Proceedings of the 3rd Conference on Fully Actuated System Theory and Applications
May 10-12, 2024, Shenzhen, China

2024 3rd Conference on Fully Actuated System Theory and Applications (FASTA) | 979-8-3503-7369-1/24/$31.00 ©2024 IEEE | DOI: 10.1109/FASTA61401.2024.10595168

Innovation-trigger-based sequential fusion Kalman filter for CPSs with multiplicative noises against deception attacks

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Abstract: For the CPSs with multiplicative noises against deception attacks, the sequential fusion filters under the innovation-triggered mechanism are presented. The random deception attacks occur in the transmitted measurements, the innovation-triggered mechanism is introduced to ease off the network transmission pressure, the fusion center makes two-sensor fusion estimation under three different fusion algorithms according to the arriving order of the local estimators. The results indicate that three sequential fusion algorithms can effectively ease off random attack effection and computational pressure, and the accuracy of sequential state fusion filter is higher than those of SCI fusion filter and SICI fusion filter. A simulation example verifies the effectiveness of above sequential fusion algorithms.

Key Words: Sequential fusion, CPSs, Innovation-triggered mechanism, Multiplicative noises, Deception attacks

1 Introduction

With the rapid development of modern control technology and communication technology, the cyber-physical systems (CPSs) solve the requirements of information and networking in the new era that the traditional single-point technology can not meet[1]. However, in many practical applications, the system model is uncertain due to modeling errors, random disturbances and other reasons. Commonly, the uncertainties include those formed by uncertain noise variance and random disturbance (such as multiplicative noise). These system uncertainties will cause the filtering performance deteriorate, and ever lead to divergence[2]. On the other side, during the communication process, the communication channel often suffers from the network attacks. The common network attacks include DoS attacks, deception attacks and so on, among which the deception attacks are the most common, since the deception attacks have strong concealment and are not easy to be found during transmission. In [3], the state estimator for CPSs with deception attacks is discussed, and the deception attacks are brought by a set of random variables to model measurements of attacks, that is, when the probability of the Bernoulli variable is 1, the system is attacked and the measurement is converted into an attack sequence, otherwise equal to the original measurement.

In order to ease off the transmission pressure of the CPSs and the pressure of network transmission, the event-triggered mechanisms can be introduced in the CPSs. Commonly, the event-triggered mechanisms include Send-on Delta and the innovation-triggered mechanisms. The triggered mechanism is considered in the CPSs in [4], where if the current measurements and the previous measurements exceed the threshold, the measurements at this moment will be transmitted.

This work is supported by the National Natural Science Foundation of China (NSFC-61503125), Natural Science Foundation of Heilongjiang Province (QC2013C062), Fundamental Research Support Program for Outstanding Young Teachers of Heilongjiang Province(YQJH2023139), and Science Foundation of Distinguished Young Scholars of Heilongjiang University(JCL201104).

In recent years, with the rapid development of navigation, target tracking and signal processing, the multi-sensor information fusion estimation method has been widely concerned and used. Commonly the state fusion methods include the centralized fusion and the distributed fusion methods. The centralized fusion method is performed by senting the measurement data of all sensors to the fusion center to form the centralized state space model, without any data loss, so it has the optimality. In the distributed fusion process, the local state estimation of each sensor is send to the fusion center for fusion according to certain fusion rule, which ease off the computational pressure of the fusion center. The Common distributed fusion mathods include ones weighted by matrices[5], diagonal matrices[6], scalars[7] and covariance intersection (CI) fusion[8] method. CI fusion do not need to know cross-covariance matrices between these local estimations, which ease off the computational pressure.

Since the sensor information is transmitted to the fusion center at different times, the fusion center is always idle and waiting for all sensors' information, so the concept of the sequential fusion is presented. The more commonly sequential fusion method includes the sequential state fusion(SSF), sequential covariance intersection(SCI) fusion[9] and sequential inverse covariance intersection(SICI) fusion[10]. SCI fusion algorithm is a recursive CI fusion of two sensors, which is fused according to the arrival sequence of the sensors, obviously reducing the computational burden of the fusion center. Similarly, both SICI fusion and SSF algorithms adopt multiplestage two-sensor fusion strategy respectively. In [11], the SSF estimation of sensor networks is presented, where the local estimates are weighted fused by matrices one by one according to the order of the data arriving at the fusion center, and the SSF has better estimation accuracy and lower computational complexity.

At present, the sequential fusion estimation for CPSs is rarely studied. For CPSs, a sequential fusion estimation with multiplicative noises against deception attacks is presented under the innovation-triggered mechanism, where the random deception attacks attack the transmitted measurements, the innovation-triggered mechanism launches the observed data transmission after the attacks, and the measurement datas are sequentially fused by SSF, SCI and SICI fusion algorithms, according to the arrival order in the fusion center. These sequential fusion estimators have good tracking performance and estimation accuracy.

The structure of the paper is summarized as follows: in Section 2, by the fictitious noise technology, the CPSs model is transform to a new model. In Section 3, the local optimal filter with multiplicative noises against deception attacks under the innovation-triggered mechanism is designed. The SSF filter weighted by matrices is presented in Section 4, and then the SCI and SICI fusion filters are presented in Section 5. In Section 6, the accuracy relationship between the three sequential fusion algorithms are shown. The effectiveness of the three sequential fusion algorithms is verified by a typical CPSs simulation examples in Section 7. Finally, the conclusion is drawn in Section 8.

