Kalman Filter With Innovation-Based Triggering

Huaimin Li, Gang Sun, Jia Zhao, Heng Li
School of Computer and Information Engineering
Fuyang Normal University
Fuyang, China
e-mail: lihuaimin.2008@163.com, lism56800679@163.com

***Abstract*—Due to the constraints of the bandwidth and energy, the communication rate of sensors-to-estimator may be required to be reduced to save communication resources and energy in network control systems (NCSs). This paper studies the remote state estimation triggered by the innovation to meet expected estimation performance in order to improve the performance of the whole system under reducing communication rate. We propose an event-trigger based on measurement innovation, which decide on how information could be sent to remote estimator for estimation. Then under the Gaussian assumption of the predicted conditional probability density, a minimum mean squared error (MMSE) Kalman filter with innovation-based triggering is derived based on the Bayes Rule which realizes the tradeoff between communication rate and estimation quality. Furthermore, it provides the solution to the average communication rate under a given threshold and the optimal threshold value in the case of known communication rate. A numerical example is simulated to verify the effectiveness and correctness of the designed filter.**

Keywords-Kalman filter; innovation-based triggering; minimum mean squared error; network control systems

I. INTRODUCTION

The network control systems (NCSs) have received extensive attention and in-depth research in both academia and industry. [1]-[6] Since the NCSs have many advantages, such as flexible architecture, low installation and maintenance cost, higher resource utilization, the NCSs are widely used in various fields of production and life, including public transportation, mobile sensor networks, and healthcare. [3] However, due to the limitations of bandwidth and energy in a shared communication network, in which the phenomenon of packets loss and delays are inevitable under the periodic data transfer mechanism, so that the performance of the system is compromised. Therefore, in order to improve the performance of the whole system, it is necessary to reduce the communication rate from the sensor to the data processing center as far as possible under the premise of meeting the accuracy requirement of the system for the remote state estimation problem.

Nowadays, the problem of remote estimation remains a significant challenge, especially the state estimation of dynamical systems based on measurements that are received from sensors. It is well known that Kalman filtering is an attractive option for resolving state estimation problems in engineering practice. But the standard Kalman filtering method uses full measurement information at every time. However, the standard Kalman filtering methods and theories have been greatly challenged in networked systems, aperiodic or event-based strategies for networked systems have attracted much attention and been combined with Kalman filtering theory, which is an effective way to solve the problem of remote state estimation. Closely related works are given by Trimpe et al. [7]-[9]. [7] focused on comparing and analyzing the relative merits and applicability of popular event triggering mechanism (including send-on-delta[10], measurement-based triggering[8], variance-based triggering[9], and relevant sampling[11]) and pointed out event triggering mechanism in which considering real-time measurement can improve estimation performance under the same communication rate. In [8], it is proposed that the state estimation with variance-based triggering is a direct extension of the classic Kalman filter to a distributed estimation problem. The proposed method allows the designer to minimize the communication cost of the sensor to the estimator by selecting the appropriate thresholds for each sensor to meet the estimated quality. The idea of using innovation as a trigger mechanism was also pursued in [12]. By adopting an approximation technique from nonlinear filtering, [12] derived a simple form of an accurate MMSE estimator, and implemented the trade-off between the sensor-to-estimator communication rate and the remote estimation quality.

Based on the above research, as for the discrete-time linear stochastic systems, we put forward the Kalman filter on the basis of innovation event where measurement data is transmitted only when innovation-based events indicate that the measurement data is required to meet constraints on the estimator performance. It is clear that there are two main factors in remote state estimation based on event-triggering: one is the design of an event triggering mechanism and the appropriate threshold. The other is the design of the estimation algorithm. First of all, we design an event-trigger based on measurement innovation, which makes a decision depending on the result of comparing the Euclidean norm of normalized innovation with the set threshold. Then the approximation MMSE Kalman filter is derived under the assumption of the predicted distribution that is to be Gaussian. Because the filter algorithm of this paper considers the additional information that is implied by the triggering event, the estimation performance improves under the same communication rate.

