EE203 - Electrical Circuits Laboratory

Experiment - 7 Laboratory Report
RC and RL Circuits

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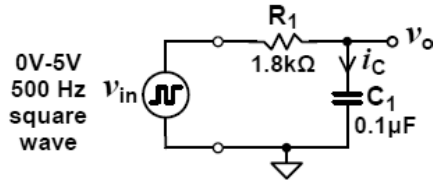
Preliminary Work

Note: Assume that capacitor voltage and inductor current reach their final steady state values before the next v_{in} transition, since τ is much shorter than period of v_{in} waveform in all questions given below.

1.a) Calculate the time constant $\tau = R_1C_1$ for the circuit given below and write $v_o(t)$ as a function of time after v_{in} switches between **0 V** and **5 V**.



1.b) Draw $v_{in}(t)$ and $v_o(t)$ on the same plot. Calculate the output voltages at $t = \tau$ on the rising and falling v_o waveforms and mark these voltage levels on the plot.

$\tau = R_1 . C_1$, and we know $R_1 = 1.8k\Omega$ $C_1 = 0.1\mu F$

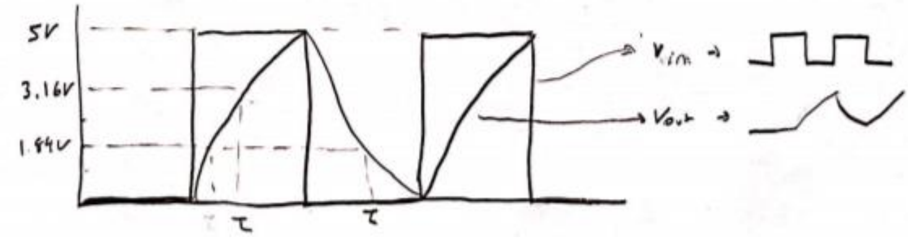
so $\tau = 1.8k \times 0.1\mu F = 0.18\ ms$

$V_{in} = 0V$, : $V_o(t) = V_o e^{-t/\tau}$

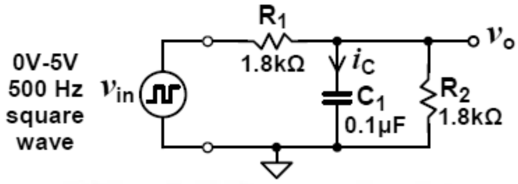
$V_{in} = 5V$, $V_o(t) = V_o(1 - e^{-t/\tau})$

$V_{in} = 0V$, $\rightarrow V(t) = V_o e^{-t/\tau} = 5V . e^{-1} = 1.84V$ ($V_{\tau fall}$)

$V_{in} = 5V$, $\rightarrow V(t) = V_o(1 - e^{-t/\tau}) = 5(1 - e^{-1}) = 3.16V$ ($V_{\tau rise}$)



2. Calculate the time constant τ and the output function $v_o(t)$ after v_{in} switches between **0 V** and **5 V** in the following circuit. Find the output voltages at **t =** τ on the rising and falling v_o waveforms.



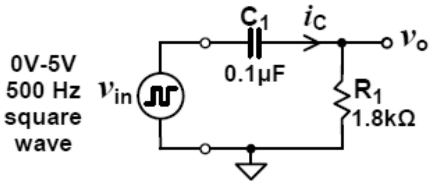
$\tau = (R_1 \parallel R_2)\, C_1 = \frac{1.8k}{2} \times 0.1\mu F = 0.09ms$

For $t<0$ using voltage dividing, $V_o = \frac{5V}{2} = 2.5V$

$V_{in} = 5V \longrightarrow V(t) = 2.5(1-e^{-t/\tau}) = 2.5(1-e^{-1}) = 1.58V\,(V_{\tau rise})$

$V_{in} = 0V \longrightarrow V(t) = 2.5(e^{-t/\tau}) = 2.5(e^{-1}) = 0.92V\,(V_{\tau fall})$

3. Calculate the time constant τ and the output function $v_o(t)$ after v_{in} switches between **0 V** and **5 V** in the following circuit. Find the output voltages at **t =** τ on the rising and falling v_o waveforms.



$\tau = R_1 C_1$, and we know $R_1 = 1.8\,k\Omega$, $C_1 = 0.1\,\mu F$

So that $\tau = 0.18ms$

In this situation, capacitor in front of resistance so that we have two situation.

For charging:

$V_o(t) = V_{in} - V_{in}(1-e^{-t/\tau})$
$= 5 - 5(1-e^{-1}) = 1.84V$

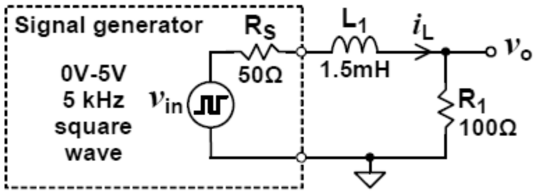
V(T_{rise})=1.84V

For discharging:

$V_o(t) = V_{in} - V_{in}(e^{-t/\tau})$
$= 0 - 5(e^{-1}) = -1.84V$

V(T_{fall})=-1.84V

4.a) Calculate the time constant $\boldsymbol{\tau} = \mathbf{L_1/(R_S+R_1)}$ for the circuit given below and write $\boldsymbol{v_o(t)}$ as a function of time after $\boldsymbol{v_{in}}$ switches between **0 V** and **5 V**. Note that output resistance $\mathbf{R_S = 50\ \Omega}$ of the signal generator should be taken into account since it is comparable to $\mathbf{R_1}$.



4.b) Draw $\boldsymbol{v_{in}}$, $\boldsymbol{i_L}$, and $\boldsymbol{v_o}$ on the same plot. Calculate $\boldsymbol{i_L}$ and $\boldsymbol{v_o}$ at $\mathbf{t} = \boldsymbol{\tau}$ after $\boldsymbol{v_{in}}$ switches between **5 V** and **0 V**, and mark these signal levels on the plot.

