例 1.4

$V(x_i) = \sigma^2 = E(x_i^2) - \mu^2 = E(x^2) - E^2(x)$

$V(\bar{x}) = \dfrac{\sigma^2}{n} = E(\bar{x}^2) - \mu^2$

$E(\hat{\theta}_1) = E\left(\dfrac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}\right) = \dfrac{1}{n}E\left(\sum_{i=1}^{n}x_i^2 - n\bar{x}^2\right)$

$\qquad\qquad = \dfrac{1}{n}(n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2) = \dfrac{n-1}{n}\sigma^2$

$E(\hat{\theta}_2) = E\left(\dfrac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}\right)$

$= \dfrac{1}{n-1}E\left(\sum_{i=1}^{n}x_i^2 - n\bar{x}^2\right) = \dfrac{1}{n-1}(n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2) = \sigma^2$

$\hat{\theta}_2 = \sum_{i=1}^{n}(x_i-\bar{x})^2 \Big/ (n-1)$ 為母體變異數 $\sigma^2$ 不偏估計量

$\hat{\theta}_1 = \sum_{i=1}^{n}(x_i-\bar{x})^2 \Big/ n$ 、、 偏誤估計量