Weather derivatives and weather derivative pricing

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Weather derivatives are a new financial instrument that has been traded in the financial markets since 1996. They exist to allow businesses and other entities such as government organisations to hedge themselves against the possible adverse effects of weather. Traditional insurance contracts that protect against material loss due to weather have existed for many years, and much property insurance includes protection against weather related damage. The difference between such traditional weather insurance and weather derivatives is that the final settlement payment of a weather derivative is calculated from measured values of the weather itself rather than on an estimate of the loss sustained due to the weather. This means that weather derivatives are a much more general tool than traditional insurance because they can be used to cover the adverse effects of weather on intangibles such as profits and volume of sales. They are thus ideal for smoothing out the effects of weather on any business, and are often used to cover weather risks that are very close to the mean. In summary: any business that swings from profit to loss in successive years due to fluctuations in the weather can remove this volatility by using weather derivatives.
Since different businesses experience different and often opposing weather risks, one of the economic roles that the weather derivatives market plays is to help such businesses insure each other by swapping risks. As an example two of the largest purchasers of weather hedges in Europe are a gas supply company and a group representing construction workers. The gas supply company doesnt like warm winters because they sell less gas, while the construction workers dont like cold winters because they cant work outside. Both parties buy weather protection and in both cases the protection is significantly cheaper because of the existence of the other hedger.

1 The weather derivatives market

The weather derivatives market consists of the companies that trade weather derivatives. It starts with the hedgers: those companies, like the gas supply company and the construction workers described above, who buy weather derivatives to hedge the weather risk in their businesses. The hedgers buy weather derivatives from a group we will collectively refer to as the speculators. The speculators are made up of banks, energy companies, insurance companies, reinsurance companies and hedge funds. Their defining feature is that they are involved in trading weather derivatives in order to make a profit rather than to hedge their risks. In fact, there is significant overlap between the hedgers and the speculators, especially amongst energy companies: a number of energy companies are involved in the market both because they want to hedge their risk, and because they believe they can make money as speculators. Other speculators, in particular the hedge funds, are involved in trading weather derivatives purely and simply to make a profit. The market in which speculators sell weather derivatives to hedgers is known as the primary weather derivatives market. In addition to the primary market there is also a lively secondary market in which the speculators trade between themselves. As in many markets the primary and secondary markets work hand in hand. The primary market serves needs of end-users and brings risk and premium into the whole market. The secondary market serves the economic function of dispersing the risk amongst the speculators and of allowing price discovery on a number of the commonly traded indices.

2 Paradigms for pricing weather derivatives

A discussion of equity derivatives pricing usually starts with an explanation of how the price of forwards is determined by a static hedging argument from the price of the underlying equity. This is then followed by an explanation of how the price of options is determined by a dynamic hedging argument from the price of the underlying equity (the Black Scholes model) or from the price of forwards (the Black model). The

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price of options is given by the discounted expected payoff on the option plus the discounted expected profit on the hedging. Together these form the cost of replicating the payoff of the option, and are equivalent to the discounted expected payoff of the option alone if calculated in the risk neutral measure. The methods used in weather derivative pricing, however, are distinctly different from these standard theories. Black-Scholes and Black do play a role, after suitable modifications, as we shall see below, but are not central. In fact, our starting point is based on principles more usually associated with fundamental analysis of equities or with insurance pricing than with derivative pricing. We will not, at least initially, consider the cost of hedging or replication, but instead will consider the distribution of all possible outcomes of each contract.

A fundamental principle is that a future cash flow is considered less valuable if it is uncertain, and the more uncertain it is, the less value it has. For instance, 1m euros plus or minus 500k euros is considered less valuable than 1m euros with no uncertainty. One simple way to capture this idea illustratively is to consider the value of an uncertain cashflow to be given in terms of the mean and the standard deviation of that cashflow. Ignoring discounting this gives:

Value = mean lambda * standard deviation

The value of lambda is subjective: a small value of lambda (perhaps 0.01) indicates a low level of aversion to risk, while a large value (perhaps 0.5) indicates a high level of risk aversion.

Academic economists like to capture this idea of risk aversion more completely than this simple formulation by using utility functions and the assumption of marginal decreasing utility. However as far as the author is aware utility functions are never used by practitioners. For this reason we will not consider them, although we note that the definition of value given above can be considered as an expected utility.

The value of uncertain cash flows is relevant to the pricing of weather derivatives because the sellers of weather contracts are committing themselves to an uncertain cashflow in the future. For instance, a company selling a weather derivative with an expected payoff of 1m euros and a payoff standard deviation of 500k euros would typically want to set the premium of such a contract such that by selling the contract they add value to their business. If we apply the simple formula for value given above, and arbitrarily select a value of lambda of 0.2, then we conclude that the premium has to be higher than the expected payoff by 100k euros i.e. the premium should be at least 1.1m euros.