2 Problem Formulation

2.1 System Model

Consider the linear discrete CPSs with multiplicative noises:

$$x(l+1)=\left(\Phi+\sum_{s=1}^{q}\xi_s(l)\,\Phi_s\right)x(l)+\Gamma w(l), \tag{1}$$

$$y_i(l)=H_i x(l)+v_i(l),\, i=1,\cdots,N, \tag{2}$$

where l is the discrete time, $x(l)\in R^n$ is the state, $w(l)\in R^r$ is the process noise, $\xi_s(l)\in R^1$ is the multiplicative noises, $y_i(l)\in R^{m_i}\,(i=1,2,\cdots,N)$ is the measurement of sensor i, $v_i(l)\in R^{m_i}$ is the measurement noise, Φ,Φ_s,Γ and H_i are known constant matrices, and N is the number of sensors.

Assumption 1. $w(l)$, $v_i(l)$ and $\xi_s(l)$ are uncorrelated Gaussian white noises with zero means and variences as Q_w, R_i and $\sigma^2_{\xi_s}$ respectively

$$\begin{aligned}\mathrm{E}\left\{\begin{bmatrix} w(l)\\ v_i(l)\\ \xi_s(l)\end{bmatrix}\begin{bmatrix} w^{\mathrm{T}}(j) & v_l^{\mathrm{T}}(j) & \xi_k^{\mathrm{T}}(j)\end{bmatrix}\right\}\\ =\begin{bmatrix} Q_w & 0 & 0\\ 0 & R_i\delta_{il} & 0\\ 0 & 0 & \sigma^2_{\xi_s}\delta_{sk}\end{bmatrix}\delta_{lj}\end{aligned} \tag{3}$$

where $\delta_{ll}=1,\delta_{lj}=0\,(l\neq j)$.

Assumption 2. $x(0)$ is independent of $w(l)$, $v_i(l)$ and $\xi_s(l)$, i.e.,

$$\begin{aligned}\mathrm{E}\left[x(0)\,w^{\mathrm{T}}(l)\right]=0,\mathrm{E}\left[x(0)\,v_i^{\mathrm{T}}(l)\right]=0,\\ \mathrm{E}\left[x(0)\,\xi_s^{\mathrm{T}}(l)\right]=0,\forall l\end{aligned} \tag{4}$$

with

$$\mathrm{E}[x(0)]=x_0,\mathrm{E}\left[(x(0)-x_0)(x(0)-x_0)^{\mathrm{T}}\right]=P_0 \tag{5}$$

When the measurement $y_i(l)$ is transmitted to the fusion center, the new measurement suffering the deception attacks can be expressed by

$$y_i^a(l)=-y_i(l)+\zeta_i(l), \tag{6}$$

where $\zeta_i(l)\in R^{m_i}$ is the attack signal injected by the attacker, which is a zero mean, Gaussian white noise with variance $\Xi_i>0$.

Usually a set of Gaussian random variables $\{\alpha_i(l)\}$ are generated to represent the attack phenomenon, which bypasses the χ^2 detector with

$$\Pr\{\alpha_i(l)=1\}=\sigma_i,\Pr\{\alpha_i(l)=0\}=1-\sigma_i, \tag{7}$$

Then, denote the final measurement $z_i(l)$ as

$$\begin{aligned}z_i(l)&=y_i(l)+\alpha_i(l)\,y_i^a(l)\\ &=(1-\alpha_i(l))\,y_i(l)+\alpha_i(l)\,\zeta_i(l).\end{aligned} \tag{8}$$

From (8), it can be seen that the measurement is attacked when $\alpha_i(l)=1$, and when $\alpha_i(l)=0$, the attacks is unsuccessful.

Our aim is to design the sequential state fusion filter, SCI and SICI fusion filters for CPSs with multiplicative noises against deception attacks under the innovation-triggered mechanism, respectively.

2.2 System Model Transformation

Applying the fictitious noises technique, the CPSs system (1) and (2) are transformed into

$$x(l+1)=\Phi x(l)+w_a(l), \tag{9}$$

where the fictitious process noise is

$$w_a(l)=\sum_{s=1}^{q}\xi_s(l)\,\Phi_s x(l)+\Gamma w(l), \tag{10}$$

Its variance is

$$Q_a(l)=\sum_{s=1}^{q}\sigma^2_{\xi_s}(l)\,\Phi_s X(l)\,\Phi_s^{\mathrm{T}}+\Gamma Q_w(l)\,\Gamma^{\mathrm{T}}, \tag{11}$$

where $X(l)=\mathrm{E}\left[x(l)\,x^{\mathrm{T}}(l)\right]$ is the second moment of the state. From the CPSs model (1) and (2), it yields

$$X(l)=\Phi X(l-1)\,\Phi^{\mathrm{T}}+\sum_{s=1}^{q}\sigma^2_{\xi_s}(l)\,\Phi_s X(l)\,\Phi_s^{\mathrm{T}}+\Gamma Q_w(l)\,\Gamma^{\mathrm{T}}, \tag{12}$$

whose initial value is $X(0)=x_0x_0^{\mathrm{T}}+P_0$.

