EE203 - Electrical Circuits Laboratory

Experiment - 5 Laboratory Report
Thevenin and Norton Equivalent Circuits

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Preliminary Work

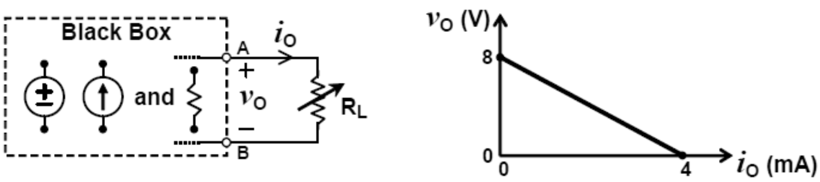
1.a) Why shouldn't you measure the short circuit current of a regulated voltage supply with an ampermeter? (Hint: What happens when you short circuit an ideal voltage source?)

The resistance is known to be zero during a short circuit. When we use Ohm's law, dividing a number by a value approaching zero gives a value approaching infinity. When a voltage source short-circuit, voltage sources do not provide infinite current. Hence, the source burns or sparks from excess current.

b) Why shouldn't you measure the open circuit voltage of a regulated current supply with a voltmeter? (Hint: What happens when you open circuit an ideal current source?)

The open circuit voltage shows the maximum voltage of a voltage source. Since it does not drop any voltage through a load, like what will happen when it is connected to a load, the open circuit voltage of a voltage source reflects the full voltage value.

2. Following figure shows the v_O versus i_O plot of a black box obtained while the load resistance varies between $\mathbf{R_L = \infty}$ and $\mathbf{R_L = 0\ \Omega}$.



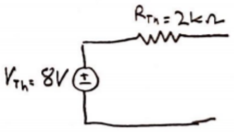
Calculate the equivalent circuit parameters, $\mathbf{V_{th}}$, $\mathbf{R_{th}}$, and $\mathbf{I_{nt}}$ for the black box, and draw the Thevenin and Norton equivalent circuits.

When $i_o = 0$, $V_o = 8V$ which is equal to V_{Th}

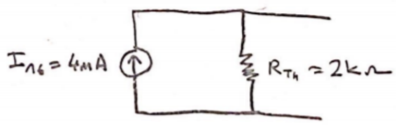
When $V_o = 0$, $i_o = 4mA$ which is equal to I_{Nt}

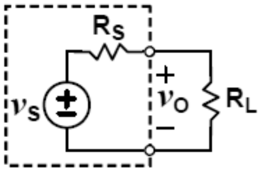
$R_{Th} = \frac{V_{Th}}{I_{Nt}} = \frac{8}{4\times10^{-3}} = 2k\Omega$

Thevenin Circuit



Norton Circuit





3. A Thevenin equivalent circuit is given on the right.
a) Find the $\mathbf{R_L}$ value that makes $v_O = v_S/2$.
b) Calculate the power $\mathbf{P_L}$ dissipated in $\mathbf{R_L}$ when $v_O = v_S/2$.
c) Derive an expression that gives $\mathbf{P_L}$ as a function of v_S, $\mathbf{R_S}$, and an arbitrary value of $\mathbf{R_L}$.
d) Find the $\mathbf{R_L}$ that makes $\mathbf{P_L}$ maximum using the expression found in **(c)**.
e) Compare the $\mathbf{R_L}$ values found in **(a)** and **(d)**.

a) $V_o = V_s \cdot \frac{R_L}{R_s+R_L}$ → to obtain $V_o = \frac{V_s}{2}$

→ $\frac{V_s}{2} = V_s \cdot \frac{R_L}{R_s+R_L}$ → $\frac{1}{2} = \frac{R_L}{R_s+R_L}$ $R_s + R_L = 2R_L$

