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Mathematical Methods in the Applied Sciences. 2021. Vol.44. No.14. P.11514-11525. 7mm { Predator-Prey Models with Memory and Kicks: } 3mm { Exact Solution and Discrete Maps with Memory} \\ 7mm { Vasily E. Tarasov} \\ 3mm { Skobeltsyn Institute of Nuclear Physics, \\ Lomonosov Moscow State University, Moscow 119991, Russia} \\ {E-mail: tarasov@theory.sinp.msu.ru} \\ { Faculty "Information Technologies and Applied Mathematics", \\ Moscow Aviation Institute (National Research University), Moscow 125993, Russia } \\ In this paper, we proposed new predator-prey models that take into account memory and kicks. Memory is understood as the dependence of current behavior on the history of past behavior. The equations of these proposed models are generalizations of the Lotka-Volterra and Kolmogorov equations by using the Caputo fractional derivative of non-integer order and periodic kicks. This fractional derivative allows us to take into account memory with power-law fading. The periodic kicks, which are described by Dirac delta-functions, take into account short duration of interaction between predators and prey. For the proposed equations, which are fractional differential equations with kicks, we obtain exact solutions that describe behaviors of predator and prey with power-law fading memory. Using these exact solutions, we derive, without using any approximations, new discrete maps with memory that represent the proposed predator-prey models with memory. MSC: 26A33; 34A08 \\ PACS: 45.10.Hj; 05.45.-a; 05.90.+m \\ Keywords: fractional dynamics; discrete map with memory; fractional differential equation; fractional calculus; processes with memory; predator-prey model; Lotka-Volterra equations; Kolmogorov equations; Predator-prey models take into account interactions of two or more types of populations. These models describe dynamics of populations that interact, thereby affecting each other's growth rates. The first predator-prey model was proposed by Alfred J. Lotka to describe autocatalytic chemical reactions. Then this model was extended to biological systems by Alfred J. Lotka in, Vito Volterra in, and Andrey N. Kolmogorov in (see also). The predator-prey models are actively used not only in biology, but also in economic sciences. For example, the predator-prey economic model was proposed by Paul A. Samuelson, who is the first American to win the Nobel Memorial Prize in Economics. Mathematically, the Lotka-Volterra and Kolmogorov models are represented by a system of two nonlinear first-order differential equations. These equations describe the dynamics of biological (or economic) systems, in which two species interact: one as a predator and the other as a prey. The Lotka-Volterra equations can be considered as a special case of equations of the Kolmogorov model, which describes dynamics of systems with predator-prey interactions, competition, disease, and mutualism. It is known that the differential equations of first-order and any integer-order cannot take into account memory effects. Memory is understood as a property of processes that describes the dependence of the current state of the process on the history of state changes in the past time interval (for example, see). Adaptations developed by prey to counter predators facilitate the development of mechanisms by predators to overcome these adaptations. In general, these adaptation processes can be based on the presence of some type of memory in prey and predators. The description of life, social and economic processes should take into account that the behavior of actors may depend on the history of previous changes in these processes. It is strange to neglect memory in these processes, since the most important actors are animals or people with memory. To describe this type of behavior, we cannot use differential equations of integer orders and we need use mathematical tools that allow us to take into account the presence of memory in life, social and economic processes. The most important tool that allows us to describe processes with memory is the theory of integral-differential operators that form a calculus. Such operators are called the fractional derivatives and integrals, and their calculus is called fractional calculus. Fractional differential equations of non-integer orders with respect to time are a powerful tool for describing processes with memory in various sciences, including physics, economics, and other sciences. The first attempt to account for memory in the predator-prey model was adopted in works in 1994. However, in these works, in fact, not memory was taken into account, but an exponentially distributed lag. Then, taking into account memory effects in predator-prey model, which is described by generalization of the Lotka-Volterra equations, was proposed in the works in 2009 and 2011 by using fractional differential equations of non-integer orders. In recent years, memory models have been actively investigated (for example, see). In all these papers, numerical simulations of the proposed predator-prey model are used. Exact solutions of generalized predator-prey model with memory are not proposed. The approach to nonlinear dynamics, which is based on discrete maps, is important to study systems described by differential equations. Discrete maps gives a much simpler formalism, which is useful in computer simulations. In nonlinear dynamics and theory of deterministic chaos, it is well-known that discrete maps can be derived from differential equations of integer orders with periodic kicks (for example, see Sections 5.2 and 5.3 in [pp.60-68]{Zaslavsky2}, and Section 1.2 in [pp.16-17]{Schuster}). In the proposed paper, we generalize this approach to derivation of the discrete maps for on the case of fading memory in a dynamical system. The discrete maps with memory are considered, for example, in papers. Unfortunately, all these discrete maps with memory were not derived from any fractional differential equations. For the first time, discrete maps with memory were obtained from fractional differential equations in works (see also and). The proposed discrete maps with memory are exact discrete solutions of fractional differential equations with kicks. No approximations were used when obtaining these maps with memory (for details see [pp.409-453]{Springer2010}). Then, the approach, which was suggested in, has been applied in works. Computer simulations of some discrete maps with memory were suggested in papers,, where new types of chaotic behavior and new kinds of attractors have been found. To obtain new type of discrete maps with memory from these equations, we use the equivalence of