Substituting (2) into (8) yields the actual sensor output

$$\begin{aligned}z_i(l)=(1-\sigma_i)\,H_i x(l)+(1-\alpha_i(l))\,v_i(l)\\ +\alpha_i(l)\,\zeta_i(l)-(\alpha_i(l)-\sigma_i)\,H_i x(l).\end{aligned} \tag{13}$$

By using the fictitious noise technique, the observed equation can be transformed as

$$z_i(l)=\Pi_i x(l)+V_i(l), \tag{14}$$

where the new observation matrices are defined as

$$\Pi_i=(1-\sigma_i)\,H_i, \tag{15}$$

and the fictitious measurement noises are defined as

$$\begin{aligned}V_i(l)=(1-\alpha_i(l))\,v_i(l)+\alpha_i(l)\,\zeta_i(l)\\ -(\alpha_i(l)-\sigma_i)\,H_i x(l).\end{aligned} \tag{16}$$

It can be easily verified that the fictitious measurement noise is a whise noise with

$$\mathrm{E}\left[V_i\left(l\right)\right]=0, \tag{17}$$

and the variance matrice

$$\begin{aligned} R_{V_i}\left(l\right) &= \mathrm{E}\left[V_i\left(l\right)V_i^{\mathrm{T}}\left(l\right)\right] \\ &= \left(1-\sigma_i\right)R_i\left(l\right)+\sigma_i\Xi_i+\sigma_i\left(1-\sigma_i\right)H_iX\left(l\right)H_i^{\mathrm{T}}. \end{aligned} \tag{18}$$

Remark 1. Since $\Gamma Q\Gamma^{\mathrm{T}}\geqslant 0$, a sufficient condition for the existence of a unique semidefinite solution to the generalized Lyapunov equation (12) is that the spectral radius $\rho\left(\Phi_\delta\right)<1$ of the matrix $\Phi_\delta=\Phi\otimes\Phi+\sum_{s=1}^{q}\sigma_{\xi_s}^2\Phi_s\otimes\Phi_s$, where the symbol $\otimes$ represents Kronecker product, so Φ_δ is a stable matrix.

2.3 The Innovation-triggered Mechanism

Since the energy constraints are prevalent in CPSs, the innovation-triggered mechanism is used to design the estimator[12], the innovation can be denoted as a one-step prediction error of the measurement, i.e.

$$e_i\left(l\right)=y\left(l\right)-\hat{y}_i\left(l|l-1\right), \tag{19}$$

and its covariance is denoted as $Q_{e_i}\left(l\right)=\mathrm{E}\left\{e_i\left(l\right)e_i^{\mathrm{T}}\left(l\right)\right\}$. Standardizing the innovation, it yields

$$\bar{e}_i\left(l\right)=\frac{e_i\left(l\right)}{\sqrt{Q_{e_i}\left(l\right)}} \tag{20}$$

The innovation-triggered threshold value is $\theta_i>0$, and the Bernoulli variable $\gamma_i\left(l\right)$ is usually introduced to describe the sensor transmission states. Then, $\gamma_i\left(l\right)=1$ represents $\bar{e}_i\left(l\right)$ falls outside the threshold value θ_i, and the measurement contains a lot of new information, so data transmission is required. On the contrary, $\gamma_i\left(l\right)=0$ represents the measuremants should be sent, i.e.

$$\gamma_i\left(l\right)=\begin{cases}1, & ||\bar{e}_i\left(l\right)||_2\geqslant\ \theta_i\\ 0, & \quad other\end{cases} \tag{21}$$

Remark 2. The innovation $e_i\left(l\right)$ and its covariance $Q_{e_i}\left(l\right)$ defined here, are given later in the calculation of the local optimal filter in Theorem 1.

3 Local Optimal Filter

Theorem 1. The innovation-trigger-based local Kalman filters for the CPSs with multiplicative noises against deception attacks (1) and (13) are obtained under Assumption 1 and 2

$$\hat{x}_i\left(l|l\right)=\hat{x}_i\left(l|l-1\right)+\gamma_i\left(l\right)K_i\left(l\right)e_i\left(l\right), \tag{22}$$

$$e_i\left(l\right)=z_i\left(l\right)-\left(1-\sigma_i\right)H_i\hat{x}_i\left(l|l-1\right), \tag{23}$$

$$\begin{aligned} Q_{e_i}\left(l\right) &= \left(1-\sigma_i\right)^2H_iP_i\left(l|l-1\right)H_i^{\mathrm{T}}+\left(1-\sigma_i\right)R_i\left(l\right) \\ &\quad +\sigma_i\Xi_i+\sigma_i\left(1-\sigma_i\right)H_iX\left(l\right)H_i^{\mathrm{T}}, \end{aligned} \tag{24}$$

$$K_i\left(l\right)=\left[\left(1-\sigma_i\right)P_i\left(l|l-1\right)H_i^{\mathrm{T}}\right]Q_{e_i}^{-1}\left(l\right), \tag{25}$$

$$P_i\left(l|l\right)=P_i\left(l|l-1\right)-\gamma_i\left(l\right)K_i\left(l\right)Q_{e_i}\left(l\right)K_i^{\mathrm{T}}\left(l\right), \tag{26}$$

$$\begin{aligned} P_i\left(l+1|l\right) &= \Phi P_i\left(l|l\right)\Phi^{\mathrm{T}}+\sum_{s=1}^{q}\sigma_{\xi_s}^2\left(l\right)\Phi_sX\left(l\right)\Phi_s^{\mathrm{T}} \\ &\quad +\Gamma Q_w\left(l\right)\Gamma^{\mathrm{T}}, \end{aligned} \tag{27}$$