$\boxed{R_L = R_S}$

b) $P_L = \frac{(V_o)^2}{R_L} = \frac{\left(\frac{V_s}{2}\right)^2}{R_L} = \frac{(V_s)^2}{4R_L}$

c) $P_L = \frac{(V_o)^2}{R_L}$, $V_o = V_s \cdot \frac{R_L}{R_s+R_L}$

$$\Rightarrow \frac{\left(V_s \cdot \frac{R_L}{R_s+R_L}\right)^2}{R_L} = \frac{V_s^2 \cdot R_L^2}{(R_s+R_L)^2} \cdot \frac{1}{R_L} = \frac{V_s^2 \cdot R_L}{(R_s+R_L)^2}$$

d) To find maximum P_L → $\frac{d}{dR_L}\left(\frac{V_s^2 \cdot R_L}{(R_s+R_L)^2}\right)$ should be equal to zero

$\frac{d}{dR_L}\left(\frac{V_s^2 \cdot R_L}{(R_s+R_L)^2}\right) = -\frac{V_s^2(R_L-R_s)}{(R_L+R_s)^3} = 0$ → to get this equation R_L must be equal R_s

$\boxed{R_L = R_s}$

e) We can see our R_L values are equal in step a and step d and R_L equal to R_s. So we can say R_L value which makes $V_o = V_s/2$, give us maximum P_L value.

4. Analyze the following circuit to determine the Thevenin equivalent circuit as seen by $\mathbf{R_L}$ between **A** and **B** terminals. There are two alternative methods that can be used to calculate the Thevenin voltage $\mathbf{V_{th}}$ for $\mathbf{R_L = \infty}$ and the short circuit current $\mathbf{I_{sc}}$ for $\mathbf{R_L = 0\ \Omega}$.

Method 1: Nodal analysis

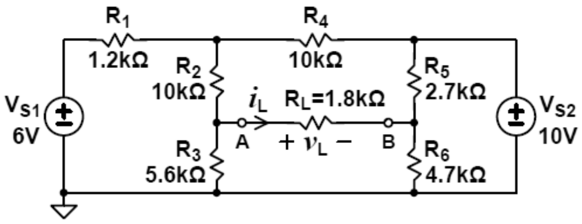
How many node equations are required for analysis of the circuits obtained when $\mathbf{R_L = \infty}$ and $\mathbf{R_L = 0\ \Omega}$? Are there any circuit elements that can be omitted without affecting the Thevenin or Norton equivalent resistance? If so, why?

The branch with the voltage source will be short-circuit, the branch with the current source will be open-circuit. For $R_L = \infty$ we need 2 node equations for and $R_L = 0\ \Omega$ we need just one node equation**.** R_L can be omitted without affecting Thevenin or Norton equivalent resistance because to find V_{th} ,R_{th} or I_{nt} we do not use it.

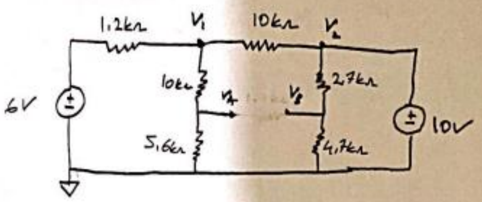
Method 2: Superposition

Calculate the open circuit voltage between **A** and **B** terminals for the two voltage sources. Is it necessary to calculate a different Thevenin resistance for the two voltage sources? If so, why?

When performing superposition, there is no need to calculate the resistance for two different voltage sources separately because to find R_{th} we do not use voltage sources.



a) Determine the Thevenin equivalent circuit (calculate $\mathbf{V_{th}}$ and $\mathbf{R_{th}}$) seen by the load resistor $\mathbf{R_L}$.



Using Nodal Analysis

Node at V_1 :

$$\frac{V_1 - 6}{1.2k\Omega} + \frac{V_1}{10k\Omega + 5.6k\Omega} + \frac{V_1 - V_2}{10k} = 0$$

1200	15600	10000	100
12	156	100	2
6	78	50	2
3	39	25	3
1	13	25	13
			25

390,000

$$\rightarrow V_1 \left[\frac{1}{1200} + \frac{1}{15600} + \frac{1}{10000}\right] - \frac{V_2}{10000} = \frac{6}{1200}$$

$$\rightarrow V_1 \left(\frac{325 + 25 + 39}{390.000}\right) - \frac{39V_2}{390000} = \frac{325.6}{390000}$$

$$\rightarrow V_1 (9,97 \times 10^{-4}) - V_2 (10^{-4}) = 50 (10^{-4})$$

We know $V_2 = 10V$ so $V_1 (9,97) - 10 = 50$

$V_1 = 6.018V$

Using voltage dividing rule

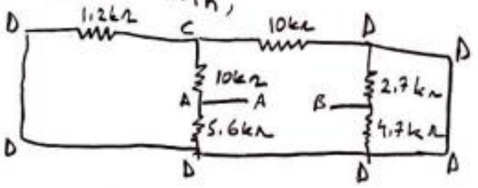
$$V_A = V_1 \left(\frac{5600}{10000 + 5600}\right) = 6.018 \times \frac{5600}{15600} = 2,16V$$

