Simulation of a Rotor with Gear Train Driving a Propeller

by

Kevin Moran

Date : November 1st, 2020

University of Southern California
Aerospace and Mechanical Engineering Department
AME 302 : System Dynamics

1 Transfer Function Analysis

The transfer function for the angular velocity of the propeller, ω_p, can be written as

$$\frac{\Omega(s)}{T(s)} = \frac{Ncs + Nk}{s^3\sigma_3 + s^2\sigma_2 + s\sigma_1 + \sigma_0} \qquad \text{(Eq. 1)}$$

where

$$\begin{aligned}
N &= N_1 N_2 \\
\sigma_3 &= N J_R J_p \\
\sigma_2 &= J_p c + J_p N^2 b_1 + J_r N^2 b_2 + J_r N^2 c \\
\sigma_1 &= J_p k + b_2 c + J_r N^2 k + N^2 b_1 b_2 + N^2 b_1 c \\
\sigma_0 &= N^2 b_1 k + b_2 k
\end{aligned}$$

and the variables J_r, J_p, b_1, b_2, c, k, N_1, *and* N_2 are system parameters. During this simulation, the stiffness coefficient of the flexible rod, k, was changed during each iteration, ranging from 5 to 1000 (non-dimensional values).

1.1 Stability, Frequency, and Lower Order Approximation

During each iteration, the poles of the transfer function were determined using the MATLAB Control System Toolbox and it was found that varying the stiffness coefficient had little affect on the real components of the poles. In each case, all of the real components satisfied the stability requirements and demonstrated the requirement for the existence of a steady state (i.e., $Re[p_i] < 0$). Furthermore, comparing the real components of the polls shows that the system can be approximated as a 2nd order system. Regardless of the stiffness coefficient, the real components of the poll were estimated to be -.00173 (multiplicity of 2) and -.0128.

The imaginary component, however, significantly changed during each iteration. The magnitude of the imaginary component of the poles increased along with the stiffness coefficient. Furthermore, since the damped frequency of a system is related to the imaginary component of the poles, it was also anticipated that the frequency of oscillation for the propeller to also increase.

1.2 Impulse Function Steady State

By using the final value theorem, we can determine the steady state for the impulse input as,

$$\omega(t_{ss}) = \lim_{s\to 0} s\Omega(s) = \lim_{s\to 0} \frac{400(Ncs^2 + Nks)}{s^3\sigma_3 + s^2\sigma_2 + s\sigma_1 + \sigma_0} = 0 \qquad \text{(Eq. 2)}$$

which also supports the notion that a damped system starting with a finite amount of energy eventually returns to rest.

1.3 Step Function Steady State

Once again, applying the final value theorem to the system with a step function input yields

$$\omega(t_{ss}) = \lim_{s\to 0} s\Omega(s) = \lim_{s\to 0} \frac{400(Ncs + Nk)}{s^3\sigma_3 + s^2\sigma_2 + s\sigma_1 + \sigma_0} = \frac{400N}{N^2 b_1 + b_2} \approx 189 \qquad \text{(Eq. 3)}$$

After applying the limit, it can be shown that the parameter k cancels out in the numerator and denominator.

