EE203 - Electrical Circuits Laboratory

Experiment - 4 Simulation
Linearity and Superposition

Objectives

1. Observe linear response of resistive circuits.
2. Observe applications of superposition theorem.

Background

Linearity

Linearity is a property that is a common attribute of electrical components, circuits, and other systems. Mathematical methods used for analysis of ordinary circuits are based on the assumption of linear response. Mathematically, a system has linear response if the following two properties are satisfied:

1. **Homogeneity (scaling)** property requires that if the output of system for an input, $\mathbf{x}(\mathbf{t})$, is given by

$$\mathbf{y}(\mathbf{t}) = \mathbf{F}\{\mathbf{x}(\mathbf{t})\},$$

then the system output to $\mathbf{x(t)}$ multiplied by a real constant $\mathbf{k}$ is

$$\mathbf{k\,y}(\mathbf{t}) = \mathbf{F}\{\mathbf{k\,x}(\mathbf{t})\}.$$

2. **Additivity** property requires that if the outputs of system for the inputs, $\mathbf{x_1(t)}$ and $\mathbf{x_2(t)}$, are given by

$$\mathbf{y_1}(\mathbf{t}) = \mathbf{F}\{\mathbf{x_1}(\mathbf{t})\} \text{ and } \mathbf{y_2}(\mathbf{t}) = \mathbf{F}\{\mathbf{x_2}(\mathbf{t})\},$$

then the system output to sum of these inputs is

$$\mathbf{y_1}(\mathbf{t}) + \mathbf{y_2}(\mathbf{t}) = \mathbf{F}\{\mathbf{x_1}(\mathbf{t}) + \mathbf{x_2}(\mathbf{t})\}.$$

These two properties have similar implications, and in practice, a system that satisfies one of these properties also satisfies the other property. It is possible to find an exceptional mathematical function that is homogeneous but not additive, or a function that is additive but not homogeneous. As long as we are dealing with real hardware, an electronic component is either considered as a linear device or it has none of these properties.

If a circuit with multiple inputs is made of linear components, then the circuit response can be calculated as sum of the responses to individual inputs. Similarly, we can calculate the output of a linear circuit for a complex input signal that can be expressed as a combination of simple input functions. If all input functions generate known outputs, then we can combine these known outputs to obtain the complete circuit response.

In reality, all electronic devices present some nonlinear behavior, and manufacturers of electronic components specify linearity of their products within certain tolerance limits. Even a simple resistor can have a nonlinear voltage-current relationship. A resistor warms up as it conducts current, and its resistance increases at higher temperatures. Consequently, voltage-to-current ratio increases when higher currents flow through the resistor. Hopefully, such nonlinear effects are not significant for most of the passive components in ordinary applications, and there are several design methods that minimize the errors resulting from nonlinear behavior of components.

Linearity requirements for electronic circuits are determined according to the particular application. As an example, consider an amplifier that is required to have a fixed offset voltage at its output. The amplifier output given by

$$v_{out}(t) = V_{os} + G\, v_{in}(t)$$

does not satisfy the mathematical linearity requirements. On the other hand, this amplifier is a linear device, if $G\, v_{in}(t)$ is considered as the useful output signal, knowing that the V_{os} offset is just a reference voltage added intentionally to the output.

There are some applications that specifically require nonlinear behavior. A typical example is the logarithmic volume control of audio devices that compensates for logarithmic sensitivity of human ear to sound intensity. If a person rates the relative intensity of two sound waves as $S_2 = 2\, S_1$, then the actual relation between these intensities is given by $\log(S_2) = 2 \log(S_1)$. Properly designed audio devices have logarithmic potentiometers or waveform calculations to establish a logarithmic volume control range corresponding to a linear scale. In another example, logarithmic and exponential response of some components are utilized in design of modulator circuits that multiply two input signals.

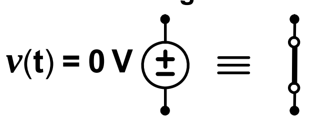
Superposition

Superposition is a calculation method that simplifies circuit analysis when there are multiple independent voltage or current sources in a linear circuit. According to the superposition theorem, responses resulting from individual sources are calculated one by one, and then these responses are added up to obtain the complete circuit response. All circuit components must have a linear voltage-current relationship for this addition to produce the correct circuit response.

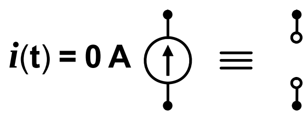
One of the independent sources remains active as we calculate its contribution to circuit response. All other voltage or current sources are "***disabled***" or "***turned off***" by setting their output to **0 V** or **0 A**. Disabling an independent source on a circuit branch should not change that branch voltage or current resulting from the active source.

