

A New Functional Link Neural Network Model with Extended Legendre Polynomials for Digital Predistortion for LTE Applications

Huadong Wang*

School of Optoelectronic Engineering
Chongqing University of Communication and Post
Chongqing, China
e-mail: wanghuadong@cqupt.edu.cn

Lijuan Zhang

School of Optoelectronic Engineering
Chongqing University of Communication and Post
Chongqing, China
e-mail: 1098581109@qq.com

Abstract—In this letter, a functional link neural network model is represented to describe the nonlinear characteristic of the wide-band power amplifiers. In the model, an extended Legendre polynomial is used to describe the nonlinearity characteristics of the radio frequency power amplifier. Compared with traditional artificial neural network models, the new model has simpler model structure and is more easily identified. The final experiment shows that the new model exhibits better performance than the traditional memory polynomial model in the DPD system as the adjacent channel power ratio improvement of the DPD system with the new model is more than -55 dB, 5 dB better than that of the DPD with the MP model.

Keywords—behavioral model; digital predistortion; Legendre polynomial; neural network

I. INTRODUCTION

Due to the requirements on both of the linearity and the efficiency for the wireless transmitter, the digital predistortion (DPD) technology has widely been used in current communication system, which in turn promotes the research on the behavioral models of the power amplifier [1,2]. All behavioral models can be divided into two kind models: the polynomial based models, which include the MP model, the general memory polynomial (GMP) and the pruning Volterra series model [3,4], and ANN models, which include the multilayer feedforward neural networks, such as multilayer perceptron (MLP), and radial basis function (RBF) network behavioral model [2,5].

Although the complex polynomial based models such as pruning Volterra series or PLUME model [6], can precisely describe the output characteristic of the power amplifier, the identification process of these models are very difficult since the coefficients numbers of these models are usually very large. The numerical instability caused by higher order polynomial is another hard problem which restricts the performance of the polynomial based models [7]. Compared with the polynomial based models, the identification algorithms of the traditional ANN models, such as genetic algorithm or error back propagation (BP) algorithm, are time-consuming, and usually not suitable for the DPD system with high speed requirement.

In this letter, a new model which is called functional link neural network with extended Legendre polynomials (FLNNEL) is represented. The functional link neural

network is used to describe the nonlinearity characteristic of the power amplifier and an extended Legendre polynomial is used for the function expansion in the model. Furthermore, the extended Legendre polynomials exhibits higher numerical stability than the power series during the model identification process, which can enhance the performance of the DPD system.

II. THE FLNNEL BEHAVIORAL MODEL

The FLNN, initially proposed by Pao [8], is a single-layer NN structure capable of performing function approximation and pattern classification. The block diagram of an m-dimensional input of FLNN model without any hidden layer is shown in the Fig. 1.

The output signal of the FLNN model $y(n)$ can be described by a group nonlinear functions $\phi_i(n)$, in which $x(n)$ is the input signal vector of the model whose definition is given in the equation (1), and M is memory depth of the model. Clearly the FLNN is a flat net and the need of the hidden layer is removed. The precision of the FLNN model can be adjusted by increasing or decreasing the number of the $\phi_i(n)$ according to the requirement of the model performance.

$$X(n) = [x(n), x(n-1), x(n-2), \dots, x(n-M)] \quad (1)$$

The output of the FLNN model in the Fig. 1 can also be written as (2), where $\omega_i(n)$ is the weight factor of the branch i at input signal.

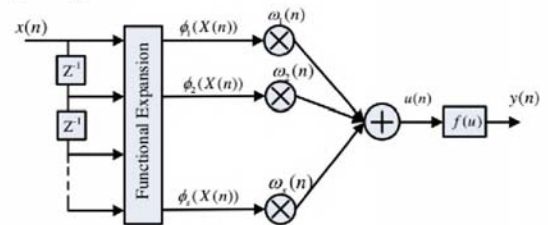


Figure 1. The diagram of the functional link neural network.

$$y(n) = f\left(\sum_{i=1}^s \phi_i(X(n))\omega_i(n)\right) \quad (2)$$

To simplify the FLNN model, the $f(*)$ in the model is approximated by a linear function, and $\omega_i(n)$ is assumed to be invariable parameter. Thus equation (2) can be rewritten as:

$$y(n) = \sum_{i=1}^s \phi_i(X(n))\omega_i \quad (3)$$

The associated Legendre polynomials is a widely used orthogonal polynomials whose expression are shown in Tab. 1. However, traditional Legendre polynomials is memoryless function and it must be modified before it has been used to describe the system with multi input.

