1. Aşağıda verilen sürekli zaman işaretlerin Fourier dönüşümlerini bulunuz.

1a. $x(t) = e^{at}u(-t)$

1a. $X(\omega) = \int_{-\infty}^{\infty} e^{at}u(-t)e^{-j\omega t}dt = \int_{-\infty}^{0} e^{(a-j\omega)t}dt = \frac{1}{a-j\omega}e^{(a-j\omega)t}\Big|_{-\infty}^{0} = \frac{1}{a-j\omega}$

1b. $x(t) = -u(t+1) + 2u(t) - u(t-1)$

1b.

$$X(\omega) = \int_{-\infty}^{\infty}[-u(t+1) + 2u(t) - u(t-1)]e^{-j\omega t}dt$$

$$= \int_{-1}^{\infty} -u(t+1)e^{-j\omega t}dt + \int_{0}^{\infty} 2u(t)e^{-j\omega t}dt + \int_{1}^{\infty} -u(t-1)e^{-j\omega t}dt$$

$$X(\omega) = \frac{1}{j\omega}e^{-j\omega t}\Big|_{-1}^{\infty} - \frac{2}{j\omega}e^{-j\omega t}\Big|_{0}^{\infty} + \frac{1}{j\omega}e^{-j\omega t}\Big|_{1}^{\infty} = -\frac{1}{j\omega}e^{j\omega} + \frac{2}{j\omega} - \frac{1}{j\omega}e^{-j\omega}$$

$$= \frac{2}{j\omega} - \frac{2}{j\omega}\left(\frac{e^{j\omega} + e^{-j\omega}}{2}\right) = \frac{2}{j\omega}(1 - \cos\omega)$$

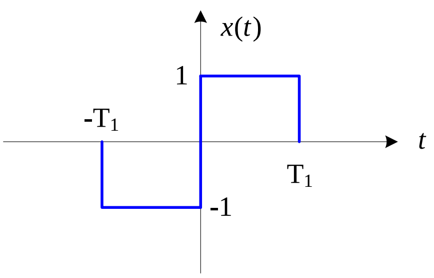
1c. $x(t) = -2u(t+1) - 2u(t-1)$

1c.

$$X(\omega) = \int_{-\infty}^{\infty}[-2u(t+1) - 2u(t-1)]e^{-j\omega t}dt = \int_{-1}^{\infty} -2u(t+1)e^{-j\omega t}dt + \int_{1}^{\infty} -2u(t-1)e^{-j\omega t}dt$$

$$X(\omega) = \frac{2}{j\omega}e^{-j\omega t}\Big|_{-1}^{\infty} + \frac{2}{j\omega}e^{-j\omega t}\Big|_{1}^{\infty} = -\frac{2}{j\omega}e^{j\omega} - \frac{2}{j\omega}e^{-j\omega} = -\frac{4}{j\omega}\left(\frac{e^{j\omega} + e^{-j\omega}}{2}\right) = -\frac{4}{j\omega}\cos\omega$$

2. Aşağıdaki şekilde verilen $x(t)$ sürekli zaman işaretin Fourier dönüşümünü bulunuz.



2.

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = -\int_{-T_1}^{0} e^{-j\omega t}dt + \int_{0}^{T_1} e^{-j\omega t}dt = \frac{1}{j\omega}\cdot e^{-j\omega t}\Big|_{-T_1}^{0} - \frac{1}{j\omega}\cdot e^{-j\omega t}\Big|_{0}^{T_1}$$

$$= \frac{1}{j\omega}\cdot(1 - e^{j\omega T_1} - e^{-j\omega T_1} + 1) = \frac{2}{j\omega}\cdot\left[\frac{2 - (e^{j\omega T_1} + e^{-j\omega T_1})}{2}\right] = \frac{2}{j\omega}\cdot[1 - \cos\omega T_1]$$

3. Aşağıdaki şekilde verilen sürekli zaman $x(t)$ işaretinin Fourier dönüşümünü bulunuz.

$$x(t) = \begin{cases} 0 & |t| > T_1 \\ \cos\pi t & |t| \le T_1 \end{cases}$$

3.

$$x(t)=\frac{1}{2}e^{j\pi t}+\frac{1}{2}e^{-j\pi t}$$

$$X(\omega)=\int_{-\infty}^{\infty}x(t)e^{-j\omega t}dt=\int_{-T_1}^{T_1}\left(\frac{1}{2}e^{j\pi t}+\frac{1}{2}e^{-j\pi t}\right)e^{-j\omega t}dt=\frac{1}{2}\int_{-T_1}^{T_1}e^{j(\pi-\omega)t}dt+\frac{1}{2}\int_{-T_1}^{T_1}e^{-j(\pi+\omega)t}dt$$

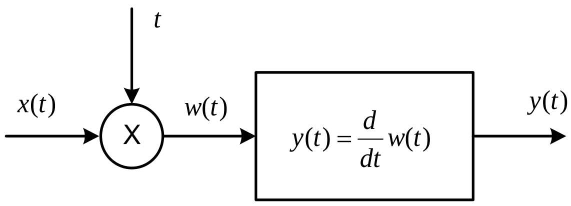
$$=\frac{1}{2}\cdot\frac{1}{j(\pi-\omega)}\cdot e^{j(\pi-\omega)t}\Big|_{-T_1}^{T_1}-\frac{1}{2}\cdot\frac{1}{j(\pi+\omega)}\cdot e^{-j(\pi+\omega)t}\Big|_{-T_1}^{T_1}$$

$$=\frac{1}{2}\cdot\frac{1}{j(\pi-\omega)}\cdot\left[e^{j(\pi-\omega)T_1}-e^{-j(\pi-\omega)T_1}\right]-\frac{1}{2}\cdot\frac{1}{j(\pi+\omega)}\cdot\left[e^{-j(\pi+\omega)T_1}-e^{j(\pi+\omega)T_1}\right]$$

$$=\frac{1}{2}\cdot\left\{\frac{2j}{j(\pi-\omega)}\cdot\left[\frac{e^{j(\pi-\omega)T_1}-e^{-j(\pi-\omega)T_1}}{2j}\right]+\frac{2j}{j(\pi+\omega)}\cdot\left[\frac{e^{j(\pi+\omega)T_1}-e^{-j(\pi+\omega)T_1}}{2j}\right]\right\}$$

$$=\frac{1}{(\pi-\omega)}\sin(\pi-\omega)T_1+\frac{1}{(\pi+\omega)}\sin(\pi+\omega)T_1$$

4. Sürekli zaman işaret $x(t)$' nin Fourier dönüşümünün $X(\omega)$ olduğu biliniyorsa aşağıdaki sistemle elde edilen $y(t)$ işaretinin Fourier dönüşümü $X(\omega)$ cinsinden nedir?



4.

$$w(t)=tx(t)$$

$y(t)=\dfrac{d}{dt}(w(t))=\dfrac{d}{dt}(tx(t))=x(t)+t\dfrac{d}{dt}x(t)$ olur.

Tablodan aşağıdakiler yazılabilir.

1.Yol

$z(t)=\dfrac{d}{dt}x(t)$ dersek $Z(\omega)=j\omega X(\omega)$

$y(t)=x(t)+tz(t)$ olur. $Y(\omega)=X(\omega)+j\dfrac{d}{d\omega}Z(\omega)=X(\omega)+j\left(jX(\omega)+j\omega\dfrac{d}{d\omega}X(\omega)\right)$

$$Y(\omega)=X(\omega)-X(\omega)-\omega\frac{d}{d\omega}X(\omega)=-\omega\frac{d}{d\omega}X(\omega)$$

2. Yol

$$w(t)=tx(t)$$

$W(\omega)=j\dfrac{d}{d\omega}X(\omega)$ olur.

$y(t)=\dfrac{d}{dt}w(t)$ dir.

$$Y(\omega)=j\omega W(\omega)=j\omega j\frac{d}{d\omega}X(\omega)=-\omega\frac{d}{d\omega}X(\omega)$$

5. Frekans spektrumu $2\pi\delta(\omega-\omega_0)$ şeklinde verilen sürekli zaman işareti ters Fourier dönüşümü ile bulunuz.