$\tau = \dfrac{L_1}{R_s+R_1}$, and we know $R_s = 50\Omega$, $R_1 = 100\Omega$, $L_1 = 1.5mH$

$\tau = \dfrac{1.5mH}{150} = \dfrac{15\times10^{-4}}{150} = 0.01\ ms$

$V_{in} = i_L(R_s + R_1) + L\dfrac{di_L}{dt} \rightarrow V_{in}\left(\dfrac{R_1}{R_s+R_1}\right) = V_o + \dfrac{dV_o}{dt}$

$V_{in} = 5V$ so $\rightarrow \dfrac{10}{3} = V_o + \dfrac{dV_o}{dt}$

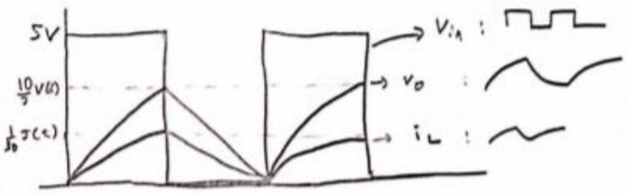
$\rightarrow V(t) = \dfrac{10}{3}(1 - e^{-t/\tau}) \xrightarrow{i_L = \frac{V_o}{R_1}} i(t) = \dfrac{1}{30}(1 - e^{-t/\tau})$

$V_{in} = 0V \rightarrow V(t) = \dfrac{10}{3}e^{-t/\tau} \xrightarrow{i_L = \frac{V_o}{R_1}} i(t) = \dfrac{1}{30}(e^{-t/\tau})$

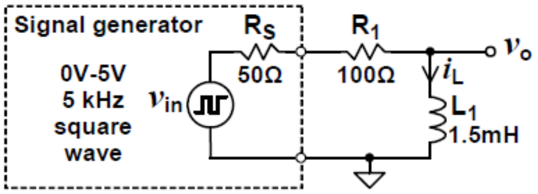
We assume $t = \tau$ so

at $V_{in} = 5V \rightarrow i(t) = 21.07mA$, $V(t) = 2.11V$ ($V_{t_{Rise}}$)

at $V_{in} = 0V \rightarrow i(t) = 12.3mA$, $V(t) = 1.23V$ ($V_{t_{Fall}}$)



5. Calculate and plot $\boldsymbol{i_L}$ and $\boldsymbol{v_o}$ as $\boldsymbol{v_{in}}$ switches between **0 V** and **5 V** in the following circuit. Mark the $\boldsymbol{i_L}$ and $\boldsymbol{v_o}$ values at $\mathbf{t} = \boldsymbol{\tau}$ on the plot. Simulate the circuit in LTspice and verify your calculations.



$$\tau = \frac{L_1}{R_s+R_1} = \frac{1.5\times10^{-3}}{50+100} = \frac{150\times10^{-5}}{150} = 0.01ms$$

For charging

$$V_o(t) = L\frac{di(t)}{dt} = L\cdot\frac{V_{in}}{R_{eq}}\times e^{-t/\tau}\cdot\frac{1}{\tau}$$

$\tau = \frac{L}{R_{eq}}$ $t=\tau$

$$= V_{in}\times e^{-1} = \frac{5}{e} = 1.84V \rightarrow V_o(T_{rise})$$

$$i_L(t) = \frac{V_{in}}{R}\times(1-e^{-t/\tau}) = \frac{1}{30}(1-e^{-t/\tau}) = \frac{1-e^{-1}}{30} = 0.021A$$

For discharging

$$V_o(t) = L\frac{di_L(t)}{dt} = -L\frac{V_{in}}{R_{eq}}\cdot e^{-t/\tau}\cdot\frac{1}{\tau}$$

$\tau = \frac{L}{R_{eq}}$, $t=\tau$

$$= -V_{in}\cdot e^{-1} = \frac{-5}{e} = -1.84V \rightarrow V_o(T_{rise})$$

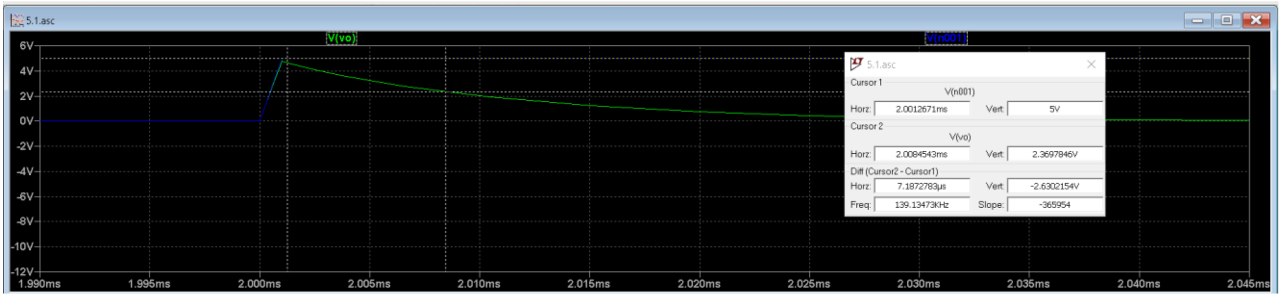
$$i_L(t) = \frac{V_{in}}{R}(e^{-t/\tau}) = \frac{1}{30e} = 0.012A$$

Using this formula,

$$v_O(T_{1/2}) = V_L + (V_H - V_L)/2.$$

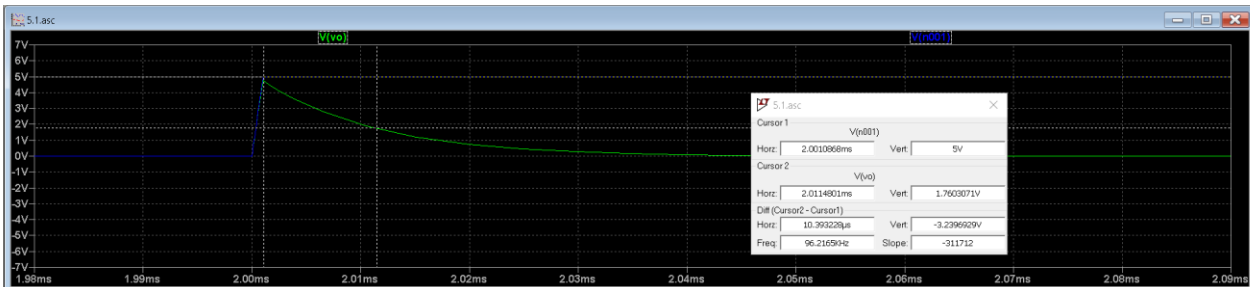
Then $\tau_{rise} = T_{1/2}/ln(2) = 1.443\ T_{1/2}$ and $\tau_{fall} = 1.443\ T_{1/2}$.