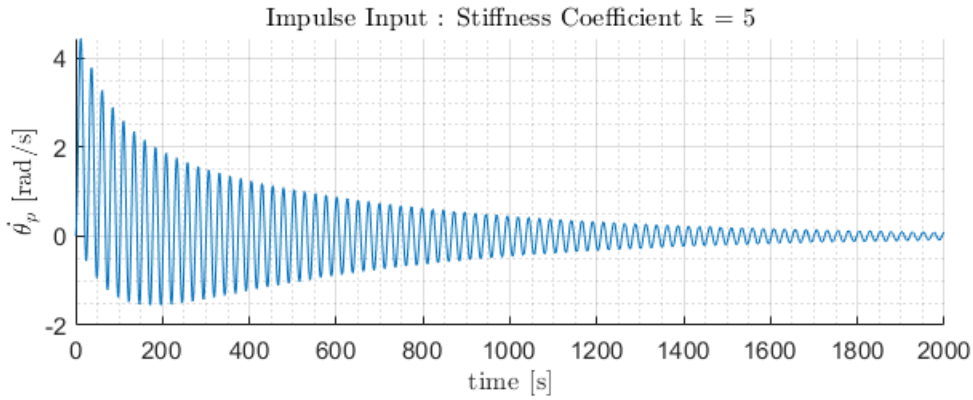
2 Simulation Results

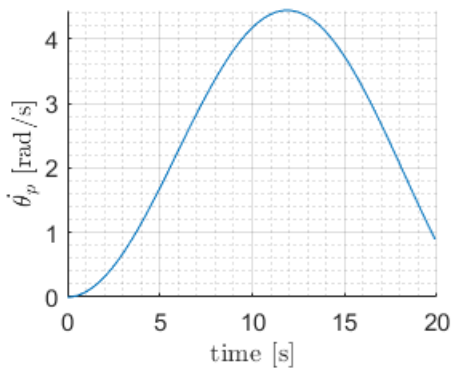
The following pages contain graphs that represent the results from the simulation. Graphs are primarily grouped by stiffness coefficients and illustrate the results through the entire time domain. Additionally, two subplots illustrate time domains of interest.

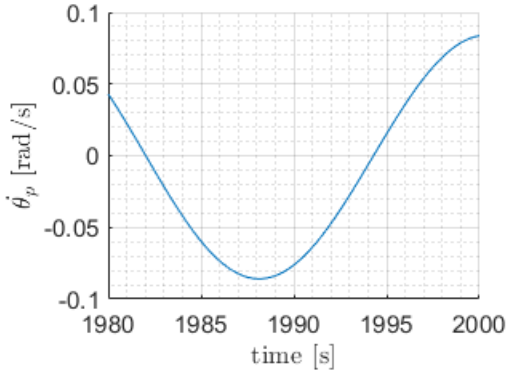
In each case, the impulse and step responses of the system agreed with the results of the final value theorem. Furthermore, as the stiffness coefficient increases, the frequency of the oscillations also increase and support the anticipated results from the transfer function poles.

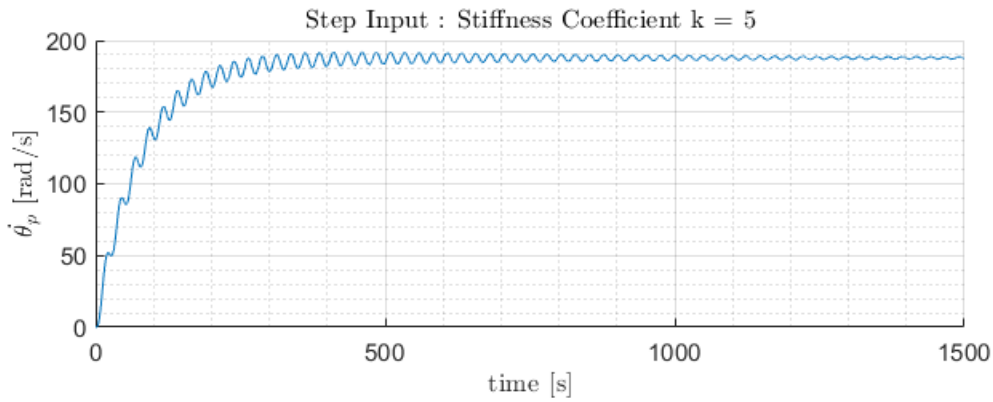
k	f_{impl} [Hz]	f_{step} [Hz]	$Imp\ A_{t0:t20}$	$Imp\ A_{t1980:t2000}$	$Step\ A_{t460:t480}$	$Step\ A_{t1480:t1500}$
5	0407	.0408	-	-	-	-
15	.0706	.0707	7.05	.171	4.95	.892
50	.129	.129	4.75	.171	2.71	.49
100	.182	.182	4.76	.171	1.92	.346
500	.407	.408	4.78	.171	.85	.154
1000	.576	.576	4.76	.170	.605	.109

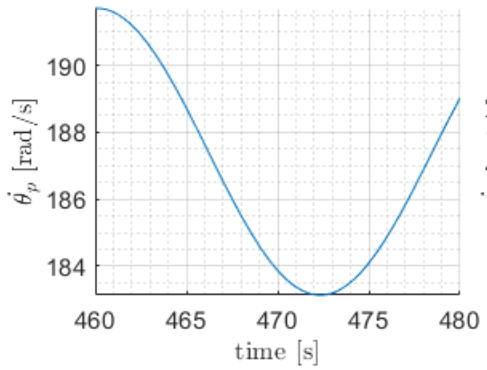
Table 1: Oscillation frequency for impulse and step responses. Frequency was determined by taking the time difference between local maximas and taking the inverse of the average time difference. The subscripts denotes the time intervals and any entry with a dash indicated that there wasn't enough data to make accurate calculations.

