EE203 - Electrical Circuits Laboratory

Experiment - 3
AC Measurements and Power

Objectives

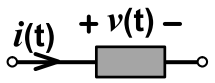
1. Practice power calculations and simulations with time-varying signals.
2. Learn significance of average and RMS amplitudes.

Background

Power Calculations

Power calculated for a component is the energy transferred to or from the component per unit time (1 Watt = 1 Joule/second). **Instantaneous power** is given by the voltage across the component multiplied by the current through the component as a function of time.

$$p(t) = v(t)\, i(t)$$



Current direction should be from positive to negative side of the voltage polarity marked on the component as shown above. Sign of the calculated power has the following meanings.

- $p(t) > 0$ **:** Component is ***sinking*** or ***taking in*** power. Energy is being dissipated (converted into heat) in the component, or it is being stored by the component. Common components that are capable of storing electrical energy are the reactive components (capacitors and inductors) and rechargeable batteries.
- $p(t) < 0$ **:** Component is ***sourcing*** or ***giving out*** power. Component can be a power source such as an electrochemical battery or electromechanical generator, or it can be an active device that transfers power from a source. It may also be a reactive component returning the previously stored energy.

Voltage on a simple resistor, $v_R(t) = R\, i_R(t)$, is proportional to the current. If $v_R(t)$ is positive then $i_R(t)$ is positive, or if $v_R(t)$ is negative then $i_R(t)$ is also negative. Therefore, the power calculated for a resistor always has a positive sign, meaning that a resistor can only dissipate power. On the other hand, if a component is sourcing or giving out power, then the voltage and the current on that component have opposite signs.

Average power is the instantaneous power averaged over time. Average power for periodic signals can be found by integrating the instantaneous power over a single period. The energy calculated as a result of the integration is divided by the period to find the average power:

$$P_{avg} = \frac{1}{T_{prd}} \int_{t=0}^{T_{prd}} p(t)\, dt$$

Average and RMS Amplitudes

Average amplitude of a periodic signal is the time-varying amplitude averaged over one period. Average value gives the DC content in a signal.

$$V_{avg} = \frac{1}{T_{prd}} \int_{t=0}^{T_{prd}} v(t)\, dt$$

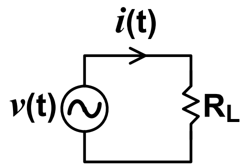
RMS (root-mean-square) amplitude is the square root of average value of square of the time varying amplitude in one period.

$$V_{rms} = \sqrt{\frac{1}{T_{prd}} \int_{t=0}^{T_{prd}} v^2(t)\, dt}$$

The average power is not related to the average amplitude, but the RMS amplitude of a signal is a good indicator of the power content of a signal. In case of a simple resistive component, the average dissipated power can be calculated easily in terms of the RMS voltage or RMS current measured on the component:

$$P_{avg} = \frac{1}{T_{prd}} \int_{t=0}^{T_{prd}} p(t)\, dt = \frac{1}{T_{prd}} \int_{t=0}^{T_{prd}} v(t)\, i(t)\, dt = \frac{1}{T_{prd}} \int_{t=0}^{T_{prd}} v(t)\, \frac{v(t)}{R_L} dt$$

$$P_{avg} = \frac{1}{R_L} \frac{1}{T_{prd}} \int_{t=0}^{T_{prd}} v^2(t)\, dt = \frac{1}{R_L} V_{rms}^2 \ , \quad \text{or} \quad P_{avg} = I_{rms}^2 R_L$$



On the other hand, the average amplitude of a periodic signal cannot be used in power calculations. In the simplest case, average amplitude of a sinusoidal signal is always zero, which is meaningless in power calculations.

DC and AC Measurements

AC amplitude specifications and measurements are given as RMS values by default. When the AC voltage on a power outlet is specified as **50 Hz**, **220 V-rms**, the corresponding time varying signal is **311 V *sin*(2π 50 t)**. The type of amplitude specification is usually written after the physical unit to avoid uncertainty. **220 V**rms, **311 V**peak, and **622 V**p-p are all valid amplitude specifications for the same sinusoidal waveform.

