

2. Öğretim I. grup I. Kısa Sınav / Metalurji

Soru $2x_1 + 2x_2 - x_3 + 11x_4 = 8$

$-x_1 + x_3 + x_4 = -6$

$-2x_1 + x_2 + 3x_3 + 12x_4 = -15$

lineer denklemler sistemini çözünüz

Gözüm $\left[\begin{array}{cccc|c} 2 & 2 & -1 & 11 & 8 \\ -1 & 0 & 1 & 1 & -6 \\ -2 & 1 & 3 & 12 & -15 \end{array} \right] \xrightarrow{R_1+R_3 \rightarrow R_3} \left[\begin{array}{cccc|c} 2 & 2 & -1 & 11 & 8 \\ -1 & 0 & 1 & 1 & -6 \\ 0 & 3 & 2 & 23 & -7 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cccc|c} -1 & 0 & 1 & 1 & -6 \\ 2 & 2 & -1 & 11 & 8 \\ 0 & 3 & 2 & 23 & -7 \end{array} \right]$

$\left[\begin{array}{cccc|c} -1 & 0 & 1 & 1 & -6 \\ 2 & 2 & -1 & 11 & 8 \\ 0 & 3 & 2 & 23 & -7 \end{array} \right] \xrightarrow{-R_1 \rightarrow R_1} \left[\begin{array}{cccc|c} 1 & 0 & -1 & -1 & 6 \\ 2 & 2 & -1 & 11 & 8 \\ 0 & 3 & 2 & 23 & -7 \end{array} \right] \xrightarrow{-2R_1+R_2 \rightarrow R_2} \left[\begin{array}{cccc|c} 1 & 0 & -1 & -1 & 6 \\ 0 & 2 & 1 & 13 & -4 \\ 0 & 3 & 2 & 23 & -7 \end{array} \right]$

$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -1 & 6 \\ 0 & 2 & 1 & 13 & -4 \\ 0 & 3 & 2 & 23 & -7 \end{array} \right] \xrightarrow{-R_2+R_3 \rightarrow R_3} \left[\begin{array}{cccc|c} 1 & 0 & -1 & -1 & 6 \\ 0 & 2 & 1 & 13 & -4 \\ 0 & 1 & 1 & 10 & -3 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{cccc|c} 1 & 0 & -1 & -1 & 6 \\ 0 & 1 & 1 & 10 & -3 \\ 0 & 2 & 1 & 13 & -4 \end{array} \right] \xrightarrow{-2R_2+R_3 \rightarrow R_3} \left[\begin{array}{cccc|c} 1 & 0 & -1 & -1 & 6 \\ 0 & 1 & 1 & 10 & -3 \\ 0 & 0 & -1 & -7 & 2 \end{array} \right]$

$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -1 & 6 \\ 0 & 1 & 1 & 10 & -3 \\ 0 & 0 & -1 & -7 & 2 \end{array} \right] \xrightarrow{-R_3 \rightarrow R_3} \left[\begin{array}{cccc|c} 1 & 0 & -1 & -1 & 6 \\ 0 & 1 & 1 & 10 & -3 \\ 0 & 0 & 1 & 7 & -2 \end{array} \right] \xrightarrow{-R_3+R_2 \rightarrow R_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 6 & 4 \\ 0 & 1 & 0 & 3 & -1 \\ 0 & 0 & 1 & 7 & -2 \end{array} \right] \xrightarrow{R_3+R_1 \rightarrow R_1} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 6 & 4 \\ 0 & 1 & 0 & 3 & -1 \\ 0 & 0 & 1 & 7 & -2 \end{array} \right]$

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 6 & 4 \\ 0 & 1 & 0 & 3 & -1 \\ 0 & 0 & 1 & 7 & -2 \end{array} \right] \Rightarrow \begin{cases} x_1 + 6x_4 = 4 \\ x_2 + 3x_4 = -1 \\ x_3 + 7x_4 = -2 \end{cases} \Rightarrow \begin{cases} x_4 = t \text{ seçelim} \\ x_1 = 4 - 6t \\ x_2 = -1 - 3t \\ x_3 = -2 - 7t \end{cases}$

Gözümle soru 2 tane olup,

olur.

$(x_1, x_2, x_3, x_4) = (4 - 6t, -1 - 3t, -2 - 7t, t)$ şeklindedir.

2. Öğretim 2. grup / Metalurji / Lineer Cebir / I. KS.

Soru

$$6x_1 + x_2 + 31x_3 + 8x_4 = 23$$

$$4x_1 + x_2 + 21x_3 + 6x_4 = 15$$

$$x_1 + 5x_3 + x_4 = 4$$

lineer denklem sisteminin
çözümünü.

Çözüm

$$\left[\begin{array}{cccc|c} 6 & 1 & 31 & 8 & 23 \\ 4 & 1 & 21 & 6 & 15 \\ 1 & 0 & 5 & 1 & 4 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{cccc|c} 1 & 0 & 5 & 1 & 4 \\ 4 & 1 & 21 & 6 & 15 \\ 6 & 1 & 31 & 8 & 23 \end{array} \right] \xrightarrow{\begin{array}{l} -4R_1 + R_2 \rightarrow R_2 \\ -6R_1 + R_3 \rightarrow R_3 \end{array}}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 5 & 1 & 4 \\ 0 & 1 & 1 & 2 & -1 \\ 0 & 1 & 1 & 2 & -1 \end{array} \right] \xrightarrow{-R_2 + R_3 \rightarrow R_3} \left[\begin{array}{cccc|c} 1 & 0 & 5 & 1 & 4 \\ 0 & 1 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$x_1 + 5x_3 + x_4 = 4$$

$$x_2 + x_3 + 2x_4 = -1$$

\Rightarrow

$$x_3 = s \in \mathbb{R}$$

$$x_4 = t \in \mathbb{R}$$

olsun. \Rightarrow

$$x_1 = 4 - 5s - t$$

$$x_2 = -1 - s - 2t$$

Böylece genel çözüm;

$$(x_1, x_2, x_3, x_4) = (4 - 5s - t, -1 - s - 2t, s, t); s, t \in \mathbb{R}$$

şeklinde dir.