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In 1989, Manin published Reflections on Arithmetical Physics, where he predicted an increasingly prominent role for number theory within theoretical physics. At that time, early results in String Theory, including Manin's work on the partition function of the Polyakov string (,), showed that arithmetic concepts like Faltings' height had physical relevance. There has been since then a wealth of examples in which this prediction came true: Grothendieck motives and periods, various classes of modular forms, and -adic geometry have all found direct applications to physics models, as did many other number theoretic objects. Ideas from physics have, in turn, suggested new strategies in classical number theory problems. One of the most interesting ideas that Manin articulated in revolves around the adelic product formula involving the -adic non-Archimedean norms and the Archimedean absolute value of rational numbers through the simple relation This formula is seen as the prototype example illustrating a general strategy, aimed at replacing concepts usually formulated in terms of the real numbers, where the Archimedean absolute value lives, by an adelic counterpart, where the product over primes of the non-Archimedean norms is defined. Using the fact that, generally, these adelic objects turn out to have simpler properties makes it then possible to establish results that can be transferred back to the real Archimedean place by this type of adelic formula. He argued that such a strategy should be especially beneficial in physics problems, where he postulated a form of “complementarity" between our usual formulation of the fundamental models of physics in terms of real variables and an equivalent adelic counterpart. In our joint work, we first explored this notion in the context of the holographic AdS/CFT correspondence in string theory in, a set of ideas that I more recently returned to in work with students and other collaborators,. In this paper, I would like to introduce a different context in which the same procedure of recasting a real variables formulation of a physical problem in terms of Manin's idea of an “adelic shadow" can be applied: the long-range percolation models in statistical physics. I will show how one can use an adelic formulation to establish a direct geometric relation between the long-range percolation models on ordinary lattices and on hierarchical lattices. In this paper we consider two different types of long range percolation models. The first is long range percolation on a lattice. In addition to the case we will consider lattices that are rings of integers of number fields, viewed as embedded in a Euclidean space via the MInkowski embedding determined by the Archimedean places of the number field. The long range percolation model on a lattice is a random graph with as set of vertices, where a pair is connected by an edge with probability for a fixed parameter and varying inverse-temperature parameter. A main question in such percolation model is the geometry of clusters (connected components) or the resulting random graph and changes occurring at phase transitions. In particular one is interested in questions such as the behavior at criticality of the 2-point function, given by the connection probability for, the critical exponent and its relations to other critical exponents such as the one ruling the volume tail behavior, and understanding the dependence of the model behavior and the critical exponents on the parameter. Similarly, one is interested in the behavior of the n-point functions given by the probability that all are in the same cluster for a set of vertices. For a general overview of various aspects of these percolation models on lattices and the main questions about them see for instance. The other class of long range percolation models we consider here are those on hierarchical lattices. Here instead of a lattice one considers an abelian group of the form, where for some integers and. and connection probabilities, with the ultrametric for, with. Unlike ordinary lattices, the hierarchical lattice has a self-similar fractal-like geometry,. Recent work of Hutchcroft shows that the model on hierarchical lattices can sometimes be used to obtain results on the behavior of the long-range percolation on ordinary lattices, see for instance, and in particular estimates on the critical exponents. In this paper we consider the abstract geometric question of understanding the relation between long range percolation on hierarchical lattices and on ordinary lattices. We approach the question of their relation from a different perspective, based on the idea that real variables physical systems have adelic counterparts, and we describe a direct relation between the long range effects of these two types of models through an intermediate arithmetic picture model based on global fields (function fields and number fields) and the respective adeles rings. Our goal here is to present this arithmetic relation. A typical way in which relations between two different geometries are established, in the context of algebraic or arithmetic geometry, is by realizing them as different special fibers of some fibration. This same idea underlies, for instance, Arakelov geometry where an arithmetic curve is seen as a fibration over the Archimedean and non-Archimedean places of a number field, see for instance the discussion in and on the fiber at infinity of Arakelov geometry. We follow a similar reasoning here and we realize both the long-range percolation models on ordinary lattices and those on hierarchical lattices as “fibers at infinity" of adelic constructions, in number fields and function fields, respectively, where the fibers over the individual finite places, give