EE203 - Electrical Circuits Laboratory

Experiment - 5 Simulation
Thevenin and Norton Equivalent Circuits

Objectives

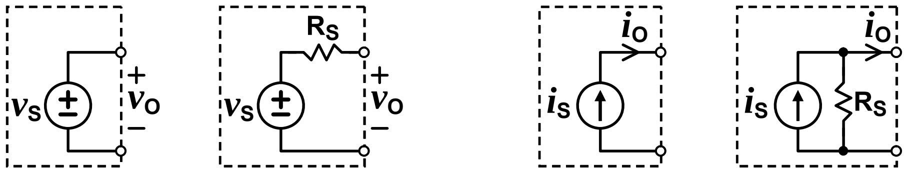
1. Apply analysis methods based on Thevenin and Norton equivalent circuits.
2. Practice methods for evaluation of equivalent circuit parameters.

Background

Voltage and Current Sources

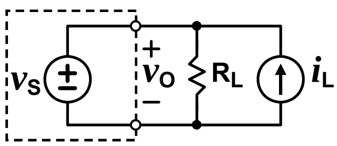
Ideal voltage and current sources are useful for analyzing circuits where load conditions have no significant effect on the source outputs. Ideal sources should be replaced by more realistic models that include ***output resistances*** as shown below, when the effect of load conditions cannot be ignored.

Ideal voltage source | **Voltage source with output resistance** | **Ideal current source** | **Current source with output resistance**



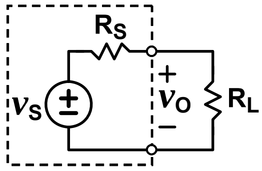
Output impedance or ***source impedance*** are other technical terms that can be used in place of output resistance. The term, ***impedance***, has a more general meaning that includes the contribution of reactive components for AC signals.

Current flowing through an ideal voltage source has no effect on its output voltage. Differences in behaviors of the ideal voltage source and the source model with output resistance are summarized in the following.



Ideal voltage source:

- $v_O = v_S$ always, regardless of $\mathbf{R_L}$ load resistance.
- v_O is not affected by any i_L load current that flows through the voltage source.



Voltage source model with $\mathbf{R_S}$ output resistance:

- $\mathbf{R_L}$ and $\mathbf{R_S}$ form a voltage divider at the voltage source output:

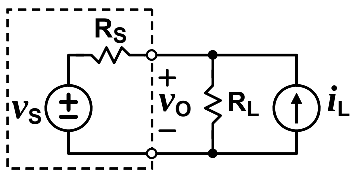
$$v_O = v_S \frac{R_L}{R_S + R_L}$$

The voltage source behaves like an ideal source as $\mathbf{R_S}$ approaches to zero:

$$v_O \simeq v_S \text{ when } R_S << R_L$$

- When there is an additional i_L load current, it also affects v_O:

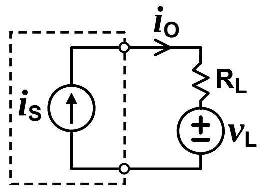
$$v_O = v_S \frac{R_L}{R_S + R_L} - i_L \frac{R_S\, R_L}{R_S + R_L}$$



In case of a current source model, the $\mathbf{R_S}$ output resistance is added in parallel to the current source. This parallel output resistance forms a current divider when it is combined with a load resistor. Addition of $\mathbf{R_S}$ to an ideal current source has consequences similar to the behavior of voltage sources.

Ideal current source:

- $i_O = i_S$ always, regardless of $\mathbf{R_L}$ resistance.
- i_O is not affected by any v_L load voltage that changes voltage across the current source.



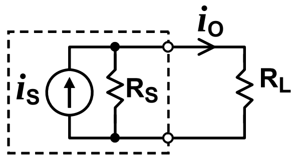
Current source model with $\mathbf{R_S}$ output resistance:

- $\mathbf{R_L}$ and $\mathbf{R_S}$ form a current divider at the current source output:

$$i_O = i_S \frac{R_S}{R_S + R_L}$$

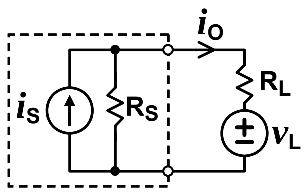
The current source behaves like an ideal source as $\mathbf{R_S}$ approaches to infinity:

$$i_O \simeq i_S \quad \text{when} \quad R_S >> R_L$$



- When there is an additional v_L load voltage, it also affects i_O:

$$i_O = i_S \frac{R_S}{R_S + R_L} - v_L \frac{1}{R_S + R_L}$$

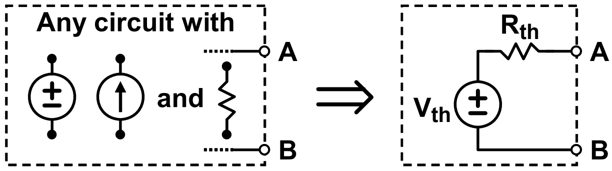


In practice, all signal sources have output resistances, and inputs of all electronic circuits have some loading effect when they are connected to these sources. For example, an ordinary laboratory signal generator has **50 Ω** output resistance. Oscilloscope probes with **1 MΩ** input resistance do not have a significant loading effect on the signal generator output, since $\mathbf{R_L = 1\ M\Omega}$ is much bigger than $\mathbf{R_S = 50\ \Omega}$. On the other hand, signal generator output decreases significantly when it is connected to a small resistor such as $\mathbf{R_L = 100\ \Omega}$, since this resistance is comparable to **50 Ω**.

If multiple circuits are connected one after another, then output of a circuit drives input of the following circuit, and input resistance of this circuit becomes load resistor for the driver. The signals at the output-to-input connections can be found easily if the input and output resistances of the circuits are known. **Thevenin's theorem** and **Norton's theorem** provide very useful methods to obtain equivalent circuits in the form of simple voltage or current source models.

Thevenin's Theorem

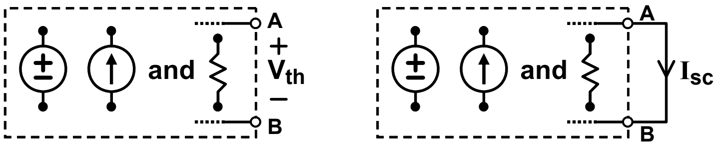
Any two-terminal linear circuit that contains voltage sources, current sources, and resistors can be replaced by an equivalent Thevenin voltage source, $\mathbf{V_{th}}$, in series with an equivalent Thevenin resistance, $\mathbf{R_{th}}$, as shown in the figure given below.



Thevenin voltage source, $\mathbf{V_{th}}$, and Thevenin resistance, $\mathbf{R_{th}}$, are found by following these steps:

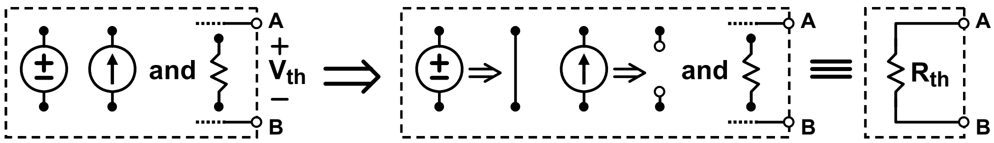
1. The equivalent Thevenin voltage $\mathbf{V_{th}}$ is the output voltage obtained when **A** and **B** terminals of the circuit are open circuited.

2. The equivalent Thevenin resistance $\mathbf{R_{th}}$ is given by the open circuit output voltage $\mathbf{V_{th}}$ divided by the short circuit current $\mathbf{I_{sc}}$ obtained when **A** and **B** terminals are connected together.



$$R_{th} = \frac{V_{th}}{I_{sc}}$$

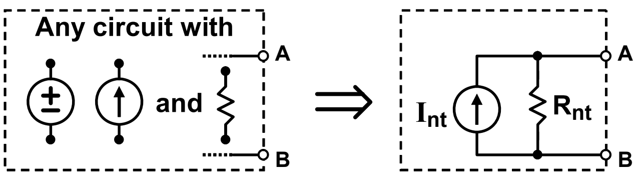
As an alternative method, the Thevenin resistance $\mathbf{R_{th}}$ can be calculated as the equivalent resistance between **A** and **B** terminals calculated when all voltage sources are replaced by a short circuit, and all current sources are replaced by an open circuit as shown below.



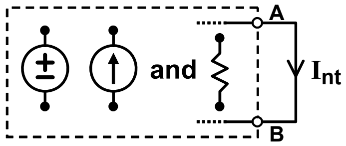
The Thevenin equivalent circuit behaves like the original circuit in terms of the voltage and current at the **A–B** terminals for any linear load condition. However, the power dissipated in the original circuit may not be the same as the power dissipated in the Thevenin equivalent circuit.