- An ideal voltage source maintains its output voltage while it conducts any amount of current, and it should still conduct any current when its voltage output is set to **0 V**. The circuit element that has this behavior is nothing but a short circuit. Therefore, an independent voltage source is replaced by a short circuit when it is disabled.
- An ideal current source maintains its output current regardless of the voltage across it, and it should still allow any voltage when its current output is set to **0 A**. The circuit element that has this behavior is nothing but an open circuit. Therefore, an independent current source is replaced by an open circuit when it is disabled.

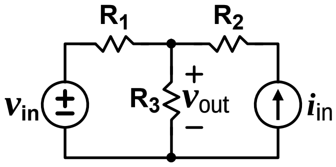
Disabled voltage source:

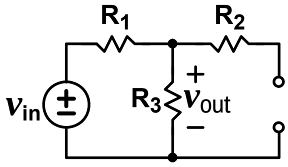


Disabled current source:



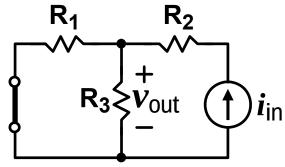
The sample circuit given below has one voltage source and one current source. The voltage across **R₃** can be found by adding the individual contributions resulting from one of the sources while the other source is turned off.





Calculate v_{out1} as a result of v_{in}, while i_{in} is disabled

$$v_{out1} = \frac{R_3}{R_1 + R_3} v_{in}$$



Calculate v_{out1} as a result of i_{in}, while v_{in} is disabled

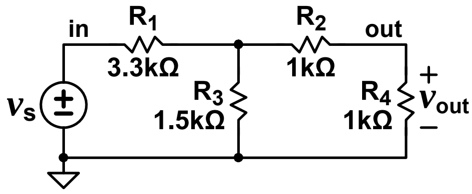
$$v_{out2} = \frac{R_1 R_3}{R_1 + R_3} i_{in}$$

The complete circuit response is given by the summation of the two results.

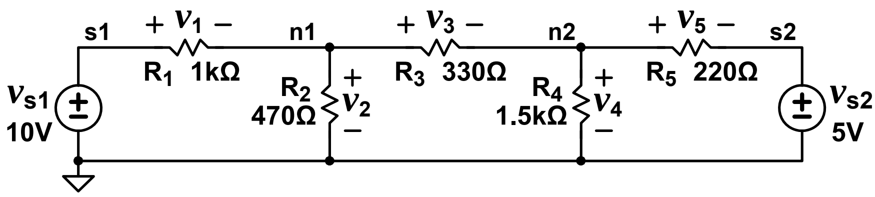
$$v_{out} = v_{out1} + v_{out2} = \frac{R_3}{R_1 + R_3} v_{in} + \frac{R_1 R_3}{R_1 + R_3} i_{in}$$

Preliminary Work

1. Calculate a proportionality factor **K** that gives the v_{out}/v_s ratio in the following circuit.



2.1 Calculate v_1, v_2, v_3, v_4, and v_5 for the circuit shown below using nodal or mesh analysis. Enter the results in the table given below.

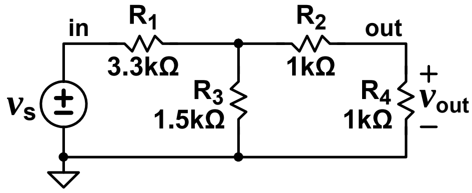


	v_1	v_2	v_3	v_4	v_5
Calculated value (**V**):					
Simulation result (**V**):					

2.2 Draw the same circuit schematic in LTspice, and enter the simulation results in the table given above. Save the schematic file since it will be used in the procedure section. If there is any significant difference between your calculations and the simulation results, then make the necessary corrections to obtain consistent results.

Procedure

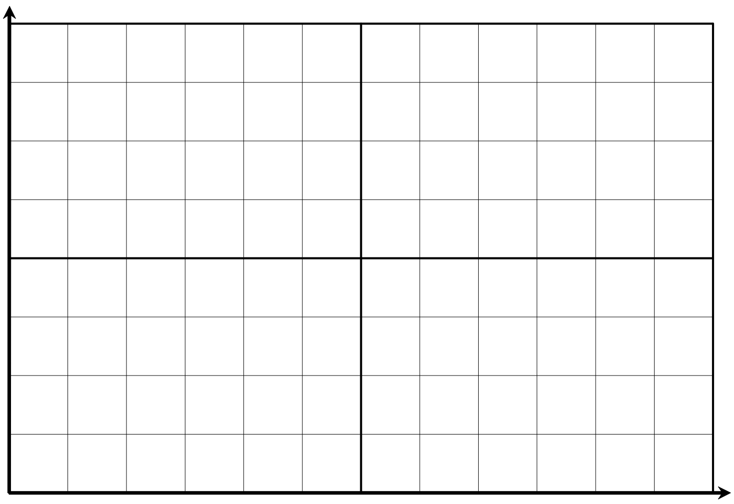
1.1 Make the following circuit schematic on LTspice.