So, $V_O(T_{1/2})=V_L+(V_H-V_L)/2 =0+(4.76-0)/2=2.38V$, our $T_{1/2}$= 7.20μs



$T_{rise}= T_{1/2}/ln(2)=1.443*T_{1/2}=10.39$ μs $T_{fall}= T_{1/2}/ln(2)=1.443*T_{1/2}=10.39$ μs

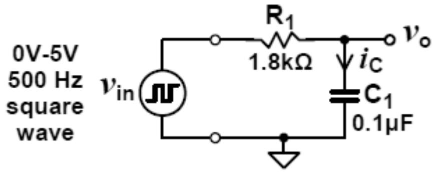
If we check for our T_{rise} and T_{fall} on the graph, we can find $V_O(T_{rise})$ and $V_O(T_{fall})$ values.



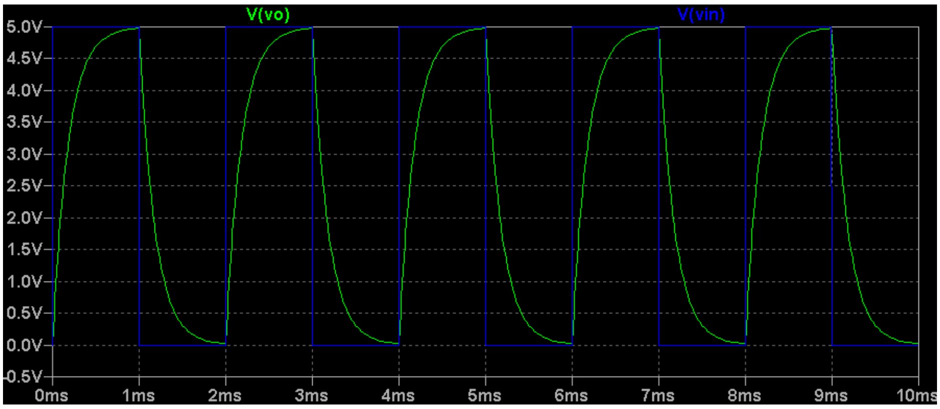
$V_O(T_{rise})$= 1.76 and $V_O(T_{Fall})$= -1.75V .Our percetage error is fewer than 5. This little bit error can be because of decimal rounding or cursor error while measuring.

Procedure

1. Build the circuit given on the right and set v_{in} source to obtain **500 Hz** square wave that switches between **0 V** and **5 V**.



1.1 Display v_{in} and v_o waveforms and plot them.



1.2 Record the high and low voltage levels at v_o.

V_H =4.9807178V **V_L = 19.931142mV**

1.3 Measure the time constant on rising and falling v_o waveforms. Zoom into the v_o waveform to obtain the best possible timing accuracy.

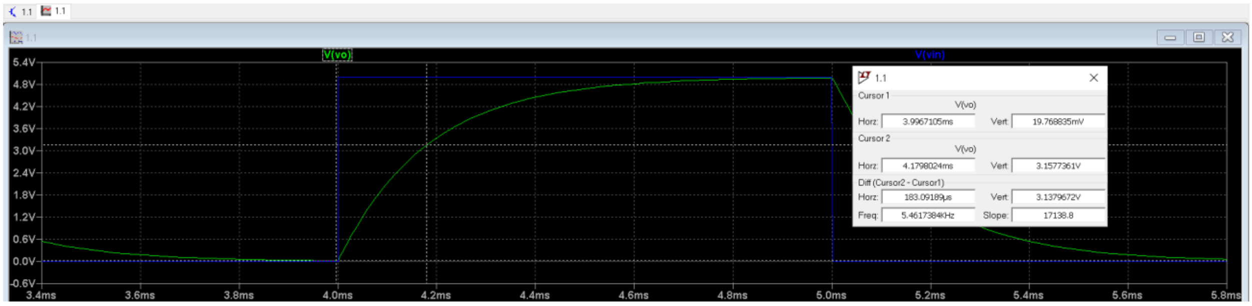
τ_{rise} = 183.09µs **τ_{fall} = 178.16µs**

On rising exponential: $v_o(\tau_{rise}) = V_L + (V_H - V_L)(1 - e^{-1})$

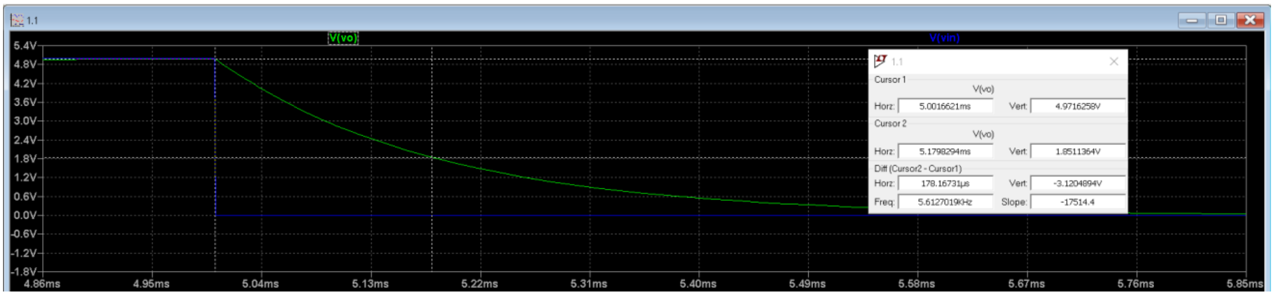
$= V_L + 0.632\,(V_H - V_L)$=3.15V

On falling exponential: $v_o(\tau_{fall}) = V_L + (V_H - V_L)e^{-1}$

$= V_L + 0.368\,(V_H - V_L)$=1.85V



τ_{rise} = 183.09µs



τ_{fall} = 178.16µs

1.4 Compare the measured values with those calculated in the preliminary work.

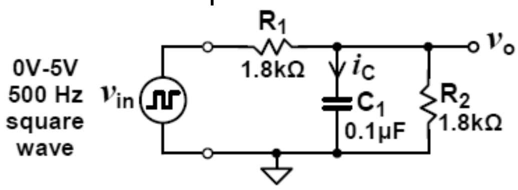
	$v_O(\tau_{rise})$	$v_O(\tau_{fall})$	τ
Calculated values in preliminary work	3.16V	1.84V	0.18ms
Measured values	3.15V	1.85V	0.181ms(average of τfall and τrise)
%Error	0.3	0.5	0.5

Our percentage of error is fewer than %1 for every values. This little bit error can be because of decimal rounding or cursor error while measuring.

