EE433 INTRODUCTİON TO DIGITAL SIGNAL PROCESSING

Computer Homework Part 1

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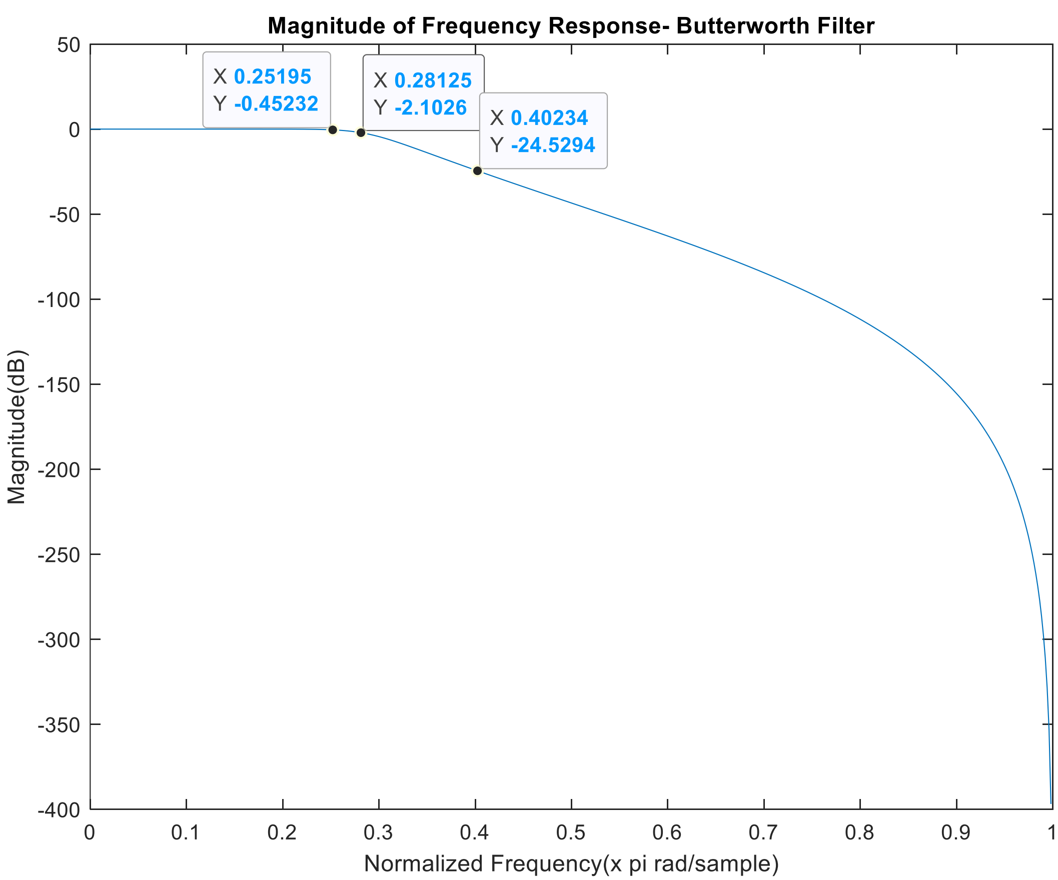
Question 1

Design a Butterworth filter with following specifications using impulse invariance method. Check the achieved gains at the critical frequencies. Plot the magnitude of frequency response.

$$0.9 \leq |H(e^{j\omega})| \leq 1, \qquad 0 \leq \omega \leq 0.25\pi$$

$$|H(e^{j\omega})| \leq 0.1, \qquad 0.4\pi \leq \omega \leq \pi$$

Simulation



Calculation:

$$|H_c(j\Omega)|^2 \; \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}$$

$$\rightarrow 1 + \left(\frac{0.25\pi}{\Omega_c}\right)^{2N} = \left(\frac{1}{0.9}\right)^2 \qquad \rightarrow 1 + \left(\frac{0.4\pi}{\Omega_c}\right)^{2N} = \left(\frac{1}{0.1}\right)^2$$

$$1 + \left(\frac{(0.25\pi)^{2N}}{\Omega_c}\right)^{2N} = \frac{100}{81} \qquad 1 + \left(\frac{0.4\pi}{\pi_c}\right)^{2N} = 100$$

