BANA4095: Decision Models – Spring 2021
Integer Optimization – Part 2

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Outline

- Modeling Fixed Costs
- Supply Chain Network Design
- Generating Alternate Solutions

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Fixed Costs

- Modeling both fixed and variable costs associated with a decision
  - *F* = fixed cost component
  - *c* = variable unit cost component
  - *x* = volume/quantity of activity (fractional variable)
  - *y* = yes/no decision (binary variable)
  - *M* = upper bound on value of *x*
- Total cost = *Fy* + *cx*
- Linking constraint: $x \le My$ $\quad y = 0 \Rightarrow x = 0$

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Threshold Levels

- A decision variable may have to fall between a minimum and a maximum value, otherwise it must be zero.
- Example: Quantity Discounts
  - *x* = volume/quantity of activity (fractional variable)
  - *y* = yes/no decision (binary variable)
  - *m* = lower bound on value of *x*
  - *M* = upper bound on value of *x*
- Constraints: $x \le My$
  $x \ge my$

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Example: Supply Chain Network Design

The Martin-Beck Company operates a plant in St. Louis with an annual capacity of 30,000 units. Product is shipped to regional distribution centers located in Boston, Atlanta, and Houston. Because of an anticipated increase in demand, Martin-Beck plans to increase capacity by constructing a new plant in one or more of the following cities: Detroit, Toledo, Denver, or Kansas City.

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Example: Supply Chain Network Design

The estimated annual fixed cost and the annual capacity for the four proposed plants are as follows:

Proposed Plant	Annual Fixed Cost	Annual Capacity
Detroit	$175,000	10,000
Toledo	$300,000	20,000
Denver	$375,000	30,000
Kansas City	$500,000	40,000

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Example: Supply Chain Network Design

The company's long-range planning group developed forecasts of the anticipated annual demand at the distribution centers as follows:

Distribution Center	Annual Demand
Boston	30,000
Atlanta	20,000
Houston	20,000

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Example: Supply Chain Network Design

The shipping cost per unit from each plant to each distribution center is shown below.

	Distribution Centers		
Plant Site	Boston	Atlanta	Houston
Detroit	5	2	3
Toledo	4	3	4
Denver	9	7	5
Kansas City	10	4	2
St. Louis	8	4	3

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Example: Supply Chain Network Design

- Decision Variables

y_1 = 1 if a plant is constructed in Detroit; 0 if not
y_2 = 1 if a plant is constructed in Toledo; 0 if not
y_3 = 1 if a plant is constructed in Denver; 0 if not
y_4 = 1 if a plant is constructed in Kansas City; 0 if not

x_{ij} = the units shipped (in 1000s) from plant i to distribution center j , with i = 1, 2, 3, 4, 5 and j = 1, 2, 3

Example: Supply Chain Network Design

- Problem Formulation

$$\begin{aligned}
\text{Min}\quad & 5x_{11} + 2x_{12} + 3x_{13} + 4x_{21} + 3x_{22} + 4x_{23} + 9x_{31} + 7x_{32} + 5x_{33} + 10x_{41} + 4x_{42} \\
& + 2x_{43} + 8x_{51} + 4x_{52} + 3x_{53} + 175y_1 + 300y_2 + 375y_3 + 500y_4
\end{aligned}$$

s.t.

$x_{11} + x_{12} + x_{13} - 10y_1$	$\leq$	0	Detroit capacity
$x_{21} + x_{22} + x_{23} - 20y_2$	$\leq$	0	Toledo capacity
$x_{31} + x_{32} + x_{33} - 30y_3$	$\leq$	0	Denver capacity
$x_{41} + x_{42} + x_{43} - 40y_4$	$\leq$	0	Kansas City capacity
$x_{51} + x_{52} + x_{53}$	$\leq$	30	St. Louis capacity
$x_{11} + x_{21} + x_{31} + x_{41} + x_{51}$	$=$	30	Boston demand
$x_{12} + x_{22} + x_{32} + x_{42} + x_{52}$	$=$	20	Atlanta demand
$x_{13} + x_{23} + x_{33} + x_{43} + x_{53}$	$=$	20	Houston demand

