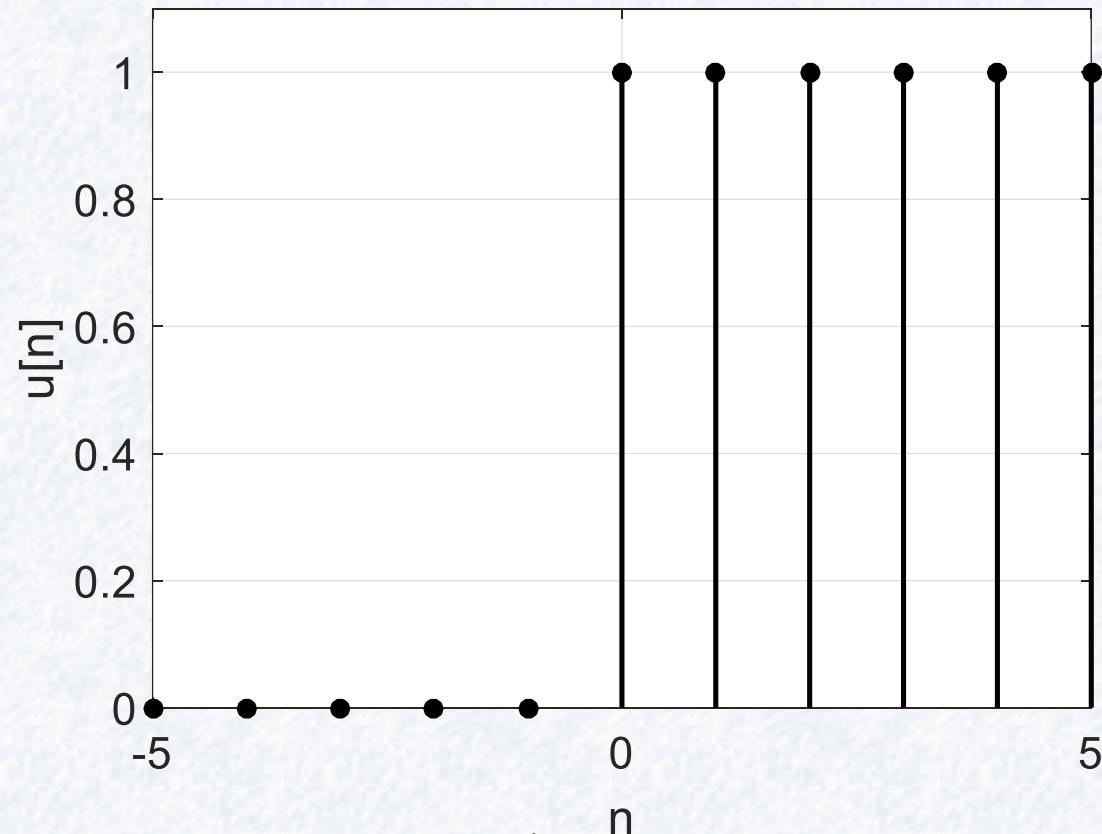
- $\delta[n - 2]$





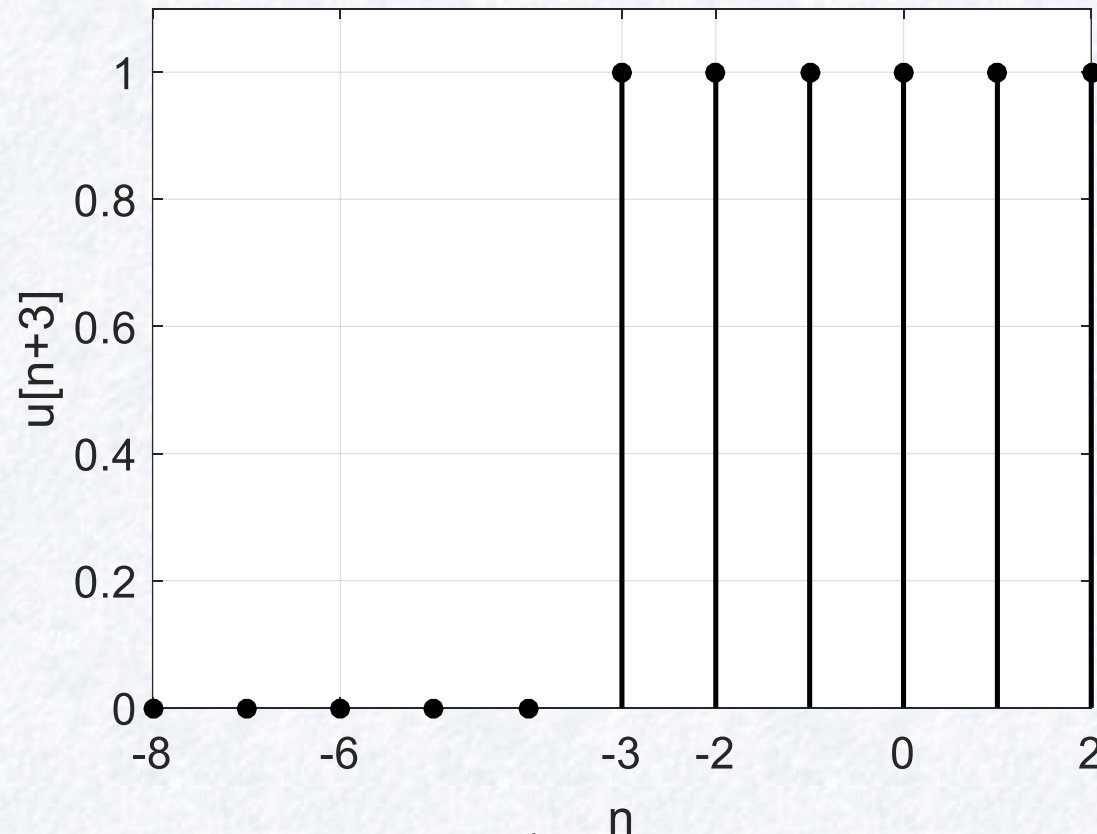
# Birim Basamak İşareti

- $u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$



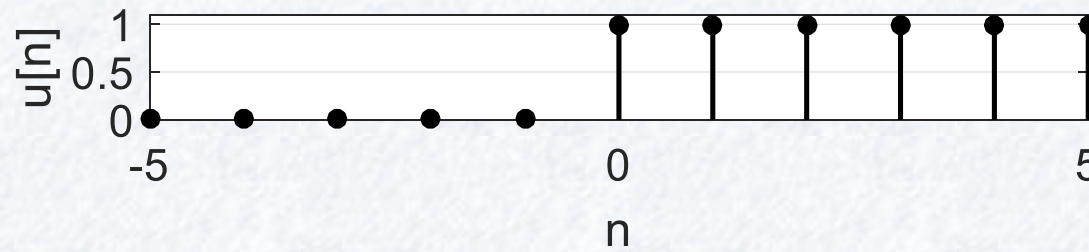
# Birim Basamak İşareti

- $u[n + 3] = ?$