in both cases equivalent systems. In the case of percolation on ordinary lattices an intermediate step is necessary in the construction we present, where the usual percolation model is realized as a special value at of a -parameter deformation determined by the power mean function, whose value at is the fiber at infinity (the contribution of the Archimedean places) of the adelic model. We summarize these construction that establish the relation between the two types of percolation models in terms of a diagram of the form: On the left side of the diagram, is a function field and is its adelic percolation model (introduced in) with its component at a point at of the algebraic curve and the components at the finite places, which form the finite adeles part. The component at the point at infinity is the hierarchical lattice percolation model, while the adelic product formula establishes an equivalence in long range behavior between this component at the point at infinity and the finite adeles part. On the right side of the diagram, is a number field with its adelic percolation model over (a region of) the ring of integers, with and, respectively, the controbutions of the non-Archimedean places and the Archimedean places, again related by the adelic product formula. In this case, each non-Archimedean component at one of the places is of the same type as the individual components and, while the Archimedean part is equivalent to what we call toric percolation model on a lattice (introduced in), which in turn is related by the power mean deformation to the ordinary lattice percolation. We explain the details of the various parts of this diagram through the coming sections. The more precise statement of the result illustrated in diagram is given in Theorem. In this section we present a one-parameter family of long-range percolation models on a lattice, which interpolate and generalize the long-range percolation on. The reason for introducing this range of models will become clear in the following section where we geometrically compare them to long-range percolation models on hierarchical lattices, using non-archimedean and adelic geometry. Given a lattice (in particular), consider the long-range percolation model, namely the random graph where two points are joined by an edge with inclusion probability where is a thermodynamic parameter (inverse temperature) and is a fixed parameter, and with the function with the Euclidean norm in. The power mean function is well known in information theory, since the Rényi entropy is obtained as for, see for a discussion of its properties and use in information. A main property of that will be useful in the following is monotonicity (see Theorem 4.2.8 of). Namely, the function satisfies, for all and, This property implies, as a special case, the arithmetic-geometric mean inequality when comparing and. We have seen in that the usual function of long-range percolation on a lattice can be seen as a special case of the power mean long-range percolation on a lattice, for and the uniform. We now show another special case that will be useful in the following. First we extend slightly the definition of to the case where is replaced by with for and, for, by simply setting with and the real and complex absolute value, and with a probability. (Note: we write here two separate terms in and because we will later take and to be two complex conjugate embeddings of a number field in.) We extend this, as before, to Without loss of generality we can assume that and write The expression has a geometric interpretation in terms of toric volume element. Let be the maximal torus in the affine space, the complement of the divisor of coordinate hyperplanes, with the multiplicative group,. The Haar measure on the torus is the product of the Haar measures on, with the volume form The associated volume element is In the case where instead of we consider, we can take as volume element Transversality of a lattice, as in Definition, is equivalently stated as. Thus, the volume form of is well defined at all non-zero elements of the lattice. The -mixed transversality condition for is the requirement that the volume element is finite at all points of. By, and we have Lemma shows that we can view long-range toric percolation on a lattice as a special case of power mean long-range percolation at. This then implies that one can use the behavior of the power mean when changing the parameter to compare the toric percolation to power mean percolation at other values of. A first observation is that the arithmetic-geometric mean inequality implies an estimate relating the inclusion probabilities of the toric percolation model and of the usual percolation model. For, the arithmetic-geometric mean inequality implies that so that the inclusion probabilities satisfy Since the long-range toric percolation on a transverse lattice has a higher probability of including an edge than the usual long-range percolation on the same lattice, the critical probability satisfies, since a positive probability of an infinite cluster in the ordinary case implies that the toric case probability is also positive. The -mixed case can be treated similarly. In this case, the arithmetic-geometric mean inequality can be combined with the inequality of Hölder means to get Thus, we obtain so that the inclusion probabilities satisfy As observed in, the arithmetic-geometric mean inequality is a special case of the monotonicity of the power mean function. Thus, we can view the comparison of Lemma as a special case of a stronger comparison result. The proof is analogous to Lemma, using the monotonicity instead of the arithmetic-geometric mean inequality. We will return to discuss this family of percolation models on lattices in. We first need to discuss the other type of