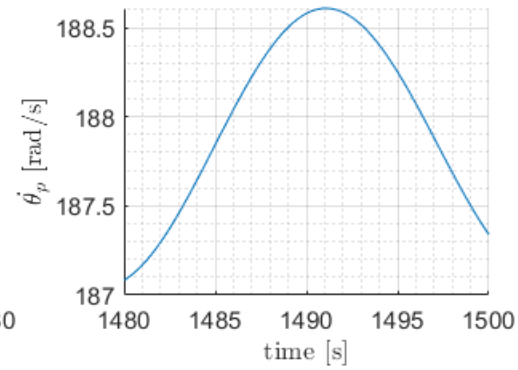

Impulse Input : Stiffness Coefficient k = 5
$\dot{\theta}_p$ [rad/s]
time [s]

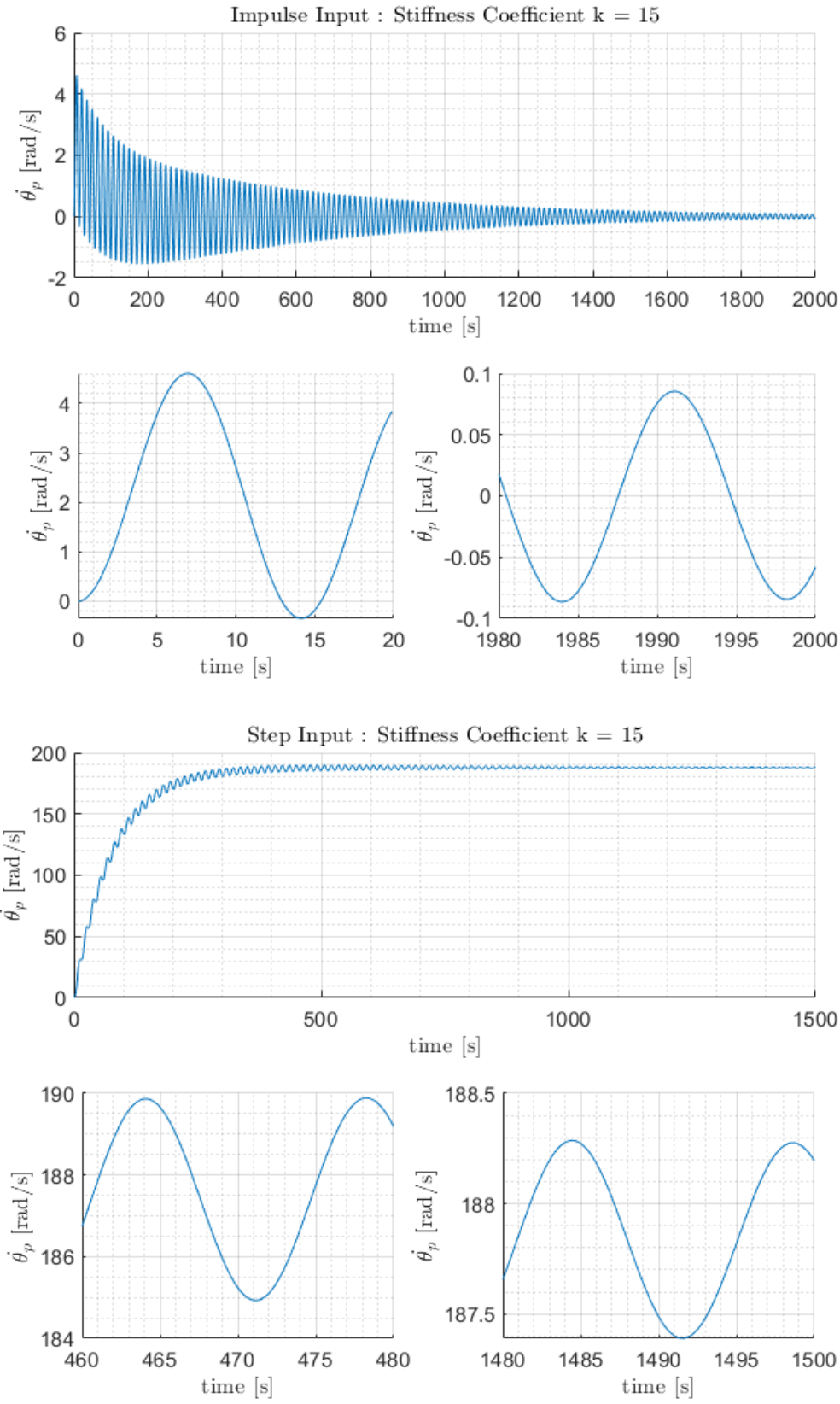

$\dot{\theta}_p$ [rad/s]
time [s]

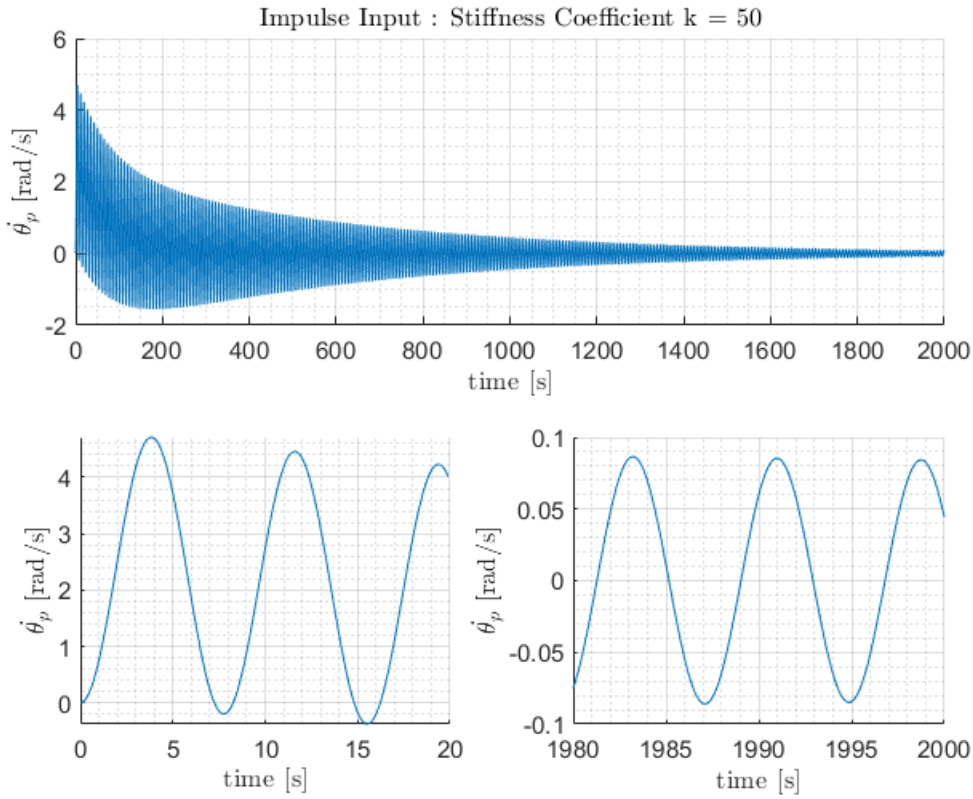

$\dot{\theta}_p$ [rad/s]
time [s]