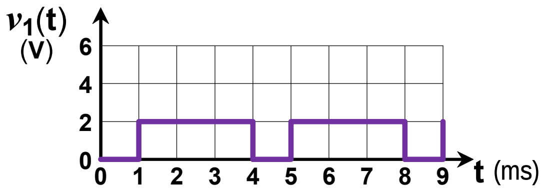
Multimeters have different settings for DC and AC measurements. If the DC measurement option is selected, a common multimeter ignores the AC content of the signal and it shows the DC or average amplitude of the signal. Similarly, if AC measurement option is selected, then DC level of the signal is ignored, and RMS value of the AC signal is displayed. There are two exceptions for these generalizations:

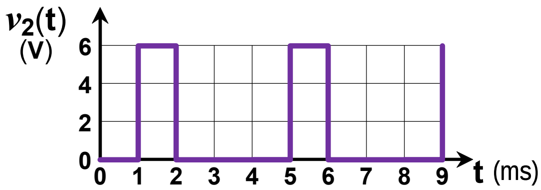
- **Very simple, cheap multimeters** make AC voltage measurements assuming that the measured signal is sinusoidal only. These multimeters may display incorrect results if the measured signal is not sinusoidal, or if the DC level of the signal is not zero.
- **True-RMS multimeters** can display the RMS value of the complete signal including both DC and AC components.

True RMS value of a signal is related to the DC level and AC RMS value measured with a common multimeter as follows:

$$V_{true\text{-}rms}^2 = V_{DC}^2 + V_{ACrms}^2$$

The main reason for measuring AC signal amplitudes as RMS values is to allow easy calculation of average power based on the measurements. The simple example given below clarifies the usage of RMS voltage in power calculations. The two waveforms have the same period and the same average voltage values, but the average power dissipated on the load resistor $\mathbf{R_L}$ is different. The RMS voltages on the other hand, can be used to calculate the average power in both cases. These waveforms are ideal pulse functions, and integration of instantaneous voltage and power can be done easily by calculating the rectangular area under the pulses.





$$V_{1avg} = \frac{1}{4\,\text{ms}} 2\,\text{V} \times 3\,\text{ms} = 1.5\,\text{V}$$

$$V_{1rms} = \sqrt{\frac{1}{4\,\text{ms}} (2\,\text{V})^2 \times 3\,\text{ms}} = \sqrt{3}\,\text{Vrms}$$

$$P_{1avg} = \frac{1}{T_{prd}} \int_{t=0}^{T_{prd}} p_1(t)\,dt = \frac{1}{T_{prd}} \int_{t=0}^{T_{prd}} \frac{(v_1(t))^2}{R_L} dt$$

$$P_{1avg} = \frac{1}{4\,\text{ms}} \frac{(2\,\text{V})^2}{R_L} 3\,\text{ms} = \frac{3}{R_L}\ \text{W}$$

$\mathbf{P_{1avg}}$ given by the RMS voltage $\mathbf{V_{1rms}}$:

$$P_{1avg} = \frac{V_{1rms}^2}{R_L} = \frac{(\sqrt{3}\,\text{V})^2}{R_L} = \frac{3}{R_L}\ \text{W}$$

$$V_{2avg} = \frac{1}{4\,\text{ms}} 6\,\text{V} \times 1\,\text{ms} = 1.5\,\text{V}$$

$$V_{2rms} = \sqrt{\frac{1}{4\,\text{ms}} (6\,\text{V})^2 \times 1\,\text{ms}} = 3\,\text{Vrms}$$

$$P_{2avg} = \frac{1}{T_{prd}} \int_{t=0}^{T_{prd}} p_2(t)\,dt = \frac{1}{T_{prd}} \int_{t=0}^{T_{prd}} \frac{(v_2(t))^2}{R_L} dt$$

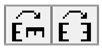
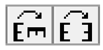
$$P_{2avg} = \frac{1}{4\,\text{ms}} \frac{(6\,\text{V})^2}{R_L} 1\,\text{ms} = \frac{9}{R_L}\ \text{W}$$

$\mathbf{P_{2avg}}$ given by the RMS voltage $\mathbf{V_{2rms}}$:

$$P_{2avg} = \frac{V_{2rms}^2}{R_L} = \frac{(3\,\text{V})^2}{R_L} = \frac{9}{R_L}\ \text{W}$$

Power content of the two waveforms are different although they have the same $\mathbf{V_{1avg} = V_{2avg}}$ average voltage value. On the other hand, correct average power dissipation values can be obtained in terms of the RMS voltages $\mathbf{V_{1rms}}$ and $\mathbf{V_{2rms}}$.

Preliminary Work

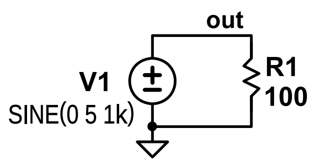
Note: Make sure that current waveforms have correct polarities in LTspice. Verify the current direction looking at the current probe  that appears when the cursor moves over a component. If it is necessary to reverse the current direction, then grab the component with  tool and move it over  buttons. Click on  buttons to change the component orientation until you obtain the correct polarity.