978-1-5386-1306-1/17/$31.00 ©2017 IEEE

Notation: The main notational conventions in this article are used

$E[\cdot \mid \cdot]$	Conditional mean	$\Pr(\cdot)$	probability of a random event
X^{T}	transpose of matrix X	$\lvert \cdot \rvert$	Euclidean norm for vectors
X^{-1}	inverse of matrix X	*	a placeholder for the same as preceding term
$N(\mu,\sigma^2)$	Gaussian distribution with mean μ and covariance matrix σ^2		

II. Problem Formulation

Consider the following discrete-time linear stochastic system

$$x_{k+1} = \mathrm{A}x_k + w_k \tag{1}$$

$$y_k = \mathrm{H}x_k + v_k \tag{2}$$

Where, $x_k \in R^n$ is the system state vector, $y_k \in R^m, (m=1)$ is the sensor measurement, $w_k \in R^n$ is the process noise and $v_k \in R^m$ is measurement noises. A, H, are known constant matrices with suitable dimensions. We introduce a random variable γ_k , let $\gamma_k = 1$ or 0. If $\gamma_k = 1$ denotes y_k is transmitted from sensor to a remote estimator. Thus, the information set of the estimator at time k is given as $L_k := \{\gamma_0 y_0, \cdots, \gamma_k y_k\}$. We have definition as following

$$\hat{x}_k^- := \mathrm{E}[x_k \mid L_{k-1}], \quad \tilde{x}_k^- := x_k - \hat{x}_k^-, \quad \mathrm{P}_k^- := \mathrm{E}[\tilde{x}_k^- \tilde{x}_k^{-\mathrm{T}} \mid L_{k-1}] \tag{3}$$

$$\hat{x}_k := \mathrm{E}[x_k \mid L_k], \quad \tilde{x}_k := x_k - \hat{x}_k, \quad \mathrm{P}_k := \mathrm{E}[\tilde{x}_k \tilde{x}_k^{\mathrm{T}} \mid L_k] \tag{4}$$

$$\varepsilon_k := y_k - \mathrm{E}[y_k \mid L_{k-1}], \ \mathrm{P}_k^{\varepsilon} := \mathrm{E}[\varepsilon_k \varepsilon_k^{\mathrm{T}} \mid L_{k-1}] = \mathrm{E}[(\varepsilon_k)^2 \mid L_{k-1}] \tag{5}$$

Where $\hat{x}_k^-$ and $\hat{x}_k$ are respectively referred to as the prior and posterior minimum mean square error estimates. ε_k is the measurement innovation, which indicates the accuracy of the predicted estimation.

Assumption 1: w_k and v_k are uncorrelated Gaussian white noises with zero mean and variances $Q_w \geq 0$ and $Q_v > 0$ respectively. (A, H) and $(\mathrm{A}, \sqrt{Q_w})$ are observable and controllable, respectively.

Assumption 2: The initial state x_0 is assumed to be Gaussian with $\mathrm{E}[x_0] = 0$ and $\mathrm{E}[x_0 x_0^{\mathrm{T}}] = \mathrm{P}_{x_0}$, uncorrelated with w_k and v_k for all k .

Assumption 3: The predicted distribution is Gaussian, i.e. $p_{x_k}(x \mid L_{k-1}) \sim N(\hat{x}_k^-, \mathrm{P}_k^-)$

In this paper, we assume that there are no packets loss and delays in network transmission, and the remote state estimation center is well equipped.

III. Event-Based Kalman Filter

In this section, we focus on designing an innovation event triggered mechanism and corresponding based on the innovation event Kalman filter.

A. Event Triggered Mechanism

We adopt the following innovation event triggered mechanism

$$\gamma_k = \begin{cases} 0 & if \ \lvert \bar{\varepsilon}_k \rvert \leq \delta \\ 1 & otherwise \end{cases} \tag{6}$$

Where, $\bar{\varepsilon}_k = \varepsilon_k \varphi_k$, $\varphi_k = (\mathrm{P}_k^{\varepsilon})^{-1/2}$ and $\delta \geq 0$ is a specified threshold. When y_k is taken, the sensor will decide whether y_k is sent to a remote estimator for correction prior estimate. From (6) we see that, when $\gamma_k = 0$, the estimator can infer that. $\bar{\varepsilon}_k \in [-\delta, \delta]$. Unlike [7], we take advantage of this extra information to further improve the estimation accuracy of the remote estimator. Herein, we redefine the set L_k that is received information by the remote estimator as $L_k := \{\gamma_0 y_0, \cdots, \gamma_k y_k\} \bigcup \{\gamma_0, \cdots, \gamma_k\}$, and have the average sensor communication rate as

$$\gamma := \limsup_{t \to +\infty} \frac{1}{t+1} \sum_{k=1}^{t} \mathrm{E}[\gamma_k] \tag{7}$$

B. Preliminary Lemmas

In order to derive the main conclusions, the relevant lemmas are given in this section.

Lemma 1: If $z \in R$ and $z \sim N(0, \sigma^2)$, set $\Delta = \delta\sigma$, then $\mathrm{E}[z^2 \| z \mid \leq \Delta] = \sigma^2(1 - \lambda(\delta))$.