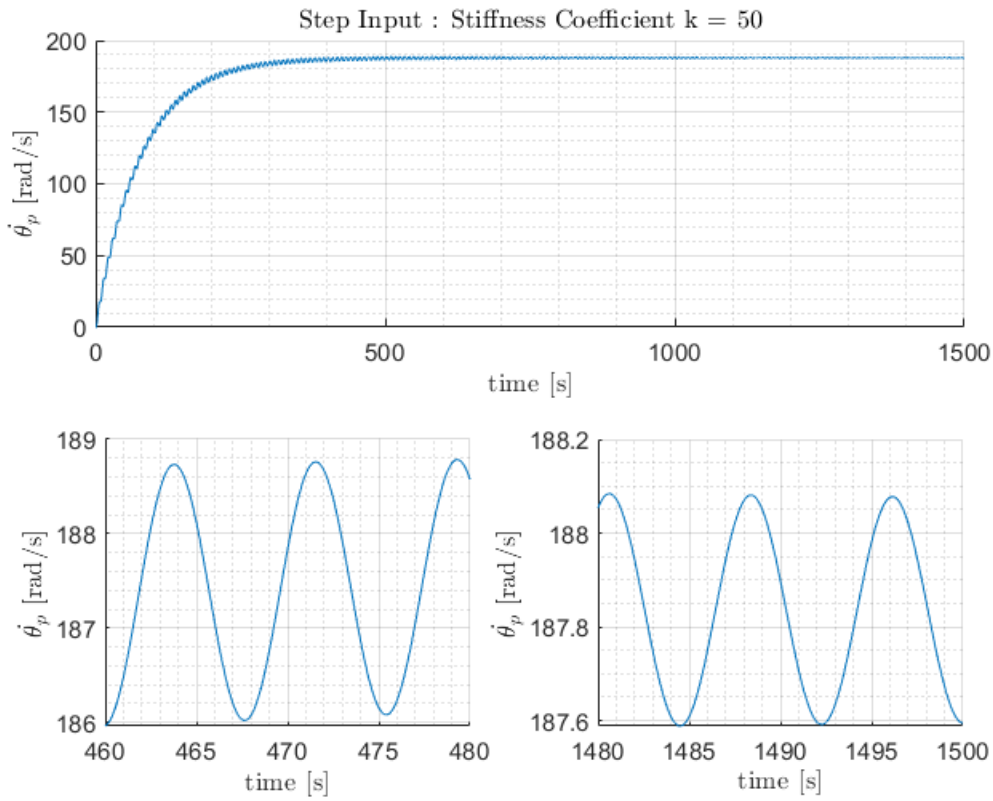

Step Input : Stiffness Coefficient k = 5
$\dot{\theta}_p$ [rad/s]
time [s]

