No:/Ad Soyad:

31.03.2017

SAÜ 2016-2017 BAHAR DÖNEMİ MÜH. FAK. GIDA MÜH. BÖLÜMÜ LİNEER CEBİR DERSİ ARASINAV SORULARI

Sınav Süresi: 80 dakika

1) a) $A=\begin{bmatrix} x & 3 \\ a+y & a+b \end{bmatrix}$, $B=\begin{bmatrix} -6 & y \\ 2 & -4 \end{bmatrix}$ ve $A+B=3I$ ise, A matrisinin elemanları toplamı kaçtır?(10p)

b) $A=\begin{bmatrix} -m & \sqrt{3} \\ -\sqrt{3} & m \end{bmatrix}$ matrisi involutif (yani $A^2=I$) ise, $m=?$(15p)

2) $A=\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 2 & 2 & -3 \end{bmatrix}$ matrisinin tersini hesaplayınız.(25p)

3) $y-x=6$, $z-y=3$ olmak üzere $\det\begin{bmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{bmatrix}=?$(Determinantı Sarrus kuralı kullanmadan hesaplayınız.)(25p)

4) $\begin{bmatrix} 3 & 2 & -1 & 4 \\ 2 & -1 & 3 & -1 \\ 1 & -5 & 7 & -6 \\ 3 & 1 & -1 & 4 \end{bmatrix}$ matrisinin rankını hesaplayınız.(25p)

31.03.2017
Cuma

SAÜ GIDA MÜH. LİNEER CEBİR DERSİ ARASINAV SORULARI

<u>Soru 1</u> a) $A=\begin{bmatrix} x & 3 \\ a+y & a+b \end{bmatrix}$, $B=\begin{bmatrix} -6 & y \\ 2 & -4 \end{bmatrix}$ ve $A+B=3I$ ise, A matrisinin elemanları toplamı kaçtır?

<u>Çözüm</u>. $A+B=\begin{bmatrix} x-6 & 3+y \\ a+y+2 & a+b-4 \end{bmatrix} = \underbrace{3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}} = 3I$ ise, $\begin{cases} x-6=3 \\ 3+y=0 \\ a+y+2=0 \\ a+b-4=3 \end{cases}$ olmalıdır.

Buradan $x=9$, $y=-3$, $a=1$ ve $b=6$ bulunur. Böylece A matrisinin elemanları toplamı:

$$x+3+a+y+a+b = 9+3+1+(-3)+1+6 = 17$$

dir.

b) $A=\begin{bmatrix} -a & \sqrt{3} \\ -\sqrt{3} & a \end{bmatrix}$ matrisi involutif (yani $A^2=I$) ise, $a=$?

<u>Çözüm</u>. $A^2=I$ ise, $\begin{bmatrix} -a & \sqrt{3} \\ -\sqrt{3} & a \end{bmatrix}\begin{bmatrix} -a & \sqrt{3} \\ -\sqrt{3} & a \end{bmatrix} = \begin{bmatrix} a^2-3 & 0 \\ 0 & a^2-3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ yani

$a^2-3=1$ olmalıdır. Buradan $a^2=4$ yani $a=\mp 2$ bulunur.

<u>Soru 2</u>. $A=\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 2 & 2 & -3 \end{bmatrix}$ matrisinin tersini bulunuz

<u>Çözüm</u>. $\det\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 2 & 2 & -3 \end{bmatrix} = \det\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 2 & -9 \end{bmatrix} = \det\begin{bmatrix} 1 & -1 \\ 2 & -9 \end{bmatrix} = -9+2=-7 \neq 0$

$c_{11}=(-1)^{1+1}\det\begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix}=-1$ $\quad c_{21}=(-1)^{2+1}\det\begin{bmatrix} 0 & 3 \\ 2 & -3 \end{bmatrix}=6$ $\quad c_{31}=(-1)^{3+1}\det\begin{bmatrix} 0 & 3 \\ 1 & -1 \end{bmatrix}=-3$

$c_{12}=(-1)^{1+2}\det\begin{bmatrix} 0 & -1 \\ 2 & -3 \end{bmatrix}=-2$ $\quad c_{22}=(-1)^{2+2}\det\begin{bmatrix} 1 & 3 \\ 2 & -3 \end{bmatrix}=-9$ $\quad c_{32}=(-1)^{3+2}\det\begin{bmatrix} 1 & 3 \\ 0 & -1 \end{bmatrix}=1$

