Available online at www.sciencedirect.com
SCIENCE DIRECT®
Pattern Recognition Letters 26 (2005) 1577–1586
Pattern Recognition Letters
www.elsevier.com/locate/patrec

SVD-based digital image watermarking scheme

Chin-Chen Chang [a], Piyu Tsai [b], Chia-Chen Lin [c,*]

[a] Department of Computer Science and Information Engineering, National Chung Cheng University, Chiayi 62107, Taiwan, ROC
[b] Department of Computer Science and Information Engineering, National United University, Miaoli 360, Taiwan, ROC
[c] Department of Computer Science and Information Management, Providence University, Taichung, Hsien 43301, Taiwan, ROC

Received 28 December 2003; received in revised form 23 November 2004
Available online 17 February 2005
Communicated by E. Backer

Abstract

In the past couple of years, several digital watermarking schemes have been proposed and based on DCT, DFT, and DWT transformations. In this paper, a singular value decomposition (SVD)-based watermarking scheme is proposed. SVD transformation preserves both one-way and non-symmetric properties, usually not obtainable in DCT and DFT transformations. In the proposed scheme, both of the *D* and *U* components are explored for embedding the watermark.

Experimental results show that the quality of the watermarked image is good and that there is strong resistance against general image processing. Furthermore, the extracted watermark can still be easily identified after tampering.

Keywords: Image watermarking; Singular value decomposition

1. Introduction

With the advance of editing software and the popularity of the Internet, illegal operations, such as duplication, modification, forgery, and others in digital media, have become easy, fast, and difficult to prevent. These illegal operations not only infringe upon the property rights of the authors of the digital media but also reduce motivation for their creation. Therefore, the protection of the intellectual property rights of digital media has become an urgent matter.

Of all the methods that have been proposed to protect the intellectual property rights of digital images, digital watermarking schemes are the most commonly used. In digital watermarking schemes, some types of digital data, such as logos, labels, or names (called watermarks), representing an author's ownership, are embedded in the image (called the host image). Generally, registration with an authentication center is necessary and

* Corresponding author. Tel.: +886 4 26328001x13410; fax: +886 4 26324045.
E-mail addresses: ccc@cs.ccu.edu.tw (C.-C. Chang), pytsai@nuu.edu.tw (P. Tsai), mhlin3@pu.edu.tw (C.-C. Lin).

0167-8655/$ - see front matter
doi:10.1016/j.patrec.2005.01.004

helps to solve ownership disputes by enabling the owner of the disputed media to be identified. If necessary, the embedded watermark in a host image can be used to verify ownership.

Basically, a set of basic requirements is evaluated for a watermarking scheme to be effective. These requirements can be organized and described as follows:

Undeletable. An embedded watermark is difficult to detect and remove by an unauthorized party. In other words, the watermark is resistant against general image processing and tampering.

Perceptually visible. The difference between the original and watermarked images cannot be distinguished by the human eye. This means that there is not enough distortion of a watermarked image to prevent inspiration by an unauthorized party.

Unambiguous. An embedded watermark extracted from a watermarked image must be clear enough for ownership to be determined. In other words, the extracted watermark cannot be distorted to such an extent that the original watermark cannot be identified.

Several watermarking schemes have been proposed. These schemes can be classified into two categories: spatial domain watermarking schemes and frequency domain watermarking schemes. In related spatial domain watermarking schemes (Celik et al., 2002; Chang et al., 2003; Lu et al., 2000; Van Schyndel et al., 1994), the least significant bit (LSB) schemes (Celik et al., 2002; Van Schyndel et al., 1994) modify the low-order bits of pixels of the host image to embed the watermark. Chang et al. (2003) used the human visual effects to adaptively adjust the bits used for embedding a watermark. The number of bits used for embedding a watermark in their scheme was determined by the visual effect of the pixel values on the host image. In Lu's method (2000), a watermark was embedded in the indices encoded by vector quantization (VQ) in which the original VQ indices were modified.