Proof. When $\gamma_i\left(l\right)=1$, denote the measurement $Z_i\left(l\right)$ received by estimator at all instants as

$$Z_i\left(l\right)=\left\{\gamma_i\left(1\right)z_i\left(1\right),\gamma_i\left(2\right)z_i\left(2\right),\cdots,\gamma_i\left(l\right)z_i\left(l\right)\right\}, \tag{28}$$

Applying to Projective Theorem, the local filters can be calculated as

$$\begin{aligned} \hat{x}_i\left(l|l\right) &= \mathrm{E}\left[x\left(l\right)|Z_i\left(l\right)\right] \\ &= \hat{x}_i\left(l|l-1\right)+\gamma_i\left(l\right)K_i\left(l\right)e_i\left(l\right), \end{aligned} \tag{29}$$

where $K_i\left(l\right)=\mathrm{E}\left[x\left(l\right)e_i^{\mathrm{T}}\left(l\right)\right]\left\{\mathrm{E}\left[e_i\left(l\right)e_i^{\mathrm{T}}\left(l\right)\right]\right\}^{-1}$. By using (19), the innovation $e_i\left(l\right)$ can be written as

$$e_i\left(l\right)=\left(1-\sigma_i\right)H_i\tilde{x}_i\left(l|l-1\right)+V_i\left(l\right), \tag{30}$$

From (30), the innovation covariance can be calculated as

$$\begin{aligned} Q_{e_i}\left(l\right) &= \left(1-\sigma_i\right)^2H_iP_i\left(l|l-1\right)H_i^{\mathrm{T}}+\left(1-\sigma_i\right)R_i\left(l\right) \\ &\quad +\sigma_i\Xi_i+\sigma_i\left(1-\sigma_i\right)H_iX\left(l\right)H_i^{\mathrm{T}}, \end{aligned} \tag{31}$$

Since $x\left(l\right)=\hat{x}_i\left(l|l-1\right)+\tilde{x}_i\left(l|l-1\right)$ and the orthogonality between $\hat{x}_i\left(l|l-1\right)$ and $\tilde{x}_i\left(l|l-1\right)$, from (1) and (30), the filter gain matrices $K_i\left(l\right)$ can be obtained as

$$K_i\left(l\right)=\left(\left(1-\sigma_i\right)P_i^{\mathrm{T}}\left(l|l-1\right)H_i^{\mathrm{T}}\right)Q_{e_i}^{-1}\left(l\right), \tag{32}$$

Using (1) and (29) yields the filtering error $\tilde{x}_i\left(k|k\right)$ as

$$\tilde{x}_i\left(l|l\right)=\tilde{x}_i\left(l|l-1\right)-\gamma_i\left(l\right)K_i\left(l\right)e_i\left(l\right), \tag{33}$$

Because of the orthogonality between $\tilde{x}_i\left(l|l-1\right)$ and $w\left(l\right)$ and the orthogonality between $\tilde{x}_i\left(l|l-1\right)$ and $v_i\left(l\right)$, from (33), the filter error variance $P_i\left(l|l\right)$ is derived as follows

$$P_i\left(l|l\right)=P_i\left(l|l-1\right)-\gamma_i\left(l\right)K_i\left(l\right)Q_{e_i}\left(l\right)K_i^{\mathrm{T}}\left(l\right), \tag{34}$$

From (33), it yields the prediction error variance $P_i\left(l+1|l\right)$

$$\begin{aligned} P_i\left(l+1|l\right) &= \Phi P_i\left(l|l\right)\Phi^{\mathrm{T}}+\sum_{s=1}^{q}\sigma_{\xi_s}^2\left(l\right)\Phi_sX\left(l\right)\Phi_s^{\mathrm{T}} \\ &\quad +\Gamma Q_w\left(l\right)\Gamma^{\mathrm{T}}, \end{aligned} \tag{35}$$

When $\gamma_i\left(l\right)=0$, the local filter values are same as the local predictor values of the last instant, i.e.

$$\hat{x}_i\left(l|l\right)=\hat{x}_i\left(l|l-1\right),P_i\left(l|l\right)=P_i\left(l|l-1\right), \tag{36}$$

The proof of theorem 1 is completed.