5. $x(t)=\dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega=\dfrac{1}{2\pi}\int_{-\infty}^{\infty} 2\pi\delta(\omega-\omega_0)e^{j\omega t}d\omega=e^{j\omega_0 t}$

6. Frekans spektrumu $X(\omega)=\pi[\delta(\omega-6\pi)+\delta(\omega-4\pi)+\delta(\omega+4\pi)+\delta(\omega+6\pi)]$ şeklinde verilen periyodik işaretin

a. Temel frekansını bulunuz.

b. Fourier seri katsayılarını bulunuz.

c. Zaman domeni ifadesi $x(t)$' yi yazınız.

6.

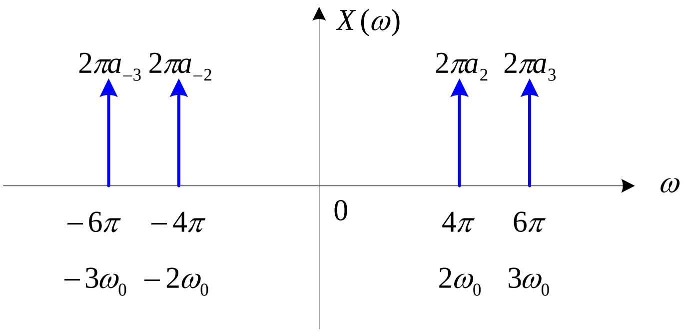
$$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega=\frac{1}{2\pi}\int_{-\infty}^{\infty} \pi[\delta(\omega-6\pi)+\delta(\omega-4\pi)+\delta(\omega+4\pi)+\delta(\omega+6\pi)]e^{j\omega t}d\omega$$

$$=\frac{1}{2\pi}\int_{-\infty}^{\infty} \pi\delta(\omega-4\pi)e^{j\omega t}d\omega+\frac{1}{2\pi}\int_{-\infty}^{\infty} \pi\delta(\omega+4\pi)e^{j\omega t}d\omega$$

$$+\frac{1}{2\pi}\int_{-\infty}^{\infty} \pi\delta(\omega-6\pi)e^{j\omega t}d\omega+\frac{1}{2\pi}\int_{-\infty}^{\infty} \pi\delta(\omega+6\pi)e^{j\omega t}d\omega$$

$$x(t)=\frac{1}{2}e^{j4\pi t}+\frac{1}{2}e^{-j4\pi t}+\frac{1}{2}e^{j6\pi t}+\frac{1}{2}e^{-j6\pi t}=\frac{1}{2}e^{j\underbrace{2}_{k}(\underbrace{2\pi}_{\omega_0})t}+\frac{1}{2}e^{-j\underbrace{2}_{k}(\underbrace{2\pi}_{\omega_0})t}+\frac{1}{2}e^{j\underbrace{3}_{k}(\underbrace{2\pi}_{\omega_0})t}+\frac{1}{2}e^{-j\underbrace{3}_{k}(\underbrace{2\pi}_{\omega_0})t}$$

$$x(t)=\frac{e^{j4\pi t}+e^{-j4\pi t}}{2}+\frac{e^{j6\pi t}+e^{-j6\pi t}}{2}=\cos(4\pi t)+\cos(6\pi t)$$



$$x(t)=\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

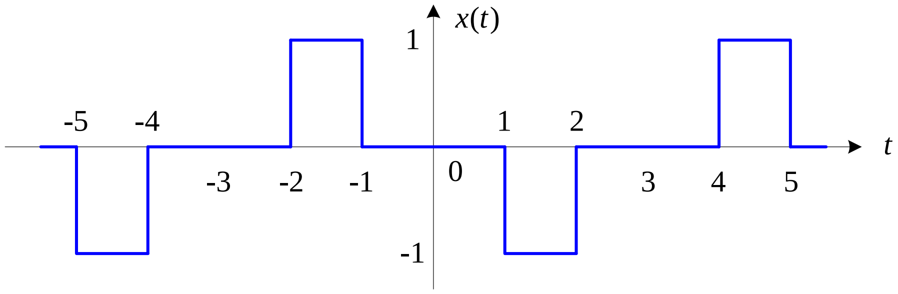
$$X(\omega)=\sum_{k=-\infty}^{\infty} 2\pi a_k\delta(\omega-k\omega_0)$$

a. $\omega_0=2\pi$

b. $a_2=a_{-2}=a_3=a_{-3}=\dfrac{1}{2}$

c. $x(t)=\cos(4\pi t)+\cos(6\pi t)$

7. Aşağıdaki şekilde verilen $x(t)$ periyodik işaretin Fourier açılımını (katsayılarını) bulunuz.



7.

$$x_k = \frac{1}{P}\int_{-P/2}^{P/2} x(t)e^{-jk\omega_0 t}dt \qquad P=6 \qquad \omega_0 = \frac{\pi}{3}$$

$$x_k = \frac{1}{6}\int_{-3}^{3} x(t)e^{-jk\omega_0 t}dt = \frac{1}{6}\left[\int_{-2}^{-1} e^{-jk\omega_0 t}dt - \int_{1}^{2} e^{-jk\omega_0 t}dt\right] = \frac{1}{6}\left[-\frac{1}{jk\omega_0}e^{-jk\omega_0 t}\Big|_{-2}^{-1} + \frac{1}{jk\omega_0}e^{-jk\omega_0 t}\Big|_{1}^{2}\right]$$

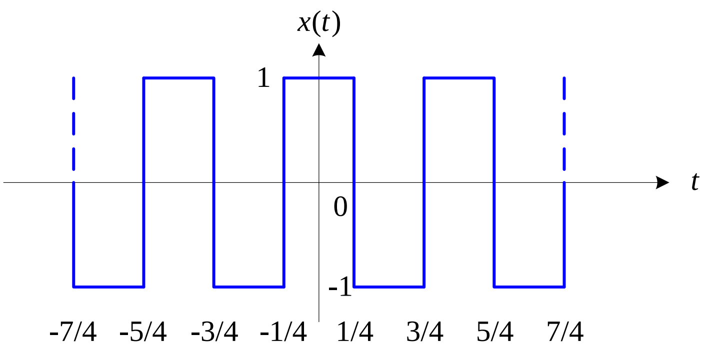
$$= \frac{1}{j6k\omega_0}\left[-e^{-jk\omega_0 t}\Big|_{-2}^{-1} + e^{-jk\omega_0 t}\Big|_{1}^{2}\right] = \frac{1}{j6k\omega_0}\left[-e^{jk\omega_0} + e^{j2k\omega_0} + e^{-j2k\omega_0} - e^{-jk\omega_0}\right]$$

$$= \frac{2}{j6k\omega_0}\left[\frac{e^{j2k\omega_0} + e^{-j2k\omega_0}}{2} - \frac{e^{jk\omega_0} + e^{-jk\omega_0}}{2}\right] = \frac{2}{j6k\omega_0}\left[\cos 2k\omega_0 - \cos k\omega_0\right]$$

$$\omega_0 = \frac{\pi}{3} \text{ için} \qquad x_k = \frac{1}{j3k\frac{\pi}{3}}\left[\cos 2k\frac{\pi}{3} - \cos k\frac{\pi}{3}\right] = \frac{1}{jk\pi}\left[\cos\frac{2k\pi}{3} - \cos\frac{k\pi}{3}\right]$$

$$x_k = \begin{cases} \dfrac{1}{jk\pi}\left[\cos\dfrac{2k\pi}{3} - \cos\dfrac{k\pi}{3}\right] & k \neq 0 \\ 0 & k = 0 \end{cases}$$

8. Aşağıdaki şekilde verilen $x(t)$ periyodik işaretin Fourier açılımını (katsayılarını) bulunuz.



8.

