BANA4095: Decision Models – Spring 2021
Linear Optimization - Part 1

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1

Outline

- Linear Optimization Concepts
- Excel Solver Modeling Tips
- General Categories of LP Models
- Veerman Furniture Example
- Interpreting & Applying LP Solutions

2

Types of Optimization Models

Type	Variables	Relationships
Linear Program (LP)	Continuous	Linear
Nonlinear Program (NLP)	Continuous	Nonlinear or Linear
Integer Program (IP)	Integer	Linear
Mixed Integer-Linear (MILP)	Integer or Continuous	Linear
Mixed Integer Nonlinear (MINLP)	Integer or Continuous	Nonlinear or Linear

3

What is a Linear Program

- Special class of optimization models
  - » Objective function and all constraints are linear expressions
    - *a* and *b* are scalars (numbers); *x* and *y* are variables
      - $ax + b$ OR $ax + by$
      - NOT $ax^2 + bxy$
  - » Decision variables are continuous/fractional
- Easiest class of optimization models to solve
  - » Simplex Algorithm (1947, George Dantzig)
  - » Always finds an optimal solution; can find an optimal solution relatively quickly even for very large problems.
- Is the Armstrong Bike problem a linear program?

4

Example – Sidneyville Desk Mfg.

- Allocation/Product Mix Problem
- Produces two types of desk
- Using three types of wood in every desk
  (measured in board feet, b.f.)

Type	Profit/desk
Rolltop	$115
Regular	$90

	Amount Used		Amount
Wood	Rolltop	Regular	Available
Pine	10	20	200
Cedar	4	16	128
Maple	15	10	220

Sidneyville Linear Programming (LP) Formulation

$\max \quad 115x_1 + 90x_2$ Maximize Total Profit

s.t. $10x_1 + 20x_2 \le 200$ Pine

$4x_1 + 16x_2 \le 128$ Cedar

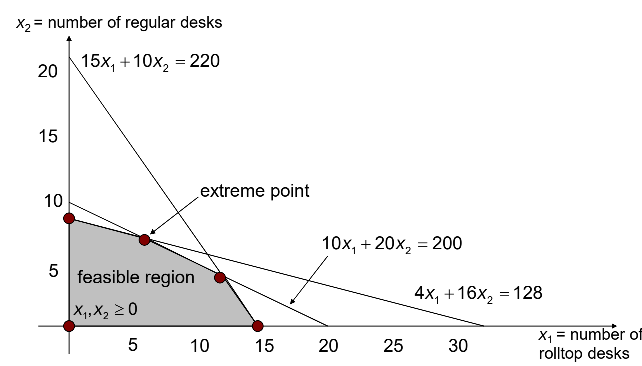
$15x_1 + 10x_2 \le 220$ Maple

$x_1, x_2 \ge 0$ Non-negative

Feasibility

- A feasible solution is a combination of decision variable values that satisfy all of the constraints
- The feasible region is the set of all feasible solutions
- Extreme points are the "corners" of the feasible region

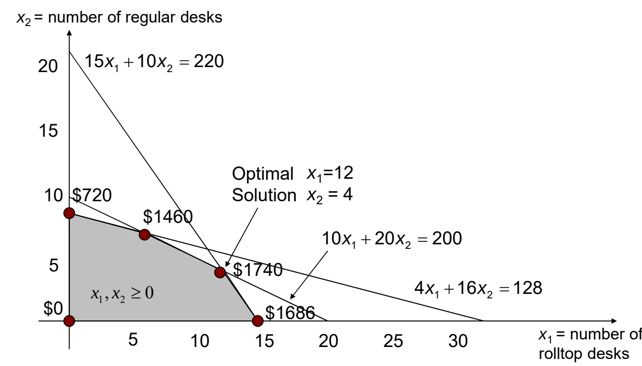
Sidneyville Feasible Region



Optimality

- An optimal solution is a feasible solution that achieves the best possible objective function value within the feasible region
  - » No other feasible solution has a better objective value
  - » There may be multiple optimal solutions
- In an LP, at least one of the extreme points is an optimal solution
  - » Graphical method
  - » Simplex method (Excel Solver)

Sidneyville Graphical Solution



Simplex Method

- Invented by George Dantzig in 1947
- The Simplex Method uses linear algebra to “pivot” from one vertex to another until it stops at an optimal vertex.
- The gradients of the objective function and the constraints are used to determine the search direction
- The gradients of the current active/binding constraints are used to compute the current vertex
- The algorithm stops when the gradient of the objective function can be expressed as a linear combination of the gradients of the active/binding constraints.