4 SSF Filter

Theorem 2. The innovation-trigger-based SSF algorithm is obtained for CPSs with multiplicative noises against deception attacks (1) and (13) under Assumption 1 and 2

$$\hat{x}_i^{SSF}(l|l) = (1-\gamma_i(l))\hat{x}_{i-1}^{SSF}(l|l) + \gamma_i(l)\Big(\varOmega_1^{(i)}\hat{x}_{i-1}^{SSF}(l|l) + \varOmega_2^{(i)}\hat{x}_{i+1}(l|l)\Big), \tag{37}$$

$$P_i^{SSF}(l|l) = \Big(e^{\mathrm{T}}(\varSigma_i(l|l))^{-1}e\Big)^{-1}, \tag{38}$$

$$\hat{x}^{SSF}(l|l) = \hat{x}_N^{SSF}(l|l), \tag{39}$$

$$P^{SSF}(l|l) = P_N^{SSF}(l|l), \tag{40}$$

and it yields the SSF predictor

$$\hat{x}^{SSF}(l+1|l) = \Phi\hat{x}^{SSF}(l|l), \tag{41}$$

$$P^{SSF}(l+1|l) = \Phi P^{SSF}(l|l)\Phi^{\mathrm{T}} + \sum_{s=1}^{q}\sigma_{\xi_s}^2(l)\Phi_s X(l)\Phi_s^{\mathrm{T}} + \Gamma Q_w(l)\Gamma^{\mathrm{T}}, \tag{42}$$

where the weights are given as

$$\begin{bmatrix} \varOmega_1^{(i)} \\ \varOmega_2^{(i)} \end{bmatrix} = (\varSigma_i(l|l))^{-1}e\Big[e^{\mathrm{T}}(\varSigma_i(l|l))^{-1}e\Big]^{-1}, \tag{43}$$

$$\varSigma_i(l|l) = \begin{bmatrix} P_{i-1}^{SSF} & P_{\left(\substack{SSF\\i-1}\right),i+1} \\ P_{i+1,\left(\substack{SSF\\i-1}\right)} & P_{i+1} \end{bmatrix}, \tag{44}$$

and where $P_{i+1,\left(\substack{SSF\\i-1}\right)} = P^{\mathrm{T}}_{\left(\substack{SSF\\i-1}\right),i+1}$, and the cross covariance between the fusion filter error and the (i+1)-th sensor filter error is computed as

$$P_{\left(\substack{SSF\\i-1}\right),i+1}(l|l) = (1-\gamma_i(l))P_{\left(\substack{SSF\\i-2}\right),i+1}(l|l) + \gamma_i(l)\Big(\varOmega_1^{(i-1)} \times P_{\left(\substack{SSF\\i-2}\right),i+1}(l|l) + \varOmega_2^{(i-1)}P_{i,i+1}(l|l)\Big), \tag{45}$$

where the cross covariance between the (i)-th filter error and the (i+1)-th sensor filter error is computed as

$$P_{i,i+1}(l|l) = P_{i,i+1}(l|l-1) + \gamma_i(l)\gamma_{i+1}(l)K_i(l)Q_{e_{i,i+1}}(l) \times K_{i+1}^{\mathrm{T}}(l) - (1-\sigma_{i+1})\gamma_{i+1}(l)P_{i,i+1}(l|l-1) \times H_{i+1}^{\mathrm{T}}K_{i+1}^{\mathrm{T}}(l) - (1-\sigma_i)\gamma_i(l)K_i(l)H_i \times P_{i,i+1}(l|l-1), \tag{46}$$

and the local prediction error cross covariance matrices and the innovation error cross covariance matrices are given as

$$P_{i,i+1}(l+1|l) = \Phi P_{i,i+1}(l|l)\Phi^{\mathrm{T}} + \sum_{s=1}^{q}\sigma_{\xi_s}^2(l)\Phi_s X(l)\Phi_s^{\mathrm{T}} + \Gamma Q_w(l)\Gamma^{\mathrm{T}}, \tag{47}$$

$$Q_{e_{i,i+1}}(l) = (1-\sigma_i)(1-\sigma_{i+1})H_i P_{i,i+1}(l|l-1)H_{i+1}^{\mathrm{T}}, \tag{48}$$

Proof. When $\gamma_i(l) = 1$, applying the fusion rule with matrix weights is presented in [5], the optimal fusion filter and variance is given by the sequential approach as

$$\hat{x}_i^{SSF}(l|l) = \varOmega_1^{(i)}\hat{x}_{i-1}^{SSF}(l|l) + \varOmega_2^{(i)}\hat{x}_{i+1}(l|l), \tag{49}$$

$$P_i^{SSF}(l|l) = \Big(e^{\mathrm{T}}(\varSigma_i(l|l))^{-1}e\Big)^{-1}, \tag{50}$$

where $\varOmega_1^{(i)} + \varOmega_2^{(i)} = I$, and using (1) and (49) yield the fusion filter error as

$$\tilde{x}_i^{SSF}(l|l) = \varOmega_1^{(i)}\tilde{x}_{i-1}^{SSF}(l|l) + \varOmega_2^{(i)}\tilde{x}_{i+1}(l|l), \tag{51}$$

By (30), the innovation covariance $Q_{e_{i,i+1}}(l) = \mathrm{E}\left[e_i(l|l)e_{i+1}^{\mathrm{T}}(l|l)\right]$ is given as

$$Q_{e_{i,i+1}}(l) = (1-\sigma_i)(1-\sigma_{i+1})H_i P_{i,i+1}(l|l-1)H_{i+1}^{\mathrm{T}}, \tag{52}$$

