BANA4095: Decision Models – Spring 2021
Linear Optimization, Part 3

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Outline

- Python Data Structures
  - » Lists, Dictionaries, Tuples
  - » For loops and `zip()`
- Pythonic Code for Linear Optimization
- Optimization Parameter Analysis
- Types of LP Models
  - » Allocation
  - » Covering
  - » Proportion/Blending

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Simple Sidneyville LP Model

- Requires excessive lines of code
- Time consuming to construct and maintain
- Should use Python data structures and loops to generalize the code and make it more efficient to construct and maintain

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`list` Data Type

- Convenient data structure for storing sequential set of items
  - » Numeric values, strings, *etc*.
- We can use lists to conveniently store the parameters, decision variables, and constraints in an optimization model
  - » `profit = [115.00, 90.00]`
  - » `products = ['Roll_Top','Regular']`
- List elements can be assigned and accessed individually by referencing their index
  - » `profit[0]` is `115.00`
  - » `product[1]` is `'Regular'`

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dictionary Data Type

- Used to store a sequence of items
- Generalizes the `list` data type
- Can use almost any data type to index items in the dictionary not just integers
- An item in a dictionary is called a *key-value* pair

```
resource = {'Pine':200, 'Cedar':128, 'Maple':220}
resource.keys() = ['Pine','Cedar','Maple']
resource.values() = [200,128,220]
resource['Maple'] = 220
```

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tuple Data Type

- Used to store an *immutable* sequence of values
- Indexed using integers like a `list`

```
t = (0, 200, [10,20])
t[1]=200
t[2]=[10,20]
```

- Can be used to make multiple simultaneous assignments

```
(lb,ub,lst) = t
lb=0, ub=0, lst=[10,20]
```

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Python Data Structures

Type	Mutable	Indices
List	Yes	Integers
Dictionary	Yes	General
Tuple	No	Integers

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Pythonic Linear Optimization Code

- Provides a more general and efficient approach for constructing and solving linear optimization models
- Uses Python data structures to store model information
  - » Only need to edit these input data structures to change the model or to create a completely new model
- Uses for-loops to efficiently construct the model and display its optimal solution

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Linear Optimization Data Structures

- Uses dictionaries and tuples to efficiently store the model data
- `inf` is a model constant representing infinity
- `variables` dictionary

name / lower bound / upper bound / objective coefficient

```
variables = {'Roll_Top':  (0,  inf,  115),
             'Regular':   (0,  inf,   90)}
```

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Linear Optimization Data Structures

- `inf` is a model constant representing infinity
- `constraints` dictionary

name / lower bound / upper bound / coefficients

```
constraints = {'Pine':   (-inf, 200, [10,20]),
               'Cedar':  (-inf, 128, [4,16]),
               'Maple':  (-inf, 220, [15,10]}
```

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`zip()` Function

- A built-in function that takes two or more sequences and returns a list of tuples where each tuple contains one element from each sequence

```
vars = ['Roll_Top','Regular']
coeffs = [10,20]

zip(vars,coeffs) = [('Roll_Top',10),('Regular',20)]
```

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Multiple Iteration Variables in a For-Loop

- Using a tuple as the iteration variable in a for-loop enables iteration/looping over multiple sequences simultaneously
- The `zip()` function can be used to combine sequences into a single list of tuples as the iteration list in the for-loop

```
a_lst = [3,5,7,11]
b_lst = [2,4,6,8]
for (a,b) in zip(a_lst,b_lst):
        print(a+b)

5, 9, 13, 19
```

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Python Optimization Code Steps

- Import Glop package
- Input parameters
- Create model object
- Create variable objects
- Create objective function and coefficients
- Create constraints and coefficients
- Solve the problem
- Check solution status
- Display optimal value
- Display optimal solution
- Display constraint values

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Important Glop Object Methods

- Model Object
  `.NumVar(), .Objective(), .Constraint(), .variables(), .constraints(), .ComputeConstraintActivities(), .Clear()`
- Objective Object
  `.SetMaximization(), .SetMinimization(), .SetCoefficient(), .Value()`
- Variable Objects
  `.name(), .lb(), .ub(), .solution_value(), .reduced_cost()`
- Constraint Objects
  `.SetCoefficient(), .name(), .lb(), .ub(), .dual_value()`

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Optimization Parameter Analysis