$$x_k = \frac{1}{P}\int_{-P/2}^{P/2} x(t)e^{-jk\omega_0 t}dt \qquad P=1 \qquad \omega_0 = 2\pi$$

$$x_k = \frac{1}{1}\int_{-1/2}^{1/2} x(t)e^{-jk\omega_0 t}dt = -\int_{-1/2}^{-1/4} e^{-jk\omega_0 t}dt + \int_{-1/4}^{1/4} e^{-jk\omega_0 t}dt - \int_{1/4}^{1/2} e^{-jk\omega_0 t}dt$$

$$= \frac{1}{jk\omega_0} e^{-jk\omega_0 t}\Big|_{-1/2}^{-1/4} - \frac{1}{jk\omega_0} e^{-jk\omega_0 t}\Big|_{-1/4}^{1/4} + \frac{1}{jk\omega_0} e^{-jk\omega_0 t}\Big|_{1/4}^{1/2}$$

$$= \frac{1}{jk\omega_0}\left\{\left[e^{jk\frac{\omega_0}{4}} - e^{jk\frac{\omega_0}{2}}\right] - \left[e^{-jk\frac{\omega_0}{4}} - e^{jk\frac{\omega_0}{4}}\right] + \left[e^{-jk\frac{\omega_0}{2}} - e^{-jk\frac{\omega_0}{4}}\right]\right\}$$

$$= \frac{1}{jk\omega_0}\left\{\left[2(e^{jk\frac{\omega_0}{4}} - e^{-jk\frac{\omega_0}{4}})\right] - \left[e^{jk\frac{\omega_0}{2}} - e^{-jk\frac{\omega_0}{2}}\right]\right\}$$

$$= \frac{2j}{jk\omega_0}\left\{\left[2\frac{(e^{jk\frac{\omega_0}{4}} - e^{-jk\frac{\omega_0}{4}})}{2j}\right] - \left[\frac{e^{jk\frac{\omega_0}{2}} - e^{-jk\frac{\omega_0}{2}}}{2j}\right]\right\} = \frac{2}{k\omega_0}\left[2\sin\frac{k\omega_0}{4} - \sin\frac{k\omega_0}{2}\right]$$

$\omega_0 = 2\pi$ için $\quad x_k = \frac{1}{k\pi}\left[2\sin\frac{k\pi}{2} - \sin k\pi\right] = \frac{2}{k\pi}\sin\frac{k\pi}{2} \qquad x_k = \begin{cases} \frac{2}{k\pi}\sin\frac{k\pi}{2} & k \text{ tek ise} \\ 0 & k \text{ çift ise} \end{cases}$

9. $f_c = 10$ kHz frekansla örneklendiğinde $x(n) = \sin(n\frac{\pi}{4})$ ayrık zaman işareti veren $x_a(t)$ analog işaretini bulunuz.

9. $x(t) = \sin\omega_0 t = \sin(2\pi f_0 n T_c) = \sin(n\frac{\pi}{4}) \qquad 2\pi f_0 n T_c = n\frac{\pi}{4} \qquad 2\pi f_0 n \frac{1}{10000} = n\frac{\pi}{4}$

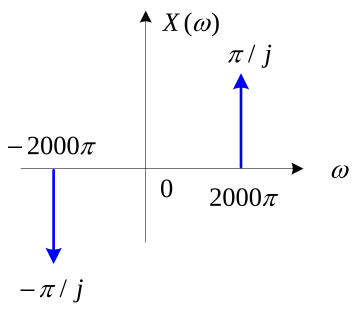
$f_0 = 1250\,Hz = 1.25\,kHz \qquad x_a(t) = \sin(2\pi 1250 t) = \sin(2500\pi t)$

10. $x_a(t) = \sin(2500\pi t)$ analog işareti $f_c = 2.5$ kHz frekans ile örneklendiğinde elde edilen ayrık zaman işareti bulunuz.

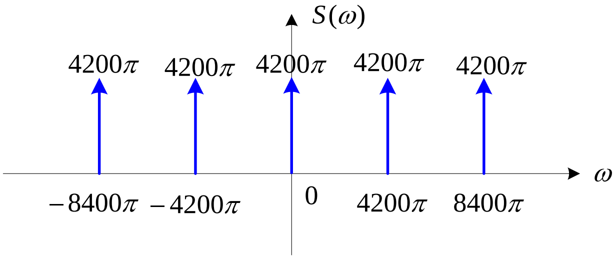
10. $x_a(t) = \sin(2\pi f_o t) \quad x(n) = \sin(2\pi 1250 n T_c) \quad x(n) = \sin(2500\pi n / 2500) \qquad x(n) = \sin(n\pi)$

11. $f_0 = 1$ kHz olmak üzere, $x(t) = \sin\omega_0 t$ analog işaretinin $T_s = \frac{1}{2100}$ sn aralıklarla örneklenmesi ile elde edilen $x(nT)$ işaretinin frekans spektrumunu çiziniz.

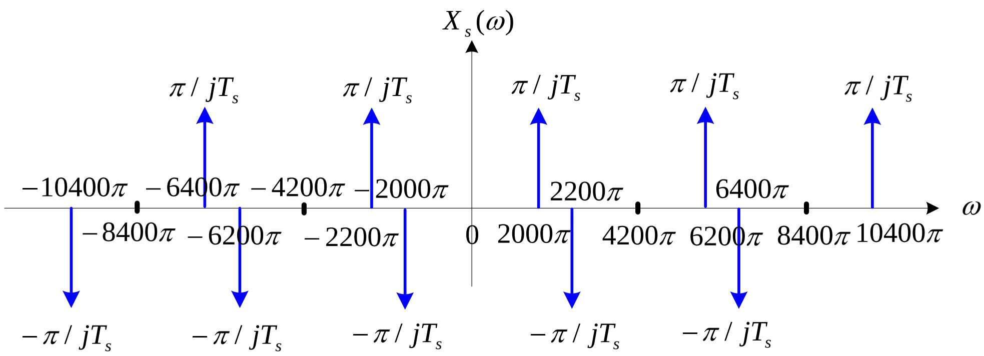
11. $x(t)$ işaretinin frekans spektrumu $X(\omega)$ aşağıdaki gibi verilir.



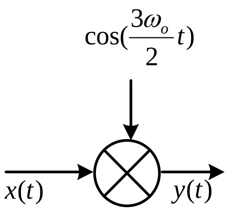
$\omega_s = \dfrac{2\pi}{T_s} = 4200\pi$ açısal frekansında örnekleme yaptığımızda $S(\omega)$ işareti aşağıdaki gibi verilir.



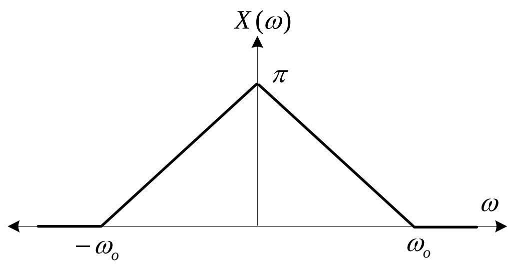
$$X_s(\omega) = \frac{1}{2\pi}\left(\underbrace{X(\omega)}_{\pi/j} * \underbrace{S(\omega)}_{4200\pi}\right) = 2100\pi/j = \pi/jT_s$$



12a. Şekil 1'deki prosesteki $x(t)$ nin Fourier dönüşü şekil 2'deki $X(\omega)$ şeklinde olduğuna göre Fourier özellik tablosunu kullanarak $Y(\omega)$'yı çiziniz.

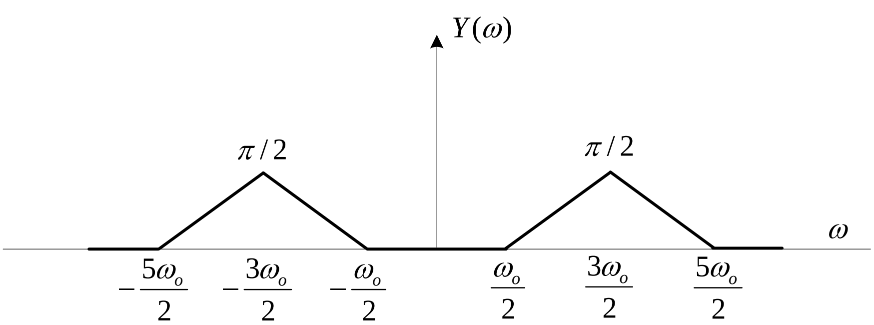


Şekil 1



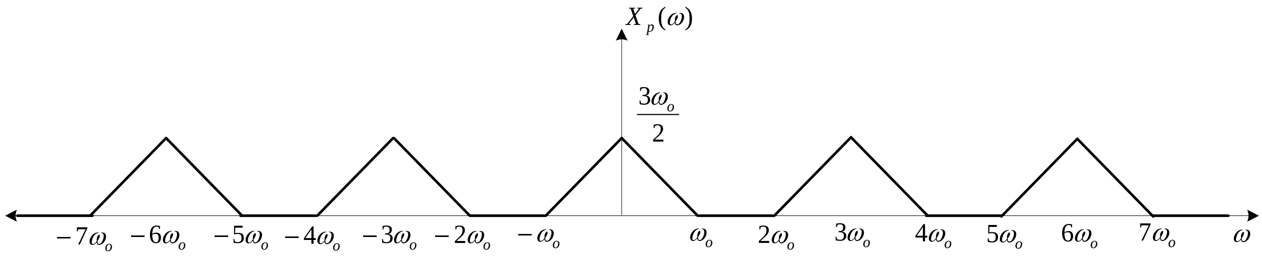
Şekil2

12a.