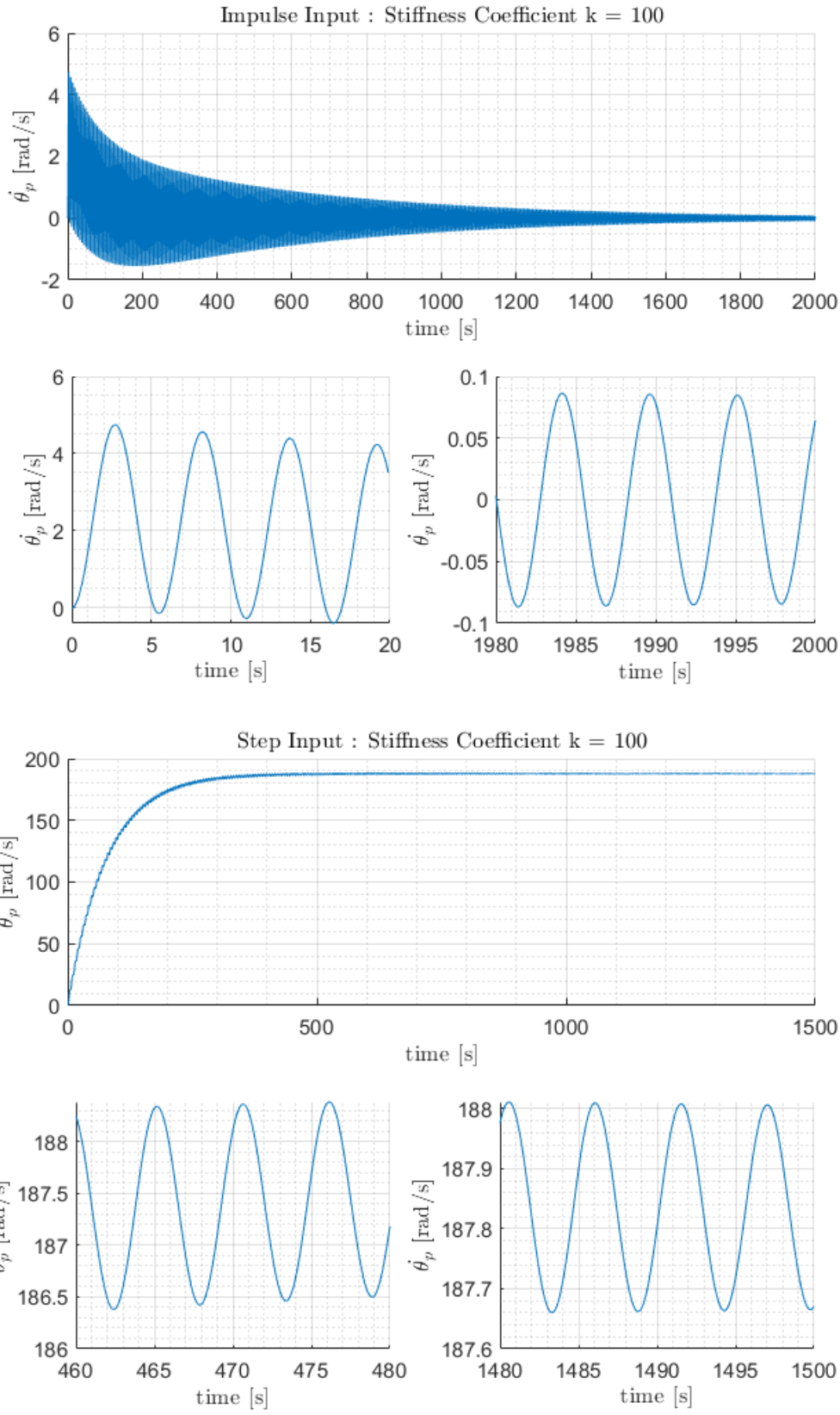

$\dot{\theta}_p$ [rad/s]
time [s]

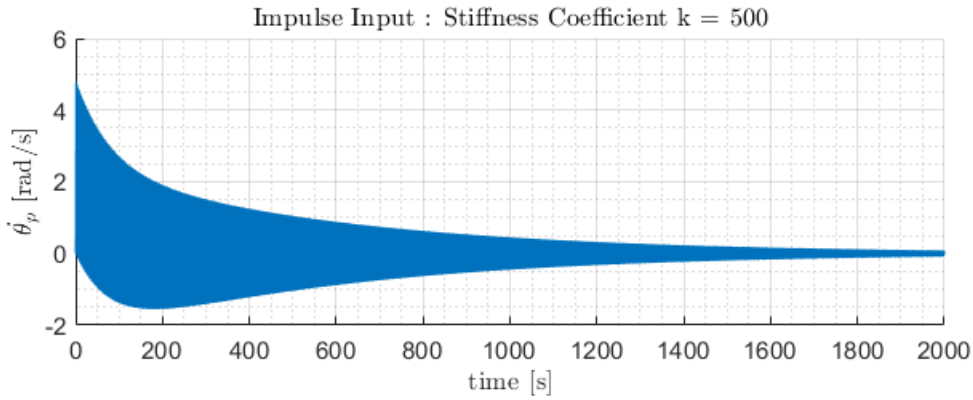

$\dot{\theta}_p$ [rad/s]
time [s]

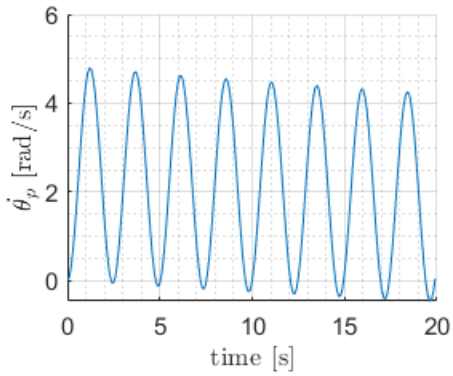

Impulse Input : Stiffness Coefficient k = 15
$\dot{\theta}_p$ [rad/s]
time [s]
Step Input : Stiffness Coefficient k = 15