1.5 Apply **5 Vp-p** sinusoidal waveform as v_{in} and record the peak-to-peak output voltage at the following frequencies.

v_{in} frequency	v_O (Vp-p)
500 Hz	4.3442159V
5 kHz	839.95468mV
50 kHz	81.698527mV

2. Add $\mathbf{R_2 = 1.8\ k\Omega}$ in parallel to the capacitor on the circuit used in step **1**.



2.1 Record the high and low voltage levels at v_O, and repeat the time constant measurements.

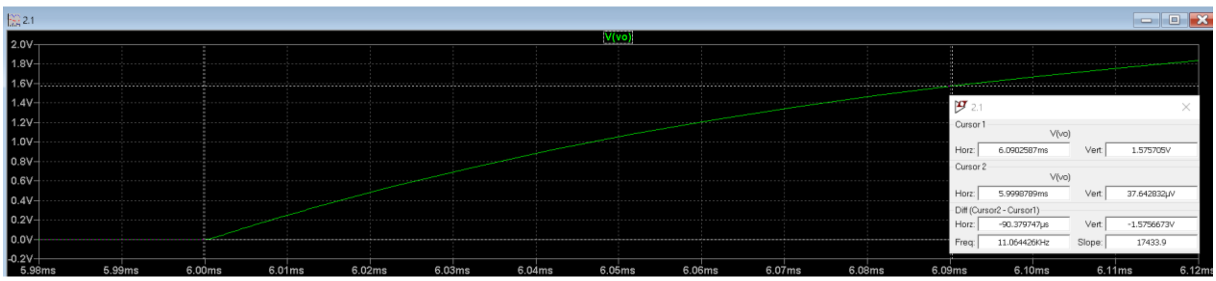
$V_H = 2.5$ $V_L = 38.67\mu V$

On rising exponential: $\mathbf{v_O(\tau_{rise}) = V_L + (V_H - V_L)(1 - e^{-1})}$

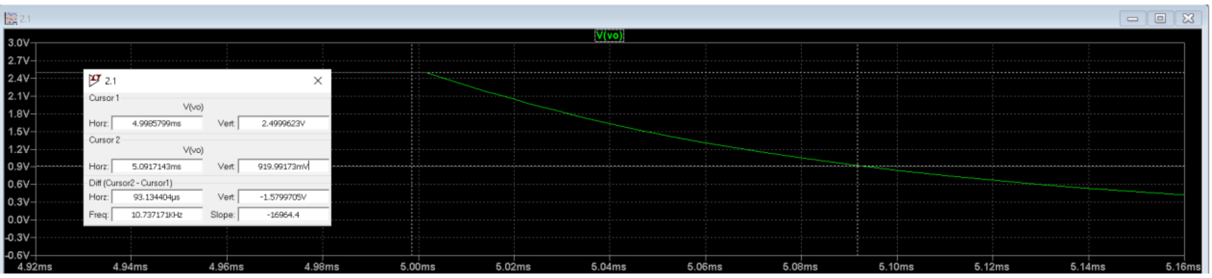
$= V_L + 0.632\ (V_H - V_L) = 1.575V$

On falling exponential: $\mathbf{v_O(\tau_{fall}) = V_L + (V_H - V_L)e^{-1}}$

$= V_L + 0.368\ (V_H - V_L) = 0.92V$



$\tau_{rise} = 90.38\ \mu s$



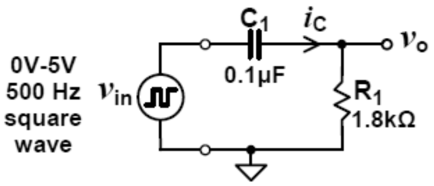
τ_{fall} = 93.13μs

2.2 Compare the measured values with those calculated in the preliminary work.

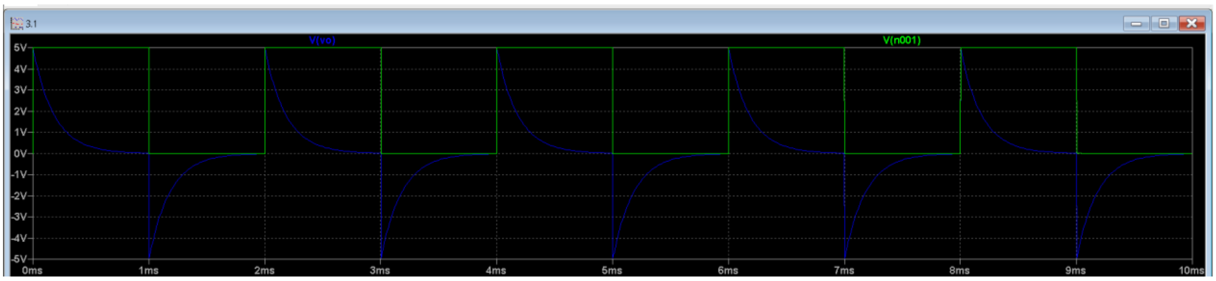
	$v_o(\tau_{rise})$	$v_o(\tau_{fall})$	τ
Calculated values in preliminary work	1.58V	0.92V	0.09ms
Measured values	1.575V	0.92V	0.091ms(average of τ_{fall} and τ_{rise})
%Error	0.3	0.0	1

Our percentage of error is fewer than %2 for every values. This little bit error can be because of decimal rounding or cursor error while measuring.

3. Set up the circuit given below.



3.1 Display v_{in} and v_o waveforms and plot them. Measure initial voltage, final voltage and time constant of the v_o waveform when v_{in} **= 0 V** and v_{in} **= 5 V**. Indicate the measured values on the plot.



