

Probability Theory

Homework 7

Ben Kallus

Due Friday, October 30

1. a.

$$P_{A,B}(a,b) = \begin{cases} \frac{1}{9} & a=0, b=0, \\ \frac{2}{9} & a=0, b=1, \\ \frac{2}{9} & a=1, b=0, \\ \frac{2}{9} & a=1, b=1, \\ \frac{1}{9} & a=2, b=0, \\ \frac{1}{9} & a=0, b=2, \\ 0 & \text{else.} \end{cases}$$

b.

$$\begin{aligned} \mathbb{P}(B < A) &= \frac{2}{9} + \frac{1}{9} \\ &= \frac{1}{3}. \end{aligned}$$

c.

$$\begin{aligned} \mathbb{E}[A^B] &= \sum_{a=0}^2 \sum_{b=0}^2 P_{A,B}(a,b) \cdot a^b \\ &= \frac{1}{9} \cdot 0^0 + \frac{2}{9} \cdot 0^1 + \frac{2}{9} \cdot 1^0 + \frac{2}{9} \cdot 1^1 + \frac{1}{9} \cdot 2^0 + \frac{1}{9} \cdot 0^2 \\ &= \frac{1}{9} + \frac{2}{9} + \frac{2}{9} + \frac{1}{9} \\ &= \frac{2}{3}. \end{aligned}$$

d.

$$p_A(a) = \begin{cases} \frac{4}{9} & a = 0, \\ \frac{4}{9} & a = 1, \\ \frac{1}{9} & a = 2, \\ 0 & \text{else.} \end{cases}$$