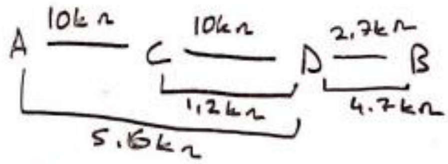
$$V_B = V_2 \left(\frac{4700}{2700 + 4700}\right) = 10 \times \frac{4700}{7400} = 6.35V$$

and we know $V_{Th} = V_A - V_B = 2,16 - 6,35 = -4,19V$

minus say us direction

To find R_{Th},





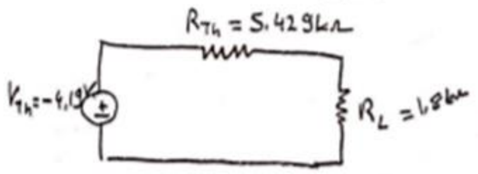
$$R_{Th} = ((10k\Omega + (10k\Omega \,//\, 1.2k\Omega)) \,//\, 5.6k\Omega) + (2.7k\Omega \,//\, 4.7k\Omega)$$
$$= ((10k\Omega + 1.07k) \,//\, 5.6k\Omega) + 1.71k\Omega$$
$$= 3.72\,k\Omega + 1.71k\Omega = 5.429\,k\Omega$$

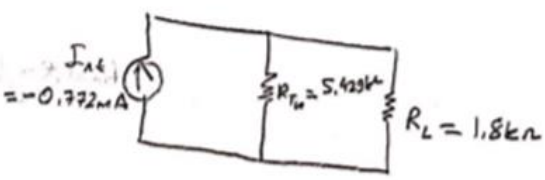
b) Determine the Norton equivalent circuit ($\mathbf{I_{nt}}$ and $\mathbf{R_{nt}}$) seen by $\mathbf{R_L}$.

$$R_{Th} = R_{nt} = 5.429\,k\Omega$$
$$I_{nt} = \frac{V_{Th}}{R_{nt}} = \frac{-4.19V}{5.429k\Omega} = -0.772mA$$

↳ minus show us direction

c) Draw the Thevenin and Norton equivalent circuits using the values calculated in **(a)** and **(b)**.



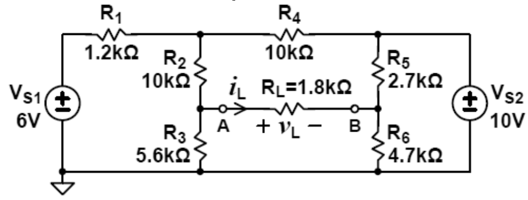


d) Calculate $\boldsymbol{v}_{\mathbf{L}}$ and $\boldsymbol{i}_{\mathbf{L}}$.

$$R_{tot} = 5.429k\Omega + 1.8\,k\Omega = 7.229\,k\Omega$$
$$I_L = \frac{V_{Th}}{R_{tot}} = -0.58\,mA = -580\,\mu A$$
$$V_L = I_L \cdot R_L = -0.58mA \times 1.8\,k = -1.04\,V$$