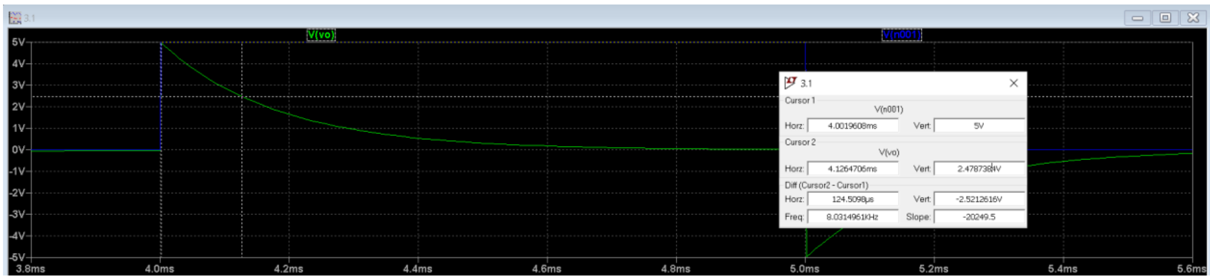
v_{in} **= 0 V** initial voltage V_o= -4.9505163V, final voltage V_o = - 19.691172mV

v_{in} **= 5 V** initial voltage V_o= 4.9584966V, final voltage V_o = 19.282073mV

$v_o(T_{1/2}) = V_L + (V_H - V_L)/2$.

Then $\tau_{rise} = T_{1/2}/ln(2) = 1.443\ T_{1/2}$ and $\tau_{fall} = 1.443\ T_{1/2}$.

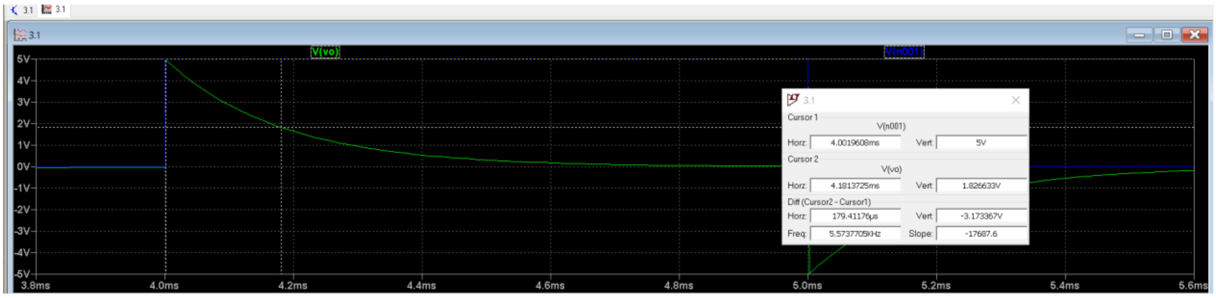
Using this formula,



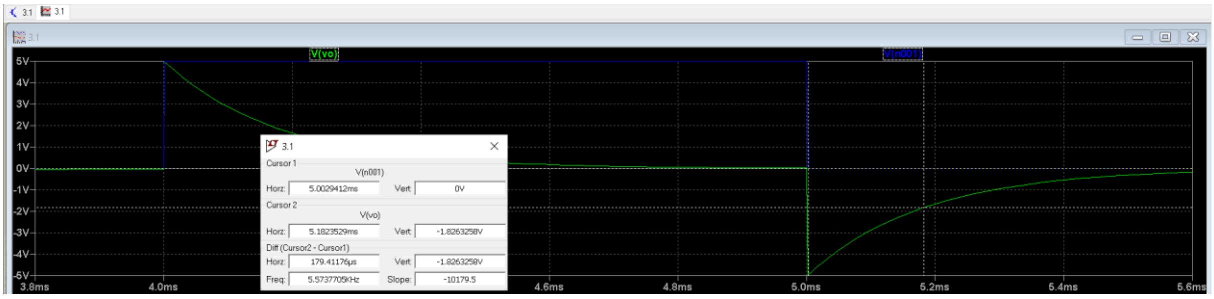
So our $T_{1/2}$= 124.5µs

T_{rise}= $T_{1/2}/\ln(2)=1.443*T_{1/2}$=179.65 µs T_{fall}= $T_{1/2}/\ln(2)=1.443*T_{1/2}$=179.65 µs

If we check for our T_{rise} and T_{fall} on the graph, we can find $V_O(T_{rise})$ and $V_O(T_{fall})$ values.



$V_O(T_{rise})$= 1.83V



$V_O(T_{fall})$= -1.83V

3.2 Compare the measured values with those calculated in the preliminary work.

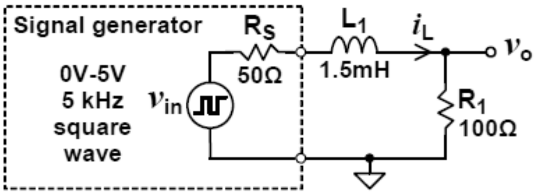
	$v_O(\tau_{rise})$	$v_O(\tau_{fall})$	τ
Calculated values in preliminary work	1.84	-1.84V	0.18ms
Measured values	1.83	-1.82V	0.179 ms (average of τ_{fall} and τ_{rise})
%Error	0.5	1	0.6

Our percentage of error is fewer than %2 for every values. This little bit error can be because of decimal rounding or cursor error while measuring.

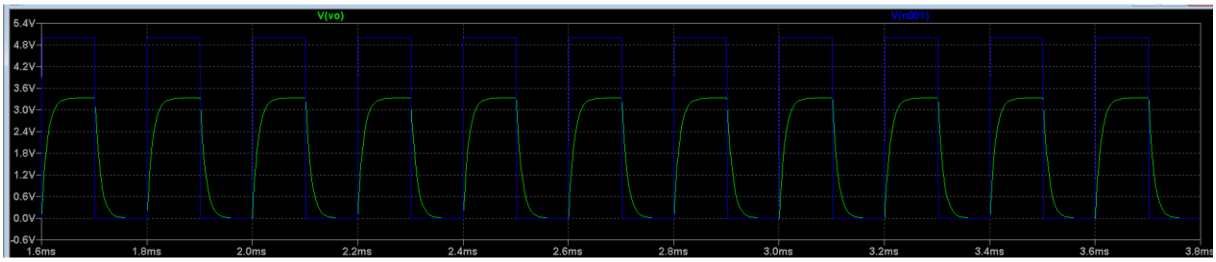
3.3 Apply **5 Vp-p** sinusoidal waveform as v_{in} and record the peak-to-peak output voltage at the following frequencies.

v_{in} frequency	v_o (Vp-p)
500 Hz	2.455681V
5 kHz	4.8958253V
50 kHz	4.9709042V

4. Set up the circuit given below. Adjust the signal source to obtain **5 kHz**, **0-5 V** square wave. The source resistance $\mathbf{R_S = 50\Omega}$ is added to obtain a realistic model of the laboratory signal generator.