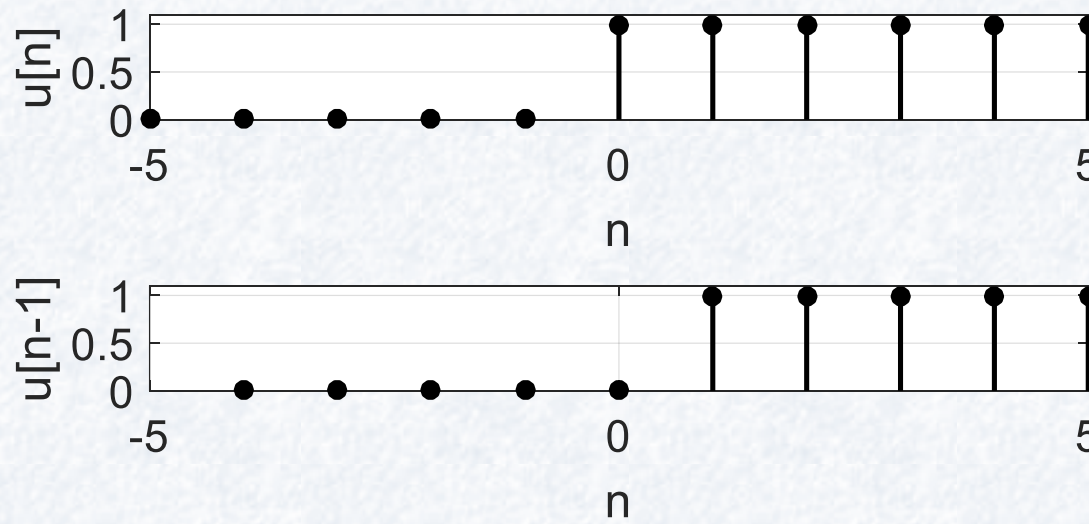
# Birim Basamaktan Birim Darbe

- $\delta[n] = ?$



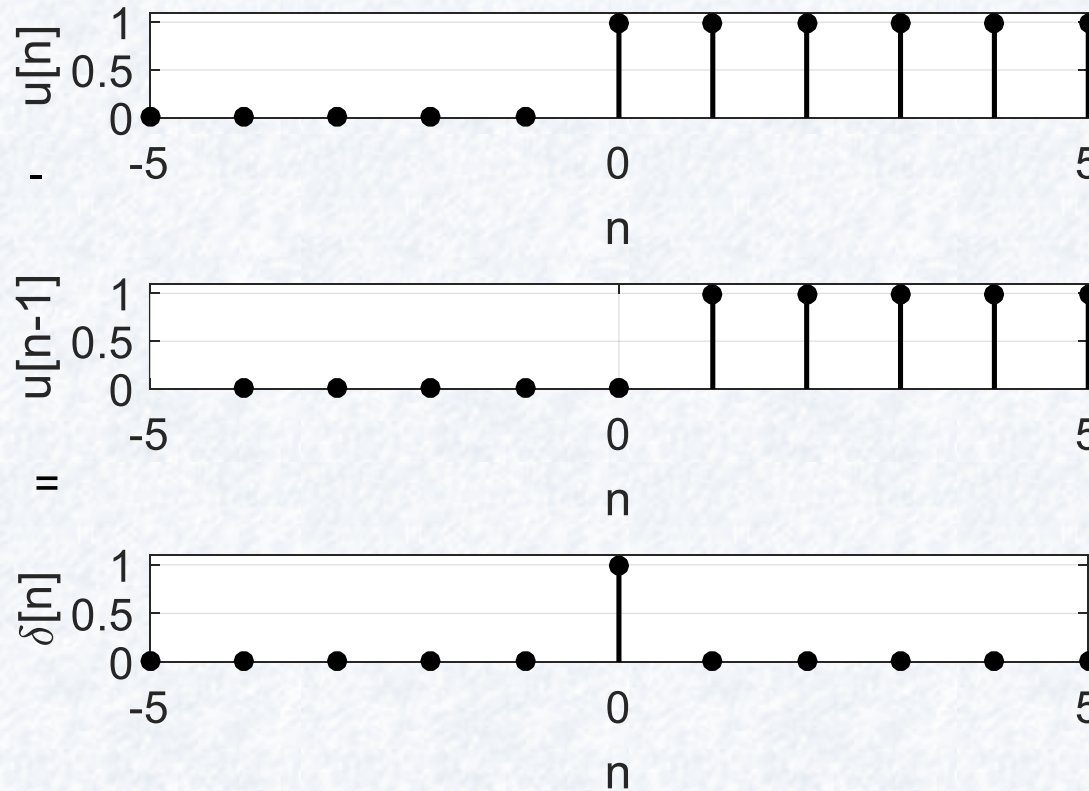
# Birim Basamaktan Birim Darbe

- $\delta[n] = ?$



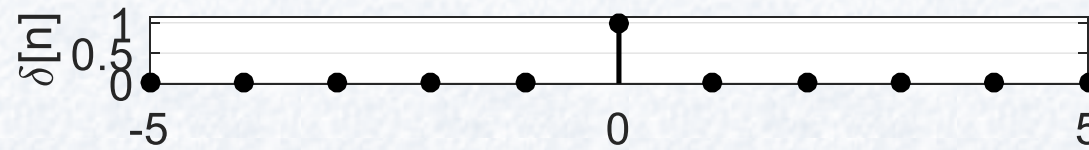
# Birim Basamaktan Birim Darbe

- $\delta[n] = u[n] - u[n-1]$