Using (30) and (33) yield the filter error cross covariance $P_{i,i+1}(l|l) = \mathrm{E}\left[\tilde{x}_i(l|l)\tilde{x}_{i+1}^{\mathrm{T}}(l|l)\right]$ between i-th sensor and (i+1)-th sensor is given as

$$P_{i,i+1}(l|l) = P_{i,i+1}(l|l-1) + K_i(l)Q_{e_{i,i+1}}(l)K_{i+1}^{\mathrm{T}}(l) - (1-\sigma_{i+1})P_{i,i+1}(l|l-1)H_{i+1}^{\mathrm{T}}K_{i+1}^{\mathrm{T}}(l) - (1-\sigma_i)K_i(l)H_i P_{i,i+1}(l|l-1), \tag{53}$$

From (33) and (51), the cross covariance $P_{\left(\substack{SSF\\i-1}\right),i+1}(l|l) = \mathrm{E}\left[\tilde{x}_{i-1}^{SSF}(l|l)\tilde{x}_{i+1}^{\mathrm{T}}(l|l)\right]$ between the fusion filter error and the (i+1)-th sensor filter error is obtained as

$$P_{\left(\substack{SSF\\i-1}\right),i+1}(l|l) = \varOmega_1^{(i-1)}P_{\left(\substack{SSF\\i-2}\right),i+1}(l|l) + \varOmega_2^{(i-1)}P_{i,i+1}(l|l), \tag{54}$$

When $\gamma_i(l) = 0$, the fusion cross covoriance are same as the local predictor values of the last instant, i.e.

$$P_{i,i+1}(l|l) = P_{i,i+1}(l|l-1), \tag{55}$$

$$P_{\left(\substack{SSF\\i-1}\right),i+1}(l|l) = P_{\left(\substack{SSF\\i-2}\right),i+1}(l|l), \tag{56}$$

$$\hat{x}_i^{SSF}(l|l) = \hat{x}_{i-1}^{SSF}(l|l), \tag{57}$$

The proof of theorem 2 is completed.

5 SCI and SICI Fusion Filter

Lamma 1[13] The innovation-trigger-based SCI fusion algorithm is obtained for CPSs with multiplicative noises against deception attacks (1) and (13) under Assumptions 1 and 2

$$\hat{x}_i^{CI}(l|l) = (1-\gamma_i(l))\hat{x}_{i-1}^{CI}(l|l) + \gamma_i(l)P_{i-1}^{CI}(l|l)\Big[w_{CI}^{(i-1)} \times \left(P_{i-1}^{CI}(l|l)\right)^{-1}\hat{x}_{i-1}^{CI}(l|l) + \Big(1-w_{CI}^{(i-1)}\Big) \times P_i^{-1}(l|l)\hat{x}_i(l|l)\Big], \tag{58}$$

$$P_i^{CI}(l|l) = (1-\gamma_i(l))P_{i-1}^{CI}(l|l) + \gamma_i(l)\Big[w_{CI}^{(i-1)} \times \left(P_{i-1}^{CI}(l|l)\right)^{-1} + \Big(1-w_{CI}^{(i-1)}\Big)P_i^{-1}(l|l)\Big]^{-1}, \tag{59}$$

$$\hat{x}^{SCI}(l|l) = \hat{x}_{N-1}^{CI}(l|l), \tag{60}$$

$$P^{SCI}(l|l) = P_{N-1}^{CI}(l|l), \tag{61}$$

where the initial values are $P_i^{CI}(0|0) = P_0, \hat{x}_i^{CI}(0|0) = x_0$, and the minimization performance index with the optimal weighting coefficient $w_{CI}^{(i-1)}$ is as follows

$$\begin{aligned} \min_{w_{CI}^{(i-1)}\in[0,1]} \mathrm{tr}P_i^{CI}(l|l) = &\min_{w_{CI}^{(i-1)}\in[0,1]} \mathrm{tr}\left\{(1-\gamma_i(l))P_{i-1}^{CI}(l|l)\right. \\ &+\gamma_i(l)\left[w_{CI}^{(i-1)}\left(P_{i-1}^{CI}(l|l)\right)^{-1}\right. \\ &\left.\left.+\left(1-w_{CI}^{(i-1)}\right)P_i^{-1}(l|l)\right]^{-1}\right\}, \end{aligned} \tag{62}$$

Lamma 2[13] The innovation-trigger-based SICI fusion algorithm is obtained for CPSs with multiplicative noises against deception attacks (1) and (13) under Assumptions 1 and 2

$$\begin{aligned} \hat{x}_i^{ICI}(l|l) = &(1-\gamma_i(l))\hat{x}_{i-1}^{ICI}(l|l) + \gamma_i(l)\left[K^{ICI}(l)\right. \\ &\left.\times\hat{x}_{i-1}^{ICI}(l|l) + L^{ICI}(l)\hat{x}_i(l|l)\right], \end{aligned} \tag{63}$$

$$\begin{aligned} P_i^{ICI}(l|l) = &(1-\gamma_i(l))P_{i-1}^{ICI}(l|l) + \gamma_i(l)\left\{\left(P_{i-1}^{ICI}(l|l)\right)^{-1}\right. \\ &+(P_i(l|l))^{-1} + \left[w_{ICI}^{(i-1)}\ P_{i-1}^{ICI}(l|l)\right. \\ &\left.\left.+\left(1-w_{ICI}^{(i-1)}\right)P_i^{-1}(l|l)\right]^{-1}\right\}, \end{aligned} \tag{64}$$