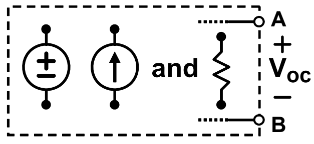
Norton's Theorem

Any two-terminal linear circuit that contains voltage sources, current sources, and resistors can be replaced by an equivalent Norton current source, $\mathbf{I_{nt}}$, in parallel with an equivalent Norton resistance, $\mathbf{R_{nt}}$, as shown in the figure given below.



The equivalent Norton current, $\mathbf{I_{nt}}$, is the short circuit output current obtained when **A** and **B** terminals of the circuit are connected together. The equivalent Norton resistance $\mathbf{R_{nt}}$ is equal to the Thevenin resistance $\mathbf{R_{th}}$ and it can be found by using the same methods described for the Thevenin equivalent circuit.





$$R_{nt} = \frac{V_{oc}}{I_{nt}}$$

Conversion between the two equivalent circuit representations can be obtained simply by using the following relations.

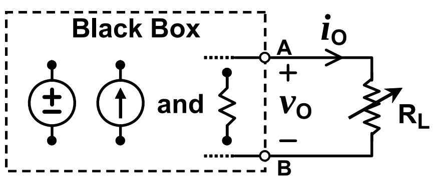
$$R_{th} = R_{nt} = \frac{V_{th}}{I_{nt}}$$

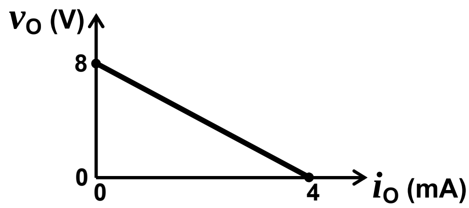
The Thevenin and Norton equivalent circuits behave the same way when a load resistor $\mathbf{R_L}$ is connected to **A** and **B** terminals. Voltage across $\mathbf{R_L}$ and current through $\mathbf{R_L}$ are same as the voltage and current that would be obtained with the original circuit in both cases.

Preliminary Work

1.a) Why shouldn't you measure the short circuit current of a regulated voltage supply with an ampermeter? (Hint: What happens when you short circuit an ideal voltage source?)
b) Why shouldn't you measure the open circuit voltage of a regulated current supply with a voltmeter? (Hint: What happens when you open circuit an ideal current source?)

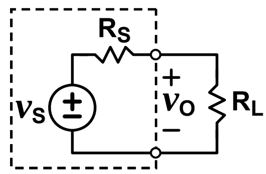
2. Following figure shows the v_O versus i_O plot of a black box obtained while the load resistance varies between $\mathbf{R_L = \infty}$ and $\mathbf{R_L = 0\ \Omega}$.





Calculate the equivalent circuit parameters, $\mathbf{V_{th}}$, $\mathbf{R_{th}}$, and $\mathbf{I_{nt}}$ for the black box, and draw the Thevenin and Norton equivalent circuits.
Note: The term "***black box***" refers to a circuit with unknown contents that can only be analyzed by measuring circuit response at its terminals.

3. A Thevenin equivalent circuit is given on the right.
a) Find the $\mathbf{R_L}$ value that makes $v_O = v_S/2$.
b) Calculate the power $\mathbf{P_L}$ dissipated in $\mathbf{R_L}$ when $v_O = v_S/2$.
c) Derive an expression that gives $\mathbf{P_L}$ as a function of v_S, $\mathbf{R_S}$, and an arbitrary value of $\mathbf{R_L}$.

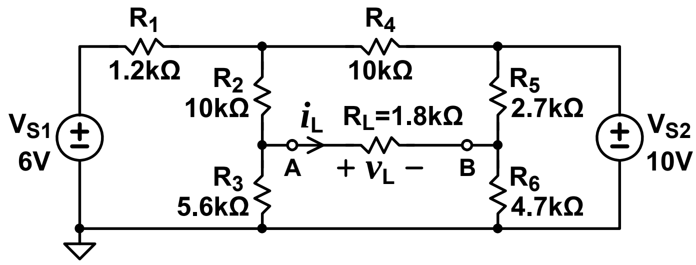


d) Find the $\mathbf{R_L}$ that makes $\mathbf{P_L}$ maximum using the expression found in **(c)**.
e) Compare the $\mathbf{R_L}$ values found in **(a)** and **(d)**.