$x_{ij} \geq$ for all i and j; $y_1, y_2, y_3, y_4 = 0, 1$

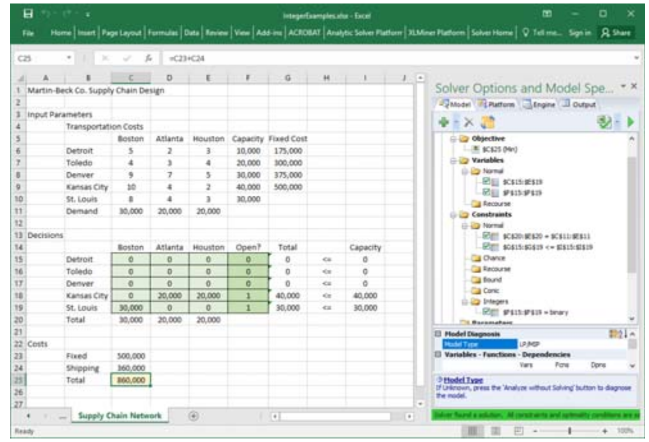
Example: Supply Chain Network Design

- Optimal Solution

Construct plant in Kansas City (y_4 = 1).

Ship 20,000 units: Kansas City to Atlanta (x_{42} = 20),
Ship 20,000 units: Kansas City to Houston (x_{43} = 20),
Ship 30,000 units: St. Louis to Boston (x_{51} = 30).

Total cost: $860,000 including fixed cost of $500,000.



Martin-Beck Co. Supply Chain Design

Input Parameters

Transportation Costs

	Boston	Atlanta	Houston	Capacity	Fixed Cost
Detroit	5	2	3	10,000	175,000
Toledo	4	3	4	20,000	300,000
Denver	9	7	5	30,000	375,000
Kansas City	10	4	2	40,000	500,000
St. Louis	8	4	3	30,000	
Demand	30,000	20,000	20,000		

Decisions

	Boston	Atlanta	Houston	Open?	Total		Capacity
Detroit	0	0	0	0	0	<=	0
Toledo	0	0	0	0	0	<=	0
Denver	0	0	0	0	0	<=	0
Kansas City	0	20,000	20,000	1	40,000	<=	40,000
St. Louis	30,000	0	0	1	30,000	<=	30,000
Total	30,000	20,000	20,000				

Costs

Fixed	500,000
Shipping	360,000
Total	860,000

Extensions and Analysis

- Logical Constraints
  - » Exactly one plant must be located in Detroit or Toledo
- Optimization Parameter Analysis
  - » St. Louis Capacity: 30,000 to 70,000
  - » Kansas City Fixed Cost: 500,000 to 600,000

Generating Alternate Solutions

- Often helpful to generate alternate optimal and even suboptimal alternate solutions for managerial consideration
  - » Model doesn't include every factor that impacts the decision
  - » Some parameters are inaccurate or uncertain
- Add a new constraint (cut) to the model to eliminate the current optimal solution from the feasible region

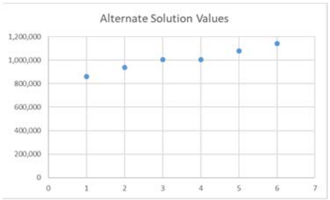
Generating Alternate Solutions

- Constraint/Cut Coefficients
  - » If x_i = 1 then a_i = 1; if x_i = 0 then a_i = -1
  - » M is the number of x_i = 1 in the current optimal solution
  - » New cut constraint: $a_1x_1 + a_2x_2 + \cdots + a_nx_n \le M - 1$
  - » At least one of the x_i = 1 values must change to x_i = 0 or at least one of the x_i = 0 values must change to x_i = 1
- Example: x = (1, 1, 1, 0) → M = 3 and a = (1, 1, 1,-1)

  New constraint: $x_1 + x_2 + x_3 - x_4 \le 2$
- Use this technique to find the next best solution for the supply chain network example

Generating Alternate Solutions

	Cut1	Cut2	Cut3	Cut4	Cut5
	-1	1	1	-1	-1
	-1	-1	-1	1	1
	-1	1	-1	1	-1
	1	-1	1	-1	1
	1	1	1	1	1
LHS	-2	2	0	2	0
	<=	<=	<=	<=	<=
RHS	1	2	2	2	2
Soln1	Soln2	Soln3	Soln4	Soln5	Soln6
0	1	1	0	0	1
0	0	0	1	1	1
0	1	0	1	0	1
1	0	1	0	1	0
1	1	1	1	1	1
860,000	940,000	1,005,000	1,005,000	1,080,000	1,140,000



Summary

- Modeling Fixed Costs
- Supply Chain Network Design
- Generating Alternate Solutions

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