The reason these arguments are not usually applied in equity derivatives pricing is that the uncertainty in the payoffs is hedged by the issuer of the contract and so the standard deviation of the payoff is zero. The risk loading term disappears and is replaced by the cost of hedging. The price of the derivative is then set at a level such that the value of the contract is zero when the cost of hedging is taken into account. This means that the price is the expected payoff of the contract plus the expected profit or loss on hedging, or simply just the expected payoff if the probability measure is chosen such that the expected profit on hedging is zero, which is the risk neutral measure mentioned above (and which is one common way used to price such contracts). In the weather derivatives case the uncertainty in the payoffs is typically not zero (although we discuss some exceptions to this below) and it may be that there is no hedging undertaken at all.

We now discuss in more detail how weather swaps and options are priced in practice. We start with a discussion of the different settlement indices that are used.

3 Weather indices

All weather derivative contracts settle on a single weather index. Any weather index can be used, but the most commonly seen are based on daily average temperatures over a month or season. There are a number of different ways that daily average temperatures are aggregated to form the settlement index. In Japan the settlement index is usually the average over the contract of the daily average temperatures, known as average of average. In Europe in summer the settlement index is usually the sum over the contract of the daily average temperatures, known as 'cumulative average temperature', or CAT. In Europe in winter the index is usually the sum over the contract of the positive difference between the daily average temperature and 18 degrees C, or zero if the daily average temperature exceeds 18 degrees C. These indices are known as heating degree days, or HDDs, and originate in the energy industry: the rationale behind the 18 degrees C cutoff is that once the temperature is over 18 degrees C then the weather no longer affects gas consumption. In the US in winter HDDs are also used but with a cutoff of 65 degrees F, and in the US in summer the settlement index is defined as the positive difference between the daily average temperature and 65 degrees F, or zero if the daily average temperature is below 65 degrees F.

These are known as cooling degree days or CDDs.
The indices described above are the standard indices and account for most trades in the secondary market and some trades in the primary market. But many other indices are also used and in the primary market indices are often designed to match the risk of the hedging entity as closely as possible. Such indices may be based on temperature, rainfall, wind or snow and may aggregate measurements over a time period, or count the number of occurrences of certain critical events.

4 Weather swaps

The simplest kinds of weather contracts are uncapped swaps. Uncapped swap contracts have a payoff that is a linear function of the weather index. The two most important parameters of a swap contract are the strike and the tick. The strike has units of the weather index. If the weather index measured during the period of the contract ends up above the strike then the seller of the contract pays the buyer of the contract an amount proportional to the difference between the weather index and the strike, and vice versa for values of the weather index below the strike. The constant of proportionality is given by the strike. There is usually no premium associated with a weather swap.
Weather swaps are of two kinds: forwards and futures. Forwards are usually traded in the OTC market through voice brokers and would typically be capped, although the caps are set at extreme levels and do not have much influence on the pricing. Futures are traded on the Chicago Mercantile Exchange (CME) and are not capped. They are futures because the rules of the CME stipulate that daily balancing payments must be made during the course of the contract to cover fluctuations in the value.
The pricing of swaps consists of deciding on an appropriate level for the strike. The seller of a contract will prefer a high strike because it shifts the odds in their favour, while the buyer will prefer a low strike. The fair strike is defined as that level at which trading the swap would give no profit or loss on average for either buyer or seller over many independent trades. For uncapped contracts the fair strike is the expected index, and for most capped contracts the caps are sufficiently far out that this is also a good approximation to the fair strike.
A speculator selling swaps, however, would typically not be interested in selling at the fair strike, but would wish to add a risk loading to compensate them for the risk they are taking on, as discussed above. The hedger would clearly prefer not to pay such a risk loading, but paying over the fair price may still make sense for the hedger because of the reduction of their business risk and the benefits that brings to them.
The fair strike is typically estimated using historical meteorological data. For instance, a contract based on average winter temperatures for Tokyo might be priced by estimating the fair strike from the average of the winter temperatures over the last 10 years. The risk loading might then be added as a fraction of the standard deviation of the index over these 10 years. So far this is very simple. However, using historical data in this way is complicated by the presence of trends. With temperature data, for instance, more or less all stations in the world show a warming trend over the last 30 to 50 years. Because of this a fair strike estimated based on averages over 50 years would be far too cold for todays climate. One can show that even a fair strike calculated on 10 years is on average too cold because of the trends. As a result, it is reasonably common practice to remove trends from the historical data. For instance, 30 years of data might be used, but with a linear trend removed. The question of exactly how to remove trends and make optimal predictions gets very complicated in detail, and there is a growing body of research looking at the statistical methods and decision rules that one can use.
The fair strikes estimated from historical data are subject to a large measure of uncertainty i.e. we can never know exactly what the fair strike is because of the limited amount of historical data available and because of uncertainty about the trend. There is thus some subjectivity in moving from averages of historical data to actual strikes one can trade in the market.