Recently, image watermarking research has moved toward embedding a watermark in transformed coefficients for robustness (Barni et al., 1998, 2001; Chu, 2003; Iwata and Shiozaki, 2001). In Barni's method (1998), Discrete Cosine Transformation (DCT) was employed in the domain transformation procedure. A watermark was embedded in the predefined medium frequency coefficients in zigzag scanning order after DCT transformation had been performed. Furthermore, the watermark strength was adapted according to human visual perceptibility to ensure the invisibility of the watermark. In Chu's method (2003), each DCT block consists of pixels, which were extracted from original pixels with interleave. The corresponding coefficients between DCT blocks were examined to determine whether they could be used to hide the watermark.

In (Barni et al.'s method, 2001), discrete wavelet transformation (DWT) was first applied to the domain transformation procedure. Then the most detailed sub-band coefficients were used to embed the watermark. The watermark strength was modulated with a mask in order to keep the modification imperceptible. The correlation between the original and the extracted watermarks was computed to identify the image copyright. In (Iwata and Shiozaki's method, 2001), the relationship of corresponding coefficients between sub-bands was explored to embed the watermark.

Most of the domain transformation watermarking schemes work with DCT and DWT. However, singular value decomposition (SVD) is one of the most powerful numeric analysis techniques and is used in various applications (Andrews and Patterson, 1976; Chung et al., 2001; Sun et al., 2002). A few proposed SVD-based watermarking of these D component coefficients obtained by SVD transformation, have been researched. In this study, features of the D and U components in embedding a watermark were explored.

The rest of this paper is organized as follows. In Section 2 SVD transformation and an SVD-based watermarking scheme are briefly described. Next, the proposed watermarking scheme is introduced in Section 3. In Section 4, the experimental results of the proposed scheme are shown. Finally, the conclusions are given in Section 5.

2. A review of related works

In this section, the powerful numerical analysis SVD transformation that is widely applied to digital image applications is first introduced. Next, an

SVD-based watermarking scheme is briefly described.

2.1. The singular value decomposition (SVD)

SVD is one of a number of effective numerical analysis tools used to analyze matrices. In SVD transformation, a matrix can be decomposed into three matrices that are the same size as the original matrix. Given a real $n \times n$ matrix A, this matrix can be transformed into three components, U, D and V, respectively, such that

$$[U \quad D \quad V] = \mathrm{SVD}(A), \quad A' = UDV^{\mathrm{T}}$$

$$= \begin{bmatrix} \boldsymbol{u}_{1,1},\ldots,\boldsymbol{u}_{1,n} \\ \boldsymbol{u}_{2,1},\ldots,\boldsymbol{u}_{2,n} \\ \vdots \\ \boldsymbol{u}_{n,1},\ldots,\boldsymbol{u}_{n,n} \end{bmatrix} \begin{bmatrix} \boldsymbol{\sigma}_{1,1},0,\ldots,0 \\ 0,\boldsymbol{\sigma}_{2,2},\ldots,0 \\ \vdots \\ 0,0,\ldots,\boldsymbol{\sigma}_{n,n} \end{bmatrix} \begin{bmatrix} \boldsymbol{v}_{1,1},\ldots,\boldsymbol{v}_{1,n} \\ \boldsymbol{v}_{2,1},\ldots,\boldsymbol{v}_{2,n} \\ \vdots \\ \boldsymbol{v}_{n,1},\ldots,\boldsymbol{v}_{n,n} \end{bmatrix}^{\mathrm{T}}$$

$$= \sum_{i=1}^{n} \boldsymbol{\sigma}_i \boldsymbol{u}_i \boldsymbol{v}_i^{\mathrm{T}},$$

where the U and V components are $n \times n$ real unitary matrices with small singular values, and the D component is an $n \times n$ diagonal matrix with larger singular value entries which satisfy $\boldsymbol{\sigma}_{1,1} \geqq \boldsymbol{\sigma}_{2,2} \geqq \cdots \geqq \boldsymbol{\sigma}_{r,r} > \boldsymbol{\sigma}_{r+1,r+1} = \cdots = \boldsymbol{\sigma}_{n,n} = 0$. A' is the reconstructed matrix after the inverse SVD transformation. Also, the relationship between A and the three matrices U, D, and V satisfies $A\,\boldsymbol{v}_i = \boldsymbol{\sigma}_i \boldsymbol{\mu}_i$, and $\boldsymbol{\mu}_i^{\mathrm{T}} A = \boldsymbol{\sigma}_i \boldsymbol{v}_i^{\mathrm{T}}$.