$$\begin{aligned} K^{ICI}(l) = &P_i^{ICI}(l|l)\left\{\left(P_{i-1}^{ICI}(l|l)\right)^{-1} - w_{ICI}^{(i-1)}\right. \\ &\left.\left[w_{ICI}^{(i-1)}P_{i-2}^{ICI}(l|l) + \left(1-w_{ICI}^{(i-1)}\right)P_i(l|l)\right]^{-1}\right\}, \end{aligned} \tag{65}$$

$$\begin{aligned} L^{ICI}(l) = &P_i^{ICI}(l|l)\left\{(P_i(l|l))^{-1} - \left(1-w_{ICI}^{(i-1)}\right)\right. \\ &\left.\left[w_{ICI}^{(i-1)}P_{i-2}^{ICI}(l|l) + \left(1-w_{ICI}^{(i-1)}\right)P_i(l|l)\right]^{-1}\right\}, \end{aligned} \tag{66}$$

$$\hat{x}_{SICI}(l|l) = \hat{x}_{N-1}^{ICI}(l|l), \tag{67}$$

$$P_{SICI}(l|l) = P_{N-1}^{ICI}(l|l), \tag{68}$$

where the initial values are $P_i^{ICI}(0|0) = P_0, \hat{x}_i^{ICI}(0|0) = x_0$, and the minimization performance index with the optimal weighting coefficient $w_{ICI}^{(i-1)}$ is as follows

$$\begin{aligned} \min_{w_{ICI}^{(i-1)}\in[0,1]} \mathrm{tr}P_i^{ICI}(l|l) = &\min_{w_{ICI}^{(i-1)}\in[0,1]} \mathrm{tr}\left\{\left(P_{i-1}^{ICI}(l|l)\right)^{-1}\right. \\ &+(P_i(l|l))^{-1} + \left[w_{ICI}^{(i-1)}P_{i-1}^{ICI}(l|l)\right. \\ &\left.\left.+\left(1-w_{ICI}^{(i-1)}\right)P_i^{-1}(l|l)\right]^{-1}\right\}^{-1}, \end{aligned} \tag{69}$$

6 Accuracy Analysis

Theorem 3. The accuracy relationship between three sequential fusion filter algorithms and the local filters is as follows

$$\mathrm{tr}P^{SSF} \leqslant \mathrm{tr}P^{SICI} \leqslant \mathrm{tr}P^{SCI} \leqslant \mathrm{tr}P_i, i = 1, 2, \cdots, N, \tag{70}$$

Proof. Under the condition of the same deception attacks, the same threshold values, and the same arriving order of local estimators, the two-sensor state fusion algorithm is optimal in the sence of LUMV criterion, so it has the highest accurcies among three fusion algorithms. It is proved in [14] that the estimation accurcy of ICI fusion algorithm is higher than that of CI fusion algorithm. All of the fused algorithms are more accurate than local filters. Therefore, under the sequential fusion form, the accury relations will still be kept, that is, (70) holds.

7 Simulation Example

Consider a three-sensor cycle-physical system

$$\begin{aligned} x(l+1) = &\left(\begin{bmatrix}0.9 & \mathrm{T}_0\\ 0 & 0.9\end{bmatrix} + \xi(l)\begin{bmatrix}0.2 & 0\\ 0 & 0.1\end{bmatrix}\right)x(l) \\ &+\begin{bmatrix}\mathrm{T}_0^2/2\\ \mathrm{T}_0\end{bmatrix}w(l), \end{aligned} \tag{71}$$

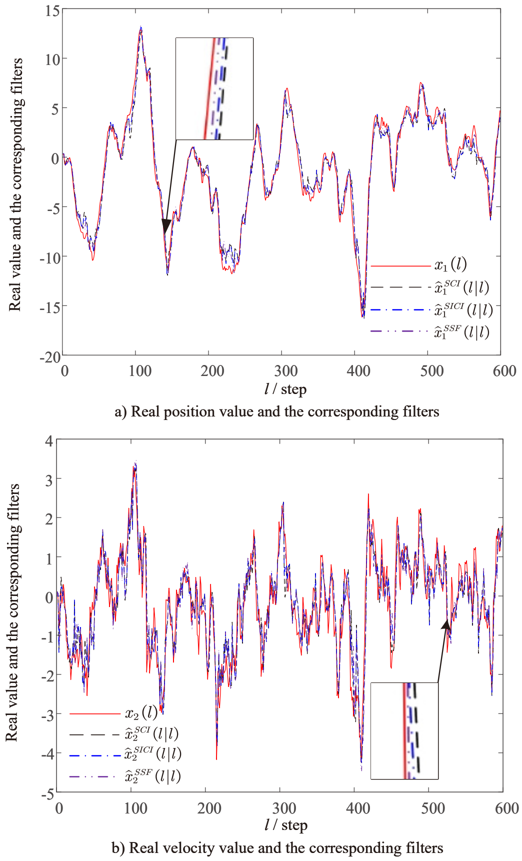
$$y_i(l) = H_i x(l) + v_i(l), \tag{72}$$

