SC42100 Networked and Distributed Control Systems

Assignment 1 - Lectures 1 and 2

Instructions:

- The assignments are individual. You may consult and discuss with your colleagues, but you need to provide independent answers.
- You may use Matlab/Python to solve the assignment. Many questions require you clearly to do so.
- Provide detailed answers, describing the steps you followed.
- Provide clear reports, typed, preferably using LaTex.
- Submit the reports digitally on Brightspace/Peer as indicated on the lectures.
- Include your code, for reproducibility of your results, in your Brightspace submission on a zip file, or through a link on your report.
- The code will *only* be used to verify reproducibility in case of doubts. The grading will be performed based on the results described in the report.

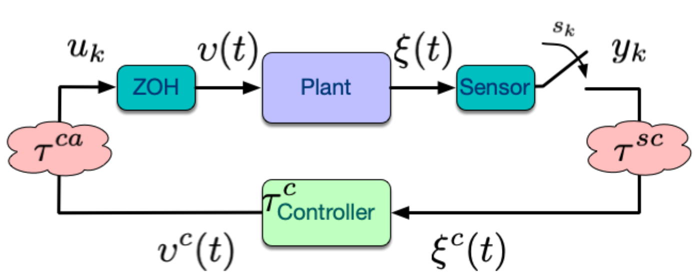

Figure 1: Schematic of a NCS including delays.

Consider a sample-data networked control system (NCS), as depicted in Figure 1. Assume that the plant dynamics are linear time-invariant given by:

$$\dot{\xi}(t) = A\xi(t) + B\upsilon(t),$$

where the control signal υ is a piece-wise constant signal resulting from the application of a controller in sampled-and-hold fashion, i.e. $\upsilon(t) = u_k,\ t \in [s_k,\ s_{k+1})$.

The system matrices are given by:

$$A = \begin{bmatrix} a-b & 0.5-c \\ 0 & 1 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

where a is given by the first digit, b by the 3rd digit, and c by the last digit of your student ID number.

We start considering the system with a constant sampling interval $h = s_{k+1} - s_k$ for all k, and assuming no delay is present $\tau = \tau^{sc} + \tau^{c} + \tau^{ca} = 0$.

Question 1: (3p)

1. (1p) Construct a linear continuous time controller $v(t) = -\bar{K}\xi(t)$ placing the poles of the continuous closed-loop at $-1+j$ and $-1-j$.
2. (2p) Construct the exact discrete-time model $x_{k+1} = F(h)x_k + G(h)u_k$, resulting from applying the controller in sample-data fashion: $u_k = -\bar{K}x_k$, $x_k := \xi(s_k)$. Provide analytical expressions for the closed-loop system matrices parametrized by h.

Next we consider the case when delays are present in the networked system. Assume that the system is affected by a **constant** small delay $\tau \in [0, h)$, and it is controlled with the same static controller you designed.

Question 2: (6p)

1. (2p) Construct the exact discrete-time model for the NCS with delays. Give explicit expressions for the system matrices.
2. (2p) Study the combinations of sampling intervals and system delays that result in an asymptotically stable closed-loop. Provide a plot illustrating the combinations of (h, τ) retaining stability.
3. (2p) Select a sampling interval h that guarantees stability under no delays. Redesign the controller to improve the robustness against delays, that is, a controller that increases the range of tolerable delays for the selected h.

Remark: When asked to redesign the controller, in Q2.3 and any subsequent question, you may go beyond merely retuning the values of $\bar{K}$.

Next we consider the case when the delays are still **constant** but larger: $\tau \in [2h, 3h)$, and the system is controlled with the same static controller you designed in Question 1.

Question 3: (7p)

1. (3p) Construct the exact discrete-time model for the NCS with delays in $[2h, 3h)$. Give explicit expressions for the system matrices.

2. (2p) Study the combinations of sampling intervals and system delays that result in an asymptotically stable closed-loop. Provide a plot illustrating the combinations of (h, τ) retaining stability. You may combine this plot with the one produced in Question 2 (there will be some empty space in the combined plot).

3. (2p) Select a sampling interval h that guarantees stability for $\tau = 2h$. Redesign the controller to improve the robustness against delays.

Finally, consider again a system with a fixed **constant** h inter-sample time, but in which the NCS is affected by **time-varying delays** taking values in the set $\mathcal{T} = \{0.5h, h, 2h\}$.

Question 4: (10p)

1. (1p) Describe the set of LMIs that need to be solved to determine if a given controller guarantees stability for this NCS.

2. (3p) Design a controller to maximize the possible selection of (constant) sampling intervals h. Describe in detail your approach.

3. (3p) Assuming now that the sequence of delays is known to be given by a periodic repeating pattern[1]: $\tau^s = (0.5h, h, 2h)^\omega$. This may be the case when the delays result from e.g. a medium access schedule. Simplify the LMIs needed to analyse the stability of the system.

4. (3p) Design a controller to maximize the possible selection of sampling intervals h, when the periodic delay sequence is as in the previous question. Describe in detail your approach and discuss the result.

[1]The notation $(abc)^\omega$ is used to denote an infinite repetition of the sequence abc, i.e. $abcabcabc\ldots$