4. Analyze the following circuit to determine the Thevenin equivalent circuit as seen by $\mathbf{R_L}$ between **A** and **B** terminals. There are two alternative methods that can be used to calculate the Thevenin voltage $\mathbf{V_{th}}$ for $\mathbf{R_L = \infty}$ and the short circuit current $\mathbf{I_{sc}}$ for $\mathbf{R_L = 0\ \Omega}$.

Method 1: Nodal analysis
How many node equations are required for analysis of the circuits obtained when $\mathbf{R_L = \infty}$ and $\mathbf{R_L = 0\ \Omega}$? Are there any circuit elements that can be omitted without affecting the Thevenin or Norton equivalent resistance? If so, why?

Method 2: Superposition
Calculate the open circuit voltage between **A** and **B** terminals for the two voltage sources. Is it necessary to calculate a different Thevenin resistance for the two voltage sources? If so, why?



a) Determine the Thevenin equivalent circuit (calculate $\mathbf{V_{th}}$ and $\mathbf{R_{th}}$) seen by the load resistor $\mathbf{R_L}$.

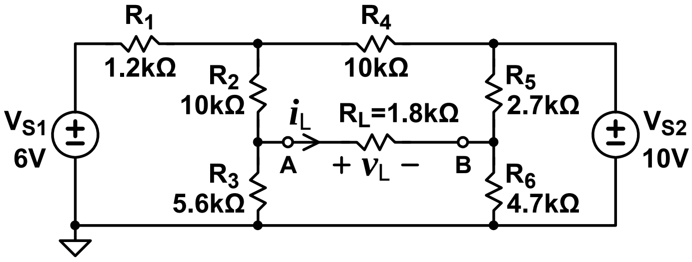
b) Determine the Norton equivalent circuit ($\mathbf{I_{nt}}$ and $\mathbf{R_{nt}}$) seen by $\mathbf{R_L}$.

c) Draw the Thevenin and Norton equivalent circuits using the values calculated in **(a)** and **(b)**.

d) Calculate v_L and i_L.

Procedure

1. Set up the circuit given below on LTspice.



1.1 Measure the load voltage v_L and load current i_L.

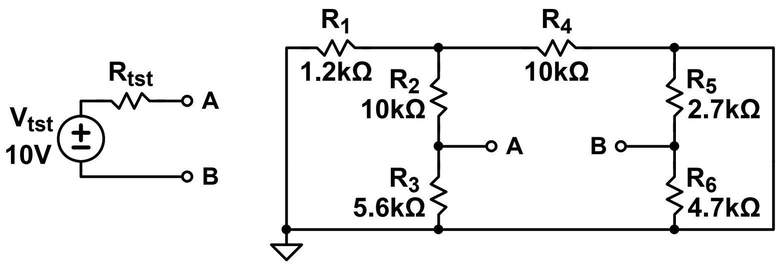
v_L = _________ i_L = _________

1.2 Remove $\mathbf{R_L}$ and measure the open circuit Thevenin voltage between **A** and **B** terminals.

$\mathbf{V_{th}}$ = _________

1.3 Replace the voltage sources with short circuits and measure the Thevenin resistance using a test voltage source $\mathbf{V_{tst}}$ as shown below. $\mathbf{V_{tst}}$ is connected to **A** and **B** terminals through the source resistance $\mathbf{R_{tst}}$. Find the Thevenin resistance $\mathbf{R_{th}}$ which is equal to the $\mathbf{R_{tst}}$ value that makes $\mathbf{V_{AB} = V_{tst}/2}$. You can start with setting $\mathbf{R_{tst}}$ value to the Thevenin resistance calculated in the preliminary work.