percolation models that we will be comparing with these models on lattices, namely the hierarchical lattice models. In the previous section we have extended the usual long-range percolation model on a lattice to a continuous family of long-range lattice percolation models, depending on a parameter, which specialize to the usual model at and also includes as a special value what we referred to as toric long-range lattice percolation model. In this section we start with a different type of long-range percolation model, where instead of a lattice, one starts with a hierarchical lattice, as in,. The long-range percolation model on the hierarchical lattice is defined in the following way (see,). For large we have Also note that the function of is integrable, namely it satisfies since. We also recall a property of the hierarchical lattice that is different from the case of ordinary lattices and that will play a role in the following (Remark 1.3 of). We reviewed the two settings we want to compare: the long-range percolation models on lattices (with the one-parameter family generalization we introduced, given by the power mean model) and the long-range percolation on the hierarchical lattice. We now build a setting, based on the theory of global and local fields, where these two types of long-range percolation models can be directly compared. We start in this section with the case of function fields and how one can reformulate the long-range percolation model on the hierarchical lattice in this setting. In we will discuss the case of number fields and the relation to ordinary lattices. We will show that both settings have non-archimedean components that provide the intermediate link between the two types of long-range percolation models via the respective adeles rings and the adelic formula for the norm. There are two types of global fields: number fields in characteristic zero, and function fields in positive characteristic. Number fields are extensions of of finite degree, while the function fields are similarly finite extensions of the field of rational functions, with and an algebraic curve over (branched cover of). There are, correspondingly, two types of non-archimedean local fields, that arise as completions of global fields. In characteristic zero these are given by -adic fields (finite extensions of the field), while in positive characteristic they are fields of formal Laurent series,. In both cases, local fields can be obtained by the same procedure. Given a discrete valuation ring (DVR), with maximal ideal for some generator, one can define on the field of fractions a discrete valuation so that. A non-Archimedean metric is defined by setting for. The completion of with respect to the topology defined by this metric is, where. For example, for and, the localization of at the prime ideal, we obtain the field of -adic numbers, and the ring of -adic integers, with the -adic metric defined as above with. Note that is a lattice in (the Archimedean completion of) but is not a lattice in the non-Archimedean completions, as elements of accumulate at in the topology induced by the -adic norm. For both types of local field, we will denote by the ring of integers, by the units, and by the residue field (a finite field) with the maximal ideas, where has a single generator (called a uniformizer). In one has a decomposition, with a unit and the valuation with for. We focus here on the positive characteristics case of Laurent series and we will return to the -adic cases in. For, the field formal Laurent series (expansion at), the valuation is given by, the smallest integer with nonzero coefficient, namely such that and we have. The ring of integers is then and the maximal ideal is, with, the constant functions. For this case of positive characteristics, to see the local fields associated to a given function fields, consider a smooth projective curve over a finite field of characteristic, with field of functions. This global field is is finite extension of the field of rational functions: the realization of as a branched cover gives a ramified extension, and extensions of the field of coefficients give unramified extensions. Consider points, with degree. Points of determine valuations (places of the global field), by taking the expansion at of functions. In the case of, for, the local field is obtained by considering the local Laurent series expansion, with ring of integers the power series expansions. The remaining point of is the point at infinity: for we have local field. The valuation for the point at infinity is for the polynomial part in of the series. For more general curves, one similarly considers an affine curve and points at infinity. For a function field and a chosen point of degree, we have Let be the completion of at this point in the metric defined by, and let be the ring of functions in regular outside of. Let be the set of places of (closed points of) and, with completion in the norm determined by, and with. The ring of adeles is the restricted product where all but finitely many of the coordinates of are in. The ring of finite adeles is, with maximal compact subring. The norms for satisfy the adelic relation: This adelic relation for function fields, and the corresponding one for number fields, will be the key property that establishes the relation between the different percolation models we are discussing. For a prime and, consider the global field of rational functions and the polynomial subring. At the point at infinity, we consider the local field, corresponding to the valuation, with the projection onto the polar part, namely the abelian subgroup. The key identity relating to is with defined as in. We then see that which then gives. With this reinterpretation, we can view the usual long-range percolation model on the