$$(0.25\pi)^{2N} \cdot \frac{81}{19} = (0.4\pi)^{2N} \cdot \frac{1}{99}$$

$$\hookrightarrow 422.06 = (1.6)^{2N} \rightarrow 2N = 13.1 \rightarrow N = 6.55$$

we have to round to be integer

$$N \approx 7$$

$$N=7 \rightarrow 1 + \left(\frac{0.25\pi}{\Omega_c}\right)^{2N} = \frac{100}{81} \rightarrow \Omega_c = 0.88$$

$$N=7 \rightarrow 1 + \left(\frac{0.4\pi}{\Omega_c}\right)^{2N} = 100 \rightarrow \Omega_c = 0.90 \rightarrow \text{limits the cut off}$$

Also, $\Omega = \omega$, $T_d = 1 \rightarrow \omega_c = 0.288\pi$

Source Matlab Code

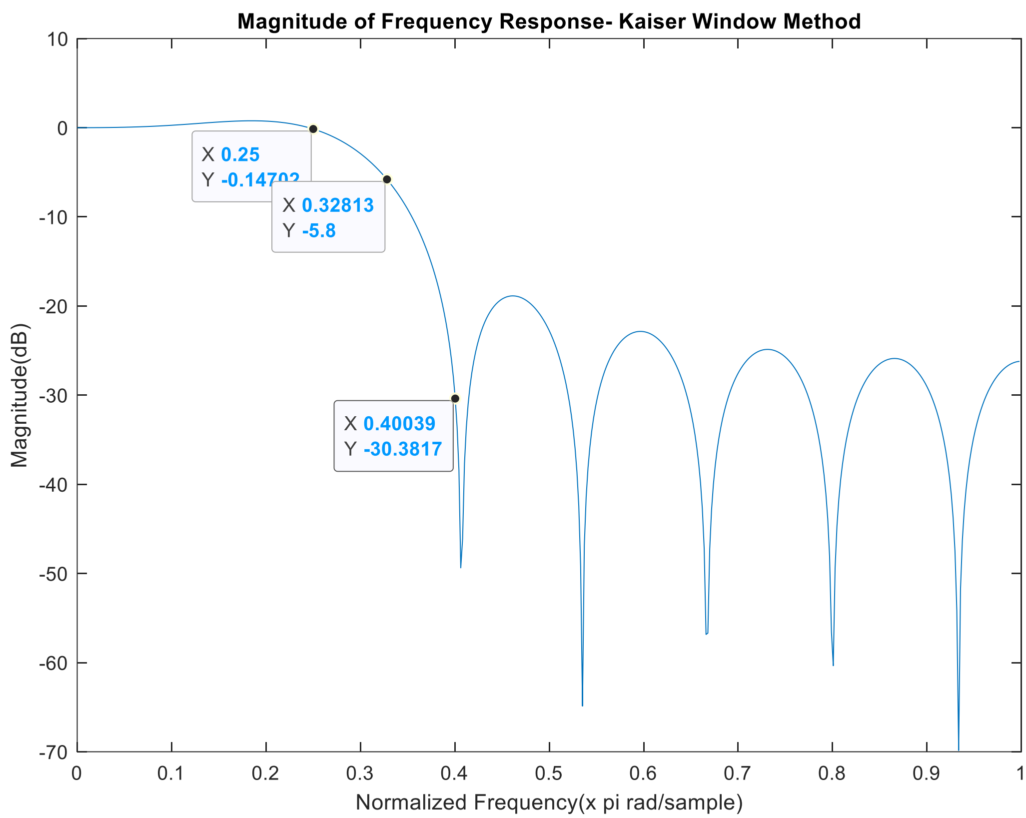
Butterworth Filter Design with **impulse invariance method**

```
[a,b] = butter(7, 0.91/pi);
[c,d] = freqz(a,b);
figure
plot(d/pi,20*log10(abs(c)));
title('Magnitude of Frequency Response- Butterworth Filter');
xlabel('Magnitude(dB)');
ylabel('Normalized Frequency(x pi rad/sample)');
```

Question 2

Utilize Kaiser window method in order to design a filter which meets the same specifications given above. Check the achieved gains at the critical frequencies. Plot the magnitude of frequency response.