TABLE I. THE LEGENDRE POLYNOMIALS

$P_0(x)$	1
$P_1(x)$	x
$P_2(x)$	$0.5(3x^2 - 1)$
$P_3(x)$	$0.5(5x^3 - 3x)$
...	...

As discussion in the [6], the enhanced memory polynomial (EMP) model given in (4) can accurately describe both the nonlinearity and the memory effect of the power amplifier. Replace the power series in (4) with the Legendre polynomials, we can obtain a new extended Legendre polynomials, which can be used to describe the nonlinear memory system. The detailed description of the basis functions $\phi_i(X(n))$ based on the extend Legendre polynomials for the function expansion of the model is given in the (5).

$$y_{emp}(n) = \sum_{m=0}^M \sum_{k=0}^{K-1} a_{m,k} x(n-m)|x(n)|^k + \sum_{m=0}^M \sum_{k=0}^{K-1} b_{m,k} x(n)|x(n-m)|^k \quad (4)$$

$$\begin{aligned} \phi_i(X(n)) = & x(n)P_0(|x(n)|), x(n)P_1(|x(n)|), \dots, x(n)P_K(|x(n)|), \\ & x(n-1)P_0(|x(n)|), \dots, x(n-1)P_K(|x(n)|), \dots \\ & x(n-M)P_0(|x(n)|), \dots, x(n-M)P_K(|x(n)|), \\ & x(n)P_0(|x(n-1)|), \dots, x(n)P_K(|x(n-1)|), \\ & x(n)P_0(|x(n-M)|), \dots, x(n)P_K(|x(n-M)|) \end{aligned} \quad (5)$$

The number of the basis functions given in (5) is $(M+1)(K+1) + MK$, where K is the maximum order of Legendre polynomials. Because all independent variables in the expressions of the Legendre polynomials in (5) is absolute of $x(n)$, the computation process of the $P_i(x)$ is real arithmetic which greatly reduces the computation complexity of the basis functions $\phi_i(X(n))$.

III. THE IDENTIFICATION OF THE FLNNEL MODEL

The weight factors of the model can be identified by solving equation (3) with RLS algorithm. Define ω^n as the weight factor after n iteration, and ω^{n+1} as the weight factor after $n+1$ iteration. Their relation can be described as the equation (6), in which ω_i^n and δ_i^n are the value and correct factor of weight factor ω_i at n iteration respectively.

$$\omega^{n+1} = \omega^n + \delta^{n+1} \quad (6)$$

Where

$$\omega^n = [\omega_1^n, \omega_2^n, \dots, \omega_s^n]^T \quad (7)$$

$$\delta^{n+1} = [\delta_1^n, \delta_2^n, \dots, \delta_s^n]^T \quad (8)$$

As discussion in [9], δ^n can be calculated by the following matrix equation with gauss elimination method.

$$R_{n+1}\delta^{n+1} = \tilde{y}_{n+1} \quad (9)$$

where R_{n+1} triangle matrix and vector \tilde{y}_{n+1} can be calculated by the following iteration process, in which λ is forgetting factor.

$$\begin{bmatrix} R_{n+1} \\ 0 \end{bmatrix} = \tilde{Q} \begin{bmatrix} \lambda R_n \\ \phi(X(n+1)) \end{bmatrix} \quad (10)$$

$$\begin{bmatrix} \tilde{y}_{n+1} \\ r(n+1) \end{bmatrix} = \tilde{Q} \begin{bmatrix} 0 \\ u(n+1) \end{bmatrix} \quad (11)$$

The definition of $u(n+1)$ is given as:

$$u(n+1) = y(n+1) - \phi(n+1) \quad (12)$$

$$\phi(X(n+1)) = [\phi_1(X(n+1)), \dots, \phi_s(X(n+1))] \quad (13)$$

A more convenient method to generate \tilde{Q} is to eliminate vector $\phi(X(n))$ directly in the augmented matrix \tilde{Q} given in (14) with Givens rotations, by which \tilde{Q} and $\phi(X(n))$ after $n+1$ iteration can be obtained at the same time.

$$\tilde{Q} = \begin{bmatrix} \lambda R_n & 0 \\ \phi(X(n+1)) & u(n+1) \end{bmatrix} \quad (14)$$

The Givens rotations can be used to trianglized the matrix \tilde{Q} , where all elements in the vector $\phi(X(n))$ are converted to 0. The Givens rotation is widely used in matrix operation for the QR decomposition. Any element in the matrix can be eliminated by multiplying a unitary matrix shown in (15). The definitions of c_1 and s_1 in the unitary matrix are given in (16). Clearly, \tilde{Q} can convert to an upper triangle matrix by multiplying a series of unitary matrices.