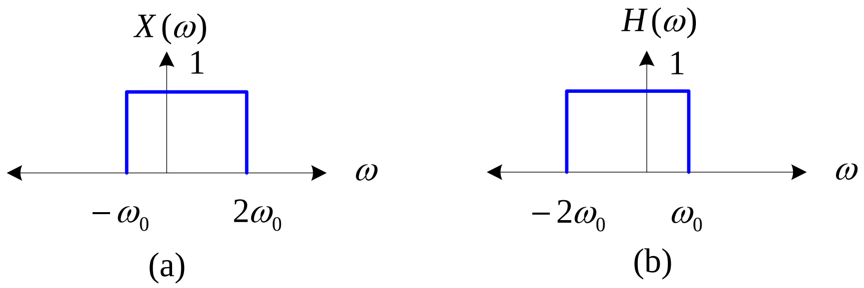


12b. a şıkkında verilen $x(t)$ işaretini $3\omega_0$ frekansı ile örneklediğimizde örneklemiş sürekli zaman işaretin frekans spektrumunu çiziniz.

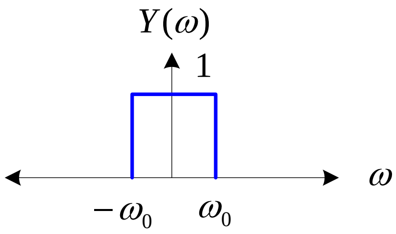
12b.



13. Spektrumu şekil (a) da verilen giriş işaretini şekil (b) deki spektruma sahip olan bir sisteme uyguladığımızda çıkışında elde edilen işaretinin $y(t)$ ifadesini bulunuz.



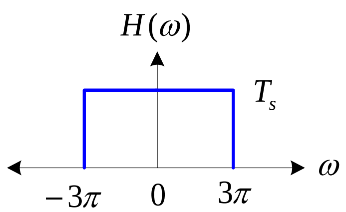
13. $Y(\omega) = X(\omega).H(\omega)$



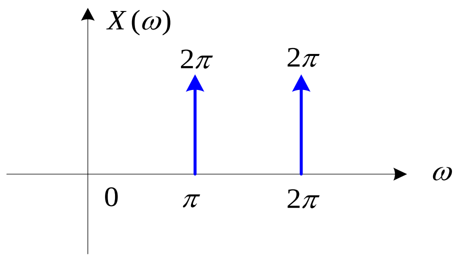
$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} e^{j\omega t}d\omega = \frac{1}{j2\pi t}e^{j\omega t}\Big|_{-\omega_0}^{\omega_0} = \frac{1}{j2\pi t}\left[e^{j\omega_0 t} - e^{-j\omega_0 t}\right]$$

$$= \frac{1}{\pi t}\left[\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}\right] = \frac{1}{\pi t}\sin\omega_0 t$$

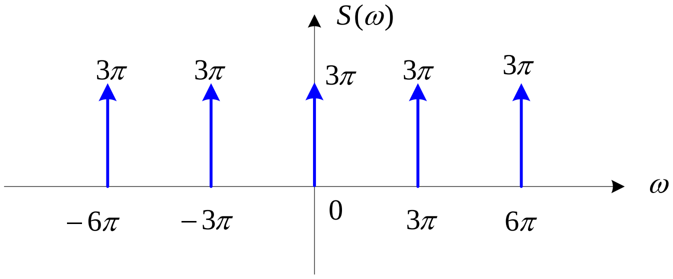
14. $x(t) = e^{j\pi t} + e^{j2\pi t}$ olarak verilen analog işaret $T_s = \frac{2}{3}sn$ ile örneklenmektedir. Örneklemeden sonra elde edilen $x_s(t)$ analog işaretinin frekans spektrumu aşağıdaki şekilde verilen sistemden geçirildiğinde elde edilen $y(t)$ işaretini bulunuz.



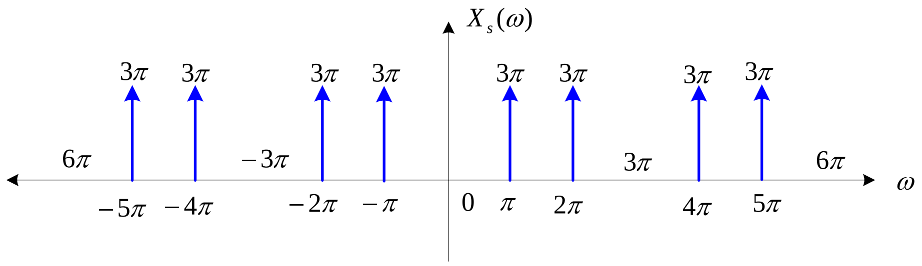
14. $x(t)$ işaretinin frekans spektrumu $X(\omega)$ aşağıdaki gibi verilir.



$\omega_s = \dfrac{2\pi}{T_s} = 3\pi$ açısal frekansında örnekleme yaptığımızda $S(\omega)$ işareti aşağıdaki gibi verilir.

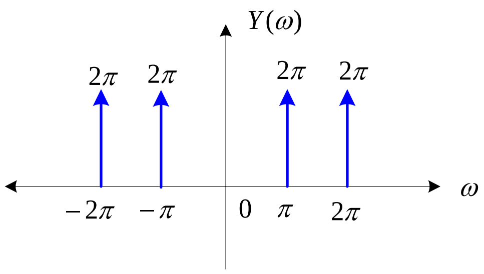


$$X_s(\omega) = \frac{1}{2\pi}\left(\underbrace{X(\omega)}_{2\pi} * \underbrace{S(\omega)}_{3\pi}\right) = 3\pi$$

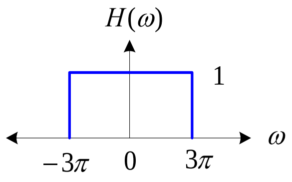


Örneklenen $X_s(\omega)$ işareti $H(\omega)$ dan geçirildiğinde $Y(\omega)$ aşağıdaki gibi elde edilir.

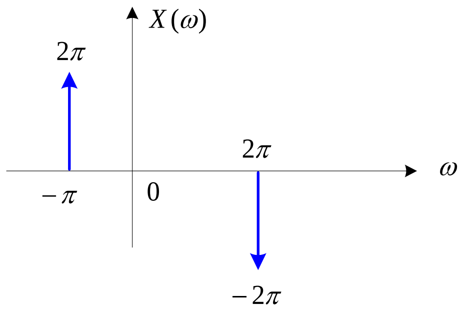
$Y(\omega) = X_s(\omega)H(\omega) = 3\pi.\dfrac{2}{3} = 2\pi$: Genlik $\qquad y(t) = 2\cos \pi t + 2\cos 2\pi t$



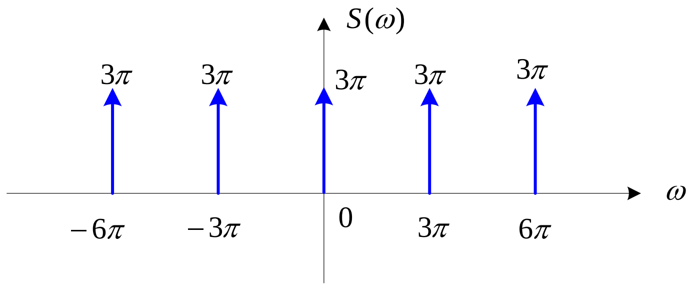
15. $x(t) = e^{-j\pi t} - e^{j2\pi t}$ olarak verilen analog işaret $T_s = \dfrac{2}{3} sn$ ile örneklenmektedir. Örneklemeden sonra elde edilen $x_s(t)$ analog işareti frekans spektrumu aşağıdaki şekilde verilen sistemden geçirildiğinde elde edilen $y(t)$ işaretini bulunuz.



