One-Dimensional Agricultural Weather Insurance Design Problem

This document describes the most simplified version of the problem, which is simpler than the two-dimensional problem that I described earlier, but it will serve as a good starting point.

In this simplified version, there is only one weather metric, say, rainfall in a specific region during a specified time interval. In existing forms of weather insurance, the farmer pays a fee (say $1) and gets a payout (to be selected based on the parameters of the situation) if the rainfall is less than a threshold (to be selected) and zero otherwise. Clearly, it is possible to scale the fee and payout proportionally for any given threshold if the farmer wants a larger insurance policy.

Let's start by generalizing this policy so that the payout is zero if the rainfall is in a "good" range in which the crop yield is expected to be "good" and increases as the rainfall deviates either further below or further above the "good" range. We can either take the "good" range as given (which might occur if there is an agricultural expert who can provide this information, or there is common knowledge among the farmers regarding what the range should be) or treat it as a decision. For now, I will assume the crop yield as a function of rainfall is deterministic, but we can introduce stochasticity later. The formulas expressing how the insurance payout depends on the rainfall outside of the good range are also choices that we can make, but probably should be aligned (although not necessarily perfectly correlated) with anticipated actual yields for the given rainfall levels. The actual yield will be zero for very small and very large rainfall values. (We can assume these thresholds are commonly known or provided by agricultural experts.) Typically, there is a maximum payout which is reached before the yield declines to zero, and the maximum payout per unit of insurance is a decision. In general, we would like to design the policy so that the insurance company (at least) breaks even in expectation and a risk averse farmer is better off (higher expected utility) if he/she buys the policy. (We may need to consider different utility functions.) We would also like to show how much better off the farmer will be with the new policy form versus the best possible policy of the form described in the previous paragraph, and how much more likely the farmer will be to buy insurance (which will contribute to economic stability in the locale).

Notation

Assume the cost of one unit of insurance is \$1 but the farmer may buy multiple units of insurance.

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\begin{aligned}
r &= \text{amount of rainfall (observed value)} \\
f(r) &= \text{probability density function of rainfall} \\
F(r) &= \text{cumulative distribution function of rainfall} \\
y(r) &= \text{yield (output per hectare) as a function of rainfall} \\
w &= \text{wholesale price of crop per unit} \\
c &= \text{cost of planting one hectare of crop} \\
R_a &= \text{rainfall below which yield is zero} \\
R_b &= \text{rainfall below which payout is at maximum (may be a decision for insurance co.)} \\
R_c &= \text{lower limit of ``good'' region for rainfall (may be a decision for insurance co.)} \\
R_d &= \text{upper limit of ``good'' region for rainfall (may be a decision for insurance co.)} \\
R_e &= \text{rainfall above which payout is at maximum (may be a decision for insurance co.)} \\
R_f &= \text{rainfall above which yield is zero} \\
x &= \text{number of hectares farmer plants (farmer's decision; there is an upper limit)} \\
z &= \text{number of \$1 insurance policies farmer purchases (farmer's decision)} \\
p(r) &= \text{payout per \$1 insurance policy if the rainfall is } r \text{ (insurance company's decision—} \\
& \text{this is a function and we may want to consider different functional forms;} \\
& \text{payout is zero for } R_c < r < R_d) \\
P &= \text{maximum payout per unit of insurance (insurance company's decision)} \\
H &= \text{maximum number of hectares the farmer can plant} \\
U(\cdot) &= \text{farmer's utility function}
\end{aligned}
$$

The insurance payout for $0 \le r \le R_b$ and $R_e \le r \le \infty$ is P per unit of insurance (i.e., the maximum), and for $R_b \le r \le R_c$ and $R_d \le r \le R_e$ (these are intermediate ranges in which there is a positive insurance payout but not the maximum) it is p(r) per unit of insurance. For $R_c \le r \le R_d$, i.e., the "good" range, the insurance payout is zero.

Note: If the farmer were risk neutral, then his/her expected profit would be linear in the number of hectares planted so we could pretend that he/she has only one hectare and we could find his/her optimal decision by determining whether it is profitable to plant one hectare and buy the corresponding optimal amount of insurance (which might be zero), but because the farmer is maximizing expected utility and there are decreasing marginal returns (with respect to utility) from each planted hectare, we cannot pretend there is only one hectare that can be planted.

Formulation

This is a two-stage game in which the insurance company, as the leader, designs the insurance policy and the farmer, as the follower, maximizes his/her expected utility for any given form of the insurance policy. However, in our setting, the insurance company does not necessarily want to maximize expected profit; instead, the company would like to (at least) break even in expectation while maximizing the expected benefit to the farmers, taking into account whether or not the famers will adopt the insurance policy, and how much insurance each will buy. Therefore, the goals and constraints are somewhat different than in a typical game theoretic model. We will eventually make adjustments to account for this, but the insurer's profit maximization problem still need to be solved to ensure that he/she can break even given the parameters of the situation.