hierarchical lattice as the component at infinity of a long-range percolation model associated to a function field. We will then equivalently write or to denote this percolation model, where the second notation suggests that, according to Proposition, we interpret this model as the component at the infinite place of a percolation model associated to the function field. Proposition shows that the long-range percolation model on the hierarchical lattice can be described in terms of the local field with the valuation, at the point at infinity. In order to extend this to a percolation model associated to the global field, or more generally, we need to consider what such a model looks like at the finite places of the global field, namely the closed points,. First observe the function defined as in has very different properties from the analogous function at the infinite place of. The function computes the position of the first coefficient at which and differ at, namely the position of the first nonzero coefficient of the expansion at of. This implies that the function is non-integrable. Indeed, fixing the order at a given closed point still leaves arbitrary possible order at other closed points of, so that has infinite multiplicity. Compare Lemma with Remark and the integrability of. One can directly see an infinite cluster in the following way. As in Proposition, we identify the hierarchial lattice, as an abelian group, with For simplicity we focus on the case and. The other places are treated analogously up to a change to coordinates. We write polynomials in the form, where and is a monic polynomial with all zeros at points, and with. We restrict to the subgraph of the random graph with vertices the with a fixed. The set of vertices of this subgraph can then be identified with the set of pairs and the inclusion probability on this subgraph can be written as When either or and in, this is Fix and consider the set, for some. The probability of having no edges between and is then equal to so there is with probability one an edge, for some in. Since we can keep repeating the same argument with a new set with, we obtain an infinite cluster. The adelic product formula gives a relation between the functions and the function of the hierarchical lattice model. The adelic product formula gives where the latter is equal to by Proposition. With the local modesl at the finite places as in Definition and the long-range percolation model on the hierarchical lattice at the infinite place, as in Proposition, we obtain the following adelic model. We can view this model as the hierarchical lattice with independent probabilities of having an edge connecting and in the -local model. Equivalently we can see this as a random graph with vertex set, where one randomly adds “adelic edges" between vertices, which means a collection of edges, included independently at randon, in each of the local model at each finite place. The existence of an edge in means the existence of an edge in each, for every finite place. The inverse parameters and are in principle independently associated to each of the local models at the closed points. In general, we can make an arbitrary choice of, with the only condition that the infinite product converges. This condition is dictated by the fact that, for any choice of, we have for all but finitely many. There is, however, a natural choice that coordinates the parameters and and is dictated directly by the geometry of the algebraic curve. Recall that the zeta function of a curve over can be written in the form of an Euler product as with the product over the closed points of (the places of). Given a finite subset of closed points of, we also write with the product over the finite places not in. For a given, for all but finitely many we have, hence. For a given we denote by the finite set of places where. Thus, when we compute as in, we have If (hence) is small for, we then obtain with as in, hence we get. The case with constant is analogous. We have For the given, we have Moreover, we have for since while for we have When is large, is small, hence for any given By Proposition, this is equal to By Lemma, this is then equal to For as in, and small on, we then have Thus, in the case of constant, we obtain Similarly, for as in, we have, for small on, so that we also obtain Note that the expressions of of are finite for any, since the zeta function is a rational function of the variable with where is a polynomial with integer coefficients, with and the number of line bundles of degree zero on the curve (see 3 of), and with the genus of the curve and with algebraic integers with. We then obtain the following rephrasing of the question on the existence of an infinite cluster in the hierarchical lattice long-range percolation model. For a given value of, the existence of an infinite cluster at given with with, in the adelic model of Definition is equivalent to the existence of a choice of infinite clusters in each local model (as in Definition) at each finite place, with inverse temperature, with the property that is still an infinite cluster. The existence of infinite clusters, for any, is guaranteed by Proposition. However, for a given collection of such infinite clusters, whether or not is also infinite depends on both and. We can use Proposition to derive conditions on the existence of an infinite cluster in the adelic model on the basis of the hierarchical lattice of Definition. The two cases follow directly from the estimates and and the conditions for as in and the estimate. We have realized the long-range percolation model on the hierarchical lattice as an adelic model over a function field, where the finite components are the simpler models. We can now return to consider the long-range percolation models on a lattice, and their