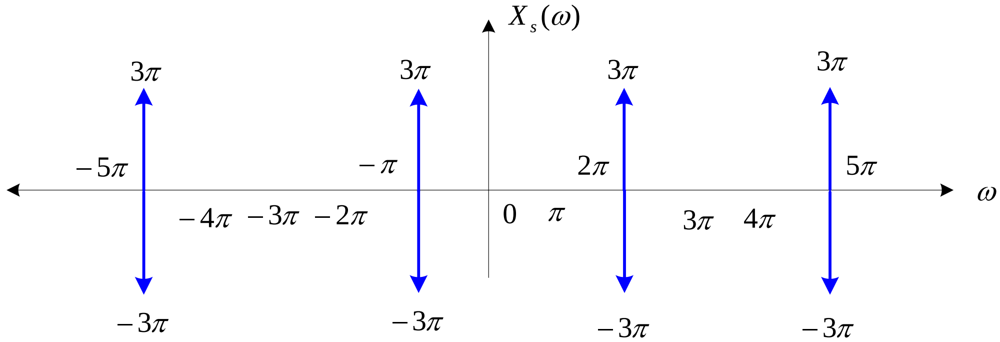
15. $x(t)$ işaretinin frekans spektrumu $X(\omega)$ aşağıdaki gibi verilir.



$\omega_s = \dfrac{2\pi}{T_s} = 3\pi$ açısal frekansında örnekleme yaptığımızda $S(\omega)$ işareti aşağıdaki gibi verilir.



$$X_s(\omega) = \frac{1}{2\pi}\left(\underbrace{X(\omega)}_{2\pi} * \underbrace{S(\omega)}_{3\pi}\right) = 3\pi$$

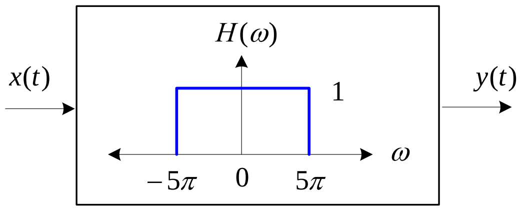


Örneklenen $X_s(\omega)$ işareti $H(\omega)$ dan geçirildiğinde $Y(\omega)$ aşağıdaki gibi elde edilir.

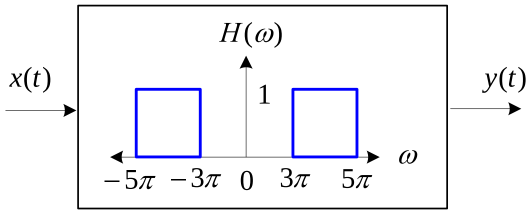
$Y(\omega) = \underbrace{X_s(\omega)}_{0}\underbrace{H(\omega)}_{1} = 0 \times 1 = 0$ Genlik $\qquad y(t) = 0$

16. Temel frekansı $\omega_0 = 2\pi$ olarak verilen $x(t)$ işaretinin Fourier seri katsayıları $a_0 = 1$, $a_1 = a_{-1} = \dfrac{1}{4}$, $a_2 = a_{-2} = \dfrac{1}{2}$ ve $a_3 = a_{-3} = \dfrac{1}{3}$ tür. $x(t)$ işaretini aşağıda spektrumları verilen sistemlere uyguladığımızda çıkışında elde edeceğimiz $y(t)$ işaretinin temel frekansını ve Fourier seri katsayılarını yazınız.

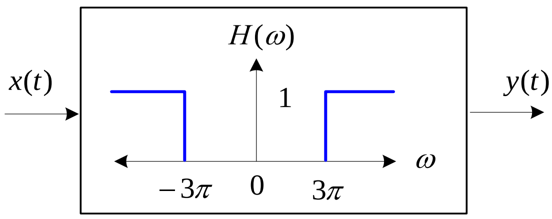
a.



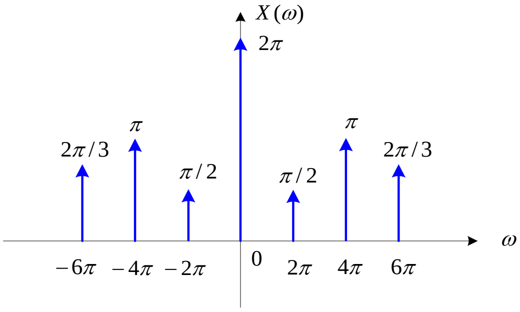
b.



c.

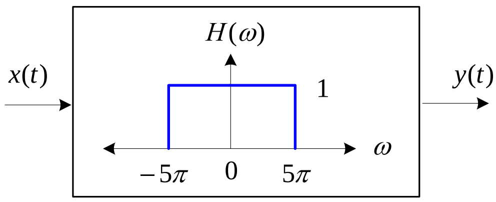


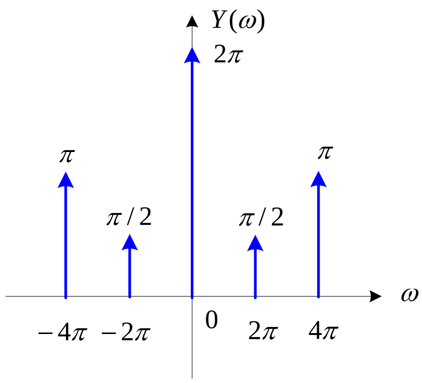
16.



$Y(\omega) = X(\omega)H(\omega)$

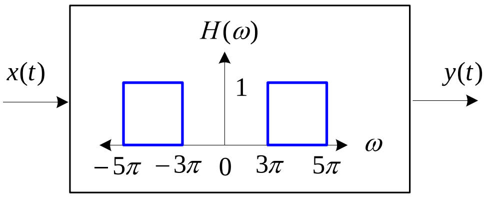
a. $\omega_0 = 2\pi$, $a_0 = 1$, $a_1 = a_{-1} = \dfrac{1}{4}$ ve $a_2 = a_{-2} = \dfrac{1}{2}$

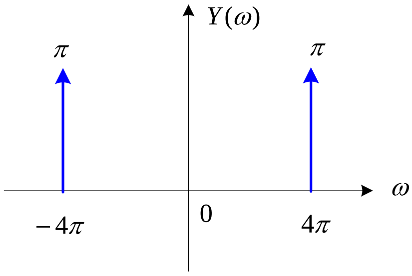




$$y(t) = 1 + \frac{1}{2}\cos 2\pi t + \cos 4\pi t$$

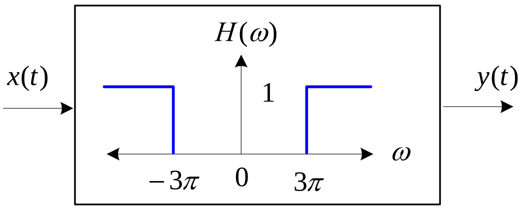
b. $\omega_0 = 4\pi$ ve $a_1 = a_{-1} = \frac{1}{2}$

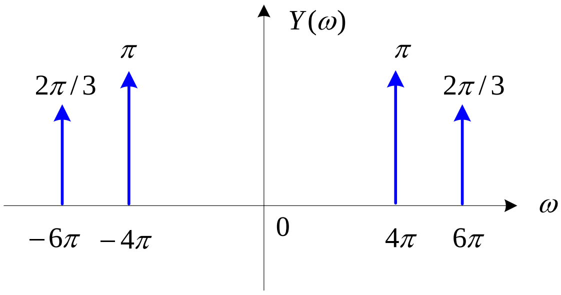




$y(t) = \cos 4\pi t$

c. $\omega_0 = 2\pi$, $a_2 = a_{-2} = \frac{1}{2}$ ve $a_3 = a_{-3} = \frac{1}{3}$

