BANA4095: Decision Models – Spring 2021
Re-Introduction to Optimization

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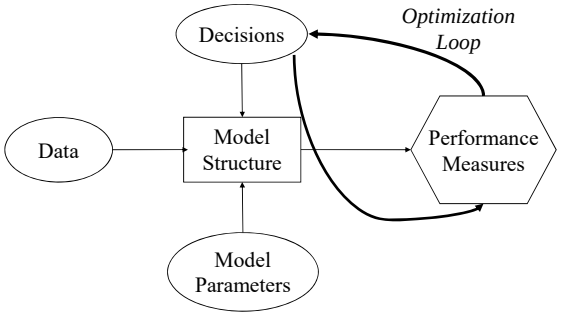
Outline

- What is Optimization?
- Key Elements of Optimization Models
- Optimization Algorithms
- Excel Solver
- Sensitivity Analysis Report

Using a Model for Experimentation



Optimization

- Finding an alternative that achieves the best possible result
  - Unconstrained Optimization
  - Constrained Optimization
- Four key elements of an optimization problem
  - Objective function
  - Decision variables
  - Constraints
  - Parameters

Optimization Elements

- Objective Function
  - » Measures the value of an alternative
  - » Basis for comparing alternatives
  - » Examples
- Decision Variables
  - » What choices/decisions affect the objective function?
  - » Values that must be chosen in order to define an alternative
  - » Consume or supply some resource

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Optimization Elements

- Constraints
  - » What resources or requirements limit the possible alternatives/decisions/choices?
  - » Determine whether a specific alternative is feasible
  - » Right-hand side: defines the amount of resource available or needed to satisfy the constraint
- Parameters
  - » Constants used in the model calculations
  - » Define relationships between decision variables, constraints, and objective function

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Example – Sidneyville Desk Mfg.

- Allocation/Product Mix Problem
- Produces two types of desk
- Using three types of wood in every desk
  (measured in board feet, b.f.)

Type	Profit/desk
Rolltop	$115
Regular	$90

	Amount Used		Amount
Wood	Rolltop	Regular	Available
Pine	10	20	200
Cedar	4	16	128
Maple	15	10	220

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Example, cont.

- What are the decision variables?
- What is the objective function?
- What are the constraints?

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Sidneyville Linear Programming (LP) Formulation

$$\max \quad 115x_1 + 90x_2$$ Maximize Total Profit

s.t. $10x_1 + 20x_2 \le 200$ Pine

$4x_1 + 16x_2 \le 128$ Cedar

$15x_1 + 10x_2 \le 220$ Maple

$x_1, x_2 \ge 0$ Non-negative

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Solver

- Free Excel add-in for optimization
- PC and Mac versions
- Located on the Data tab
- Simple user interface
- Limited problem size
- Covered in BANA 2082
- Frontline Systems
  - » Developer of Solver
  - » Also offers commercial version: Analytic Solver Platform (ASP)

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Building a Solver Optimization Model

- Build base case spreadsheet model
  - » Decision variable cells
  - » Input parameter cells
  - » Objective function formula
  - » Constraint formula(s)
- Enter Solver settings
- Solve

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Solver Settings

- Set Objective (Max/Min)
  - » Select the cell containing the objective function formula
- Changing/Decision Variable Cells
  - » Select the cells containing the decision variable values
- Subject to the Constraints
  - » Add each constraint
  - » Cell Reference is cell containing the formula (left hand side)
  - » Constraint is cell containing the limiting value (right hand side)
  - » Drop down box selects the relationship between LHS and RHS
- Check box: Make unconstrained variables non-negative
- Select a Solving Method

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Sidneyville Manufacturing

- Construct an Excel Solver optimization model for the Sidneyville example and use it to find the optimal solution

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Types of Optimization Models

Type	Variables	Relationships
Linear Program (LP)	Continuous	Linear
Nonlinear Program (NLP)	Continuous	Nonlinear or Linear
Integer Program (IP)	Integer	Linear
Mixed Integer-Linear (MILP)	Integer or Continuous	Linear
Mixed Integer Nonlinear (MINLP)	Integer or Continuous	Nonlinear or Linear

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Solver Engines/Algorithms

- Various algorithms that are used to solve different classes of optimization problems. The two basic engines in Solver are

Engine	Objective	Constraints
Simplex LP	Linear	Linear
GRG Nonlinear	Nonlinear	Nonlinear

- The specific Solver Engine is selected from the drop down list at the top of the Engine tab
- Uncheck the “Automatically Select Engine” box

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Non-Negativity

- Decision variables are frequently assumed/implied to have non-negative values. This assumption is often not explicitly given in the problem statement, but the modeler should recognize the need for this restriction even if it is not explicitly stated.
- Check box: Make Unconstrained Variables Non-Negative
  - » Constrains ALL decision variables to be non-negative
  - » If only SOME of the decision variables should be non-negative, then explicit bound constraints (>= 0) must be added to the model for the variables that require them.