the fractional differential equation with Caputo fractional derivative and the Volterra integral equation of the second king (for details see and). At the beginning, we obtain discrete memoryless maps from the first-order differential equation of Lotka-Volterra with periodic kicks in terms of interactions. Then, we obtain the exact solutions for the fractional Lotka-Volterra and the fractional Kolmogorov equations with kicks, which takes into account power-law memory. Using these exact solutions, we obtain discrete maps with memory for the proposed predator-prey models. The Lotka-Volterra model is described by two first-order nonlinear differential equations. These equations are used in biology and economics to describe the dynamics of two species, which are called a prey and a predator that interact with each other. The Lotka-Volterra equations have the form where is the number of preys, and is the number of some predators. The parameters,, are positive real numbers. The prey are assumed to have an unlimited food supply and to reproduce exponentially, unless subject to predation. The parameter is the rate of this exponential growth. The parameter represents the loss rate of the predators due to either natural death or emigration, it leads to an exponential decay in the absence of prey. The parameters and are positive real parameters that characterize the interaction of the two species. Let us assume that these interactions of the two species (prey and predator) occur for very short time interval, which can be considered as negligible time interval, almost instantaneous. We will assume that this interaction is realized in the form of sharp splashes ("kicks") that takes an infinitesimally short time. This property will be represented by series of the Dirac delta-functions. We will assume that these sharp splashes ("kicks") occur after a certain time interval. In the general case, this time interval can be considered as a random variable, and then the discrete maps can be averaged over some distribution. In our model, the parameter can be interpreted as the mean time between interactions. The parameters and characterize the intensity of this interaction, or rather the amplitudes of these sharp splashes ("kicks"). In this case, the Lotka-Volterra equations with periodic kicks can be represented in the form In these equations, perturbations by interactions are taken into account by the periodic sequences of Dirac delta-function type kicks following with period, where and are amplitudes of these kicks for prey and predator, respectively. Equations and can be written in the form Integration of and from to gives Let us use the equation which holds for, if is continuous at. Then equations and give where is the Heaviside step function, which is equal to zero when (i.e.,). We can see that solutions, are discontinuous. The product of the delta-functions and the functions, in equations and is meaningful, if and are continuous functions at the points. This is a contradiction, which can be resolved using and with ( ) instead of and to make a sense of these equations, when and,. Expressions and are exact solutions of the Lotka-Volterra equations with periodic kicks. Let us derive the discrete-time maps by using solutions and. For the left side of the -th kicks, where, equation and can be written in the form where and are defined by the expressions for. For at in equations and, we get For at in equations and, we get Subtracting equation and of the -th step from equations of the -th step, we obtain the equations of the discrete-time maps These maps can be rewritten in the form Discrete maps and describe behaviors of predator and prey without memory in the framework of proposed generalization of the Lotka-Volterra model that is described by equations and. The description of life, social and economic processes should take into account a memory i.e. that the behavior of actors may depend on the history of previous changes in processes. It is strange to neglect memory in these processes, since the most important actors are animals and people, which have a memory. To describe this type of behavior, we cannot use differential equations of integer orders and we need use mathematical tools that allow us to take into account the presence of memory in life, social and economic processes. To take into account a fading memory, we can consider integro-differential equations instead of differential equations of the integer orders. If we take into account the memory with power-law fading and periodic kicks, then we get the generalization of Lotka-Volterra model represented by the integro-differential equations where and are memory functions. The properties of this functions are described, for example, in and [pp.3-52]{BOOK-DG-2021}. For, equations and give equations and that described dynamics without memory. We can consider the power-law form of memory fading and powe-law memory functions due to the following reasons. (I) Power laws and power-law functions play an important role in different life and social sciences: Biology; Ecology; Population dynamics; Victimology; Economics; Finance. (II) The power-law memory function can be considered as an approximation of the generalized memory functions. In paper, we use the generalized Taylor series in the Trujillo-Rivero-Bonilla form for wide class of the memory functions. We proved that the equations with memory functions can be represented through the integro-differential operators with power-law kernels. As a result, we will assume the following power-law form of memory function where is the gamma function, and are the memory fading parameters for behavioral processes of prey and predator, respectively. If we take into account the memory with power-law fading by the memory functions and, then the generalized Lotka-Volterra equations with kicks and memory and take the form For, equations and gives and. The Caputo fractional derivative is defined by the equation where, and is the derivative of the integer order with. In equation, it is assumed that, i.e., the function has integer-order derivatives up to ( )-th order, which are continuous functions on the interval, and the derivative is Lebesgue summable on the interval. To obtain new type of discrete maps with memory from these equations, we the equivalence of the fractional differential equation with Caputo fractional derivative and the Volterra integral equation of the second king (for details see). To get the solution, we will use the equivalence of the fractional differential equation with Caputo fractional derivative and the Volterra integral equation of the second king (for details see and). This equivalence is based on the equality (see Lemma 2.22 in [p.