1. Read the procedure given for this experiment, and make sure that you have the required knowledge about the laboratory equipment to perform the specified steps. If necessary, refer to **EELab_GuideBeginner.pdf** to refresh your memory.

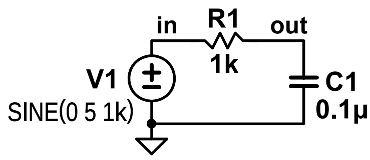
2.a) Calculate power dissipated on a **100 Ω** resistor as a function of time when the voltage across the resistor is $\mathbf{10\,V}\ \boldsymbol{sin}(\mathbf{2\pi\,1000\,t})$.

Hint: $sin^2(x) = \dfrac{1 - cos(2x)}{2}$

2.b) Verify your calculation by simulating the simple circuit given on the right on LTspice. To display the power on **R1**, Right-click on the waveform window, select the **Add Traces** option, and enter the expression `V(out) * I(R1)`. LTspice displays the power variation with a vertical scale in Watts.



2.c) Simulate the circuit shown on the right (used in Experiment-2) and display the power on **C1** by adding a trace with the expression `V(out) * I(C1)`. Identify the time intervals where **C1** stores and gives back energy.



3. Show that the RMS voltage of sinusoidal signal $V_P\, sin(\omega t)$ is given by

$$V_{rms} = \frac{V_p}{\sqrt{2}}$$

Hint: Calculate the RMS value as it is defined:

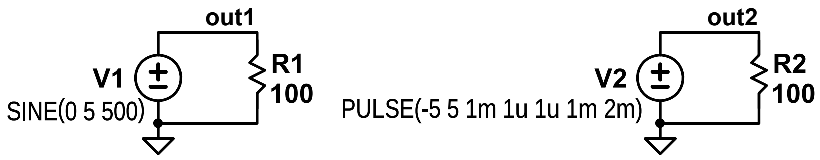
$$V_{rms} = \sqrt{\frac{2}{T}\int_{t=0}^{T_{prd}} V_P^2\, sin^2(\omega t)\, dt} = \frac{V_P}{\sqrt{2}} \quad \text{by using} \quad sin^2(\omega t) = \frac{1 - cos(2\omega t)}{2}$$

4. Calculate DC level and RMS voltage of a pulse waveform with **50 %** duty cycle, $\mathbf{V_{High} = +5\ V}$, and $\mathbf{V_{Low} = -5\ V}$.

5. Calculate DC level and true RMS voltage of a pulse waveform with **25 %** duty cycle, $\mathbf{V_{High} = +10\ V}$, and $\mathbf{V_{Low} = 0\ V}$.

Procedure

1. Make the following circuit schematic on LTspice.



- Set **V1** source to obtain **500 Hz** sinusoidal signal with **10 V**p-p amplitude and **0 V** DC offset.
- Set **V2** source to obtain **500 Hz** pulse signal with **10 V**p-p amplitude, **0 V** DC offset and **1 µs** rise and fall time.
- Set simulation time to **10 ms**.

1.1 Measure the DC and RMS values of the voltages at **out1** and **out2** nodes and record them in the following table.

1.2 Change the **V1** and **V2** source parameters to obtain 5 V DC offset. Repeat the DC and RMS voltage measurements record them in the following table.

DC offset	**DC** voltage at **out1**	**RMS** voltage at **out1**	**DC** voltage at **out2**	**RMS** voltage at **out2**
0 V				
5 V				

2.1 Change the **V2** source timing parameters to obtain a **100 Hz** pulse signal with **20 %** duty cycle. Keep the **V2** source rise time and fall time settings at **1 µs**. Set simulation time to **50 ms**. Set the **V2** source low and high voltage levels listed in the following table. Measure DC and RMS voltages for each setting and record them in the following table.

Add a new trace on the waveform window and set the waveform expression to display the power dissipation on **R2** resistor. While holding down the **CTRL** key, left-click on the power trace label to display the average power. Record the measured average power values in the table given below for part **2.4**.

Low and high voltage levels with **20 %** duty cycle	voltage readings		calculated voltages	
	DC	**RMS**	**DC**	**RMS**
0 V and **10 V**				
-5 V and **+5 V**				
-10 V and **0 V**				

2.2 Change the **V2** source timing parameters to obtain **80 %** duty cycle. Repeat the DC voltage, RMS voltage and average power measurements and record them in the following tables.