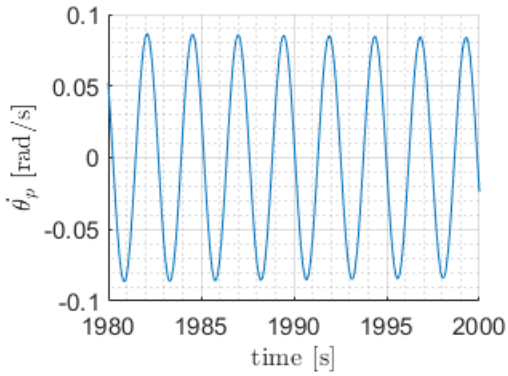

Impulse Input : Stiffness Coefficient k = 50
$\dot{\theta}_p$ [rad/s]
time [s]

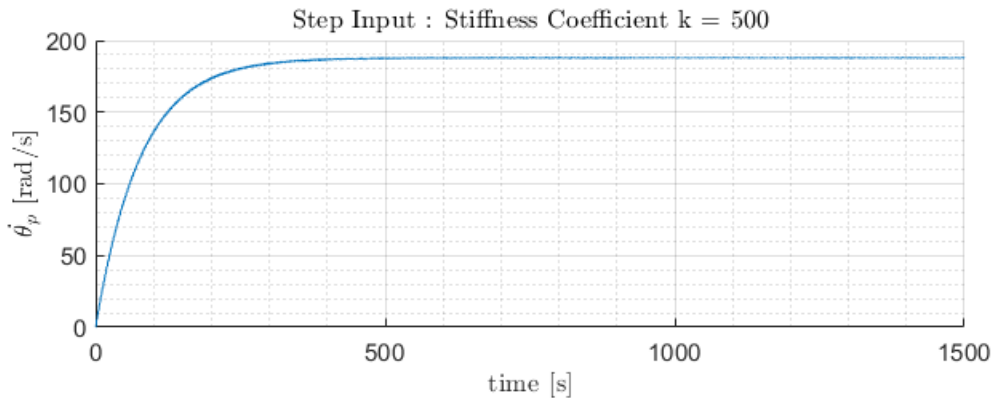

Step Input : Stiffness Coefficient k = 50
$\dot{\theta}_p$ [rad/s]
time [s]

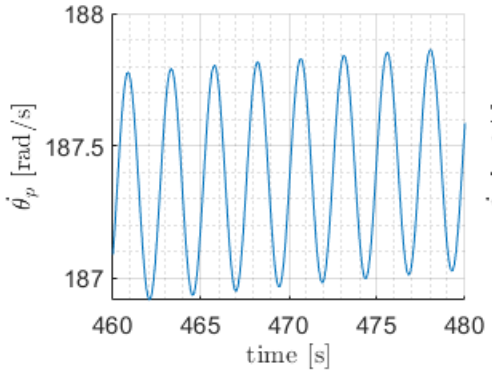

Impulse Input : Stiffness Coefficient k = 100
$\dot{\theta}_p$ [rad/s]
time [s]
Step Input : Stiffness Coefficient k = 100

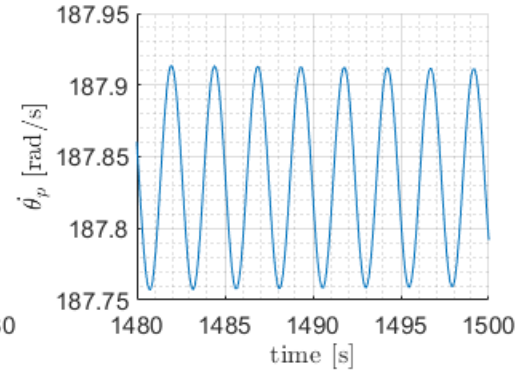

Impulse Input : Stiffness Coefficient k = 500
$\dot{\theta}_p$ [rad/s]
time [s]