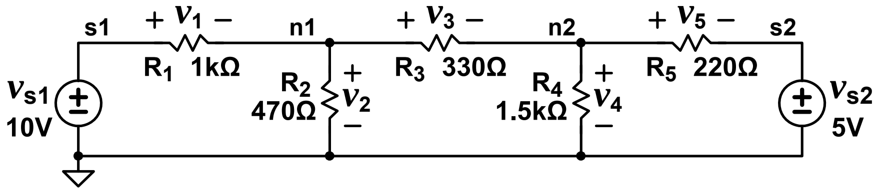
1.2 Set v_s source to obtain DC outputs from **2 V** to **10 V** in **~2 V** steps and measure v_{out} at each v_s setting. Enter the measured voltages and v_{out}/v_s, ratio in the following table.

Input v_s (V)	Output v_{out} (V)	v_{out}/v_s ratio

1.3 Set v_s source to obtain **500 Hz** triangular signal with **10 V**p-p amplitude and **0 V** DC offset. Set simulation time to **10 ms**. Display the v_s and v_{out} waveforms. Add another trace in the form of "`K * V(in)`" where `K` is the v_{out}/v_s ratio found in the preliminary work. Check if there is a perfect match between v_{out} and the "`K * V(in)`" waveform and verify the linear relationship between v_s and v_{out}.



2.1 Build the following circuit and set v_{s1} **= 10 V** and v_{s2} **= 5 V**. Make sure that you enter the proper node labels (i.e. **s1**, **s2**, **n1**, **n2**) that will help you easily identify all voltages marked on the schematic. Measure the voltages, v_1, v_2, v_3, v_4, and v_5, shown on the schematic and record their values in the first row of the table given below.



2.2 Disconnect v_{s2}, and replace it with a short circuit. Record the measured voltages in the second row of the table.

2.3 Disconnect v_{s1}, replacing it with a short circuit, and connect v_{s2} **= 5 V** source again. Record the measured voltages in the third row of the table.

Step	Source settings	v_1 (V)	v_2 (V)	v_3 (V)	v_4 (V)	v_5 (V)
2.1	v_{s1} = 10 V, v_{s2} = 5 V					
2.2	v_{s1} = 10 V, v_{s2} is disabled					
2.3	v_{s1} is disabled, v_{s2} = 5 V					
2.4	Sum of voltages measured in steps **2.2** and **2.3**					

2.4. Calculate the sum of voltages measured in steps **2.2** and **2.3** and write the results in the last row of the table. Compare these results with the voltages measured in step **2.1**.

2.5. Connect v_{s1} **= 10 V** again and set v_{s2} source to obtain **500 Hz** sinusoidal signal with **10 V**p-p amplitude and **0 V** DC offset. Measure DC offset and peak-to-peak amplitude of v_1, v_2, v_3, v_4, and v_5 on the displayed waveforms and record the results in the following table.

	v_1	v_2	v_3	v_4	v_5
DC offset (**V**):					
AC amplitude (**V**p-p):					

Questions

Q1. Compare the proportionality factor calculated in the preliminary work with the v_{out}/v_s ratios found in step **1.2** of the procedure. Calculate the proportionality factor again using the measured resistances in step **1.1**. Comment on the results.

Q2. Consider the following cases where one of the voltage supplies is changed by **1 V** in the circuit built for step **2** of the procedure.

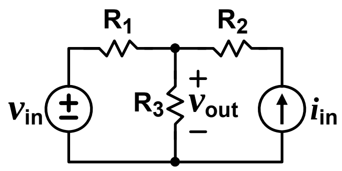
Case 1: v_{s1} is reduced to **9 V** or raised to **11 V**, while v_{s2} remains at **5 V**.

Case 2: v_{s2} is reduced to **4 V** or raised to **6 V**, while v_{s1} remains at **10 V**.

Which of these changes has in a bigger effect in v_1, v_2, v_3, v_4, and v_5? How can you make a decision looking at the voltage measurements made in steps **2.2** and **2.3** of the procedure?

Q3. How can you relate the DC offset and peak-to-peak AC voltage measurements made in step **2.5** to the voltage measurements made in steps **2.2** and **2.3** of the procedure?

Q4. The output voltage calculated for the following circuit (example given in the Background section) is independent of R_2. Explain why R_2 has no effect on v_{out}, considering its function when one of the sources is active and the other source is disabled.



$$v_{out} = \frac{R_3}{R_1 + R_3} v_{in} + \frac{R_1 R_3}{R_1 + R_3} i_{in}$$

Q5. How can you apply the superposition principle to calculate the power dissipated in a resistor, when the circuit contains independent DC and AC sources?