$\mathbf{R_{th}}$ = _________



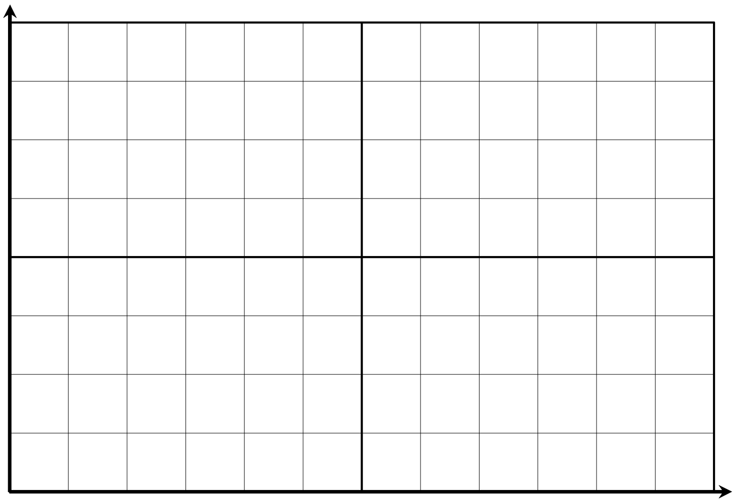
1.4 How can you determine $\mathbf{R_{th}}$ without the measurement in the step **1.3**, based on v_L, i_L, and $\mathbf{V_{th}}$ measured in steps **1.1** and **1.2**?

2. Find the equivalent circuit parameters, $\mathbf{V_{th}}$, $\mathbf{I_{nt}}$, and $\mathbf{R_{th}}$, based on a load line plot as in question **2** of preliminary work, instead of measuring the short circuit current. Connect the DC supplies and follow the steps given below to obtain the load line.

2.1 Set the $\mathbf{R_L}$ values given in the following table and record the voltage v_L and the current i_L for each $\mathbf{R_L}$ value.

R_L (kΩ)	v_L (V)	i_L (mA)
2		
4		
6		
8		
10		

2.2 Plot v_L as a function of i_L using the data obtained in step **2.1**.



2.3 Determine $\mathbf{V_{th}}$, $\mathbf{I_{nt}}$, and $\mathbf{R_{th}}$ using the graph above.

$\mathbf{V_{th}}$ = __________ $\mathbf{I_{nt}}$ = __________ $\mathbf{R_{th}}$ = __________

2.4 Compare the results obtained in part **1** and part **2.3**.

3.1 Replace $\mathbf{V_{S2}}$ with a short circuit (or set its value to **0 V**), and measure the first open circuit Thevenin equivalent voltage $\mathbf{V_{th1}}$.

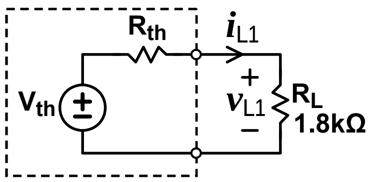
$\mathbf{V_{th1}}$ = __________

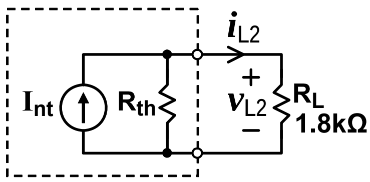
3.2 Restore $\mathbf{V_{S2}}$, replace $\mathbf{V_{S1}}$ with a short circuit, and measure the second open circuit Thevenin equivalent voltage $\mathbf{V_{th2}}$.

$\mathbf{V_{th2}}$ = __________

3.3 Describe the relationship between $\mathbf{V_{th1}}$, $\mathbf{V_{th2}}$, and $\mathbf{V_{th}}$ obtained in the previous parts of the experiment.

4. Setup the two circuits shown below. Use the $\mathbf{V_{th}}$, $\mathbf{I_{nt}}$ and $\mathbf{R_{th}}$ values obtained in step **2**.





4.1 Measure v_{L1} and i_{L1} with the Thevenin equivalent circuit:

v_{L1} = __________ i_{L1} = __________

4.2 Measure v_{L2} and i_{RL2} with the Norton equivalent circuit:

v_{L2} = __________ i_{L2} = __________

4.3 Compare v_{L1}, v_{L2}, i_{L1} and i_{L2} with the load voltage and current values previously measured in step **1**.

Questions

Q1. Compare the experimental results with those calculated in the preliminary work.

Q2. What is meant by the word "equivalent" in Thevenin and Norton equivalent circuits? Can you make power calculations for the original circuit based on its Thevenin equivalent? Why?

Q3. What are the practical applications of Thevenin's theorem? What are the advantages of using Thevenin or Norton equivalent circuits in circuit analysis?

Q4. Can you use the superposition principle to find the Thevenin equivalent when the original circuit contains voltage and current sources? Would it be possible to obtain the same Thevenin resistance for each and every one of the sources? Explain your answers.

Q5. Describe necessary measurements and calculations to determine the Thevenin equivalent of a black box by using a single fixed load resistor $\mathbf{R_L}$ and a voltmeter, without taking the risk of shorting the circuit outputs.