Simulation



Calculation

$$\omega_p = 0.25\pi \quad , \quad \omega_s = 0.4\pi \quad , \quad \delta = 0.1$$

$$\omega_c = \frac{\omega_p + \omega_s}{2} = \frac{0.25\pi + 0.4\pi}{2} = 0.325\pi$$

$$\Delta\omega = \omega_s - \omega_p = 0.4\pi - 0.25\pi = 0.15\pi$$

$$A = -20\log\delta = -20\log(0.1) = 20$$

$$M = \frac{A-8}{2.285 \cdot \Delta\omega} = \frac{12}{2.285(0.15\pi)} = 11.144 \cong 12 \rightarrow$$

$11.144 \cong 12$
we have to round to be integer.

To get less than 0.1 ripple, we increased order to 14.

Source Matlab Code

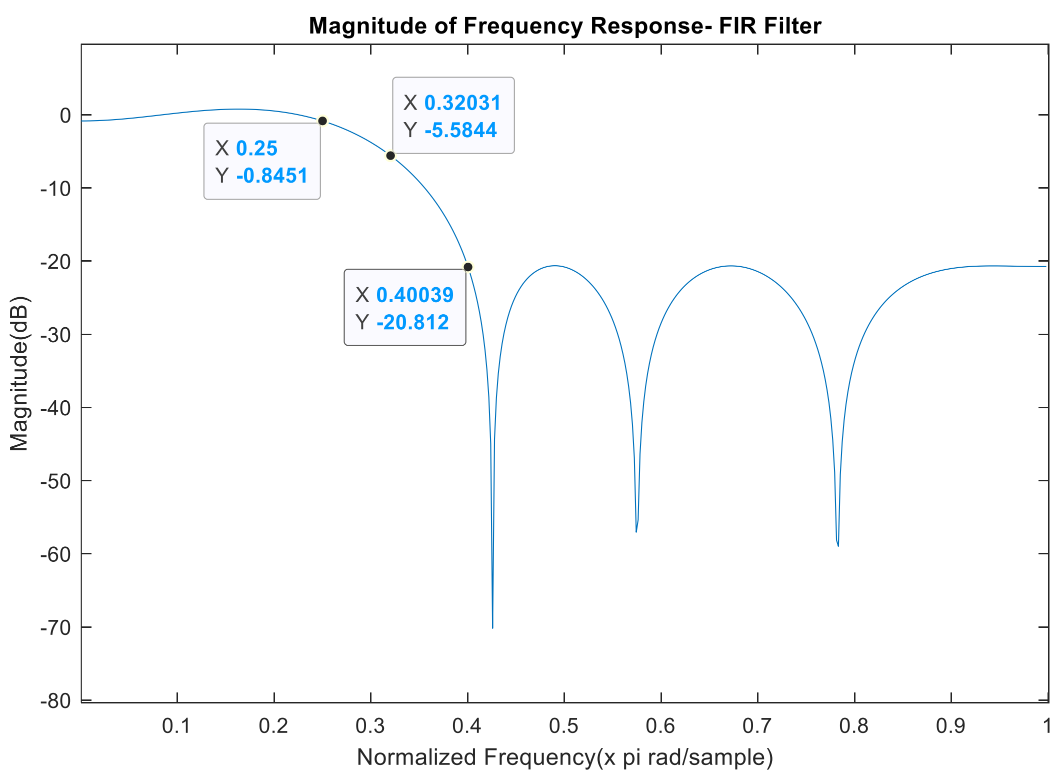
```
Kaiser Window Method to design a Filter

[k,l,alpha] = kaiserord([0.25 0.4],[1 0],[0.1 0.1],2);
h = fir1(k+2,l,kaiser(k+3,alpha));
figure
[m,n] = freqz(h,1);
plot(n/pi,20*log10(abs(m)));
title('Magnitude of Frequency Response- Kaiser Window Method');
ylabel('Magnitude(dB)');
xlabel('Normalized Frequency(x pi rad/sample)');
```

Question 3

Utilize built-in programs of MATLAB or OCTAVE to design an FIR filter which meets the same specifications given above. Check the achieved gains at the critical frequencies. Plot the magnitude of frequency response.