$$\begin{bmatrix} c_1 & 0^H & s_1^* \\ 0 & 1 & 0 \\ -s_1 & 0^H & c_1 \end{bmatrix} \quad (15)$$

$$c_1 = \frac{\sqrt{\lambda} \tilde{r}_{11}(n)}{\tilde{r}_{11}(n)}, s_1 = \frac{x^*(n)}{\tilde{r}_{11}(n)} \quad (16)$$

One special benefit of the Givens rotation is that it can be implemented in parallel, as illustrated in Fig. 2. At time $n-1$, the elements of \bar{Q} are stored in the array elements. The arriving new input data is fed from the top and propagate downward. The Givens rotation parameters are calculated in the boundary cells and propagate from left to right. The internal cells receive the rotation parameters from the left, perform the rotation on the data from the top and then pass the results to the cells at right and below.

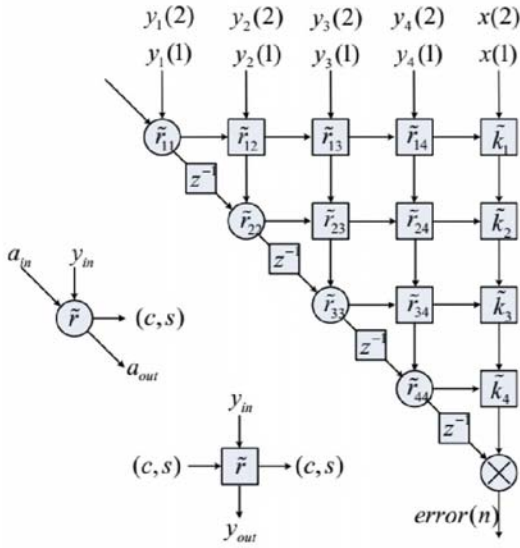


Figure 2. The implementation of the QR-RLS algorithm by systolic array architecture.

IV. SIGNAL PREPROCESS FOR MODEL IDENTIFICATION

Before the model identification, the output signal of the PA must be processed. The process program mainly includes two steps: power normalization and time alignment.

A. Power Normalization

The reason the input and output signal must be normalized before identification is discussed in the following section. First, due to the gain variation in the transmitter, the average power of the feedback signal cannot keep constant value. To build an accurate predistortion model, the effect of the gain variation must be eliminated before the DPD model identification process. Second, the average gain of the PD module should be equal to one, as the predistortion system should not change the average output power of the wireless transmitter. This means the nonlinear model should keep a unity gain at all times and the average power of $y(n)$ needs to be aligned to the average power of $x(n)$ before $y(n)$ is used to the model identification.

The calculating equation of power normalization is given as following.

$$y_{nor}(n) = y(n) \sqrt{\frac{P_x}{P_y}} \quad (17)$$

in which $y_{nor}(n)$ represents feedback signal after power normalization, where the definition of P_x and P_y is given in (18) and (19) respectively, and L is the length of $y(n)$.

$$P_x = \frac{1}{L} \sum_{l=1}^L |x(l)|^2 \quad (18)$$

$$P_y = \frac{1}{L} \sum_{l=1}^L |y(l)|^2 \quad (19)$$

B. Time Alignment

Due to the propagation in the transmitter, the feedback signal is later than the input signal when captured simultaneously in the FPGA device. It is clear that two signals must be synchronized prior to model identification and the delay between them should be estimated first. In this paper, a method called the frequency multiplication and the parabolic curve fitting (FMPCF) method [10], are used to estimate the delay between input and output signal, which exhibits better performance and lower compute complexity than traditional time-alignment algorithms.

V. THE IDENTIFICATION OF THE FLNNEL MODEL

The test setup shown in Fig. 3 is considered for the model performance evaluation purposes. A remote radio unit (RRU) is used to test the DPD performance of the model. The test signal is 20 MHz LTE signal generated from the baseband unit (BBU) and is up-converted to 2.6 GHz before being sent to the power amplifier. The Doherty power amplifier in the transceiver module is designed by MRF8P26080 LDMOS transistors with 10W maximum output power. The computer is connected with RRU through LAN for control order and data transmission.