Procedure

1. Set up the circuit given below on LTspice.



1.1 Measure the load voltage v_L and load current i_L.

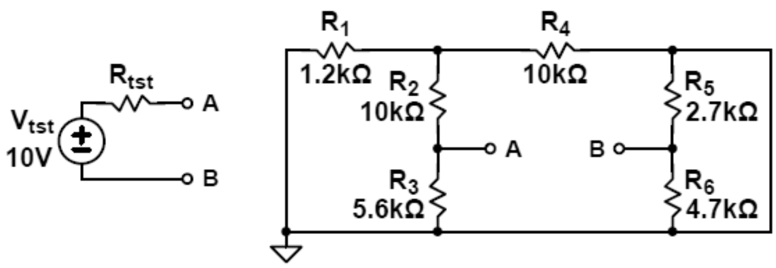
v_L = -1.0431V **i_L = -579.5μA**

1.2 Remove $\mathbf{R_L}$ and measure the open circuit Thevenin voltage between **A** and **B** terminals.

V_{th} = -4.192V

1.3 Replace the voltage sources with short circuits and measure the Thevenin resistance using a test voltage source $\mathbf{V_{tst}}$ as shown below. $\mathbf{V_{tst}}$ is connected to **A** and **B** terminals through the source resistance $\mathbf{R_{tst}}$. Find the Thevenin resistance $\mathbf{R_{th}}$ which is equal to the $\mathbf{R_{tst}}$ value that makes $\mathbf{V_{AB} = V_{tst}/2}$. You can start with setting $\mathbf{R_{tst}}$ value to the Thevenin resistance calculated in the preliminary work.

R_{th} = 5.434kΩ



1.4 How can you determine $\mathbf{R_{th}}$ without the measurement in the step **1.3**, based on v_L, i_L, and $\mathbf{V_{th}}$ measured in steps **1.1** and **1.2**?

According to circuit, we can say $R_{TH}=(V_{TH}-V_L)/i_L$ and we know their values

V_{TH}=-4.192V $V_{L=}$ -1.0431V $i_{L=}$-579.5μA

So,

$R_{TH}=(V_{TH}-V_L)/i_L$ = (-4.192V-(-1.0431V))/(-0.579.5mA)=**5.433k Ω**

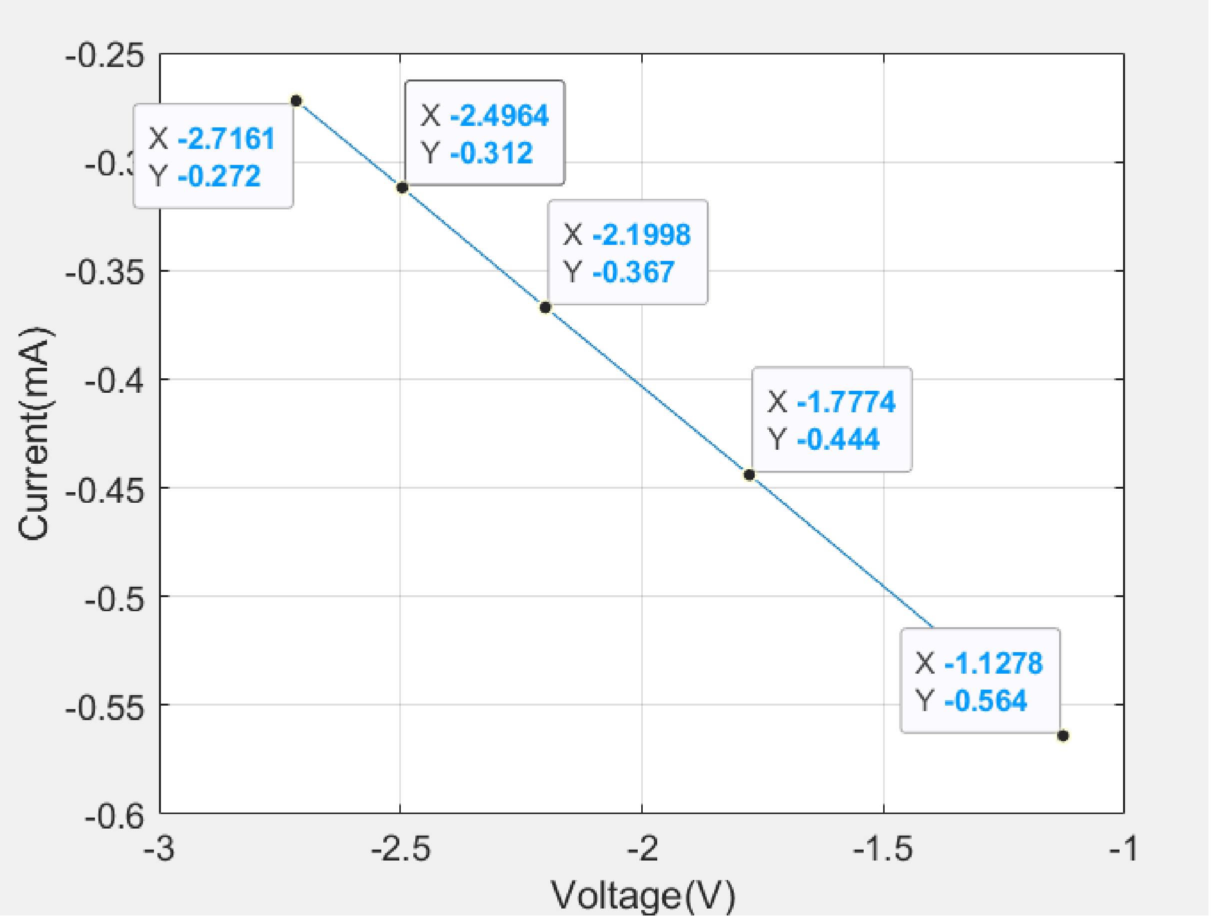
In step 1.3 our measured $\mathbf{R_{th}}$ =5.434k Ω and in step 1.4 our calculated $\mathbf{R_{th}}$=5.433k Ω , so our values are so close and our percentage of error is 0.02.