Using SVD in digital image processing has some advantages. First, the size of the matrices from SVD transformation is not fixed and can be a square or a rectangle. Second, singular values in a digital image are less affected if general image processing is performed. Third, singular values contain intrinsic algebraic image properties.

2.2. SVD-based watermarking

In 2002, Sun et al. proposed an SVD and quantization-based watermarking scheme (Sun et al., 2002). The D component with a diagonal matrix was explored. In the embedding procedure, the largest coefficients in D component were modified and used to embed a watermark. The modification was determined by the quantization mechanism. After that, the inverse of the SVD transformation was performed to reconstruct the watermarked image. Because the largest coefficients in the D component can resist general image processing, the embedded watermark was not greatly affected. Also, the quality of the watermarked image can be determined by the quantization. Thus, the quality of the watermarked image quality can be maintained.

To extract an embedded watermark, the SVD transformation was employed and the largest coefficients in the D component were examined. After that, the watermark was extracted. The watermark embedding and extracting procedures can be described as follows.

The watermark embedding procedure

Step 1. Partition the host image into blocks.
Step 2. Perform SVD transformation.
Step 3. Extract the largest coefficient $D(1,1)$ from each D component and quantize it by using a predefined quantization coefficient Q. Let $Z = D(1,1) \bmod Q$.
Step 4. For an embedded watermark bit valued of 0, if $Z < 3Q/4$, $D(1,1)$ modify to $D'(1,1) = D(1,1) + Q/4 - Z$. Otherwise, $D'(1,1) = D(1,1) + 5Q/4 - Z$.
Step 5. For an embedded watermark bit valued of 1, if $Z < Q/4$, $D(1,1)$ modify to $D'(1,1) = D(1,1) - Q/4 + Z$. Otherwise, $D'(1,1) = D(1,1) + 3Q/4 - Z$.
Step 6. Perform the inverse of the SVD transformation to reconstruct the watermarked image.

The watermark extracting procedure

Step 1. Partition the watermarked image into blocks.
Step 2. Perform SVD transformation.
Step 3. Extract the largest coefficient $D'(1,1)$ from each D component and quantize it by using the predefined quantization coefficient Q. Let $Z = D'(1,1) \bmod Q$.
Step 4. If $Z < Q/2$, the extracted watermark has a bit value of 0. Otherwise, the extracted watermark has a bit value of 1.

In this scheme, the steady property of the largest D component coefficients resists the image processing was preserved. However, the D component was a diagonal matrix in which only a small number of the coefficients could be used. In addition, the modification of the largest coefficients would cause a greater measure of image degradation.

3. The proposed watermarking scheme

In this section, an SVD-based watermarking scheme, which explores the characteristics of the D and U components, is proposed. In this scheme, both the embedding procedure and extracting procedure are included. The overview of the proposed watermarking scheme is shown in Fig. 1.

3.1. The watermark embedding procedure

To utilize the characteristics of the SVD domain for embedding a watermark, the coefficients of the D and U components were explored. In our observation, two important features of the D and U components were found. In the first feature, the number of non-zero coefficients in the D component could be used to determine the complexity of a block (matrix). Generally, the greater number of non-zero coefficients would indicate greater complexity. For a block-based watermarking scheme, a more complex block was favored for embedding a watermark with perceptibility. In the second feature, the relationship between the coefficients in the first column of the U component could be preserved when general image processing was performed. Both features were supporting the idea to develop a robust SVD-based watermarking scheme.