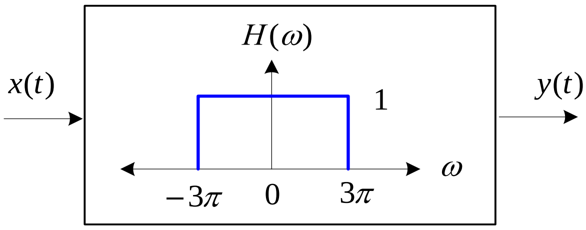




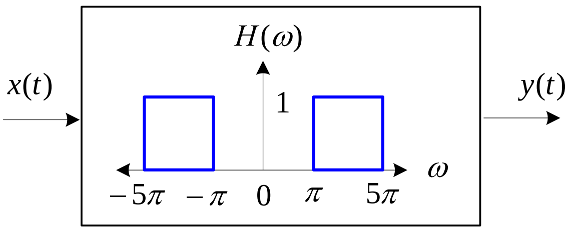
$y(t) = \cos 4\pi t + \frac{2}{3}\cos 6\pi t$

17. Temel frekansı $\omega_0 = 2\pi$ olarak verilen $x(t)$ işaretinin Fourier seri katsayıları $a_0 = 1$, $a_1 = a_{-1} = \frac{1}{4}$, $a_2 = a_{-2} = \frac{1}{2}$ ve $a_3 = a_{-3} = \frac{1}{3}$ tür. $x(t)$ işaretini aşağıda spektrumları verilen sistemlere uyguladığımızda çıkışında elde edeceğimiz $y(t)$ işaretinin temel frekansını ve Fourier seri katsayılarını yazınız.

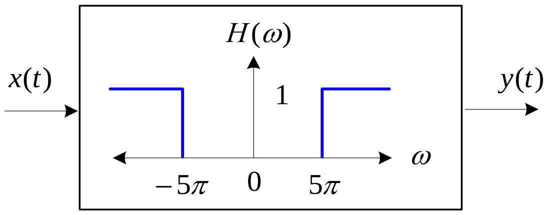
a.



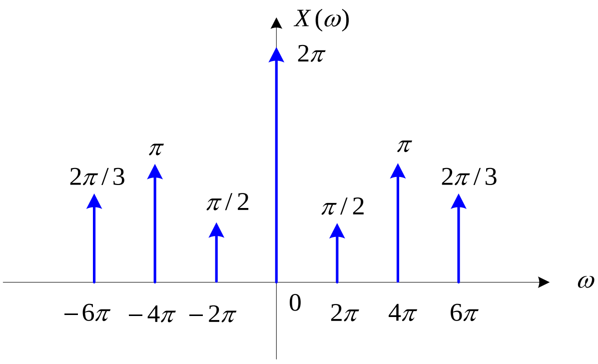
b.



c.

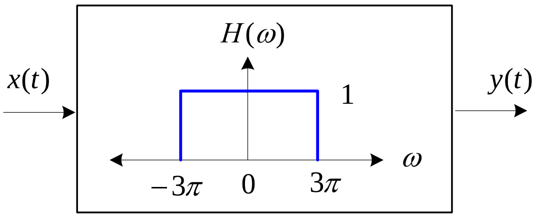


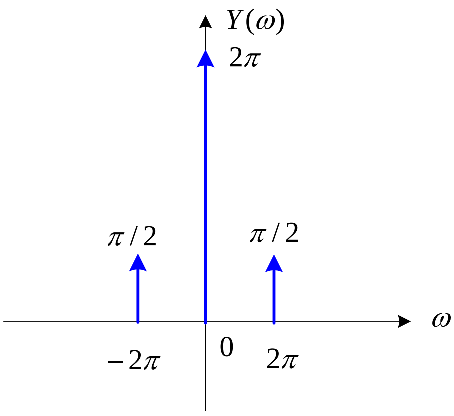
17.



$Y(\omega) = X(\omega)H(\omega)$

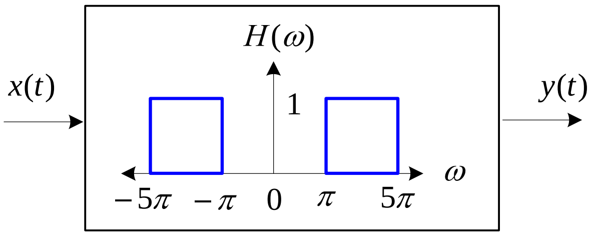
a. $\omega_0 = 2\pi$, $a_0 = 1$ ve $a_1 = a_{-1} = \dfrac{1}{4}$

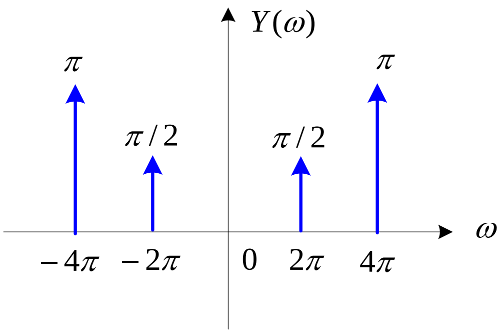




$$y(t) = 1 + \frac{1}{2}\cos 2\pi t$$

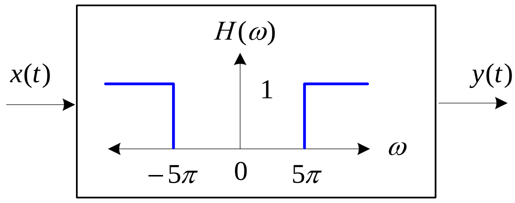
b. $\omega_0 = 2\pi$, $a_1 = a_{-1} = \dfrac{1}{4}$ ve $a_2 = a_{-2} = \dfrac{1}{2}$

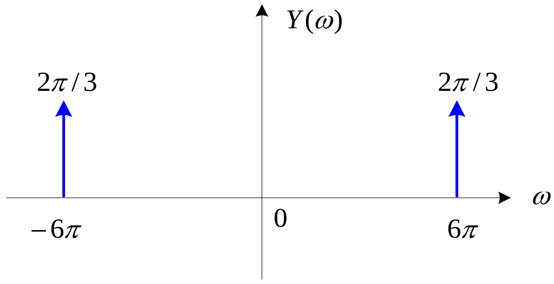




$$y(t) = \frac{1}{2}\cos 2\pi t + \cos 4\pi t$$

c. $\omega_0 = 6\pi$ ve $a_1 = a_{-1} = \frac{1}{3}$



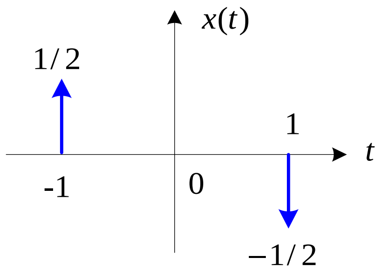


$$y(t) = \frac{2}{3}\cos 6\pi t$$

18. $T_s = \frac{1}{3}sn$ periyotla örneklendiğinde $x(n) = (-1)^n$ ayrık zaman işareti veren üç ayrı anolog işaret bulunuz.

18. $n = 0$ da $x(n) = 1$ ve $n = 1$ de $x(n) = -1$ olduğundan dolayı bu fonksiyon kosinüs biçiminde olmalıdır. Çünkü cos 0=1 dir. O halde $x(t) = \cos\omega_0 t = \cos 2\pi f n T_s = \cos 2\pi f n \frac{1}{3}$ olur. $n = 1$ de $\pi f n \frac{2}{3} = \pi$ olması için $f = \frac{3}{2}$ olmalıdır. O halde $x(t) = \cos 3\pi t$ veya $x(t) = \cos 9\pi t$ veya $x(t) = \cos 15\pi t$ olmalıdır.

19. Şekilde verilen $x(t)$ işaretinin Fourier dönüşümünü bulunuz.