# Birim Darbeden Birim Basamak

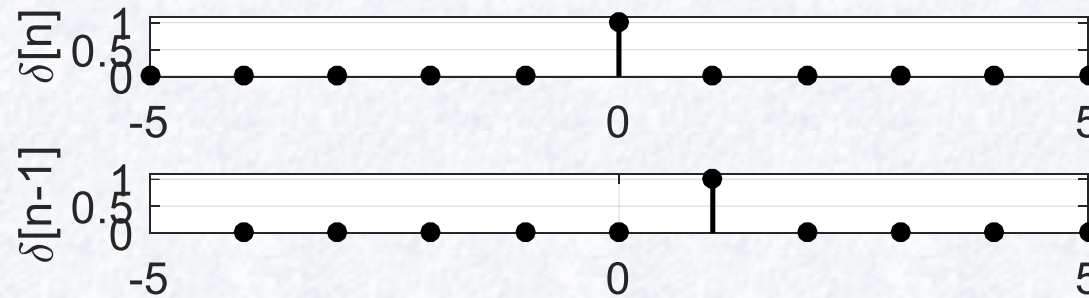
- $u[n] = ?$





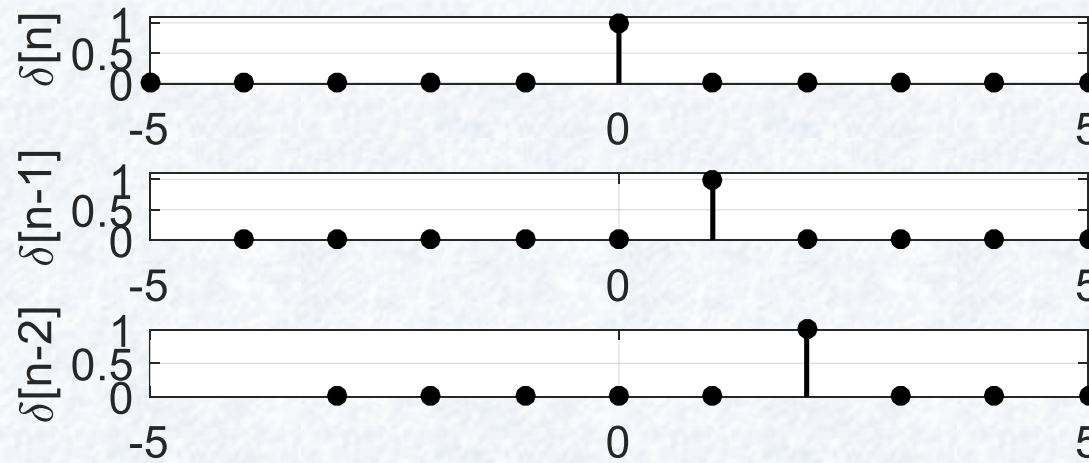
# Birim Darbeden Birim Basamak

- $u[n] = ?$



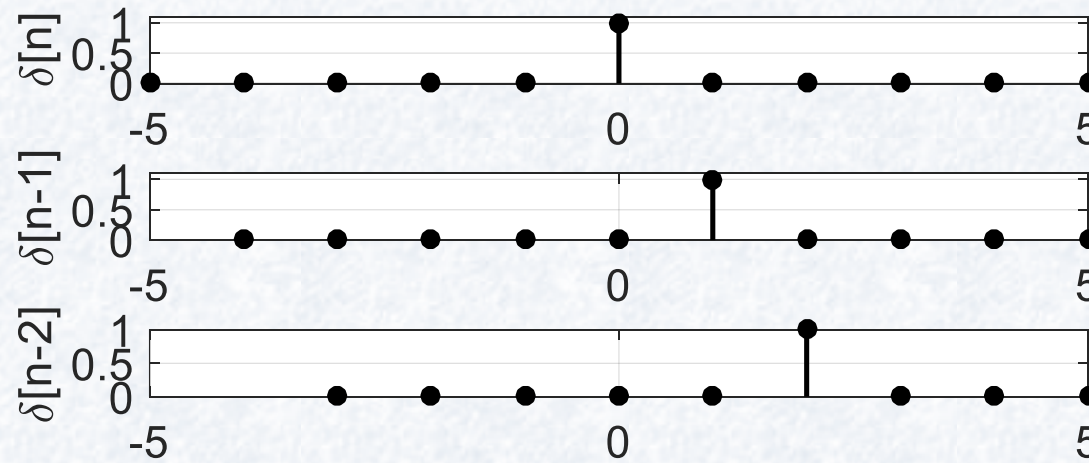
# Birim Darbeden Birim Basamak

- $u[n] = ?$



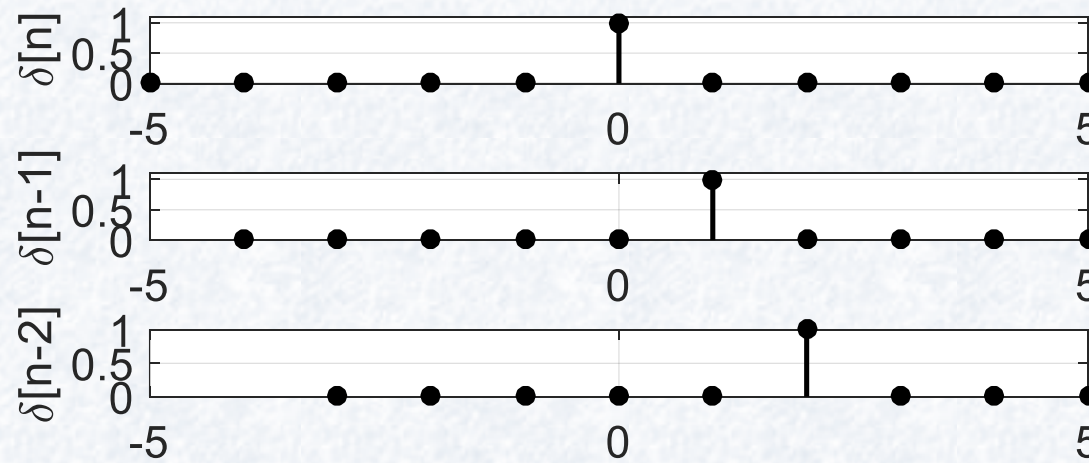
# Birim Darbeden Birim Basamak

- $u[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \dots$



# Birim Darbeden Birim Basamak

- $u[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \dots = \sum_{k=0}^{\infty} \delta(n-k)$



# Birim Darbeden Birim Basamak

- $u[n] = \sum_{k=-\infty}^n \delta[k]$