4.1 Display v_{in} and v_o waveforms and plot them. Measure initial voltage, final voltage and time constant of the v_o waveform when v_{in} **= 0 V** and v_{in} **= 5 V**. Indicate the measured values on the plot.



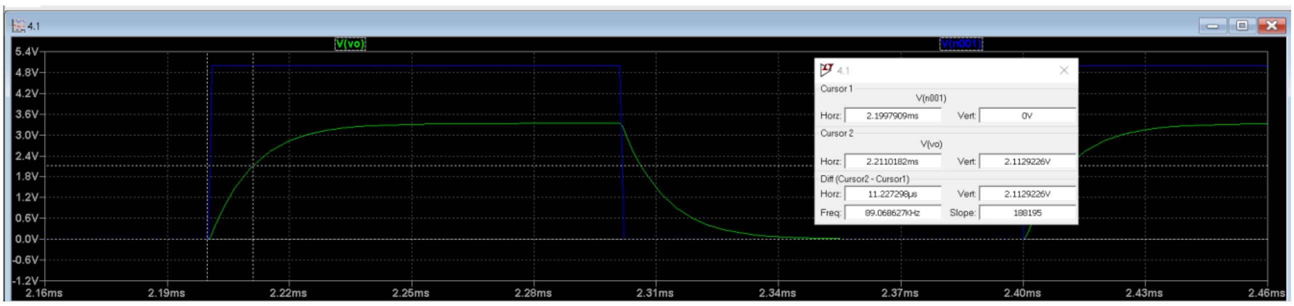
v_{in} **= 0 V** initial voltage V_o= 3.1359717V, final voltage V_o = 157.60071μV

v_{in} **= 5 V** initial voltage V_o= 166.76067mV, final voltage V_o = 3.3332094V

$\mathbf{V_H}$ = 3.3349908V $\mathbf{V_L}$ = 0V

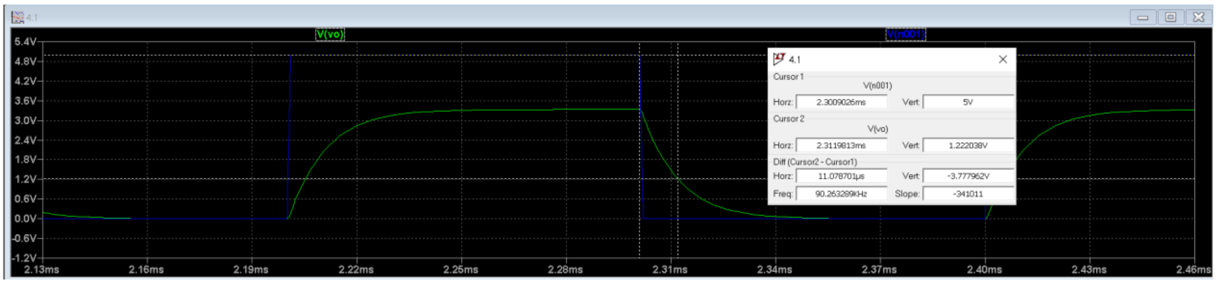
On rising exponential: $v_o(\tau_{rise}) = V_L + (V_H - V_L)(1 - e^{-1})$

$= V_L + 0.632\,(V_H - V_L) = 2.11$



On falling exponential: $v_o(\tau_{fall}) = V_L + (V_H - V_L)e^{-1}$

$= V_L + 0.368\,(V_H - V_L) = 1.22$ V



4.2 Compare the measured values with those calculated in the preliminary work.

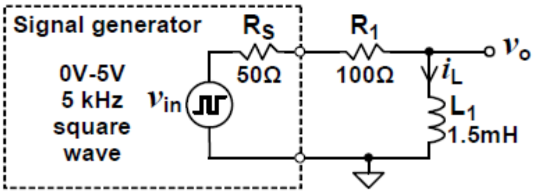
	$v_o(\tau_{rise})$	$v_o(\tau_{fall})$	τ
Calculated values in preliminary work	2.112	1.23	0.01ms
Measured values	2.11	1.22	0.011 ms (average of τ_{fall} and τ_{rise})
%Error	0.5	1	0.6

Our percentage of error is fewer than %2 for every values. This little bit error can be because of decimal rounding or cursor error while measuring.

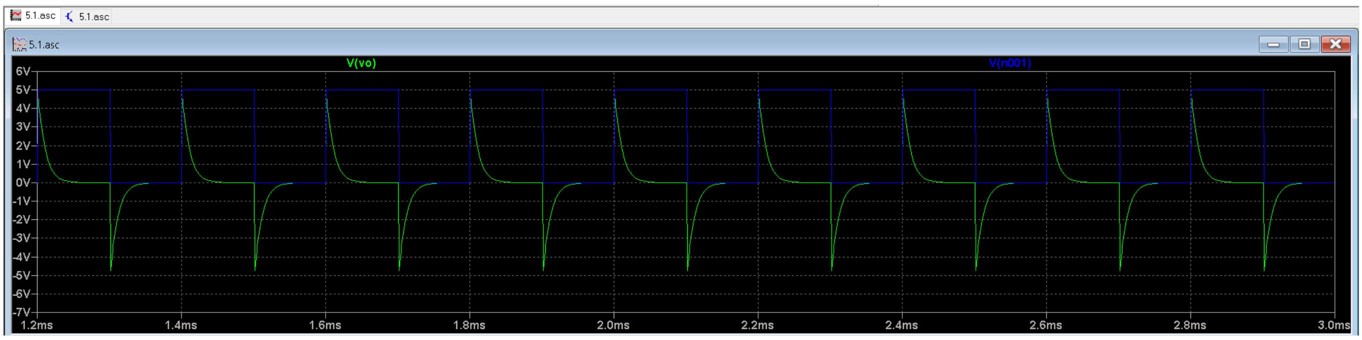
4.3 Set the v_{in} signal source to obtain **5 Vp-p** sinusoidal waveform. Record the peak-to-peak v_o output voltage at the following frequencies.

v_{in} frequency	v_o (Vp-p)
5 kHz	3.1392296V
50 kHz	934.77654mV
500 kHz	96.839093mV

5. Set up the circuit given below. Set the v_{in} signal source to obtain **5 kHz**, **0-5 V** square wave.



5.1 Display v_{in} and v_o waveforms and plot them. Measure initial voltage, final voltage and time constant of the v_o waveform when v_{in} **= 0 V** and v_{in} **= 5 V**. Indicate the measured values on the plot.