19.

$$X(\omega) = \int_{-\infty}^{\infty} x(t).e^{-j\omega t}dt = \int_{-\infty}^{\infty}\left(\frac{1}{2}\delta(t+1) - \frac{1}{2}\delta(t-1)\right).e^{-j\omega t}dt = \frac{1}{2}\int_{-\infty}^{\infty}\delta(t+1)e^{-j\omega t}dt - \frac{1}{2}\int_{-\infty}^{\infty}\delta(t-1).e^{-j\omega t}dt$$

$$= \frac{1}{2}e^{j\omega} - \frac{1}{2}e^{-j\omega} = \frac{e^{j\omega} - e^{-j\omega}}{2} = j\frac{e^{j\omega} - e^{-j\omega}}{2j} = j\sin(\omega)$$

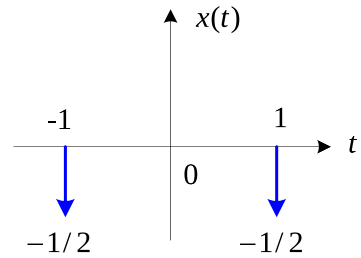
20. 19 uncu soruda verilen $x(t)$ işareti kullanılarak elde edilen $x_1(t) = \sum_{k=-\infty}^{\infty} x(t-4k)$ periyodik işaretin temel frekansını ve Fourier seri açılımını bulunuz.

20. $T = 4sn$ dir. Bu durumda temel frekans $\omega_0 = \frac{2\pi}{T} = \frac{\pi}{2}$ olarak elde edilir.

$$a_k = \frac{1}{4}\int_{-2}^{2}\left(\frac{1}{2}\delta(t+1) - \frac{1}{2}\delta(t-1)\right).e^{-jk\omega_0 t}dt = \frac{1}{4}\left(\frac{1}{2}e^{jk\omega_0} - \frac{1}{2}e^{-jk\omega_0}\right)$$

$$= \frac{1}{4}\left(\frac{e^{jk\omega_0} - e^{-jk\omega_0}}{2}\right) = \frac{1}{4}j\left(\frac{e^{jk\omega_0} - e^{-jk\omega_0}}{2j}\right) = \frac{1}{4}j\sin(k\omega_0) = \frac{1}{4}j\sin(k\frac{\pi}{2})$$

21. Şekilde verilen $x(t)$ işaretinin Fourier dönüşümünü bulunuz.



21.

$$X(\omega) = \int_{-\infty}^{\infty} x(t).e^{-j\omega t}dt = \int_{-\infty}^{\infty}\left(-\frac{1}{2}\delta(t+1) - \frac{1}{2}\delta(t-1)\right).e^{-j\omega t}dt = -\frac{1}{2}\int_{-\infty}^{\infty}\delta(t+1)e^{-j\omega t}dt - \frac{1}{2}\int_{-\infty}^{\infty}\delta(t-1).e^{-j\omega t}dt$$

$$= -\frac{1}{2}e^{j\omega} - \frac{1}{2}e^{-j\omega} = \frac{-(e^{j\omega} + e^{-j\omega})}{2} = -\cos(\omega)$$

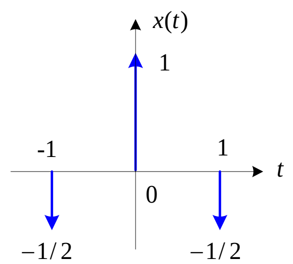
22. 21 inci soruda verilen $x(t)$ işareti kullanılarak elde edilen $x_1(t) = \sum_{k=-\infty}^{\infty} x(t-3k)$ periyodik işaretinin temel frekansını ve Fourier seri açılımını bulunuz.

22. $T = 3sn$ dir. Bu durumda temel frekans $\omega_0 = \frac{2\pi}{3}$ olarak elde edilir.

$$a_k = \frac{1}{3}\int_{-2}^{2}\left(-\frac{1}{2}\delta(t+1) - \frac{1}{2}\delta(t-1)\right).e^{-jk\omega_0 t}dt = -\frac{1}{3}\left(\frac{1}{2}e^{jk\omega_0} + \frac{1}{2}e^{-jk\omega_0}\right)$$

$$= -\frac{1}{3}\left(\frac{e^{jk\omega_0} + e^{-jk\omega_0}}{2}\right) = -\frac{1}{3}\cos(k\omega_0) = -\frac{1}{3}\cos(k\frac{2\pi}{3})$$

23. Şekilde verilen $x(t)$ işaretinin Fourier dönüşümünü bulunuz.



23.

$$X(\omega)=\int_{-\infty}^{\infty}x(t).e^{-j\omega t}dt=\int_{-\infty}^{\infty}\left(\delta(t)-\frac{1}{2}\delta(t+1)-\frac{1}{2}\delta(t-1)\right).e^{-j\omega t}dt$$

$$=\int_{-\infty}^{\infty}\delta(t).e^{-j\omega t}dt-\frac{1}{2}\int_{-\infty}^{\infty}\delta(t+1).e^{-j\omega t}dt-\frac{1}{2}\int_{-\infty}^{\infty}\delta(t-1).e^{-j\omega t}dt$$

$$=1-\frac{1}{2}e^{j\omega}-\frac{1}{2}e^{-j\omega}=1-\frac{(e^{j\omega}+e^{-j\omega})}{2}=1-\cos(\omega)$$

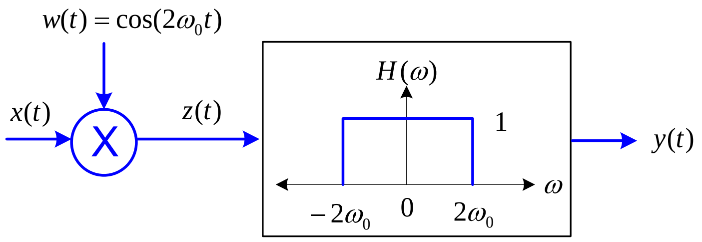
24. 23 üncü soruda verilen $x(t)$ işareti kullanılarak elde edilen $x_1(t)=\sum_{k=-\infty}^{\infty}x(t-kT_1)$ periyodik işaretin temel frekansını ve Fourier seri açılımını bulunuz.

24. Periyot T_1 olduğundan temel frekans $\omega_0=\dfrac{2\pi}{T_1}$ olarak elde edilir.

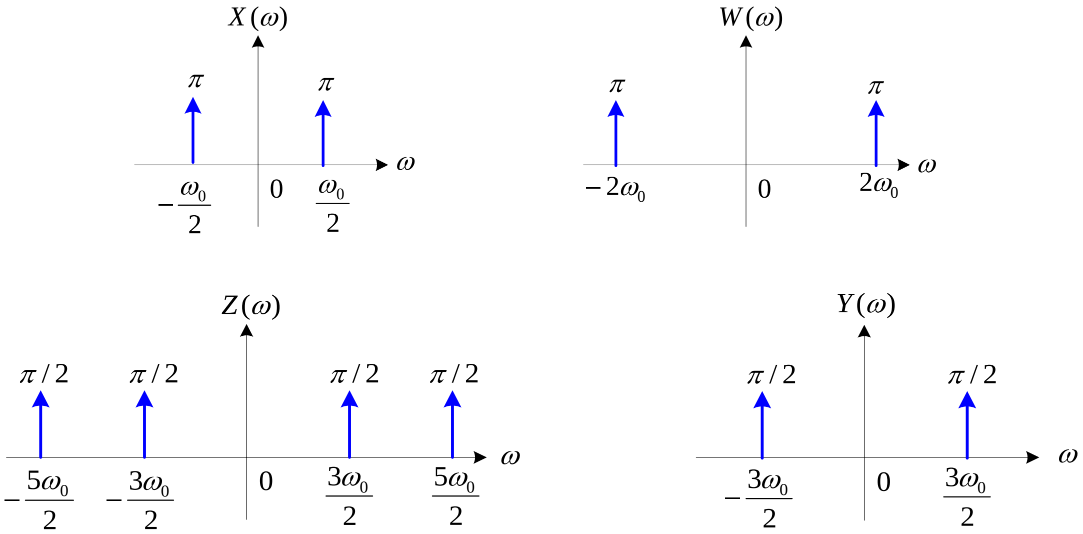
$$a_k=\frac{1}{T_1}\int_{-2}^{2}\left(\delta(t)-\frac{1}{2}\delta(t+1)-\frac{1}{2}\delta(t-1)\right).e^{-jk\omega_0 t}dt=\frac{1}{T_1}\left(1-\frac{1}{2}e^{jk\omega_0}+\frac{1}{2}e^{-jk\omega_0}\right)$$

$$=\frac{1}{T_1}\left(1-\frac{e^{jk\omega_0}+e^{-jk\omega_0}}{2}\right)=\frac{1}{T_1}\left(1-\cos(k\omega_0)\right)=\frac{1}{T_1}\left(1-\cos(k\frac{2\pi}{T_1})\right)$$

25. $x(t)=\cos\left(\dfrac{\omega_0}{2}t\right)$ olarak verilen işaret aşağıda verilen sisteme uygulandığında çıkışta elde edilecek $y(t)$ işaretini bulunuz.