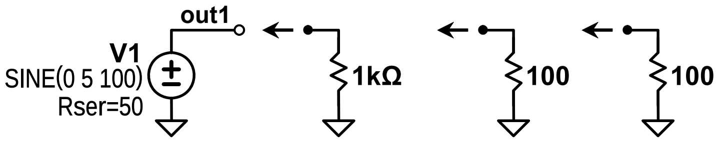
Low and high voltage levels with **80 %** duty cycle	voltage readings		calculated voltages	
	DC	**RMS**	**DC**	**RMS**
0 V and **10 V**				
-5 V and **+5 V**				
-10 V and **0 V**				

2.3 Calculate DC and RMS voltages according to the low and high voltage levels used in parts **2.1** and **2.2** and write the results in the tables above. Compare the measured voltages with the calculated values.

2.4 Calculate the average power dissipated on **R2** with the pulse waveforms used in parts **2.1** and **2.2** and compare the measured average power values.

Low and high voltage levels	average power (mW) at **20 %** duty cycle		average power (mW) at **80 %** duty cycle	
	measured	**calculated**	**measured**	**calculated**
0 V and **10 V**				
-5 V and **+5 V**				
-10 V and **0 V**				

3.1 An ordinary signal generator used in the laboratory has **50 Ω** output resistance. Right-click on the **V1** source and enter the ***Series Resistance*** parameter as **50 Ω** to obtain a more realistic model of the laboratory signal generator. Set **V1** source to obtain **100 Hz** sinusoidal signal with **10 V**p-p amplitude and **0 V** DC offset. Connect resistors to the **V1** source output as shown below, measure the output voltage for each resistance setting and write the measured voltages in the following table.



connected resistor	**peak-to-peak** voltage at **out1**	**RMS** voltage at **out1**
none		
1 kΩ		
100 Ω		
100 Ω // 100 Ω		

Note: 100 Ω // 100 Ω means two resistors connected in parallel.

3.2 Explain the changes in the measured amplitude depending on the connected resistance.

Questions

Q1. According to the measurements in parts **1.1** and **1.2,** what type of RMS values (AC-only or true RMS) are measured in LTspice?

What would happen to the average power dissipated on **R1** and **R2**, if the pulse frequency was **1 kHz** instead of **500 Hz**? Why?

Q2. The true RMS value of a signal $v(t) = V_{DC} + v_{AC}(t)$ is given by the following equation in terms of the DC level V_{DC} and the RMS value V_{ACrms} of the pure AC signal $v_{AC}(t)$. Show that this equation is valid for any periodic AC waveform under ideal conditions.

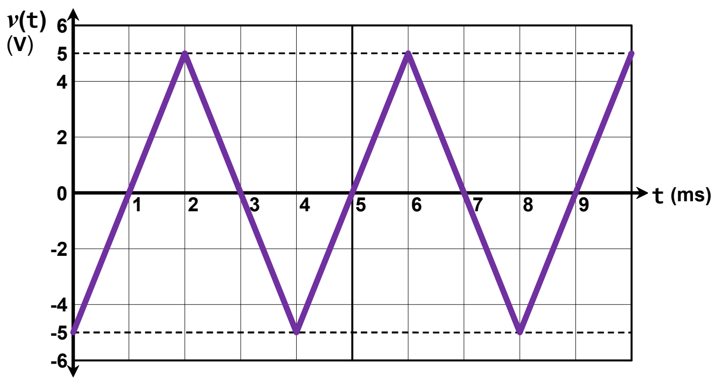
$$V_{\text{true-rms}}^2 = V_{DC}^2 + V_{ACrms}^2$$

Hint: DC (average) value of $v_{AC}(t)$ is zero by definition, and this implies

$$\frac{1}{T}\int_{t=0}^{T} v_{AC}(t)\, dt = 0$$

Q3. Describe a method to find the average power dissipated on a **1 kΩ** resistor by using a multimeter. The resistor cannot be disconnected to make measurements, and current through the resistor has both AC and DC components.

Q4.a) Calculate power dissipated on a **100 Ω** resistor as a function of time when the voltage across the resistor is a triangular function as shown below.



Q4.b) Verify your calculation by simulating a simple circuit on LTspice. Select the **PULSE** option for the voltage source and enter **Trise**, **Tfall**, **Period**, and other parameters to obtain the triangular output.

Q4.c) Calculate average power dissipated on the resistor. It is sufficient to calculate the average power for **1/4** of the period because of symmetry in the voltage waveform.