~96]{FC4}) in the from which holds for or, where. For the case,, equation takes the form for or. Here is the Riemann-Liouville fractional integral that is defined by the equation where is the gamma function, and the function satisfies the condition. For, operator gives the standard integration of the first order Let us consider the case. Then, application of the Riemann-Liouville fractional integrals of orders alpha and beta, respectively, gives the equations Fractional differential equations and contain the Dirac delta-functions. These functions are the generalized functions that are treated as functionals on a space of test functions. Therefore equations and should be considered in a generalized sense, i.e. on the space of test functions, which are continuous. The product of the delta-functions and the functions, is meaningful, if and are continuous functions at the points. Therefore we use and with ( ) instead of and to make a sense of these equations for, when and,. For in equations and, we get Using equation 2.1.16 of (see Property 2.1 in [p.71]{FC4}) for the Riemann-Liouville fractional integral where and, we get equations and in the form Expressions and are the exact solutions of the generalized Lotka-Volterra model with power-law memory and periodic kicks,. Let us derive the discrete-time maps for these solutions. For the left side of the -th kicks ( ), we can define and by equation. For at in equations and, we get For at in equations and, we get Subtracting equation and of the -th step from equation for the -th step, we obtain the equations of the discrete-time maps with memory where we use the function { V}_{ } (z) = (z+1)^{ -1} - z^{ -1} for. Discrete maps and describe behaviors of predator and prey with power-law fading memory in the framework of generalized Lotka-Volterra model with periodic kicks and power-law memory. Let us consider economic model of two interacting sectors (or two economic processes) [pp.377-381]{BOOK-DG-2021}, that is described by two first-order differential equations The functions and are interpreted as the respective growth rates of these two sectors. Equations and define the Kolmogorov predator-prey model. We should note that these equations can be written in the form Let us consider a generalization of the Kolmogorov predator-prey model by taking into account periodic kicks. Equations with periodic kicks have the form The predator-prey model with periodic kicks and power-law memory is described by the fractional differential equations For, equations and take the form of equations and. In equations and, the products of the delta-functions and the functions,, are meaningful, if, are continuous functions at the points. Therefore, we use the functions with ( ) instead of, to make a sense of these equations, when and,. By repeating the transformations of the previous section, we can get the exact solution of these fractional differential equations and discrete mappings with memory. Let us consider the case. Then, application of the Riemann-Liouville fractional integrals of orders and, respectively, gives the equations Fractional differential equations and contain the Dirac delta-functions. The product of the delta-functions and is meaningful, if the functions are continuous at the points. Therefore we use with ( ) instead of to make a sense of these equations for, when and,. For at in equations and, we get Expressions and describe solutions of equations of the generalized Kolmogorov model with power-law memory and periodic kicks. Let us derive the discrete-time maps for these solutions by using and that are defined by equation. For at in equations and, we get For at, we have Subtracting equations and of the -th step from equations and of the -th step, we obtain the equations of the discrete maps with memory where we use the function { V}_{ } (z) = (z+1)^{ -1} - z^{ -1} for. Discrete maps and describe behaviors of predator and prey with power-law fading memory in the framework of generalized Kolmogorov model with periodic kicks and power-law memory. We can consider a special case of the proposed general model. For example, we can use the expansion of the functions of two variables in the Taylor series. We can consider only the linear case where If we assume that, and relabel the parameters and such that then equations and give the equations of the new generalized Lotka-Volterra model with memory and kicks. Note that the first model, which is described by equations and, the linear terms with parameters and are outside the action of periodic kicks. Therefore, in case with, equations and do not coincide with equations of discrete maps with memory and. In this paper, new type of predator-prey models is proposed. These models take into account power-law fading memory and periodic kicks. The equations of these models are generalizations of the Lotka-Volterra and Kolmogorov equations, in which we take into account power-law memory by using the fractional derivatives of non-integer orders and periodic kicks. This derivative allows us to take into account memory with power-law fading. The periodic kicks take into account short duration of interaction between predators and prey. In this paper, we obtain exact solutions that describe behaviors of predator and prey with power-law fading memory. These exact solutions are used to get discrete maps with memory that represent the proposed predator-prey models with memory. It should be emphasized that these discrete memory maps with memory are obtained from fractional differential equations without using any approximations. In this paper, we proposed models and discrete maps with memory for the case and to simplify consideration and equations. These models can be simply generalized for all positive values of these parameters by using the methods, which are described in Chapter 18 of [pp.409-453]{Springer2010}. For this purpose it is necessary to use generalized moments for variables,, and Theorems 18.17-18.19 of [pp.442-445]{Springer2010}. We assume that proposed predator-prey models and discrete maps with memory will find many applications in the nonlinear economic and biological dynamics with memory. It is safe to hope that the proposed maps with memory can simplify simulations of the behavior of prey and predators with power-law fading memory in computer simulations. However, this modeling remains an open question hopefully it will be solved in future research. This work does not have any conflicts of interest. There are no funders to report for this submission {100} Lotka, A.J. Contribution to the theory of periodic reaction... Vol.14. No.3. P.271-274. DOI:

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