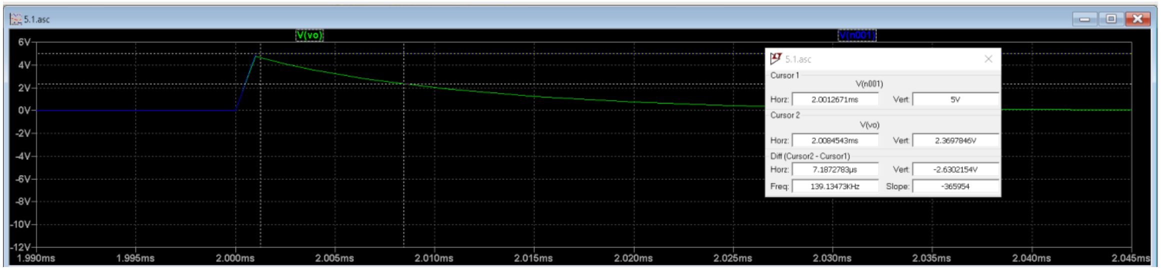


v_{in} **= 0 V** initial voltage V_o= -4.750977V, final voltage V_o = -238.89126μV

v_{in} **= 5 V** initial voltage V_o =4.752395V, final voltage V_o = 213.86307μV

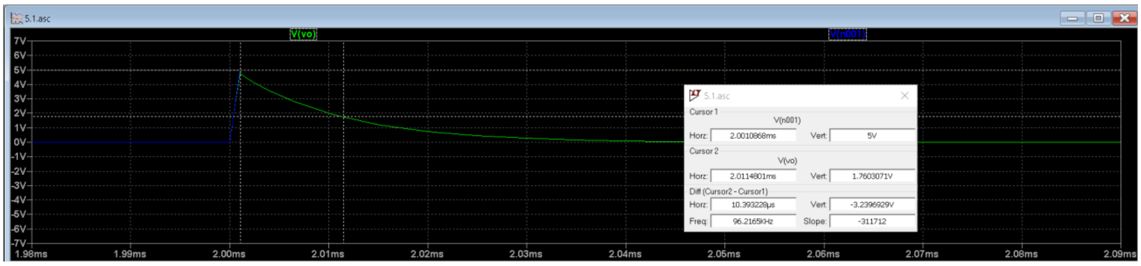
$v_o(T_{1/2}) = V_L + (V_H - V_L)/2$.

Then $\tau_{rise} = T_{1/2}/ln(2) = 1.443\ T_{1/2}$ and $\tau_{fall} = 1.443\ T_{1/2}$. So, $V_O(T_{1/2})=V_L+(V_H-V_L)/2$ =0+(4.75-0)/2=2.38V



So our $T_{1/2}$= 7.20μs, T_{rise}= $T_{1/2}$/ln(2)=1.443*$T_{1/2}$=10.39 μs T_{fall}= $T_{1/2}$/ln(2)=1.443*$T_{1/2}$=10.39 μs

If we check for our T_{rise} and T_{fall} on the graph, we can find $V_O(T_{rise})$ and $V_O(T_{fall})$ values.



$V_O(T_{rise})$= 1.76V

$V_O(T_{Fall})$= -1.75V

5.2 Compare the measured values with those calculated in the preliminary work.

	$v_O(\tau_{rise})$	$v_O(\tau_{fall})$	τ
Calculated values in preliminary work	1.84V	-1.84V	0.01ms
Measured values	1.76V	-1.75V	0.104ms(average of τ_{fall} and τ_{rise})
%Error	0.3	0.5	0.1

Our percentage of error is fewer than %1 for every values. This little bit error can be because of decimal rounding or cursor error while measuring.

5.3 Apply **5 Vp-p** sinusoidal waveform as v_{in} and record the peak-to-peak output voltage at the following frequencies.

v_{in} frequency	v_O (Vp-p)
5 kHz	1.4798484V
50 kHz	4.7454872V
500 kHz	4.9598434V

Conclusion

In this experiment, in first part we set a circuit which is given in Figure 1 and we set Vin source to obtain 500Hz square wave that switches 0V and 5V and we displayed Vin and Vo waveforms, we recorded high and low voltage levels at Vo. We measured time constant on rising and falling Vo waveforms. We obtained Trise and Tfall values. Then we apply 5Vpp sinusoidal waveform as Vin and we recorded peak-to peak output voltages for different frequency values(500Hz,5kHz,50kHz). In part 2, we added 1.8k Ω resistor in parallel to capacitor on the circuit used in part 1. Then we recorded high and voltage levels at Vo and we repeated time constant measurements. In third part, we set a given circuit but in this circuit we see capacitor has opposite direction current so that we have negative low voltage value. We measured initial voltage,final voltage and time constant of Vo. Then we added 50Ω resistance and we did same calculation. In final part, we obtain negative low voltage value because of inductor place on the circuit and we see inductor has opposite current direction. We displayed Vin and Vo waveforms. We measured initial voltage and final voltage and time constant of the Vo.

To sum up in this experiment, we learnt behavior of storage devices. We investigated RC and RL circuit response to switching voltage sources.

Questions

Q1.a) Calculate power on $\mathbf{C_1}$ as a function of time in the circuit used for procedure step **1** when $\boldsymbol{v_{in}}$ **= 0 V** and $\boldsymbol{v_{in}}$ **= 5 V**.