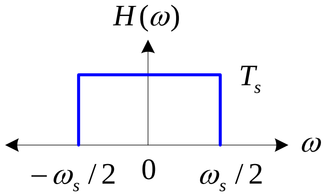
25.



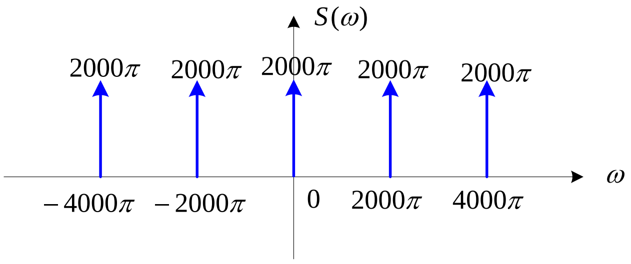
$$y(t)=\frac{1}{2}\cos\left(\frac{3\omega_0}{2}t\right)$$

26. Aşağıdaki $x(t)$ işaretleri periyodu $T_s = 1ms$ olan darbe dizisi ile örneklenmektedir. Örneklemeden sonra elde edilen $x_s(t)$ işaretleri frekans spektrumu aşağıda verilen filtreden geçirilerek $y(t)$ işaretleri elde edilmiştir. Aşağıdaki $x(t)$ işaretlerine karşılık gelen $y(t)$ işaretlerini bulunuz.

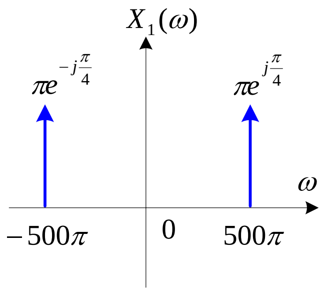
a. $x_1(t) = \cos(500\pi t + \frac{\pi}{4})$ **b.** $x_2(t) = \cos(1500\pi t + \frac{\pi}{2})$ **c.** $x_3(t) = \cos(1000\pi t + \frac{\pi}{2})$

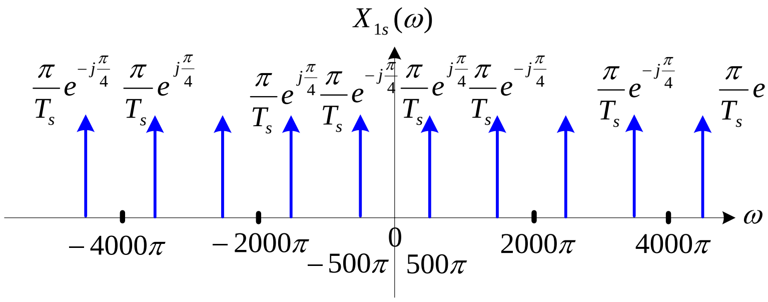


26a. $\omega_s = \frac{2\pi}{0.001} = 2000\pi$ açısal frekansında örnekleme yaptığımızda $S(\omega)$ işareti aşağıdaki gibi verilir.

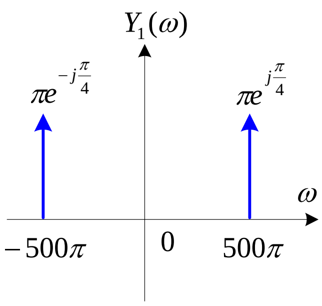


$x_1(t)$ işaretinin frekans spektrumu $X_1(\omega)$ ve örneklenmiş işaretin frekans spektrumu $X_{1s}(\omega)$ aşağıdaki gibi verilir.



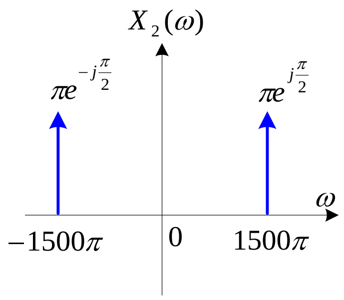


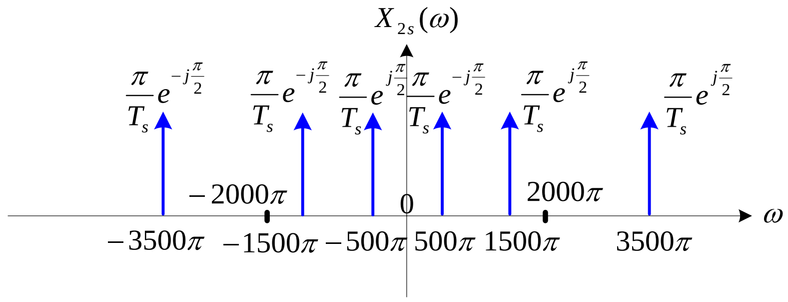
$H(\omega)$'nın band genişliği $\frac{\omega_s}{2} = 1000\pi$ ve genliği $T_s = 0.001$ dir. O halde;

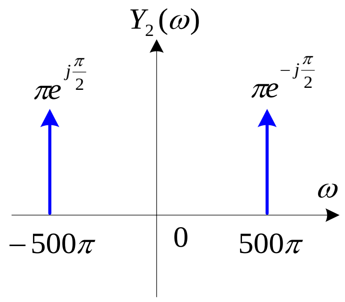


$$Y_1(\omega) = \left[\underbrace{X_{s1}(\omega)}_{\frac{\pi}{T_s}.e^{j\frac{\pi}{4}}} \underbrace{H(\omega)}_{T_s} \right] = \pi e^{j\frac{\pi}{4}} \qquad y_1(t) = \cos(500\pi t + \frac{\pi}{4})$$

26b. $x_2(t)$ işaretinin frekans spektrumu $X_2(\omega)$ ve örneklenmiş işaretin frekans spektrumu $X_{2s}(\omega)$ aşağıdaki gibi verilir.

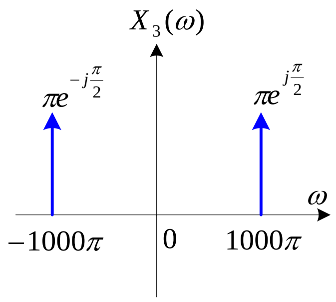


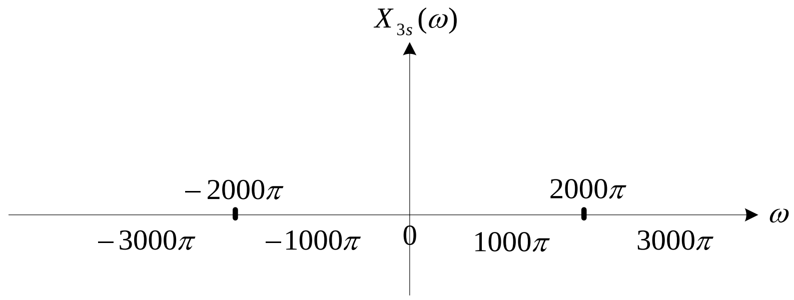




$$Y_2(\omega) = \left[\underbrace{X_{2s}(\omega)}_{\frac{\pi}{T_s}.e^{-j\frac{\pi}{2}}}\underbrace{H(\omega)}_{T_s}\right] = \pi e^{-j\frac{\pi}{2}} \qquad y_2(t) = \cos(500\pi t - \frac{\pi}{2})$$

26c. $x_3(t)$ işaretinin frekans spektrumu $X_3(\omega)$ ve örneklenmiş işaretin frekans spektrumu $X_{3s}(\omega)$ aşağıdaki gibi verilir.





$$Y_3(\omega) = \left[\underbrace{X_{3s}(\omega)}_{0}\underbrace{H(\omega)}_{T_s}\right] = 0 \qquad y_3(t) = 0$$