Farmer's Problem

Given any set of R_b, R_c, R_d and R_e, some of which may be selected by the insurance company, as well as all of the other information that we take as given, the farmer's optimization problem is:

$$\begin{aligned}
\max_{x \le H, y} \quad & \int_0^{R_a} U(-cx - z + Pz) f(r) dr + \int_{R_a}^{R_b} U(-cx - z + wy(r)x + Pz) f(r) dr \\
+ \quad & \int_{R_b}^{R_c} U(-cx - z + wy(r)x + p(r)z) f(r) dr + \int_{R_c}^{R_d} U(-cx - z + wy(r)x) f(r) dr \\
+ \quad & \int_{R_d}^{R_e} U(-cx - z + wy(r)x + p(r)z) f(r) dr + \int_{R_e}^{R_f} U(-cx - z + wy(r)x + Pz) f(r) dr \\
+ \quad & \int_{R_f}^{\infty} U(-cx - z + Pz) f(r) dr
\end{aligned}$$

Within each integral, there is a $-cx$ term for the planting cost and a $-z$ term that corresponds to the farmer's payment for the insurance (because one unit of insurance costs \$1). The other terms in the objective function depend on the rainfall amount. The terms can be explained as follows: (i) the rainfall is below R_a so the yield is zero and the farmer receives only the insurance payout Pz; (ii) the rainfall is between R_a and R_b, so the yield is positive and the farmer earns revenue from it and the yield is in a region corresponding to the maximum insurance payout; (iii) the rainfall is between R_b and R_c, so the farmer earns revenue from the yield and gets insurance payout corresponding to the realized rainfall; (iv) the rainfall is between R_c and R_d, i.e., in the good range, so the farmer earns revenue from the yield but no insurance payout; (v) the rainrall is between R_d and R_e, so the revenue and insurance payout are symmetric to those in the third term; (vi) the rainfall is between R_e and R_f, so the revenue and insurance payout are symmetric to those in the second term; and (vii) the rainfall is greater than R_f, so the yield is zero and the farmer receives the maximum insurance payout.

If we were to insert a utility function with an analytic form into the above expression, we could, in principle, optimize jointly over $x \leq H$ and z. Let the set of optimal solutions be represented by (x^*, z^*) which depend on the various problem parameters and decisions of the insurance company.

Now, realistically, a subsistence farmer will plant his/her entire field, so it would be reasonable to set $x = H$ and just optimize z. I will continue under the assumption this is valid. For any given payout function $p(r)$, what results from the optimization is an expression (formula) for $z^*(R_c, R_d)$, i.e., the optimal value of z as a function of R_c and R_d.

Insurer's Problem

I'm going to start with an expected profit maximization form of the problem because if the insurer's expected profit is positive, the company can always give some or all of the profit back to the farmer. The insurer needs to decide $P, p(r), R_c$ and R_d simultaneously, but this is challenging. For a given P and $p(r)$, the problem is easier and can be stated as:

$$\begin{aligned}\max_{R_c,R_d} \quad & z^*(R_c,R_d) - Pz^*(R_c,R_d)[F(R_b) + (1 - F(R_e))] - \int_{R_b}^{R_c} p(r)y(r)z^*(R_c,R_d)f(r)dr \\ & - \int_{R_d}^{R_e} p(r)y(r)z^*(R_c,R_d)f(r)dr\end{aligned}$$

Recall that $z^*(R_c, R_d)$ is the farmer's optimal insurance purchase quantity as a function of R_c and R_d. The terms in the objective function are (i) revenue from insurance sales (at \$1 per unit of insurance); (ii) maximum insurance payout given a purchase of $z^*(R_c, R_d)$ times the probabilty that the maximum payout is required, i.e., if the rainfall is less than R_b or greater than R_e; (iii) expected insurance payout given a purchase of $z^*(R_c, R_d)$ for rainfall outcomes between R_b and R_c; and (iv) expected insurance payout given a purchase of $z^*(R_c, R_d)$ for rainfall outcomes between R_d and R_e. Recall there is no insurance payout for rainfall outcomes between R_c and R_d.

A near-optimal solution for a given P and $p(r)$ can be found using a grid search over R_c and R_d. This still leaves the problem of optimizing P and $p(r)$

Suggestion for Initial Exploration

Because it's difficult to optimize P and $p(r)$, it would be worthwhile to first explore how–for many reasonable combinations of $R_b, R_c, R_d, R_e, y(r), f(r)$ and $U(\cdot)$–what combinations of P and $p(r)$ cause the farmer to optimally choose to buy insurance. For any such viable combinations, we would need to check whether the insurer can break even. At a very high level, the insurance transfers the risk from a risk averse farmer to a risk-neutral insurer, so there must be a way to accomplish this risk transfer so that both parties are at least a well off as they were before and at least one party is better off. The purpose of the exploration is to identify regions in the space of the various parameters and functions mentioned in the first sentence of this paragraph where it's relatively easy to find solutions that can make both parties happy.