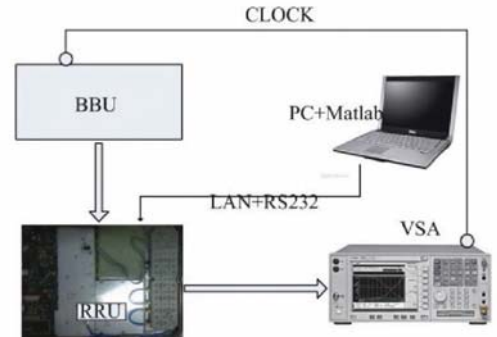


Figure 3. The test platform for the FLNNEL model.

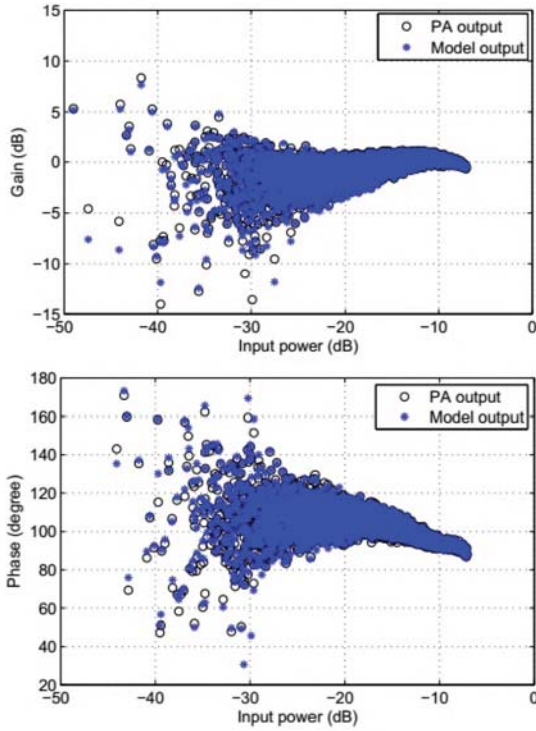


Figure 4. The AM-AM and AM-PM characteristics of the FLNNEL model and PA output.

The AM-AM and AM-PM curves of the FLNNEL model output is compared with the output signal of the power amplifiers in the Fig. 4. It is clear that the FLNNEL model can accurately describe not only the magnitude and phase distortion characteristic but also the memory effect of the power amplifier, as the model output can precisely fit the output signal of the PA. Due to the memory effect, the both of the AM-AM and AM-PM curves of the PA output expand to a wide range at the small signal region and the model output can predict PA characteristic very well.

The error power spectrums of the different models are shown in the Fig. 5. It is clear that the FLNNEL model will exhibit similar precision with the EMP model, far better than the precision of the MP model. To evaluate the performance of the extend Legendre polynomials, the error power spectrums of the CCFLNN model, which is represented in [11], is also given in the figure and it exhibits similar precision with the MP model. This result verifies the extend Legendre polynomials can obtain higher model precision than the complex Chebyshev polynomial when applied to the wide-band PA. The more detailed data of these models is given in the Table 2, which shows the normalized mean square error (NMSE) of the FLNNEL model is -45 dB, 1.5 dB higher than that of the EMP model but more than 8 dB lower than that of the MP model or the CCFLNN model. On the other hand, the coefficients number of the FLNNEL model is 25, only more than that of the CCFLNN model but less than that of the MP model or the EMP model. Clearly, the FLNNEL model can obtain a optimal tradeoff between model complexity and model precision.

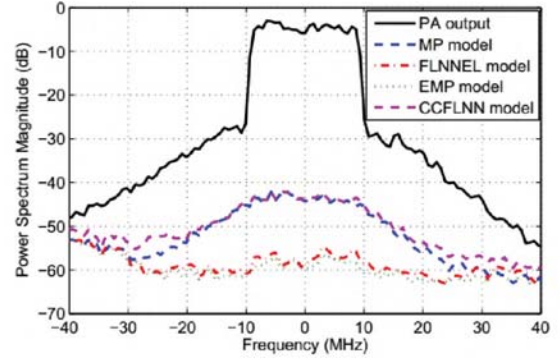


Figure 5. Error power spectrum.