one-parameter family of deformations introduced in and show how they relate to, through another adelic construction over number fields, where the percolation model at the finite places will be comparable to the local models. We now return to discuss the percolation models on lattices introduced in, and we provide a framework where these can be directly compared, through a common underlying adelic geometry, to the hierarchical lattices discussed in the previous section. In the following, we will focus our attention on lattices that arise as rings of integers, or more generally as fractional ideals inside number fields. We view them as lattices in a Euclidean space through the set of embeddings given by the archimedean places of number fields. Let denote a number field, an algebraic extension of of some degree. The degree of the extension is the dimension of as a -vector space. The ring of integers is a free -module of rank, on an integral basis. Let be the set of places of, with the set of finite (non-Archimedean) places and the set of infinite, Archimedean, places, namely the embeddings. Let denote the number of real embeddings and the number of complex conjugate pairs of complex embeddings, so that. We denote by. This satisfies and with its integral basis defines a lattice in. Similarly, any fractional ideal in gives a lattice in with. The p-adic field is the completion of in the non-archimedean metric with, for, with. The ring of integers is given by the projective limit (ordered by divisibility) As an abelian group, is Pontrjagin dual to the Prüfer -group where the direct system is ordered by the inclusions. This in turn can be identified with the group of all roots of unity, for,. More generally, let be a -adic field, that is, a finite extension of, with, and let be its ring of integers. The discrete valuation associated to the discrete valuation ring determines the -adic absolute value on as where is the ramification index of over. The ramification index has the property that. The ring of integers is characterized by and the maximal ideal of is. The ring of integers is a free -module of rank. Let's denote by a generator of the maximal ideal,. When we have residue field, and similarly for an extension with we have, with, a finite field extension of with the inertial degree. Then. If and are two -adic fields with, then one defines by the relation, and, so that and, and. This gives the compatibility of the extensions of the -adic valuation. The -adic norm on -adic fields can also be described in terms of norms of field extensions. Given of degree, viewing as an -dimensional vector space over, we have where is the -linear map given by in. Then the -adic norms in and are related by. For, the p-adic expansion of is of the form with. The dense field of rational numbers is characterized as those points that have an eventually periodic -adic expansion, and a point has a -adic expansion that is eventually zero iff it is of the form for some non-negative integers. This identifies the set of terminating sequences with with its monoid structure. Note that negative integers have infinite -adic expansion, with In fact, all negative integers have the eventually equal to. Also note that the addition on -adic expansions is not the coordinatewise addition on the: it is the addition of base expansions, where there is a carry. In the more general case of a -adic field with, with, the -adic expansion for elements in is replaced by a unique expansion for elements of the form where the are in a chosen set of representatives of, identified with the elements of the finite residue field. We still refer to as the -adic (or more appropriately -adic) expansion of. In this more general setting, characterizing terminating (eventually equal to) and eventually periodic expansions is more delicate. It is shown in that, for a number field of degree, and a principal prime ideal of, with, such that the norm satisfies, all the elements in the ring of integers of the number field have a finite or eventually periodic -adic expansion, which is a generalization of the expansion of the form, in which different choices of the generator of (differing by multiplication by units of) can be used at each term of the expansion. If is a number field and is the completion at a non-archimedean place associated to a prime of, then we have, and the generator satisfies. The -adic expansions are then of the form with in a set of representatives of for, the local field at the place determined by, and with for some in the group of units of, and for. In the cases where all the one obtains an expansion of the form, in the local field. Since with, the condition corresponds to primes with On the other hand, the principal prime ideals of are the prime ideals that split completely in the Hilbert class field, the maximal abelian unramified extension of. For number fields of class number the ring of integers is a PID and all prime ideals are principal. Thus, in general one cannot assume that the conditions of are satisfied, to ensure that all elements of have terminating or eventually periodic -expansion, at all non-Archimedean places of. Therefore, we proceed in the following way to construct adelic percolation models for number fields. We can write an element of in the form with, where is the period. This means, in terms of -expansions, where with the in the group of units of. The norm at the non-Archimedean place determined by is given by for, where, for. Thus, is the largest that divides. So if, we have and and with equality if. We now construct a -adic version of the local long-range percolation model at finite places that we introduced in in the case of function fields. This model behaves in the same way as the local models at finite places of a function field discussed in. In