$$z_i(l) = (1-\alpha_i(l))y_i(l) + \alpha_i(l)\zeta_i(l), i = 1, 2, 3. \tag{73}$$

where $\mathrm{T}_0 = 0.5, H_1 = \begin{bmatrix}0 & 1\end{bmatrix}, H_2 = \begin{bmatrix}1 & 0\end{bmatrix}, H_3 = \begin{bmatrix}1 & 1\end{bmatrix}$, $w(l)$, $\xi(l)$ and $v_i(l)$ are zero mean white noises, their variances are $Q_w = 0.21, \sigma_\xi^2 = 0.02, \Sigma_1 = 0.04, \Sigma_2 = 0.3$ and $\Sigma_3 = 0.5$, respectively. The innovation-triggered threshold $\theta_i = 0.3$. These attacked probabilities α_i are selected as $\alpha_1 = 0.1, \alpha_2 = 0.2$ and $\alpha_3 = 0.4$. $\zeta_i(l)$ are random zero-mean Gaussian white noises, and their variances are $\Xi_1 = 0.16, \Xi_2 = 0.12$ and $\Xi_3 = 0.04$ respectively.

The simulation results are shown in Fig.1, where the solid curves represent the real state values $x_1(l)$ and $x_2(l)$, the dashed curves represent the SCI fusion Kalman filter estimation $\hat{x}_1^{SCI}(l|l)$ and $\hat{x}_2^{SCI}(l|l)$ against deception attacks under the innovation-triggered mechanism, the doted curves represent the SICI fusion Kalman filter estimation $\hat{x}_1^{SICI}(l|l)$ and $\hat{x}_2^{SICI}(l|l)$ against deception attacks under the innovation-triggered mechanism, the double doted curves represent the SSF Kalman filter estimation $\hat{x}_1^{SSF}(l|l)$ and $\hat{x}_2^{SSF}(l|l)$ against deception attacks under the innovation-triggered mechanism. It can be seen that these algorithms have good tracking results.

In order to verify the effectiveness of the presented sequential fusion Kalman filters, 100 Monte-Carlo simulations are performed and the mean square relative error (MSRE) curves of the fusion filter are drawn, as shown in Fig.2, where $MSRE^{(n)}(l) = \frac{1}{100}\sum_{p=1}^{100}\left\{\left[\hat{x}_{(p)}^{(n)}(l|l) - x_{(p)}^{(n)}(l)\right]/x_{(p)}^{(n)}(l)\right\}^2$ $(n = SCI, SICI, SSF)$, the subscript p represents the p-th experiment. The doted curves represent the MSRE value of SCI



a) Real position value and the corresponding filters

b) Real velocity value and the corresponding filters

Fig. 1: Fusion filter tracking curves

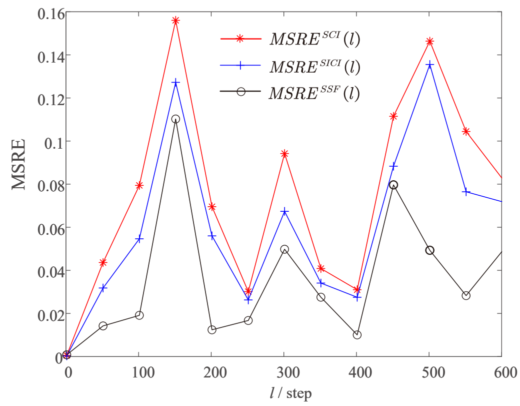


Fig. 2: The MSRE curves of different fusion algorithms

fusion filter, the solid curves represent the MSRE value of the SICI fusion filter, and the dashed curves represent the MSRE value of the SSF filter. The comparison of MSRE values of different fusion algorithms for each 100 steps from $l = 0$ to $l = 600$ is shown in Table 1. It can be seen that when the running steps are sufficient long, under the innovation-triggered mechanism the accuracy of the SCI fusion filter is the lowest, the accuracy of the SICI fusion filter is the second, and both of their accuracies are lower than that of the SSF filter.

Table 1: The comparison of MSRE values of different sequential fusion algorithms for each 100 steps from $l = 0$ to $l = 600$.

l	100	200	300	400	500	600
$MSRE^{SCI}(l)$	0.0794	0.0695	0.0942	0.0310	0.1464	0.0825
$MSRE^{SICI}(l)$	0.0546	0.0560	0.0674	0.0275	0.1356	0.0718
$MSRE^{SSF}(l)$	0.0191	0.0124	0.0498	0.0100	0.0493	0.0491

8 Conclusion

Three sequential fusion filters for CPSs with multiplicative noises against deception attacks have been presented under the innovation-triggered mechanism. The uncertainties caused by the deception attacks and multiplicative noises have been transformed into white fictitious noises by the model transformation, and the new measurements are send to the fusion center. The simulation results show that the estimation accuracy of the SICI fusion filter is higher than that of the SCI fusion filter, and lower than that of the SSF filter, which verifies the effectiveness of these above sequential fusion approaches. In addition, the accuracy analysis verified in this paper also greatly explains the accuracy relationship among the three fusion algorithms.

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