2. Find the equivalent circuit parameters, $\mathbf{V_{th}}$, $\mathbf{I_{nt}}$, and $\mathbf{R_{th}}$, based on a load line plot as in question **2** of preliminary work, instead of measuring the short circuit current. Connect the DC supplies and follow the steps given below to obtain the load line.

2.1 Set the $\mathbf{R_L}$ values given in the following table and record the voltage $\boldsymbol{v_L}$ and the current $\boldsymbol{i_L}$ for each $\mathbf{R_L}$ value.

$\mathbf{R_L}$ (kΩ)	$\boldsymbol{v_L}$ (V)	$\boldsymbol{i_L}$ (mA)
2	-1.1278V	-0.564mA
4	-1.7774V	-0.444mA
6	-2.1998V	-0.367mA
8	-2.4964V	-0.312mA
10	-2.7161V	-0.272mA

2.2 Plot $\boldsymbol{v_L}$ as a function of $\boldsymbol{i_L}$ using the data obtained in step **2.1**.



2.3 Determine $\mathbf{V_{th}}$, $\mathbf{I_{nt}}$, and $\mathbf{R_{th}}$ using the graph above.

According to graph, we can find V_{Th} and I_{nt} values from using point slope equation.

$$y_2 - y_1 = m(x_2 - x_1)$$

We chorse rondom two point to find m from graph.

$$-0.272 - (-0.312) = m(-2.7161 - (-2.4964))$$

$$0.04 = m(-0.2197) \rightarrow m = -\frac{0.04}{0.2197} = -0.182$$

To find V_{Th}, our $y=0$ and our $x = V_{Th}$

$$\underset{\downarrow\,0}{y} + 0.312 = -0.182(x + 2.4964)$$

$$x = -\frac{0.312}{0.182} - 2.4964 = -4.21 \rightarrow V_{Th} = -4.21V$$

To find I_{nt}, our $x=0$ and our $y = I_{nt}$

$$y + 0.312 = -0.182(\underset{\sim\,0}{x} + 2.4964)$$

$$y = (-0.182 \cdot 2.4964) - 0.312 = -0.7663 \rightarrow I_{nt} = -0.7663mA$$

$$R_{Th} = \frac{V_{Th}}{I_{nt}} = \frac{4.21V}{0.7663mA} = 5.494k\Omega$$

$\mathbf{V_{th}} =$ -4.21V $\mathbf{I_{nt}} =$-0.7663mA $\mathbf{R_{th}} =$ 5.494K Ω

2.4 Compare the results obtained in part **1** and part **2.3**.

	Part 1	Part 2.3	%Error
$\mathbf{V_{th}}$	-4.192V	-4.21V	0.43
$\mathbf{I_{n}}$	-0.771mA	-0.7663mA	0.61
$\mathbf{R_{th}}$	5.434KΩ	5.494KΩ	1.10

We can see our measured valued in simulation and calculated values from graph so close. Our error percentage is fewer than 1.1 for all values.