5 Weather options

Trading swaps is one way to hedge oneself against undesirable weather. However, using swaps as hedges in this way is not for everyone because the payoff at the end of the contract may involve the hedger having to make a large payment to the speculator. The alternative is to use options. These are more like insurance in that a premium is paid up front by the hedger and the speculator pays the hedger at the end of the contract if the payoff is triggered.
As with swaps, options are based on a weather index, and again any weather index can be used. A hedger can design the index to suit themselves, and then seek protection for that index from the speculators.

The most commonly seen indices are the same as those used for swaps. The structure of the payoff of an option can also be designed in any way, and again the hedger can design their own payoff structure. The most commonly seen structures are calls and puts, but collars, binaries, straddles and strangles are also used.
The fair price of an option is defined as the discounted expected payoff (under the objective measure). If the premium is set at the fair price then neither the seller nor the buyer will make a profit on average in the long run over many independent trades. As with swaps, speculators are likely to want to sell options at above the fair price in order to compensate them for the risk they are taking on. The fair price can be estimate by taking the average of historical payoffs for the option i.e. payoffs that would have occurred in previous historical years. As with swaps one can use between 10 and 50 years of data for this calculation, and as with swaps if one is using 50 years of data it would usually make sense to remove trends from the data in some way.
The method we have just described, involving calculating how a contract would have performed in previous years, is usually known as burn analysis. A slightly more complex method in which a distribution is fitted to the weather index is also often used. Such index modelling has the principal advantage that it allows the historical values of the index to be interpolated and extrapolated. This can give more reliable estimates of the fair price, the greeks and the probabilities of extreme outcomes, and also allows us to summarise the historical data in a concise way. For standard seasonal indices the normal distribution can usually be used. For monthly indices the normal distribution can often be used, but not always, and gives a poor fit for many standard summer contracts. For exotic contracts, such as contracts based on the number of days it freezes during a certain period, the normal distribution cannot usually be used and one has to use other distributions such as the negative binomial or gamma. For any contract a reasonable alternative to these parametric distributions is to use non-parametric density estimators such as the kernel distribution.
When fitting distributions one typically has to estimate the index variance: the detrending step introduces some complexity into the decision of what estimator to use because a) the number of degrees of freedom has changed and b) the parameters of the trend are uncertain and this uncertainty should be taken into account. This gets complicated for certain trend models.
When a distribution is fitted to the historical index the fair price, greeks and standard deviation of payoffs can often be calculated using closed-form solutions based on the parameters of the distribution. For instance for the normal distribution the fair price, greeks and standard deviation of the payoffs can all be written in terms of the mean and standard deviation of the index. Fair prices, greeks and standard deviations can also be calculated using Monte Carlo simulations.

6 Market based pricing

The discussion of pricing has thus far been entirely based on a discussion of how to derive prices from an analysis of fundamentals i.e. the distribution of possible outcomes of a contract. However this is not always the only available method. Certain contracts, especially temperature based monthly and seasonal swap contracts on London, Chicago and New York, are traded sufficiently frequently that one can observe the prices of these contracts varying on a daily basis. This introduces new possibilities for both swap and option contracts, as described below.
For swap contracts it means that one can estimate the value of a currently held swap position simply by looking at the market. It may be that the market value is a good estimate of the fair strike of the swap: this would likely be the case if the market is somewhat balanced in terms of supply and demand. This then avoids the need to analyse meteorological data to derive the fair strike for the swap.
For option contracts on the same indices as these frequently traded swap contracts there are two interesting possibilities. The first is that when calculating the fair price of the option contract one can use the market swap strike as an estimate of the expected index. The remaining unknown parameter is then the standard deviation, or volatility, of the weather index (assuming that we are happy to model the index as a normal distribution, which is not always the case as discussed above). This standard deviation can then be estimated either from historical data or from the market prices of other option contracts.
The second possibility is that the swap contracts can be used to partially hedge the risk in the options contracts. In the (fictional) limiting case in which the swap could be traded an infinite number of times this is then very similar to the situation in which equity forwards are used to hedge equity options, for which the price of the option is determined by the Black model. In the weather case one has to make some small adjustments to the Black model since the Black model is based on geometric Brownian motion with drift, while arithmetic Brownian motion without drift is a more appropriate model for the swap price. But the basic principle is the same: the price of the option is determined by the principle of no arbitrage

and is given in terms of the swap level.