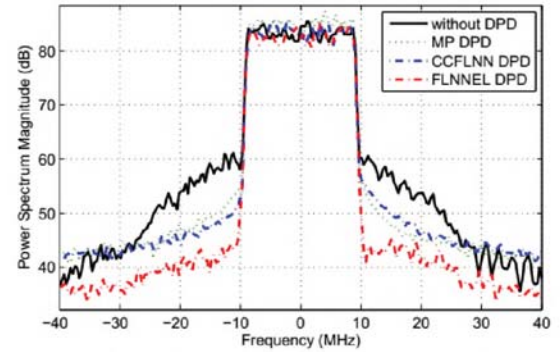


Figure 6. Output spectrums of PA after different DPD models

TABLE II. COMPARISON OF THE PERFORMANCE OF DIFFERENT BEHAVIORAL MODELS

Model	Memory Depth	Order	NMSE(dB)	Coef. Num
MP	3	7	-37.4	28
EMP	3	7	-46.5	44
FLNNEL	3	3	-45	25
CCFLNN	3	3	-36.8	16

The output spectrums of the power amplifier after the DPD with different models are compared in Fig. 6. The Adjacent Channel Leakage Ratio (ACPR) after DPD with the FLNNEL model is less than -55 dB, more than 27 dB better than that before DPD, and 5 dB lower than the ACPR after DPD with the MP model or the CCFLNN model, whose value is about -50 dB. Compared with the MP DPD, the FLNNEL DPD exhibits better performance on both precision and complexity. Compared with the CCFLNN DPD, although the FLNNEL DPD has more coefficients, it has better ACPR which satisfies the linearity requirement of the base station given in 3GPP.

VI. CONCLUSIONS

In this letter, we propose a FLNNEL model to describe the nonlinearity characteristic of the wide-band RF PA. The result verifies the new model can make better tradeoff between the model complexity and model precision than traditional models and exhibits good performance when it is applied to the DPD system, which shows that the FLNNEL

model can be an attractive solution for digital predistortion of the RF PA.

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REFERENCES

- [1] J. C. Pedro, and S. A. Maas, "A comparative overview of microwave of microwave and wireless power-amplifier behavioral modeling approaches," *IEEE Trans. Microw. Theory Tech.* vol.53, no.4, pp.1150-1163, Apr. 2005..
- [2] M. Isaksson, D. Wisell and D. Ronnow "A Comparative Analysis of Behavioral Models for RF Power Amplifiers," *IEEE Trans. Microw. Theory Tech.*, vol.54, no.1, pp.348-358, Jan. 2006.
- [3] D. Morgan, Z. Ma, J. Kim, M. Zierdt, and J. Pastalan, "A Generalized Memory Polynomial Model for Digital Predistortion of RF Power Amplifiers," *IEEE Trans. Signal Process.* vol.54, no.10, pp. 3852-3860.
- [4] B. Schubert, A. Gokceoglu, L. Anttila, and M. Valkama, "Augmented Volterra Predistortion for the Joint Mitigation of Power Amplifier and I/Q Modulator Impairments in Wideband Flexible Radio," in *Proc. IEEE Global Conf. on Signal and Information Processing (GlobalSIP 2013)*, Austin, TX, Dec. 2013.
- [5] M. Rawat, K. Rawat, and F. M. Ghannouchi, "Adaptive Digital Predistortion of Wireless Power Amplifiers/Transmitters Using Dynamic Real-Valued Focused Time-Delay Line Neural Networks," *IEEE Trans. Microw. Theory Tech.*, vol.58, no.1, pp.95-107, Jan. 2010.
- [6] M. Younes, O. Hammi, and A. Kwan, "An Accurate Complexity-Reduced "PLUME" Model for Behavioral Modeling and Digital Predistortion of RF Power Amplifiers," *IEEE Trans. Industrial Electronics* vol.58, no.4, pp.1397-1406, Jan. 2011.
- [7] F. Mkadem, A. Islam, and S. Boumaiza, "Multi-Band Complexity-Reduced Generalized-Memory-Polynomial Power-Amplifier Digital Predistortion," *IEEE Trans. Industrial Electronics* vol.64, no.6, pp. 1763-1774, Jun. 2016.
- [8] Y. H. Pao: *Adaptive Pattern Recognition and Neural Networks* Reading, MA:Addison Wesley, 1989.
- [9] X. D. Zhang: *Matrix analysis and application*, Tsinghua University Press, Beijing, 2004, pp. 242.
- [10] H. Wang, W. Xue and H. Ma, "The Fast Algorithms for the Delay Estimation in Digital Predistortion System," *IEEE Microw.Wireless Compon. Lett.* vol.25, no.3, pp.202-204, Mar. 2015.
- [11] M. Li, J. Liu, Y. Jiang and W. Feng, "Complex-Chebyshev Functional Link Neural Network Behavioral Model for Broadband Wireless Power Amplifiers," *IEEE Trans. Microw. Theory Tech.*, vol.60, no.6, pp.1979-1989, Jun. 2012.