For $V_{in} = 5V$ $\qquad \tau = 18ms$

$V_c(t) = 5V \cdot \left(1 - e^{-\frac{t}{\tau}}\right)$, $\quad i_c(t) = C \frac{d}{dt}(V_c(t)) = 0.1\mu F \cdot \frac{5V}{\tau} \cdot e^{-\frac{t}{\tau}}$

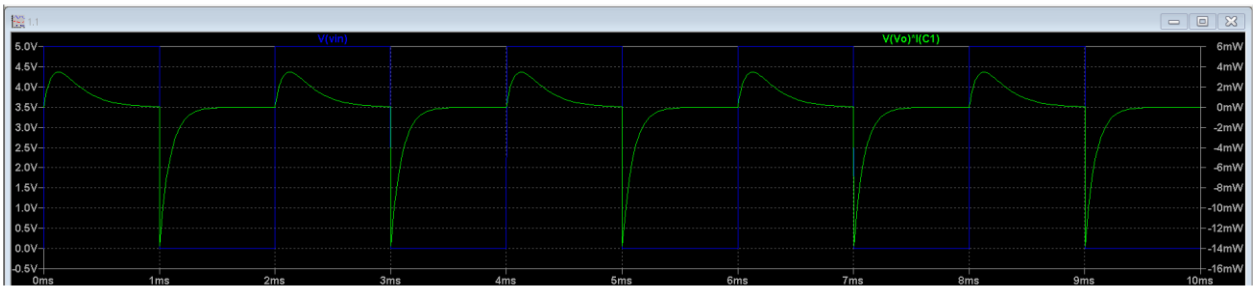
$P_c(t) = V_c(t) \cdot i_c(t) = \frac{250}{18} \cdot 10^{-3}(e^{-1} - e^{-2}) = 3.229 mW$

For $V_{in} = 0V$

$V_c(t) = 5V e^{-\frac{t}{\tau}}$, $\quad i_c(t) = C \frac{d}{dt}(V_c(t)) = -0.1\mu F \frac{5V}{\tau} e^{-\frac{t}{\tau}}$

$P_c(t) = V_c(t) \cdot i_c(t) = \frac{-250}{18} \cdot 10^{-3} \cdot e^{-2} = -1.879 mW$

b) Simulate the circuit (preliminary work part **1**) and plot power on $\mathbf{C_1}$ to verify your calculations.
Right-click on the waveform window and select **Add Traces** option. Enter an expression that multiplies output voltage with $\mathbf{C_1}$ current. Make sure that $\mathbf{C_1}$ current has the correct direction.



We can see our percentage of error is fewer than 5.

c) What is the meaning of positive power and negative power on a capacitor?

Positive power means that the capacitor stores energy. Also, negative power means that the capacitor is exert its stored energy.

Q2. Simulate the same circuit in step **1** when $\boldsymbol{v_{in}}$ is **5 Vp-p** sinusoidal source with **500 Hz**, **5 kHz**, and **50 kHz** frequency. Compare the simulation results with the measurements in procedure step **1.5**.

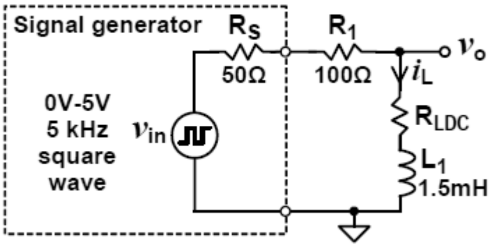
$\boldsymbol{v_{in}}$ frequency	$\boldsymbol{v_o}$ (**Vp-p**) Step1.5	$\boldsymbol{v_o}$ (**Vp-p**) Q2	%Error
500 Hz	4.34V	4.35V	0.23
5 kHz	839.95mV	838.95mV	0.11
50 kHz	81.69mV	82.69mV	1.21

Our percentage of error is fewer than %1 for every values. This little bit error can be because of decimal rounding or cursor error while measuring.

Q3. Negative voltage peaks are observed at the output in procedure steps **3** and **5**. How is it possible to obtain negative output voltages when there is no negative voltage source in a circuit?

It is according to our circuit. In step 3 and step 5 our capacitor and inductor components come before resistors. As we see our current on capacitor and inductor has opposite direction if we compare them with step 1,2 and 4. We calculated the output voltage by using the difference between input voltage and on resistor on and capacitor voltage. When our input voltage becomes zero, output voltage will be minus value.

Q4. In procedure step **5**, you observed an offset at v_O when v_{in} **= 5 V**. Calculate the final value of v_O at the end of exponential decay after including the measured DC resistance $\mathbf{R_{LDC}}$ of the inductor as shown on the right. Compare your calculation with the measured DC level in step **5.1.**

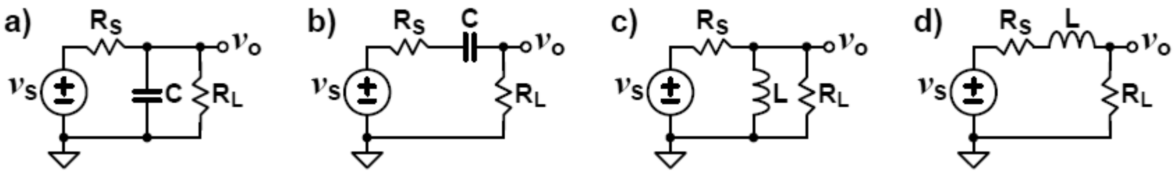


We can say v_{in} **= 5 V** initial voltage V_o =4.75V in step 5.1. Also we find for this circuit 4.72V so we can say our percentage of error is fewer than %1 for every values. This little bit error can be because of decimal rounding or cursor error while measuring.

Q5. Based on your observations in the experiment, write the expected steady state output voltage in each of the following circuits for two extreme cases:

Case 1: v_S is a constant, DC voltage source

Case 2: v_S is a high-frequency (**HF**) source such as **sin(2π f t)**, where **f ≫ 1/τ**.



DC v_S ⇒ DC v_S ⇒ DC v_S ⇒ DC v_S ⇒

HF v_S ⇒ HF v_S ⇒ HF v_S ⇒ HF v_S ⇒

Hint: Answer is either $v_O = 0$ or $v_O \simeq v_S \dfrac{R_L}{R_S+R_L}$

a) DC V_S ➔ $v_o \simeq v_s \frac{R_L}{R_S+R_L}$

HF V_S ➔ 0

b) DC V_S ➔ 0

HF V_S ➔ $v_o \simeq v_s \frac{R_L}{R_S+R_L}$

c) DC V_S ➔ 0

HF V_S ➔ $v_o \simeq v_s \frac{R_L}{R_S+R_L}$

d) DC V_S ➔ $v_o \simeq v_s \frac{R_L}{R_S+R_L}$

HF V_S ➔ 0

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Date:29/12/2020

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