

Probability Theory

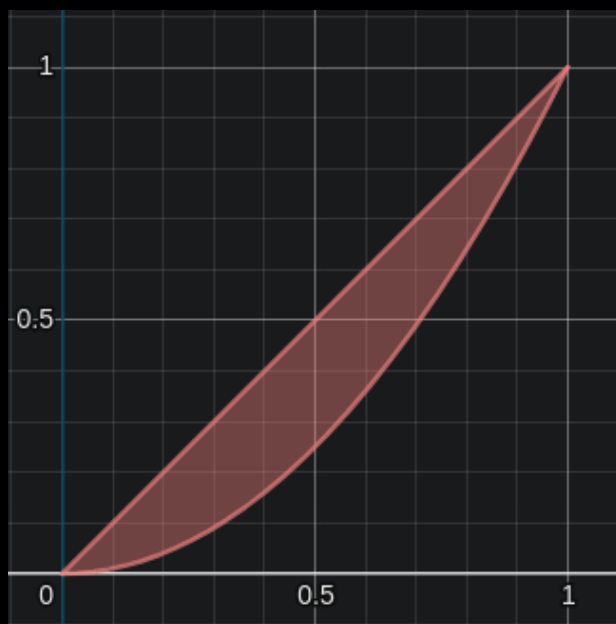
Daily Task

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October 26, 2020

$$f_{X,Y}(x,y) = \begin{cases} 6 & x^2 \leq y \leq x, \\ 0 & \text{else.} \end{cases}$$

1.



2.

$$f_X(x) = \begin{cases} 6(\sqrt{x} - x) & 0 \leq x \leq 1, \\ 0 & \text{else.} \end{cases}$$
$$f_Y(y) = \begin{cases} 6(y - y^2) & 0 \leq y \leq 1, \\ 0 & \text{else.} \end{cases}$$

X and Y are not independent, since $f_X(\frac{1}{2})f_Y(\frac{1}{2}) \approx 1.864$ and $f_{X,Y}(\frac{1}{2}, \frac{1}{2}) = 6$.

3.

$$\begin{aligned}\mathbb{E}[XY] &= \int_0^1 \int_{x^2}^x 6xy \, dy \, dx \\ &= \frac{1}{4}.\end{aligned}$$

4.

$$\begin{aligned}f_{X|Y}(x|5) &= \frac{f_{X,Y}(x, 0.5)}{f_Y(0.5)} \\ &= \frac{2f_{X,Y}(x, 0.5)}{3}\end{aligned}$$