Simulation



Source Matlab Code

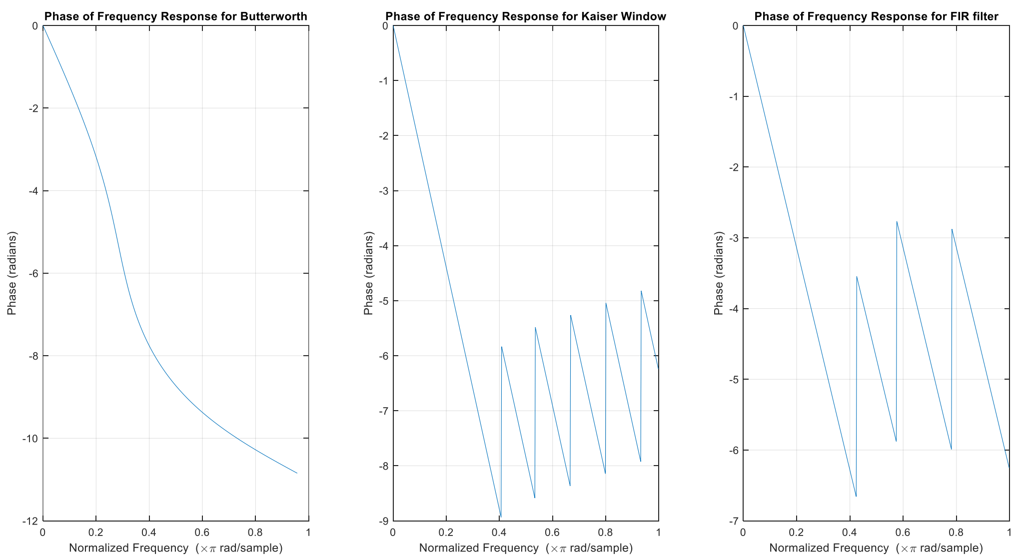
Using Parks-McClellan Algorithm program, FIR filter design.

```
y= [0 0.25 0.4 1];
z = [1 1 0 0];
t = firpm(10,y,z);
[hir,pir] = freqz(t,1);
figure
plot(pir/pi,20*log10(abs(hir)));
title('Magnitude of Frequency Response- FIR Filter');
ylabel('Magnitude(dB)');
xlabel('Normalized Frequency(x pi rad/sample)');
```

- To get less ripple, we set filter order to 10.

Question 4

(a) Plot the phases of the frequency responses corresponding to the filters designed in quesetions 1,2 and 3. What are your observations?



- We can say, In the passband intervals, the Butterworth filter has shifted away from linearity. In the passband, the Butterworth filter is mostly linear, but in the transition band, it become a nonlinear. (0.25π-0.4π).

- At pass band intervals, we noticed that the other FIR filters we installed create linear phases. The phase response of both FIR filters is linear and follows a straight line. After the main lobe, there are discontinuities on the left side of each side lobe of these linear phase filters.