particular, the function of does not satisfy the integrability condition, as in Lemma, and the critical inverse temperature is, namely there is always an infinite cluster, as in Proposition. In fact, there is a more geometric way to describe these local models, in terms of Bruhat–Tits trees. We discuss it in for the -adic case. An analogous formulation with Bruhat–Tits trees can be similarly obtained for function fields. One can associate to a -adic field its Bruhat–Tits tree. It is the homogeneous space, which can be identified with an infinite tree with vertices of valence with. The boundary at infinity of is identified with. The choice of a projective coordinate on (equivalently, the choice of three points identified with) determines a unique vertex in the interior of the tree, which is the meeting point of the infinite lines connecting these boundary points pairwise. We consider that vertex as a root of the tree. The edges adjacent to the root vertex are labelled by the elements of with, where we identify with the edge pointing in the direction of the boundary point, see Figure. We refer the reader to for a general overview of Bruhat–Tits trees and -adic geometry. The part of the boundary that is obtained by following paths from the root that starts in any of the directions in gives a copy of. After going a number of steps from the root along the path from the root vertex to, the part of the boundary that can be reached continuing with arbitrary paths away from the root gives copies of the successive, as a system of neighborhood of. The -adic valuation for is computed by the number of steps on the tree that connect the root to the vertex where the paths from the root to and to bifurcate, and -adic distance is therefore see Figure. With this geometric picture in terms of Bruhat–Tits trees, we can reinterpret the -adic hierarchical lattices described in Definition. In the Bruhat-Tits tree, at each vertex, one can identify the directions (edges incident to that vertex) with a copy of, for the residue field. Given a root vertex in the tree (determined by a choice of a projective coordinate in, as the meeting points of the geodesics connecting the points), we refer to the subtrees stemming from the vertices adjacent to (oriented away from the root) as the sectors in the directions labelled by elements of. Consider the sector of the Bruhat–Tits tree that lies in the directions in from the root vertex. This is referred to in the statistical physics literature as the Bethe tree, or Bethe lattice. It has boundary. Thus, is realized as a subset of the boundary of this sector. The sets and of terminating and eventually periodic -adic expansions, can be also identified as sets of points in the boundary, where the coefficients of the -expansion in a set of representatives of provide the path in from the root vertex to the boundary point. The identification of and a restriction to this boundary region of follows from. The interpretation in terms of paths on Bruhat–Tits trees of the local percolation models at the non-Archimedean places of a number field, or at the finite places of a function field, suggest a possible relation to the well studied percolation models on trees,. Indeed, one can reinterpret the set of terminating -series as labeling the internal vertices of the Bethe subtree of the Bruhat–Tits tree, and the function can then be used to obtain a long-range percolation model on. However, unlike in the usual percolation models on trees, here the function does not satisfy integrability, resulting in a different behavior. The argument is analogous to Lemma and Proposition for the equivalent systems for function fields, but we reformulate it here in terms of the geometry of the Bruhat–Tits tree. Fix an element, identified with a point in the bounday of the Bethe subtree of the Bruhat–Tits tree. Also fix an. The level set can be described geometrically in the following way. Consider the geodesic in the Bruhat–Tits tree from the root vertex to the boundary point. The condition that means that the path consisting of the first edges of starting at will end at a vertex of which is on the geodesic and is the first vertex of that meets this geodesic. Let denote the subtree of with root at and consisting of all vertices below in the orientation away from. It is then clear that, hence hence is non-integrable. The existence of an infinite cluster for any value of can be seen similarly. Fix an element and a subtree of such that and such that is a countable set. Such a set always exists: for example, consider the path and choose a number of steps along this path. Let be the next direction at of the next edge of the path, let be a different direction at and let be the other endpoint of the edge in the direction. Then the set has the desired properties. We then see that the probability of having no edges between and is given by Thus, it suffices to show that is divergent, which now follows from the argument above, as for all by construction. This shows that there are some edges between and. Choose a connected to by an edge, and consider the complement and a region constructed in the following way. Consider the geodesic in, and let be the direction at of the path. Choose a and let be the other endpoint of the edge at in the direction. Let. The same argument then applies to and, showing there are edges connecting them. Iterating this argument shows the existence of an infinite cluster. Note that, while the same argument applies to each, each of these graphs now lives in a different. Thus, the existence of an infinite cluster in each does not imply that the same infinite cluster would exist at all, as already observed in the function fields case. Indeed, the difference is what makes the adelic case much more interesting that the individual local