In the proposed watermarking scheme, the host image was a gray-level image. The watermark W was a binary image consisting of $w \times h$ bits, where $W = (w_1, w_2, \ldots, w_{w \times h})$ and $w_i \in (0, 1)$. The host image was first partitioned into blocks with $n \times n$ pixels. And then the blocks were transformed by SVD. The number of non-zero coefficients in the

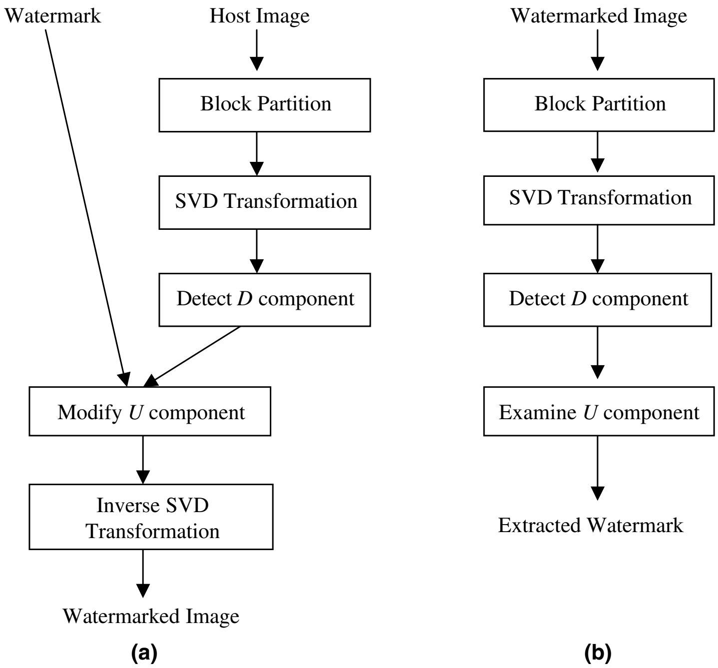


Fig. 1. An overview of the proposed watermarking scheme: (a) the watermark embedding procedure and (b) the watermark extracting procedure.

D component of each block was calculated to determine the complexity of this block. A set of greater complexity blocks was selected according to the pseudo random number generator (PRNG) and the feature of the D component. Using the PRNG increases the watermarking security. Applying the feature of the D component prevents the smooth blocks from being selected and benefits the perceptibility of the watermarked image.

On each selected block, the relationship between the first column coefficients $(\boldsymbol{\mu}_{1,1}, \boldsymbol{\mu}_{2,1}, \ldots, \boldsymbol{\mu}_{n,1})$ in the U component was examined. This relationship could be taken as the magnitude difference between the neighboring coefficients. The magnitude difference could either be a positive or non-positive value. When the positive difference was computed, a positive relationship would be assigned. Otherwise, a negative relationship would be assigned. The relationship could be preserved when general image processing was performed. In other words, when one coefficient had a larger magnitude than the other, the positive relationship was not easily affected by image processing.

An example shown in Table 1 illustrates the relationship between the U component coefficients. Table 1(a) and (b) show the original block and the JPEG compressed block, respectively. Both the SVD transformed U components of Table 1(a) and (b) are shown in Table 1(c) and (d), respectively. From Table 1(c) and (d), it can be observed that the positive relationships (i.e., $|-0.5008| - |-0.5000|$ and $|-0.5004| - |-0.4996|$) between the coordinates $(1,1)$ and $(1,2)$ were still preserved as in Table 1(c) and (d) even though the compression processing was performed.

According to the features of the U component, it would seem that if a positive relationship is found, a bit value of 1 would be hidden. Otherwise, a bit value of 0 would be embedded. From that, the coefficients of the U component might be modified for embedding a watermark. On the coefficient modification, two scenarios must be discussed. First, if the magnitude difference matches the embedding watermark (e.g. positive relationship matching a bit value of 1 or negative relationship matching a bit value of 0), the coefficients are retained. Second, if the magnitude difference does not match the embedding watermark, the coefficients must be modified. However, the modification of the U component coefficients may alter the original pixel values and degrade the quality of the watermarked image. The larger the modification of the U component, the more the distortion of image quality and the stronger the robustness. On the other hand, the smaller modification implies that a better image quality and a weaker resistance have been achieved. There is, in other words, a tradeoff between robustness and quality.