$c_{13}=(-1)^{1+3}\det\begin{bmatrix} 0 & 1 \\ 2 & 2 \end{bmatrix}=-2$ $\quad c_{23}=(-1)^{2+3}\det\begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix}=-2$ $\quad c_{33}=(-1)^{3+3}\det\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=1$

$$\mathrm{Adj}(A)=\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}^T=\begin{bmatrix} -1 & -2 & -2 \\ 6 & -9 & -2 \\ -3 & 1 & 1 \end{bmatrix}^T=\begin{bmatrix} -1 & 6 & -3 \\ -2 & -9 & 1 \\ -2 & -2 & 1 \end{bmatrix}$$

$$A^{-1}=\frac{1}{\det A}\cdot \mathrm{Adj}(A)=-\frac{1}{7}\cdot\begin{bmatrix} -1 & 6 & -3 \\ -2 & -9 & 1 \\ -2 & -2 & 1 \end{bmatrix}$$

Soru 3. $\det\begin{bmatrix} x & 1 & x \\ 2 & 3 & 4 \\ x & 5 & x \end{bmatrix}=16$ olduğuna göre $x=?$

Çözüm. $\det\begin{bmatrix} x & 1 & x \\ 2 & 3 & 4 \\ x & 5 & x \end{bmatrix}=\det\begin{bmatrix} x & 1 & 0 \\ 2 & 3 & 2 \\ x & 5 & 0 \end{bmatrix}=(-1)^{2+3}\cdot 2\cdot\det\begin{bmatrix} x & 1 \\ x & 5 \end{bmatrix}=-2\cdot(5x-x)=-8x$

$-8x=16 \Rightarrow x=4$ bulunur.

Soru 4. $\begin{bmatrix} 3 & 2 & -1 & 4 \\ 2 & -1 & 3 & -1 \\ 1 & -5 & 7 & -6 \\ 3 & 1 & -1 & 4 \end{bmatrix}$ matrisinin rankını hesaplayınız.

Çözüm. $\begin{bmatrix} 3 & 2 & -1 & 4 \\ 2 & -1 & 3 & -1 \\ 1 & -5 & 7 & -6 \\ 3 & 1 & -1 & 4 \end{bmatrix}\sim\begin{bmatrix} 1 & -5 & 7 & -6 \\ 2 & -1 & 3 & -1 \\ 3 & 1 & -1 & 4 \\ 3 & 2 & -1 & 4 \end{bmatrix}\sim\begin{bmatrix} 1 & -5 & 7 & -6 \\ 0 & 9 & -11 & 11 \\ 0 & 16 & -22 & 22 \\ 0 & 17 & -22 & 22 \end{bmatrix}\sim$

$\begin{bmatrix} 1 & -5 & 7 & -6 \\ 0 & 9 & -11 & 11 \\ 0 & 16 & -22 & 22 \\ 0 & 1 & 0 & 0 \end{bmatrix}\sim\begin{bmatrix} 1 & -5 & 7 & -6 \\ 0 & 1 & 0 & 0 \\ 0 & 9 & -11 & 11 \\ 0 & 16 & -22 & 22 \end{bmatrix}\sim\begin{bmatrix} 1 & -5 & 7 & -6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -11 & 11 \\ 0 & 0 & -22 & 22 \end{bmatrix}\sim\begin{bmatrix} 1 & -5 & 7 & -6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

olup, satır - eşelon formda tümü birden sıfır olmayan satırların sayısı 3 tür. Dolayısıyla verilen matrisin rankı 3 tür.

Soru 3. $y-x=6$, $z-y=3$ olmak üzere $\det\begin{bmatrix}1 & x & yz\\ 1 & y & zx\\ 1 & z & xy\end{bmatrix}=?$

Çözüm. $\det\begin{bmatrix}1 & x & yz\\ 1 & y & zx\\ 1 & z & xy\end{bmatrix}=\det\begin{bmatrix}1 & x & yz\\ 0 & y-x & zx-yz\\ 0 & z-y & xy-zx\end{bmatrix}=\det\begin{bmatrix}1 & x & yz\\ 0 & y-x & z(x-y)\\ 0 & z-y & x(y-z)\end{bmatrix}$

20

$=\det\begin{bmatrix}1 & x & yz\\ 0 & 6 & -6z\\ 0 & 3 & -3x\end{bmatrix}=(-1)^{1+1}\cdot 1\cdot\det\begin{bmatrix}6 & -6z\\ 3 & -3x\end{bmatrix}$

$=-18x+18z=18(z-x)=18\cdot 9=162$

$$\begin{aligned} y-x&=6\\ +\ z-y&=3\\ \hline z-x&=9\end{aligned}$$

05