Solver Engines/Algorithms

- Various algorithms that are used to solve different classes of optimization problems. The two basic engines in Solver are

Engine	Objective	Constraints
Simplex LP	Linear	Linear
GRG Nonlinear	Nonlinear	Nonlinear

- The specific Solver Engine is selected from the drop down list at the top of the Engine tab
- Uncheck the “Automatically Select Engine” box

Solver Modeling Tip: SUMPRODUCT( ) Function

- Very useful for LP modeling
- SUMPRODUCT(A1:A3, B1:B3)
  =A1*B1+A2*B2+A3*B3

A1	B1
A2	B2
A3	B3

- SUMPRODUCT(A1:C1, A2:C2)
  = A1*A2+B1*B2+C1*C2

A1	B1	C1
A2	B2	C2

- SUMPRODUCT(A1:B2,C3:D4)
  = A1*C3+B1*D3+A2*C4+B2*D4

A1	B1
A2	B2

C3	D3
C4	D4

13

Solver Modeling Tip: Range Constraints

- Can use a range of cells for left-hand and/or right/hand side of a constraint to model multiple constraints
  - » D10:D15 >= E5
    - – Each cell in the range D10:D15 must be >= E5
  - » D10:D15 >= E10:E15
    - – Each cell in the range D10:D15 must be >= the corresponding cell in the range E10:E15

14

Solver Modeling Tip: Designing LP Spreadsheets

- Organize the LP model in a series of rows
- Each column of the model corresponds to a Decision Variable
- Decision Variables and Objective Coefficients at the top
- Constraints in a separate section below the DVs and Objective
- List similar constraints together

- Don't use decision variable cell address directly in a constraint
  - » Sensitivity Report will not include information on the constraint
  - » Instead, place a formula in another cell that references the decision variable and use this cell address in the Solver constraint setting

15

Solver Modeling Tip: Debugging

- Debugging optimization models can be difficult
- Read the error message carefully
  - » It may provide a clue about which model element(s) are generating the error
- Audit all the optimization model settings
- Are all the cell addresses/ranges accurate and complete?
  - » Min/Max? Assume Non-negative?
- Review/audit all the formulas in the spreadsheet
  - » Are they correctly computing the appropriate values?

16

General Types of LP Models

- **Allocation** – allocate limited resources across different activities
- **Covering** – select decisions to meet minimum requirements
- **Blending/Proportion** – decisions are subject to one or more constraints on a proportion or a weighted average computed from the decision variables
- **Network** – optimize decisions over a network structure

17

Veerman Furniture Company makes three kinds of office furniture: chairs, desks, and tables. Each product requires some labor in the parts fabrication department, the assembly department, and the shipping department. The furniture is sold through a regional distributor, who has estimated the maximum potential sales for each product in the coming quarter. Finally, the accounting department has provided some data showing the profit contributions on each product. The decision problem is to determine the product mix—that is, to maximize Veerman's profit for the quarter by choosing production quantities for the chairs, desks, and tables. The following data summarizes the parameters of the problem:

	Hours per Unit			Labor Hours
Department	Chairs	Desks	Tables	Available
Fabrication	4	6	2	1,850
Assembly	3	5	7	2,400
Shipping	3	2	4	1,500
Demand Potential	360	300	100	
Profit	$15	$24	$18	

18

Example: Veerman Furniture Co.

- Construct a model to determine the best production quantities
- Which constraints are binding or non-binding?
- What are some managerial implications/recommendations based on the solution and the sensitivity report?

19

Interpreting the Optimal Solution

"*All models are wrong, but some are useful*."
- George Box

- Much more should be said about the solution of an optimization model than simply stating: "This is the answer . . . ."
- Interpretation and implications of the proposed solution
- Sensitivity analysis
  - » Indicates robustness of the solution
  - » Indicates risks in the solution
  - » Suggests opportunities for even further improvements in the solution

20

Theory of Constraints (TOC)

- Championed by Eliyahu Goldratt
- “The Goal”
- Productivity improvement strategy that focuses on the bottleneck or constraining resource of a process
- In optimization terminology – the binding constraint(s)
- Identify ways to increase the productivity and capacity of the bottleneck or binding constraint

- Consider the binding constraints in the optimal solution of the Veerman Furniture example

21

Summary

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22