Table 1
The relationship of SVD transformed U component coefficients between the original block and the corresponding JPEG compressed blocks

(a) The original block			
158	156	158	159
157	156	156	156
157	158	156	153
155	154	153	154
(b) The JPEG compressed block of (a)			
155	155	155	154
155	155	154	154
155	154	154	154
154	154	154	154
(c) The U component of (a)			
−0.5056	0.6127	−0.4989	0.3466
−0.5008	0.0840	0.0122	−0.8614
−0.5000	−0.7818	−0.3076	0.2101
−0.4936	0.0792	0.8102	0.3061
(d) The U component of (b)			
−0.5012	0.6532	0.2678	−0.5004
−0.5004	−0.2708	0.6545	0.4980
−0.4996	−0.6536	−0.2729	−0.4988
−0.4988	0.2699	−0.6523	0.5028

To retain the image quality and provide a stronger robustness of a watermarking scheme, the coefficient modification is further considered. For each selected greater complexity block, the magnitude difference between neighboring coefficients had to match both the embedding watermark and a predefined threshold in which to increase the watermarking robustness. In other words, when the magnitude difference matched the watermark but was smaller than the predefined magnitude difference threshold, both coefficients had to be further modified. Both coefficients modification not only reduces the image perceptibility but also enhance the robustness to resist attacks. In addition, if the second scenario described in the above accounted, the magnitude difference between two modified coefficients must greater than or equal to the predefined magnitude difference threshold. It means that the gap between two modified coefficients must larger enough to against attacks.

The example shown in Table 2 illustrates the coefficient modification. Assume the magnitude difference threshold is set as 0.002, then the coefficients of coordinates (1,2) and (1,3) shown in Table 1(c) must be modified to satisfy the threshold requirement. If the magnitude difference matches the watermark, the coefficients of coordinates (1,2) and (1,3) must be modified as $-0.5014\,(-||-0.5008| + (0.0020 - 0.0008)/2|)$ and $-0.4994(-||-0.5000| - (0.0020 - 0.0008)/2|)$, respectively. Otherwise, if the magnitude difference does not match the watermark, the coefficients of coordinate (1,2) and (1,3) must be modified to $-0.4994(-||-0.5008| - (0.0020 + 0.0008)/2|)$ and $-0.5014(-||-0.5000| + (0.0020 + 0.0008)/2|)$, respectively. The modified coefficients of the U component in above two cases (one is called "matched case" and the other is called "unmatched case") are shown in Table 2(a) and (b), respectively. The magnitude difference threshold determines both coefficients modification. It is useful to reduce the image degradation and perceptibility caused by only one coefficient modification.

The reconstruct coefficients from the modified U components shown in Table 2(a) and (b) are displayed in Table 2(c) and (d), respectively. In Table 2(c) and (d), only the coefficients in the second and third rows are modified. These modified coefficients are caused by the change of U component coefficients in coordinates (1,2) and (1,3) in the matched and unmatched cases. These modifications indicate that only partial coefficients in a reconstructed block are altered. Furthermore, the modification of the reconstructed coefficients is less than one. These results also imply that the U component's modification only caused a little influence on the watermarked image. In comparison of Table 1(b) and Table 2(c) and (d), the average difference between modified coefficients and original coefficients in the matched case is 0.18, it is 0.435 in the unmatched case, and it is 2 in the JPEG compression. In other words, the degradation caused by the proposed watermarking is less than which is caused by JPEG compression. Based on above results, it is confirmed that it is a good strategy to modify the coefficients of U component in the original image to embed a watermark.

Table 2
The modified coefficients of the U component in matched and unmatched cases and the corresponded reconstructed coefficients

(a) Modification coefficients in matched case			
−0.5056	0.6127	−0.4989	0.3466
−0.5014	0.0840	0.0122	−0.8614
−0.4994	−0.7818	−0.3076	0.2101
−0.4936	0.0792	0.8102	0.3061
(b) Modification coefficients in unmatched case			
−0.5012	0.6532	0.2678	−0.5004
−0.4994	−0.2708	0.6545	0.4980
−0.5014	−0.6536	−0.2729	−0.4988
−0.4988	0.2699	−0.6523	0.5028
(c) The reconstruct coefficients of (a)			
158.00	156.00	158.00	159.00
157.19	156.17	156.19	156.19
156.82	157.82	155.82	152.82
155.00	154.00	153.00	154.00
(d) The reconstruct coefficients of (b)			
155.00	155.00	155.00	154.00
156.57	155.57	156.57	155.57
157.44	158.44	156.44	153.44
154.00	154.00	154.00	154.00