3.1 Replace $\mathbf{V_{S2}}$ with a short circuit (or set its value to **0 V**), and measure the first open circuit Thevenin equivalent voltage $\mathbf{V_{th1}}$.

$\mathbf{V_{th1}}$ **= 1.7995V**

3.2 Restore $\mathbf{V_{S2}}$, replace $\mathbf{V_{S1}}$ with a short circuit, and measure the second open circuit Thevenin equivalent voltage $\mathbf{V_{th2}}$.

$\mathbf{V_{th2}}$ **= -5.9915V**

3.3 Describe the relationship between $\mathbf{V_{th1}}$, $\mathbf{V_{th2}}$, and $\mathbf{V_{th}}$ obtained in the previous parts of the experiment.

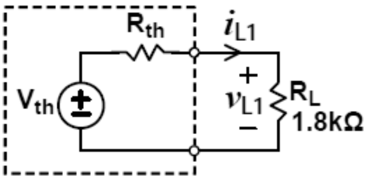
We can say $V_{th1} + V_{th2} = V_{th} = 1.7995V - 5.9915V = -4.192V$

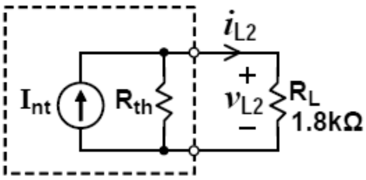
In part 1.2 from simulation our V_{th} = -4.192V

In part 2.3 from graph our V_{th} = -4.21V

From this equation in this part, we get exactly same V_{th} with from simulation(part 1.2) and we have just 0.42 percentage error between graph(part 2.3). So we can see example of superposition theorem with this part.

4. Setup the two circuits shown below. Use the $\mathbf{V_{th}}$, $\mathbf{I_{nt}}$ and $\mathbf{R_{th}}$ values obtained in step **2**.





4.1 Measure v_{L1} and i_{L1} with the Thevenin equivalent circuit:

v_{L1} = -1.0389V i_{L1} = -577.19μA

4.2 Measure v_{L2} and i_{RL2} with the Norton equivalent circuit:

v_{L2} =-1.0389V i_{L2} = -577.19μA

4.3 Compare v_{L1}, v_{L2}, i_{L1} and i_{L2} with the load voltage and current values previously measured in step **1**.

	Part 1	Part 4.1	Part 4.2	%Error between part1 and others
v_L	-1.0431V	-1.0389V	-1.0389V	0.40
i_L	-579.5μA	-577.19μA	-577.19μA	0.39

We can see our v_L values and i_L values are exactly same between part 4.1 and part 4.2 because we use same $\mathbf{V_{th}}$, $\mathbf{I_{nt}}$ $\mathbf{R_L}$ and $\mathbf{R_{th}}$ values. For Norton circuit we use $\mathbf{I_{nt}}$, $\mathbf{R_L}$, and $\mathbf{R_{th}}$ and for Thevenin circuit we use $\mathbf{R_L}$, $\mathbf{V_{th}}$ and $\mathbf{R_{th}}$ values, actually we built same circuit just one of them we have voltage source and another of them we have current source. When we compared these two values with measured values in part 1 we can see we get so close result. Our percentage of error is fewer than 1.