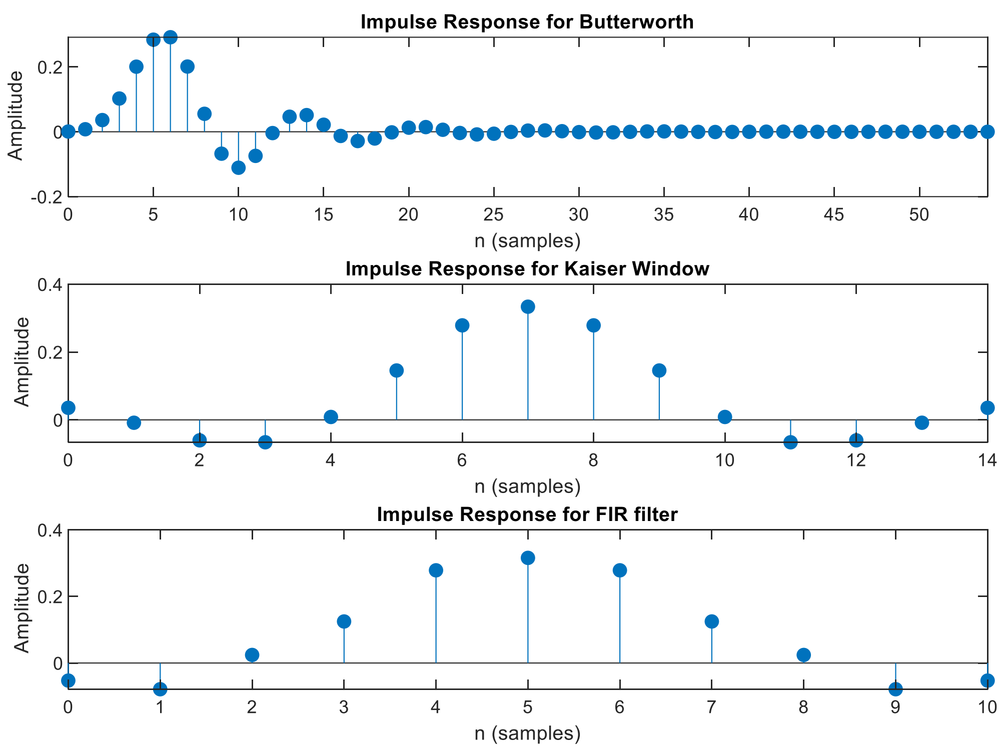
Source Matlab Code

```
Phase of Frequency Responses Of The Filters
figure
subplot(1,3,1)
phasez(a,b);
title('Phase of Frequency Response for Butterworth');


subplot(1,3,2)
phasez(h);
title('Phase of Frequency Response for Kaiser Window');


subplot(1,3,3)
phasez(t);
title('Phase of Frequency Response for FIR filter');
```

(b) Plot the impulse responses of the filters designed in questions 1,2 and 3. What are your observations?



- We can say, Butterworth filters give response quickly so that they have more efficiency and faster if we compare them with the FIR Filters. Butterworth filter is lower order, less stable and longer length than other FIR filters. Cascading processes and lower order give a permission more efficient and faster computation than FIR filters.

-We can say their high order and shortness are the main reason why FIR filters do not react quickly.

-The deficiency of feedback is why FIR filters are more stable than Butterworth filters.

-Also, Butterworth filters show recursive action.

Impulse Responses Of The Filters MATLAB Code:

```
Figure

subplot(3,1,1)
impz(a,b);
title('Impulse Response for Butterworth');

subplot(3,1,2)
impz(h,1)
title('Impulse Response for Kaiser Window');

subplot(3,1,3)
impz(t,1)
title('Impulse Response for FIR filter');
```

The Complete Source Code

```
%1
[a,b] = butter(7, 0.91/pi);
[c, d] = freqz(a,b);
figure
plot(d/pi,20*log10(abs(c)));
title('Magnitude of Frequency Response- Butterworth Filter');
ylabel('Magnitude(dB)');
xlabel('Normalized Frequency(x pi rad/sample)');


%2
[k,l,alpha] = kaiserord([0.25 0.4],[1 0],[0.1 0.1],2);
h = fir1(k+2,l,kaiser(k+3,alpha));
figure
[m,n] = freqz(h,1);
plot(n/pi,20*log10(abs(m)));
title('Magnitude of Frequency Response- Kaiser Window Method');
ylabel('Magnitude(dB)');
xlabel('Normalized Frequency(x pi rad/sample)');
%3
y= [0 0.25 0.4 1];
```

```
z = [1 1 0 0];
t = firpm(10,y,z);
[hir,pir] = freqz(t,1);
figure
plot(pir/pi,20*log10(abs(hir)));
title('Magnitude of Frequency Response- FIR Filter');
ylabel('Magnitude(dB)');
xlabel('Normalized Frequency(x pi rad/sample)');


%4
%a
figure
subplot(1,3,1)
phasez(a,b);
title('Phase of Frequency Response for Butterworth');


subplot(1,3,2)
phasez(h);
title('Phase of Frequency Response for Kaiser Window');


subplot(1,3,3)
phasez(t);
title('Phase of Frequency Response for FIR filter');


%b
figure

subplot(3,1,1)
impz(a,b);
title('Impulse Response for Butterworth');

subplot(3,1,2)
impz(h,1)
title('Impulse Response for Kaiser Window');

subplot(3,1,3)
impz(t,1)
title('Impulse Response for FIR filter');
```