3.2. The watermark extracting procedure

The watermark extracting procedure is similar to the watermark embedding procedure. The first three steps of the watermark extracting procedure is the same as the watermark embedding procedure except that the host image is replaced with the watermarked image. Once an embedded block is detected according to the feature of the D component and PRNG, the relationship of the U component coefficients is examined. If a positive relationship is detected, the extracted watermark is assigned a bit value of 1. Otherwise, the extracted watermark is given a bit value of 0. These extracted bit values form the extracted watermark. The extracted watermark can be identified by the human eye or compared with the original watermark. The example shown in Table 2 is used to illustrate the proposed extracting procedure. If the magnitude difference between coefficients in the embedded U component is positive (i.e., $0.002 = |-0.5014| - |-0.4994|$ in Table 2(a)), the watermark bit value of 1 is extracted. Otherwise, the negative relationship (i.e., $-0.002 = |-0.4994| \times |-0.5014|$ in Table 2(b)) denotes that the extracted watermark had a bit value of 0.

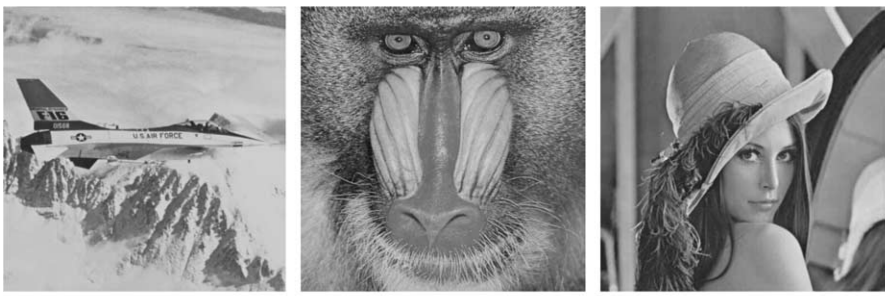
4. Experimental results

Several simulations were performed to verify the validity of the proposed watermarking scheme. A set of gray-level images of 512×512 pixels, "Airplane", "Baboon", "Lena", shown in Fig. 2, was used as host images. Two binary images, "CCU" and "IEEE", each with 32×32 bits, were used as watermarks in the simulations and are shown in Fig. 3.

(a) (b)

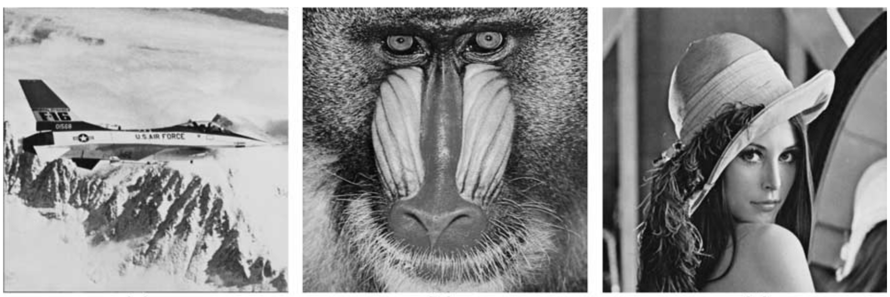
Fig. 3. Two watermark images of 32×32 bits: (a) CCU and (b) IEEE.

To select the greater complexity blocks, the host images were first partitioned into blocks of 8×8 pixels. Each block was transformed into U, D, and V components by SVD. And then, a set of blocks with the same size as the watermark was selected according to the feature of the D component and PRNG. For each embedding block, the relationship between the U component coefficients was examined and the coefficients were modified according to the watermark to be embedded. The watermarked images and the corresponding image qualities were shown in Fig. 4. It was found that the image quality measured by peak signal-to-noise ratio ($PSNR$) among the watermarked



(a) (b) (c)

Fig. 2. Three host images of 512×512 pixels: (a) Airplane; (b) Baboon and (c) Lena.



(a) (b) (c)

Fig. 4. Three watermarked images: (a) Airplane (42.92 dB); (b) Baboon (42.02 dB) and (c) Lena (46.28 dB).

Fig. 5. The extracted watermarks: (a)–(c) CCUs.