- Show how the solution of an optimization model changes as an input parameter changes
- Change the input parameter value, re-optimize the model, and report the result
- Use a for or a while loop to iterate over the range of parameter values
- Example: Sidneyville
  - » Resources
    - – Pine, Maple
  - » Objective Coefficients
    - – Profit per Desk: Rolltop

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Optimization Parameter Analysis

- Use lists to store parameter values and optimal values
- Use for-loop to iterate over the parameter values solving for and storing each optimal value
- Use `.SetCoefficient()` method to change an objective or constraint coefficient value before re-optimizing/solving
- Use `.SetLB()` or `.SetUB()` to change a constraint resource or requirement value (right hand side) before re-optimizing/solving
- Plot the results

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General Types of LP Models

- **Allocation** – allocate limited resources across different activities
- **Covering** – select decisions to meet minimum requirements
- **Blending/Proportion** – decisions are subject to one or more constraints on a proportion or a weighted average computed from the decision variables
- **Network** – optimize decisions over a network structure

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Veerman Furniture Company makes three kinds of office furniture: chairs, desks, and tables. Each product requires some labor in the parts fabrication department, the assembly department, and the shipping department. The furniture is sold through a regional distributor, who has estimated the maximum potential sales for each product in the coming quarter. Finally, the accounting department has provided some data showing the profit contributions on each product. The decision problem is to determine the product mix—that is, to maximize Veerman's profit for the quarter by choosing production quantities for the chairs, desks, and tables. The following data summarizes the parameters of the problem:

		Hours per Unit		Labor Hours
Department	Chairs	Desks	Tables	Available
Fabrication	4	6	2	1,850
Assembly	3	5	7	2,400
Shipping	3	2	4	1,500
Demand Potential	360	300	100	
Profit	$15	$24	$18	

Example: Veerman Furniture Co.

- Construct a model to determine the best production quantities
- Which constraints are binding or non-binding?
- What are some managerial implications/recommendations based on the solution and the sensitivity report?

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Dahlby Outfitters wishes to introduce packaged trail mix as a new product. The ingredients for the trail mix are seeds, raisins, flakes, and two kinds of nuts. Each ingredient contains certain amounts of vitamins, minerals, protein, and calories. The marketing department has specified that the product be designed so that a certain minimum nutritional profile is met. The decision problem is to determine the optimal product composition—that is, to minimize the product cost by choosing the amount for each of the ingredients in the mix. The following data summarizes the parameters of the problem:

			Grams per Pound			Nutritional
Component	Seeds	Raisins	Flakes	Pecans	Walnuts	Requirement
Vitamins	10	20	10	30	20	20
Minerals	5	7	4	9	2	10
Protein	1	4	10	2	1	15
Calories	500	450	160	300	500	600
Cost/pound	$4	$5	$3	$7	$6	

Example: Dahlby Outfitters

- Construct a model to determine the best quantities for the trail mix
- Which requirements are binding or non-binding?
- Which constraint has the highest per unit impact on the optimal cost?
- What are some managerial implications/recommendations based on the solution and the sensitivity report?

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Blending/Proportion Constraints

- Mixing materials with different properties to achieve a blend with a certain weighted average property
- Requiring a minimum/maximum proportion in the decisions
- Convert the nonlinear weighted average or proportion constraint into a linear constraint
- How? (requires a bit of algebra)
  - » Multiply both sides of the constraint by the denominator and then group similar terms on the left hand side leaving 0 on the right hand side
- Why bother? Why not just use the nonlinear constraint?

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The Diaz Coffee Company blends three types of coffee beans (Brazilian, Colombian, and Peruvian) into ground coffee to be sold at retail. Suppose that each kind of bean has a distinctive aroma, and the company has a chief taster who can rate this characteristic on a scale of 1 to 100. The features of the beans are tabulated as follows:

Bean	Aroma	Cost/lb.	Pounds Available
Brazilian	75	$0.50	1,500,000
Colombian	65	$0.60	1,200,000
Peruvian	85	$0.70	2,000,000

The company would like to create a blend that has an aroma rating of at least 78 and contains at least 20% Colombian beans. Its supplies of the various beans are limited, however. The available quantities are specified above. Diaz wants to make four million pounds of the blend at the lowest possible cost.

Example: Diaz Coffee

- Construct a model to determine the best quantities for the coffee blend
- Which constraints are binding and which are not?

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