7 The use of weather and seasonal forecasts

The pricing methods desribed thus far are appropriate when one is pricing a weather contract well in advance of the start of the measurement period. However as the start of the contract approaches first seasonal and then weather forecasts become available and give us extra information about the distributions of possible payoffs of contracts. One may be a little doubtful about the usefulness of weather forecasts in day to day life, but for weather derivatives pricing there is no doubt that weather forecasts provide useful information up to at least 10 days in the future, and prices in the weather market are very sensitive to such forecasts. The challenge is how to merge this information in an appropriate way with information based purely on historical weather data, and various methods have been suggested for how this should be done.

8 The management of portfolios

Selling a single weather derivative, however large, would not be a practical business plan for a speculator. The premium taken in could not possibly justify the risk of having to pay out. The only way for speculators to run a successful business is to trade a number of contracts in such a way as to reduce their risk. There are various ways that this can be done, such as:

1)Trading back to back
In this business model the speculator sells weather derivatives to hedgers, and then buys identical derivatives from other speculators at a lower price. In this way they pass the weather risk thru to others in the market and hold only credit risk themselves while still making a profit.

2)Hedging in the secondary market
As described above a speculator that sells options on indices that are frequently traded in the secondary market can then attempt to hedge their risk by trading swaps. The hedging will not be perfect, and the speculator will end up holding some weather risk themselves, but the residual weather risk may be sufficiently small that the risk-return profile is favourable.

3)Building a diversified portfolio
In this business model, the speculator sells many weather options on different and weakly correlated weather indices. This can be achieved by selling options on indices based on different regions, on different weather variables and on different time periods. The resulting portfolio can then have very favourable risk-return characteristics.

All of these different business models, and others, are used by the players in the weather market today.
Estimation of the likely performance of a weather portfolio can be performed using the same burn method as described above for single weather derivative contracts. Alternatively one can also model the multivariate distribution of the outcomes of the contracts in a portfolio. This modelling approach has the advantage that it can be used to give estimates of the tail risk in a portfolio which cannot be reliably obtained from burn. If the indices of contracts in a portfolio are all being modelled using the normal distribution then the multivariate distribution of these indices can be modelled using the multivariate normal distribution (although other multivariate distributions with normal marginals may also be appropriate, and may be more appropriate in some cases). If, however, some or all of the contracts are being modelled using non-normal distributions then the multivariate normal cannot be used directly. A relatively simple alternative method involves transforming all the distributions to normal, using the multivariate normal, and transforming back. This method is a special case of a more general methodology using copulas, which can also be applied. When modelling using multivariate distributions there are no general closed form solutions and Monte Carlo simulations must be used.

9 Risk management

Many of the speculators in the weather market need to calculate their risk positions on a daily basis. This involves considering the distribution of possible outcomes of all the individual weather derivatives that they hold, and also the distribution of outcomes for the entire portfolio. The distribution of outcomes of the portfolio is calculated as described above. One can then consider various measures of risk such as 1) the expiry value at risk: this is defined as the quantiles of the distribution of outcomes if all the contracts in a portfolio are held to expiry 2) the horizon value at risk: this is defined as the quantiles of the distribution of changes in the fair value of a portfolio over a short time horizon, typically shorter than the time it would take for all contracts to expire (and possibly as short as one or two days). It should be noted that neither of these measures is exactly the same as the market value at risk commonly used in the equity and other markets.

10 Daily modelling

In addition to the index modelling methods described above there has also been considerable interest in trying to model the daily variations of weather indices using time series models. This is not as widely used in the market as index modelling principally because it is very difficult to build reasonable statistical models. However, as such models are developed it is likely that daily modelling based methods will become more prevalent.

11 Summary

We have given an overview of the methods used for the pricing of weather derivatives. We have described how weather derivative pricing is based on a varied mix of statistical modelling, arbitrage pricing and weather forecasting. Which aspects of this mix are most important depends on the contract being priced, the state of the market, and the point in time relative to the start of the contract. As the market develops and changes this mix of different pricing paradigms is likely to change although the fundamental elements will remain the same.

12 Further reading

The author's own articles on weather derivative pricing can be obtained from http://www.stephenjewson.com. Other sources of articles are http://www.ssrn.com and http://www.artemis.bm. There are several books that cover weather derivatives from a general standpoint, and discuss not only pricing but a wide range of issues including legal and accounting aspects, and give many examples (see [1], [2], [3]). The author's own book covers pricing issues alone in detail (see [4]).

13 Bibliography

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14 Legal statement

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