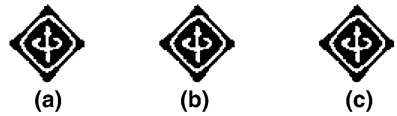


Fig. 7. The extracted watermarks: (a)–(c) IEEEs.

images was greater than 42 dB. This indicated that the proposed watermarking scheme only caused a slight image distortion. Meanwhile, by comparing Figs. 2 and 4, it seemed difficult to distinguish the difference between the host and the watermarked images by the human eye.

To identify the ownership of the watermarked image, the proposed watermark extracting procedure was performed. Fig. 5 shows the extracted watermarks. From Fig. 5, it is clearly seen that the watermarks were fully extracted. In other words, the error rate between the extracted watermark and the original watermark was almost zero. Another example shown in Figs. 6 and 7 also demonstrated the same results as shown in Figs. 4 and 5.

To evaluate the robustness of the proposed watermarking scheme, several attacks, including the JPEG compression with a quality factor (QF) of 70, an addition of 3% Gaussian noise, a cropping of one-fourth of the upper left area of the watermarked images, and sharpening, blurring and tampering processing were performed. The error rate between the extracted watermark and the original watermark was used to evaluate the robustness of the watermarking scheme. Table 3 shows the results of the watermarked images after different attacks. From Table 3, it can be seen that the error rates were almost less than 0.05. This result demonstrates the robustness of the proposed scheme against attacks. Furthermore, the watermarks shown in Fig. 9, extracted from tampered

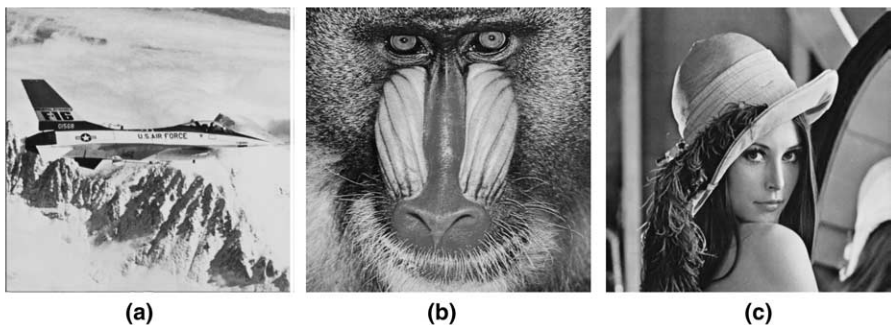


Fig. 6. The watermarked images and the corresponding qualities: (a) Airplane (41.43 dB); (b) Baboon (41.39 dB) and (c) Lena (46.52 dB).

Table 3
Results of the error ratio of the extracted watermark after different attacks by the proposed scheme

Attacks Images	JPEG QF = 70	Noise 3%	Cropping 25%	Sharpening	Blurring	Tampering
Airplane	0.0078	0.0167	0.0381	0.0001	0.0381	0.0273
Baboon	0.0195	0.0332	0.0625	0.0298	0.0576	0.0107
Lena	0.0850	0.0557	0.0479	0.0001	0.0264	0.0425
Average	0.0374	0.0352	0.0495	0.0100	0.0407	0.0268

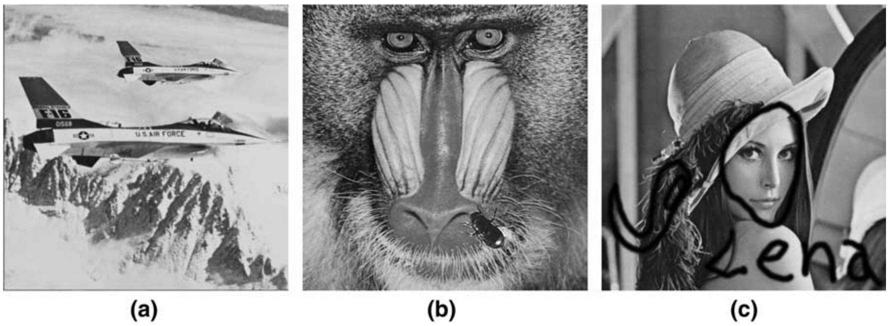


Fig. 8. The tampered watermarked images: (a) Airplane; (b) Baboon and (c) Lena.

