BANA4095: Decision Models – Spring 2021
Linear Optimization, Part 4

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Outline

- Types of LP Constraints
  - » Allocation
  - » Covering
  - » Proportion
  - » Blending

General Types of LP Models

- **Allocation** – allocate limited resources across different activities
- **Covering** – select decisions to meet minimum requirements
- **Blending/Proportion** – decisions are subject to one or more constraints on a proportion or a weighted average computed from the decision variables
- **Network** – optimize decisions over a network structure

Veerman Furniture Company makes three kinds of office furniture: chairs, desks, and tables. Each product requires some labor in the parts fabrication department, the assembly department, and the shipping department. The furniture is sold through a regional distributor, who has estimated the maximum potential sales for each product in the coming quarter. Finally, the accounting department has provided some data showing the profit contributions on each product. The decision problem is to
determine the product mix—that is, to maximize Veerman's profit for the quarter by choosing production quantities for the chairs, desks, and tables. The following data summarizes the parameters of the problem:

		Hours per Unit		Labor Hours
Department	Chairs	Desks	Tables	Available
Fabrication	4	6	2	1,850
Assembly	3	5	7	2,400
Shipping	3	2	4	1,500
Demand Potential	360	300	100	
Profit	$15	$24	$18	

Example: Veerman Furniture Co.

- Construct a model to determine the best production quantities
- Which constraints are binding or non-binding?
- What are some managerial implications/recommendations based on the solution and the sensitivity report?

Dahlby Outfitters wishes to introduce packaged trail mix as a new product. The ingredients for the trail mix are seeds, raisins, flakes, and two kinds of nuts. Each ingredient contains certain amounts of vitamins, minerals, protein, and calories. The marketing department has specified that the product be designed so that a certain minimum nutritional profile is met. The decision problem is to determine the optimal product composition—that is, to minimize the product cost by choosing the amount for each of the ingredients in the mix. The following data summarizes the parameters of the problem:

	Grams per Pound					Nutritional
Component	Seeds	Raisins	Flakes	Pecans	Walnuts	Requirement
Vitamins	10	20	10	30	20	20
Minerals	5	7	4	9	2	10
Protein	1	4	10	2	1	15
Calories	500	450	160	300	500	600
Cost/pound	$4	$5	$3	$7	$6	

Example: Dahlby Outfitters

- Construct a model to determine the best quantities for the trail mix
- Which requirements are binding or non-binding?
- Which constraint has the highest per unit impact on the optimal cost?
- What are some managerial implications/recommendations based on the solution and the sensitivity report?

Blending/Proportion Constraints

- Mixing materials with different properties to achieve a blend with a certain weighted average property
- Requiring a minimum/maximum proportion in the decisions
- Convert the nonlinear weighted average or proportion constraint into a linear constraint
- How? (requires a bit of algebra)
  - » Multiply both sides of the constraint by the denominator and then group similar terms on the left hand side leaving 0 on the right hand side
- Why bother? Why not just use the nonlinear constraint?

The Diaz Coffee Company blends three types of coffee beans (Brazilian, Colombian, and Peruvian) into ground coffee to be sold at retail. Suppose that each kind of bean has a distinctive aroma, and the company has a chief taster who can rate this characteristic on a scale of 1 to 100. The features of the beans are tabulated as follows:

Bean	Aroma	Cost/lb.	Pounds Available
Brazilian	75	$0.50	1,500,000
Colombian	65	$0.60	1,200,000
Peruvian	85	$0.70	2,000,000

The company would like to create a blend that has an aroma rating of at least 78 and contains at least 20% Colombian beans. Its supplies of the various beans are limited, however. The available quantities are specified above. Diaz wants to make four million pounds of the blend at the lowest possible cost.

Proportion Constraint

- At least 20% of the blend must be Colombian beans

B = pounds of Brazilian beans
C = pounds of Colombian beans
P = pounds of Peruvian beans

$$\text{Percent Colombian} = \frac{C}{B+C+P} \geq 0.20$$

- Nonlinear expression (division)
- Can convert nonlinear constraint to a linear constraint using some basic algebra

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Proportion Constraint

- At least 20% of the blend must be Colombian beans

$$\frac{C}{B+C+P} \geq 0.20$$

$$\frac{C}{\cancel{(B+C+P)}}\cancel{(B+C+P)} \geq 0.20(B+C+P)$$

$$C \geq 0.20(B+C+P)$$

$$C - 0.20(B+C+P) \geq \cancel{0.20(B+C+P)} - \cancel{0.20(B+C+P)}$$

$$-0.20B + 0.80C - 0.20P \geq 0$$

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Blending Constraint

- Aroma rating of the blend must be at least 78
- Weighted average of individual aroma ratings

$$\text{Blend Aroma} = \frac{75B + 65C + 85P}{(B+C+P)} \geq 78$$

- Nonlinear expression (division)
- Can convert nonlinear constraint to a linear constraint using some basic algebra

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Blending Constraint

- Aroma rating of the blend must be at least 78

$$\frac{75B + 65C + 85P}{(B + C + P)} \geq 78$$

$$\frac{75B + 65C + 85P}{\cancel{(B + C + P)}}\cancel{(B + C + P)} \geq 78(B + C + P)$$

$$75B + 65C + 85P \geq 78(B + C + P)$$

$$75B + 65C + 85P - 78(B + C + P) \geq \cancel{78(B + C + P)} - \cancel{78(B + C + P)}$$

$$-3B - 13C + 7P \geq 0$$

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Example: Diaz Coffee

- Construct a model to determine the best quantities for the coffee blend
- Which constraints are binding and which are not?

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Summary

- Types of LP Constraints
  - » Allocation
  - » Covering
  - » Proportion
  - » Blending

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