MTH 264 Project III

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1 Problems

Use the following methods to approximate definite integrals:

1. Bayesian Numerical Integral
2. Monte Carlo Integration with Trig Distribution

NOTE: $N = 1000$, repeat 5 times.

1.1 Functions

Integrate the following functions using the methods listed above.

i. $\int_0^{\pi/2} \sin x^2 dx$

ii. $\int_0^{\pi/2} \frac{x}{\sin x} dx$

iii. $\int_0^{\pi/2} \frac{e^x - 1}{\sin x} dx$

2 Solutions

2.1 Bayesian Numerical Integral

The Bayesian Numerical Integral method was used to approximate the integrals of the assigned functions. The table below lists the results of 5 runs on each function. The results are relatively consistent with each other, with small variations due to the randomness involved with Bayesian Numerical integration.

$\int_0^{\pi/2} \sin x^2 dx$	$\int_0^{\pi/2} \frac{x}{\sin x} dx$	$\int_0^{\pi/2} \frac{e^x - 1}{\sin x} dx$
0.84045118983470	1.82519731136805	2.92034205525110
0.81300986203938	1.82513807323085	2.87723163838495
0.80562048631475	1.82775260493337	2.85772229012692
0.80003778520629	1.82696575239499	2.87493520874632
0.83704060284947	1.83013519198564	2.82211105757757

2.2 Monte Carlo Integration

Monte Carlo Integration was used to approximate the integrals of the assigned functions. Instead of using a random distribution, we used a trig distribution in the algorithm. The table below lists the results of 5 runs on each function.

$\int_0^{\pi/2} \sin x^2 dx$	$\int_0^{\pi/2} \frac{x}{\sin x} dx$	$\int_0^{\pi/2} \frac{e^x - 1}{\sin x} dx$
0.95969077502208	1.93637690507613	2.99868399929463
0.96755709285013	1.94625637908162	3.19707021807748
0.97699667424379	2.00553322311456	2.86896993316738
0.97070361998135	1.94625637908162	3.14365854378979
0.95969077502208	1.96848519559397	3.18943997889353

3 Code

3.1 Bayesian Numerical Integral

```
% Problem 1
% Bayesian Numerical Integral

clear
clc
format long

% Assigned Functions
f1 = @(x) sin(x.^2);
f2 = @(x) (x./sin(x));
f3 = @(x) ((exp(x)-1)/(sin(x)));


f = f3;
a=0;
b=pi/2;

n = 1000;
dx = (b-a)/ n;

% Lower Sum
x = a + rand(n,1)*(b-a);
lowerSum = sum(f(x)) * dx;

disp(sum(lowerSum))
```

3.2 Monte Carlo Integration with Trig Distribution

```
% Problem 2
% Monte Carlo Integration with Trig Distribution
clear
clc
format long

% Assigned Functions
f1 = @(x) sin(x.^2);
f2 = @(x) (x./sin(x));
f3 = @(x) (exp(x)-1)/sin(x);


f = f1;
a=0;
b=pi/2;

n = 1000;
dx = (b-a)/n;

X = a:dx:b;
fx = f(X);
M = max((fx))+dx;

intervals = transpose((1:n)) *(pi/n);
I = sin(intervals);
rando = rand(n,1);

x = a + I*(b-a);

y = rando * M;
mask = y < f(x);

MCL_Integral = mean(mask)*(M*(b-a))
```