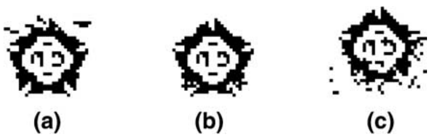


Fig. 9. The extracted watermarks: (a)–(c) CCUs.

images (Fig. 8), were still clearly identifiable even though objects were inserted.

Finally, the simulations were also performed using Sun et al.'s scheme. The error rates of the extracted watermarks after different attacks are shown in Table 4. From Table 4, it can be noted that the higher error rates were caused by image processing noise and sharpening. By comparing the average error rates between Tables 3 and 4, it can be seen that both schemes were robust against JPEG compression. To resist noise addition and sharpening attacks, Sun et al.'s scheme failed whereas the proposed scheme sustained lower error rates. The average results shown in Tables 3 and 4 illustrate that the proposed watermarking performed better than Sun et al.'s scheme.

5. Conclusions

In this paper, a new image watermarking scheme based on SVD was proposed. SVD transformation is quite different from the commonly used DCT, DFT, and DWT transformations, since non-fixed orthogonal bases and one-way non-symmetrical decomposition are employed in SVD. These properties provide the advantages of various sizes of transformation and more security. That is, a good performance of the proposed scheme both in robustness and security can be achieved.

The features of the D component and the relationship between the U component coefficients were explored in the proposed scheme providing a stronger robustness against different attacks and better image quality than Sun et al.'s. The experimental results also demonstrated the effectiveness of the proposed watermarking scheme. Furthermore, extracted watermarks from tampered images were also clearly identified.

Table 4
Results of the error ratio of the extracted watermark after different attacks by Sun et al.'s scheme

Attacks Images	JPEG QF = 70	Noise 3%	Cropping 25%	Sharpening	Blurring	Tampering
Airplane	0.0488	0.4932	0.1895	0.2080	0.0908	0.0127
Baboon	0.0918	0.4580	0.1963	0.4257	0.1992	0.0127
Lena	0.0450	0.4766	0.1923	0.2226	0.0566	0.1005
Average	0.0619	0.4759	0.1930	0.2854	0.1152	0.0419

References

Andrews, H.C., Patterson, C.L., 1976. Singular value decomposition (SVD) image coding. IEEE Trans. Comm. COM-24, 425–432.

Barni, M., Bartolini, F., Cappellini, V., Piva, A., 1998. A DCT-domain system for robust image watermarking. Signal Process. 66, 357–372.

Barni, M., Bartolini, F., Cappellini, V., Piva, A., 2001. Improved wavelet-based watermarking through pixel-wise masking. IEEE Trans. Image Process. 10, 783–791.

Celik, M., Sharma, U., Saber, G.E., Tekalp, A.M., 2002. Hierarchical watermarking for secure image authentication with localization. IEEE Trans. Image Process. 11, 585–595.

Chang, C.C., Hwang, K.F., Hwang, M.S., 2003. A digital watermarking scheme using human visual effects. Informatica 24, 505–511.

Chu, W.C., 2003. DCT-based image watermarking using subsampling. IEEE Trans. Multimedia 5, 34–38.

Chung, K.L., Shen, C.H., Chang, L.C., 2001. A novel SVD and VQ-based image hiding scheme. Pattern Recognition Lett. 22, 1051–1058.

Iwata, M., Shiozaki, A., 2001. Watermarking method for embedding index data into images utilizing features of wavelet transform. IEICE Trans. Fundam. E84-A, 1772–1778.

Lu, Z.M., Pau, J.S., Sun, S.H., 2000. VQ-based digital image watermarking method. IEE Electron. Lett. 36, 1201–1202.

Sun, R., Sun, H., Yao, T., 2002. A SVD and quantization based semi-fragile watermarking technique for image authentication. Proc. Internat. Conf. Signal Process. 2, 1952–1955.

Van Schyndel, R.G., Tirkel, A.Z., Mee, N., Osborne, C.F., 1994. A Digital Watermark. Proceedings of the IEEE International Conference on Image Processing, Austin, USA. 2, 86–90.