Conclusion

In this experiment, in preliminary work we learnt during short circuit, voltage sources can not supply infinite current so that source burns. Then we calculated equivalent circuit parameters V_{th}, R_{th}, and I_{nt} from given graph. In next step we find load resistance value which makes $V_O=V_S/2$. Then we calculated power dissipated in load resistance and we derived an expression that gives P_L as a function of V_S,R_S and R_L. .Then we found R_L value which makes P_L maximum. In next step we determined R_L,I_{nt},R_{nt},V_{th} values and we draw a Thevenin and Norton equivalent circuits and we calculated V_L an i_L. In first part we measured load voltage V_L and load current i_L values. Then we removed R_L and we measure Vth. Then we replace voltage sources with short circuits and we found R_{TH} which makes $V_{ab}=V_{tst}/2$. Then we determined R_{th}, with we substract V_{TH} and V_L then we divided result by load current and get R_{th}. In second part we measured V_L and i_L values for different R_L values(2kΩ ,4kΩ ,6kΩ ,8kΩ ,10kΩ). Then we plot V_L as a function of i_L and we determined V_{th}, Int, R_{th} using the graph. We saw our result is so close with our measured values in first part. In third part, firstly we replace V_{s2} with short circuit and we measured V_{TH1} than we restore V_{S2} and replace V_{S1} with a short circuit and measured V_{TH2} then we see sum of these two value give us V_{th} value. In fourth part we built given Norton and Thevenin circuits and we measured V_L and i_L values of these two circuits and we compared them in each other and with measured values in first part.

To sum up in this experiment, we see how we apply analysis methods based on Thevenin and Norton equivalent circuits. How we calculated R_L,I_{nt},R_{nt},V_{th}, V_L and i_L values and how we draw the Thevenin and Norton equivalent circuits. We learnt how we get maximum dissipated power on R_L. We practiced methods for evaluation of equivalent circuit parameters.

Questions

Q1. Compare the experimental results with those calculated in the preliminary work.

	V_{TH}	$R_{TH}=R_{Nt}$	I_{NT}	V_L	i_L
Calculated Values	-4.18V	5.494kΩ	-0.772mA	-1.04V	-0.580mA
Experimental Results	-4.192V	5.434kΩ	-0.771mA	-1.043V	-0.579mA
%Error	0.28	1.1	0.13	0.29	0.17

Our percentage of error is fewer than 2 for each component so we are in acceptable error range. Simulation is ideal, there is no error because of environment or measuring tool. Error can be because of decimal rounding in calculation.

Q2. What is meant by the word "equivalent" in Thevenin and Norton equivalent circuits? Can you make power calculations for the original circuit based on its Thevenin equivalent? Why?

Equivalent is an electrical part configuration that is electrically equivalent to a complex circuit that is used to simplify analysis of circuits. The Thevenin equivalent circuit is an electrical model composed of two components, which are ideal voltage source (V_{th}) and resistor (R_{th}) and Norton equivalent circuit is also an electrical model composed of two components, which are ideal current source (I_{nt}) and resistor (R_{nt}).

In thevenin or norton equivalent circuits, we can calculate the load voltage or load current for load resistance. Thus, we can make power calculations for load resistance, but we can not make power calculations for the remaining resistances because we see total of resistor in Norton or Thevenin equivalent we can not make power calculation one by one.

Q3. What are the practical applications of Thevenin's theorem? What are the advantages of using Thevenin or Norton equivalent circuits in circuit analysis?

One advantage of using Thevenin and Norton equivalent circuits in circuit analysis, they are electrically equivalent to a complex circuit and is used to simplify circuit analysis. Another advantage of them, when analyzing DC circuits so that we do not need to analyze circuits from scratch when we have a variable load, when variable change we can determine load voltage or load current easily.Also Thevenin’s theorem can be used to find maximum power transfer in special cases where $R_{th}=R_L$

Q4. Can you use the superposition principle to find the Thevenin equivalent when the original circuit contains voltage and current sources? Would it be possible to obtain the same Thevenin resistance for each and every one of the sources? Explain your answers.

We can use the superposition principle to find the Thevenin equivalent when the original circuit contains voltage and current sources. Because of we have removed all independent voltage and current sources when calculating the Thevenin resistance, we can not calculate resistance values with the superposition method.

Q5. Describe necessary measurements and calculations to determine the Thevenin equivalent of a black box by using a single fixed load resistor $\mathbf{R_L}$ and a voltmeter, without taking the risk of shorting the circuit outputs.

We can measure voltage of $\mathbf{R_L}$ and we know it becomes of our Thevenin voltage but we must considering to voltage dividing rule.

$$V_L = V_{Th} \times \frac{R_{Load}}{R_{Th} + R_{Load}}$$

So we can find the Thevenin equivalent resistance.

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Faculty of Engineering

HONOR CODE for EXAMS/HOMEWORKS

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