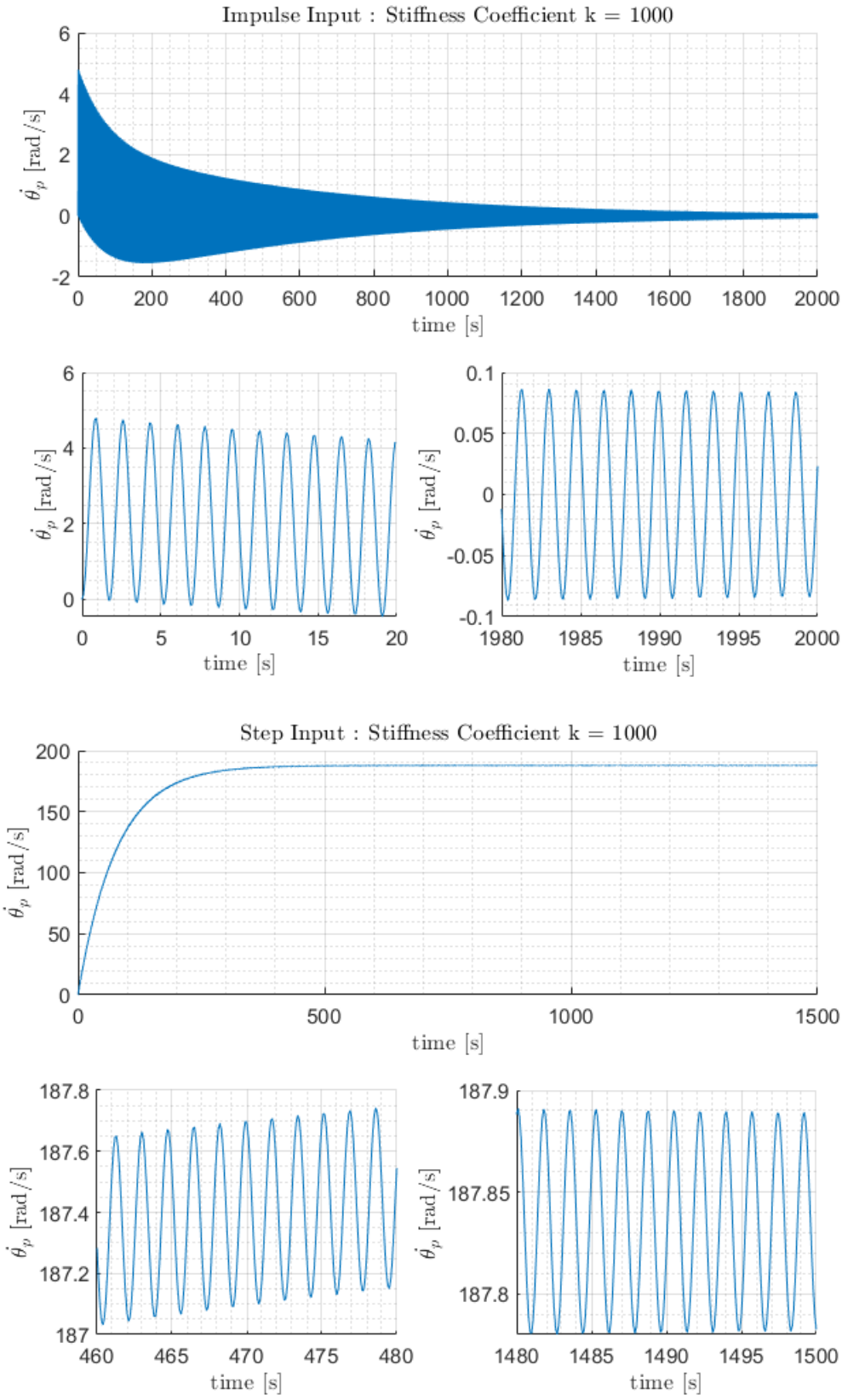

$\dot{\theta}_p$ [rad/s]
time [s]


$\dot{\theta}_p$ [rad/s]
time [s]


Step Input : Stiffness Coefficient k = 500
$\dot{\theta}_p$ [rad/s]
time [s]


$\dot{\theta}_p$ [rad/s]
time [s]


$\dot{\theta}_p$ [rad/s]
time [s]


Impulse Input : Stiffness Coefficient k = 1000
$\dot{\theta}_p$ [rad/s]
time [s]
Step Input : Stiffness Coefficient k = 1000