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Special Cases of Optimization Models

- Infeasible Model
  - » There are no feasible solutions that satisfy all of the model constraints
  - » Usually caused by one or more errors in the definition of the constraints or the formula cells referenced in the constraints
- Unbounded Solution
  - » There is no finite feasible solution that is optimal. There is always another solution that has a better objective value.
  - » Usually caused by one or more errors in the definition of the objective or the constraints or the formulas used in the objective and constraints

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Binding vs. Non-Binding Constraints

- Example: Sidneyville
- Binding Constraint
  - » Left Hand Side = Right Hand Side
- Non-Binding Constraint
  - » Left Hand Side not equal to Right Hand Side
  - » Slack = RHS – LHS
- Marginal Value of a Resource
  - » If a constraint is non-binding then the marginal value of its corresponding resource is . . . ?

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Marginal Value / Shadow Price

- The change in the optimal objective value for one additional unit of a resource.
  - » Increases for maximization problems
  - » Decreases for minimization problems
  - » For LPs the marginal value is constant over a range
- What is the “economic value” of obtaining an additional amount of this resource?
- What would you be willing to pay for an additional amount of this resource?
- Example: Sidneyville

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Solver Sensitivity Report

- Provides sensitivity information for the optimal solution
  - » Decision Variables,
  - » Objective Coefficients, and
  - » Constraints
- Decision Variable (DV) - Reduced Cost (RC)
  - » The RC for a DV is the shadow/dual price associated with the nonnegativity constraint for the DV. RC = 0 indicates the DV may be positive in an optimal solution; RC > 0 indicates the DV must be 0 in an optimal solution.
  - » If a DV = 0 and its RC = 0, there may be multiple optimal solutions.
    - – Methods to find alternate solutions

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Solver Sensitivity Report

- Objective Coefficient – Allowable Increase/Decrease
  - » Indicates the range of coefficient values over which the current solution remains optimal
- Constraint – Shadow Price
  - » The marginal value or marginal cost of the right-hand-side (RHS) of the constraint
- Constraint – Allowable Increase/Decrease
  - » Indicates the range of RHS values over which the current shadow price remains constant
- Any change outside of an allowable range requires re-optimization of the model

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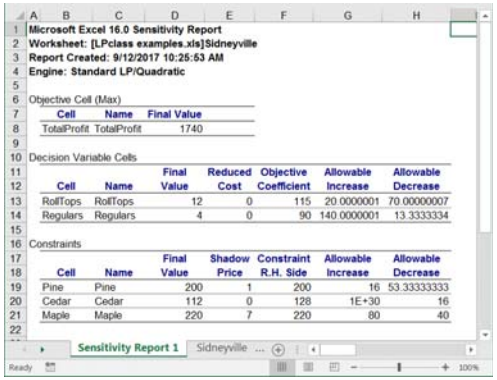
Solver Sensitivity Report



Microsoft Excel 16.0 Sensitivity Report
Worksheet: [LPclass examples.xls]Sidneyville
Report Created: 9/12/2017 10:25:53 AM
Engine: Standard LP/Quadratic

Objective Cell (Max)

Cell	Name	Final Value
TotalProfit	TotalProfit	1740

Decision Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
RollTops	RollTops	12	0	115	20.0000001	70.00000007
Regulars	Regulars	4	0	90	140.0000001	13.3333334

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
Pine	Pine	200	1	200	16	53.33333333
Cedar	Cedar	112	0	128	1E+30	16
Maple	Maple	220	7	220	80	40

Sensitivity Report 1 | Sidneyville

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Solver Sensitivity Report

- Answer the following questions using only the optimization sensitivity report on the previous slide. How does the optimal solution change when . . .

1. The profit for each Regular desk is increased by $20?
2. The sales price for each Rolltop desk increases by $25?
3. There is 15 additional board feet of Pine?
4. There is 100 additional board feet of Cedar?
5. We can purchase 40 additional board feet of Maple at a cost of $3 per board foot.

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Summary

- What is Optimization?
- Key Elements of Optimization Models
- Optimization Algorithms
- Excel Solver
- Sensitivity Analysis Report

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