BANA4095: Decision Models – Spring 2021
Monte Carlo Simulation, Part 2

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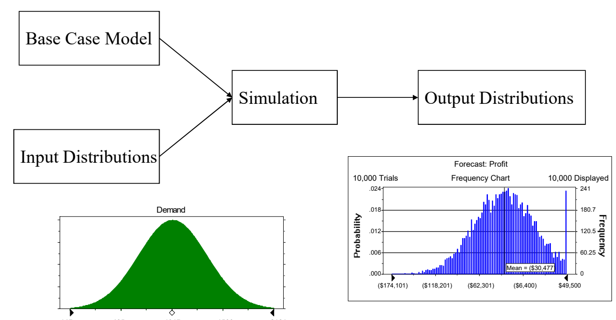
Outline

- Simulation Review
- Simulation Parameter Analysis
- Nested For-Loops
- FTBC Sim Analysis
- Selecting Probability Distributions
- Retirement Sim Analysis
- Estimating Event Probabilities

Monte Carlo Simulation with Python

- Define mathematical relationships/formulas
- Build and validate a base case model
- Specify random variables and probability distributions
- Specify simulation result(s)
- Create for-loop to generate simulation trials
  - » Code the calculations for each trial
  - » Store the result(s) of each trial
- Generate histogram to visualize results
- Compute summary statistics

Simulation Modeling Process



Simulation Parameter Analysis

- Compare simulation results across a range of different parameter values
  - » Conduct a complete simulation run (set of trials) for each parameter value
  - » REMEMBER to set the number of Trials to a reasonable value first!!
- Create additional lists to store the parameter values and summary statistic(s)
- Create an additional for loop to iterate over the parameter values (nested for-loops)
- Example – FTBC Simulation
  - » How does the average profit vary with the production quantity?

Nested For-Loops

- Used to execute a for-loop multiple times

Outer for-loop

```
for param in range(300,600):
    sample = list()
    for trial in range(trials):
        #simulate trial result
        sample.append(result)
    #compute and store statistics
```

Inner for-loop

Example: Fort Thomas Brewing Co.

Fort Thomas Brewing Company (FTBC) produces a popular Oktoberfest beer, Hochland Marzen, every year. Each year FTBC management struggles with deciding how many cases it should brew for the Oktoberfest season. The production cost for this beer is $12.00 per case and FTBC sells it for $18.00 per case. After Oktoberfest season ends, FTBC sells all the remaining Hochland beer at a price of $8.00 per case. The actual in-season demand is random and not known to FTBC when it needs to make the production quantity decision, but we assume demand is uniformly distributed from 300 to 600 cases.

- How does FTBC's profit vary with the chosen production quantity?
  - Mean? Min? Max?

Defining Random Variables (RVs)

- Specify the Distribution that will be used to generate values for the RV
- Specify the parameters of the distribution
  - » Number and type of parameters depend on the properties of the chosen distribution
  - » For example: min, max, mean, standard deviation
- Types of probability distributions
  - » Continuous
  - » Discrete

Selecting Probability Distributions

- Continuous Random Variables
  - » Can have any fractional value across a range
- Discrete Random Variables
  - » Can have any one of a discrete number of values
  - » For example: {0, 1, 2, 3, . . .}
- Subjective Data
  - » Estimates made by experts
  - » May consist of mean, minimum, 10th or 90th percentiles
  - » Usually more readily available than empirical data
- Empirical Data
  - » Numerical observations from past experience

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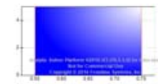
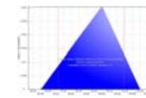
Continuous Distributions

- Uniform: `random.uniform(min,max)`
  - » Value equally likely to fall anywhere between minimum and maximum
  - » Useful when have reasonable guess about minimum and maximum but no reason to suspect any value in between is more likely than the others



- Triangular: `random.triangular(min,max,mode)`
  - » More flexible than uniform
  - » Can have a single mode anywhere in its range
  - » Need minimum, maximum, and most likely values for values



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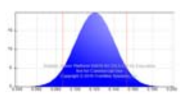
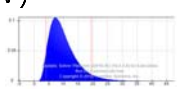
Continuous Distributions, cont.

- Normal: `random.normalvariate(mean,st_dev)`
  - » Infinite tails – negative values are possible
  - » Symmetric – most likely outcome is midpoint
  - » Appropriate for representing uncertain quantities influenced by a large number of independent factors



- Lognormal: `random.lognormvariate(mean,st_dev)`
  - » Logarithm of the variable is normally distributed
  - » Useful for continuous but nonnegative random variables



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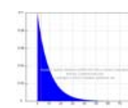
Continuous Distributions, cont.

- Exponential: `random.expovariate(lambd)`
  - » Rate: `lambd = 1/mean`
  - » Time until next event
  - » Reliability analysis – time to failure
  - » Queueing analysis – time until next customer arrival



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Discrete Probability Distributions

- Discrete Uniform: `random.randint(a,b)`
  - Equally likely integer value from a to b
- General Discrete RV: `random.choices(outcome_lst,prob_lst)`
  - Randomly selects an element from the outcomes list using the specified probabilities list
- Bernoulli: `numpy.random.binomial(1,p)`
  - 1 with prob. *p*, and 0 with prob. 1– *p*
- Binomial: `numpy.random.binomial(n,p)`
  - Number of "successes" out of *n* trials
  - Sum of *n* Bernoulli random variables

Discrete Probability Distributions

- Geometric: `numpy.random.geometric(p)`
  - Number of Bernoulli trials until the first success
- Poisson: `numpy.random.poisson(mean)`
  - Counting the number of items found in an area
  - Counting the number of events over a period of time

Retirement Simulation

A recent law school graduate would like your assistance in determining how much to save for retirement. She is planning to invest $3,000 in a tax-sheltered retirement fund at the end of each year. The rate of return each year can be modeled as a normally distributed random variable with a mean of 12 percent and a standard deviation of 2 percent.

a. If she is 30 years old now, how much money should she expect to have in her retirement fund at age 60?
b. What is the probability she will have more than $1 million in her retirement fund when she reaches age 60?
c. How much should she invest each year if she wants the mean value of her portfolio to be at least $1 million at age 60?
d. How much should she invest each year if she wants there to be a 90 percent chance of having at least $1 million in her retirement fund at age 60?

Estimating the Probability of an Event

- In some cases we want to estimate the probability of an event
  - e.g., probability that profit is greater than 0
- Use a for loop and counter or list comprehension with the appropriate event condition(s) to count the number of event occurrences in the sample and divide by the sample size

Summary

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