non-Archimedean cases, and directly related to long-range percolation on lattices, as we discuss in. We first need to discuss the corresponding local percolation model at the Archimedean places, so that the adelic formula can be applied. For a number field with, the Minkowski embedding is given by, for the set of real embeddings of and the set of complex embeddings (which come in conjugate pairs), which together give all the archimedean places of. For every, the set of infinite, archimedean, places of, we have an embedding, of which are real embeddings and are number of complex conjugate pairs of complex embeddings, with. Let be the associated Euclidean norm in or. Since is an embedding of, for we have iff. Thus, the image is a lattice in with. We can also assemble these Archimedean local models over the infinite places of. The resulting model recovers the toric percolation model of Definition on the lattice. For large, that is, for small, we have with as in. We can then similarly assemble the local models at the non-Archimedean places, as in Definition into a single model over the finite adeles of a number field and compare the resulting model to the one associated to in Definition. As in the case of function fields, we make a choice of the sequences and that is dictated by the underlying geometry. The Dedekind zeta function of a number field has an Euler product expression of the form with ranging over the non-zero prime ideals of. In the case of this is the Euler product expansion of the Riemann zeta function. The Euler product converges absolutely for, and has analytic continuation to a meromorphic function in the complex plane, with a single simple pole at. For a finite set of prime ideals of, we also write for the Dedekind zeta function with those Euler factors removed. In the analytic continuation region we take. The long range toric percolation model on the lattice with the Minkowski embedding, restricted to, behaves like the non-Archimedan adelic long-range percolation model, in the following sense. For a place of a number field corresponding to a prime of over a rational prime, we have, hence. Thus, we can write as We have, for large at, Thus, arguing as in Lemma, we obtaun, for constant, and for large, for, where Similarly, as in Lemma, for, we have, in the same range, where By Proposition we also know that we have where, for large, at the Archimedean places, we have The adelic formula for the norm of elements of then gives Thus, combining these relations we obtain that, in this range with large, for, we have, for constant, where if and if. Similarly, for we have where in this case if and if. Thus, we obtain and. Note that, unlike the case of function fields, except in the case of, where the Riemann zeta function does not have any zeros on the positive real line, the Dedekind zeta function for a number field can have a single simple zero the positive real line. Estimates of zeros-free zones are given, for instance, in. This means that, in the case of number fields, the expressions for and in, as functions of can have a singular point for a in the interval, where is the bound given in, with the absolute value of the discriminant of. Thus, in the case of the stated estimates hold in the complement of this region. As in the case of function fields, we can use the result of Theorem to obtain estimates on the critical temperature for the adelic system. The result follows from Theorem in the same way as in Proposition for the case of function fields. We can now combine all the relations between the various percolation models described in the previous sections and obtain the following statement that makes the content of the diagram more precise. Consider again the diagram of, which we now draw as where the numbered arrows correspond to the numbering of the statements. The arrow marked as is the content of Proposition. The arrows marked as and are the result of the adelic product formula, respectively as shown in Lemma and Theorem for the function field case, and in Theorem for number fields. The arrows marked are, in both cases, the restriction to the component at one of the finite places of the global field. The arrow is obtained by comparing Proposition with Lemma and Proposition, or by observing that both models have an analogous geometric description in terms of Bruhat–Tits trees of the respective local fields. The arrow marked is the equivalence shown in Proposition. The arrow is the result of Lemma and Remark. The hypotheses of the statement correspond to the hypotheses of each of these individual statements that establish the relations illustrated in the arrows of the diagram. About the arrow above, note that while local models and with residue fields of the same characteristics have similar formulation on the corresponding Bruhat–Tits trees, the adelic products of such local models look different. In the number field case of with non-Archimedean places determined by primes of, the characteristics of the residue field ranges over all the primes, hence the branching structure of the Bruhat–Tits tree correspondingly varies, while in the function fields case of the cardinality remains a single fixed, and the branching is always just by powers. Note also that the individual systems and are insensitive to different choices of, because of the non-integrability condition of Proposition and Lemma, but the corresponding adelic systems are highly sensitive to the data and. In particular, the behavior of the adelic system will be different for and as the data and reflect the different underlying arithmetic geometry of function fields and number fields. 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Źródła podobieństw:

[1] AdelicPercolationModels.tex

Porównane pliki:

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