Proof: Considering the property $p_z(z \| z \mid \leq \Delta) = p_z(t) / \int_{-\Delta}^{\Delta} p_z(t) dt$ yields

$$\mathrm{E}[z^2 \| z \mid \leq \Delta] = \frac{1}{\int_{-\Delta}^{\Delta} p_z(t) dt} \int_{-\Delta}^{\Delta} \frac{t^2}{\sqrt{2\pi}\sigma} e^{-\frac{t^2}{2\sigma^2}} dt \tag{8}$$

Let $t = y\sigma,$ and $dt = \sigma dy, \lvert y \rvert \leq \delta,$ then (8) can be rewrite as

$$\mathrm{E}[z^2 \| z \mid \leq \Delta] = \frac{1}{1 - 2\Phi(\delta)} \int_{-\delta}^{\delta} \frac{y^2}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy$$

By using the distribution integral method, we have

$$\mathrm{E}[z^2 \| z \mid \leq \Delta] = \sigma^2 [1 - \frac{2}{\sqrt{2\pi}} \delta e^{-\frac{\delta^2}{2}} (1 - 2\Phi(\delta))^{-1}] \tag{9}$$

Where,

$$\lambda(\delta) = \frac{2}{\sqrt{2\pi}} \delta e^{-\frac{\delta^2}{2}} (1 - 2\Phi(\delta))^{-1}, \quad \Phi(\delta) = \int_{\delta}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \tag{10}$$

Proof is completed.

Remark 1: $\Phi(\cdot)$ is the tail probability function of a standard Gaussian random variable, that is $\Phi(x) = 1 / \sqrt{2\pi} \int_x^{\infty} e^{-\frac{t^2}{2}} dt$.

Lemma 2[13]: Let $X \in \Re^n, Y \in \Re^m$ be jointly Gaussian distribution as following

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim N\left(\begin{bmatrix} \bar{X} \\ \bar{Y} \end{bmatrix}, \begin{bmatrix} \mathrm{P}_{XX} & \mathrm{P}_{XY} \\ \mathrm{P}_{YX} & \mathrm{P}_{YY} \end{bmatrix} \right)$$

Then edge distribution of X is $p_X(x) \sim N(\bar{X}, \mathrm{P}_{XX})$, when given $Y = y$, the conditional distribution of X is given by $p_{X|Y}(x \mid y) \sim N(\mu, \sigma^2)$, where $\mu = \bar{X} + \mathrm{P}_{XY}\mathrm{P}_{YY}^{-1}(y - \bar{Y})$, $\sigma^2 = \mathrm{P}_{XX} - \mathrm{P}_{XY}\mathrm{P}_{YY}^{-1}\mathrm{P}_{YX}$.

C. Kalman Filter with Innovation-Based Triggering

Since the predicted probability density $p_{y_k}(y \mid L_{k-1})$ belongs to the Gaussian family with mean $\mathrm{H}\hat{x}_k^-$ and covariance $\mathrm{HP}_k^-\mathrm{H}^{\mathrm{T}} + Q_v$ under the Gaussian assumption 3, thus a very simple form of the filter that similar to Kalman filter is provided.

Theorem 1: Consider the situation of remote state estimation with the event triggered mechanism (6). Under the assumption1-3, the MMSE Kalman filter with innovation-based triggering is given recursively as follows:

1) Time update

$$\hat{x}_k^- = \mathrm{A}\hat{x}_{k-1},\ \mathrm{P}_k^- = \mathrm{AP}_{k-1}\mathrm{A}^{\mathrm{T}} + Q_w \tag{11}$$

2) Measurement update

$$\hat{x}_k = \hat{x}_k^- + \gamma_k K_k \varepsilon_k,\ \mathrm{P}_k = \mathrm{P}_k^- - [\gamma_k - (1-\gamma_k)\lambda(\delta)]K_k\mathrm{HP}_k^- \tag{12}$$

Where $K_k = \mathrm{P}_k^-\mathrm{H}^{\mathrm{T}}[\mathrm{HP}_k^-\mathrm{H}^{\mathrm{T}} + Q_v]^{-1}$, recursive initial values are $\hat{x}_0 = 0,$ $\mathrm{P}_0 = \mathrm{P}_{x_0}$.

Proof: From assumption 3, we have the result is shown as follows

$$\hat{x}_k^- = \mathrm{A}\mathrm{E}[x_{k-1} \mid L_{k-1}] = \mathrm{A}\hat{x}_{k-1}$$

$$\mathrm{P}_k^- = \mathrm{E}[(\mathrm{A}\tilde{x}_{k-1} + w_{k-1})(*)^{\mathrm{T}} \mid L_{k-1}] = \mathrm{AP}_{k-1}\mathrm{A}^{\mathrm{T}} + Q_w$$

Next, we prove that the measurement update. In here, the sensor needs to determine whether the current measured information needs to be transmitted to the remote estimator to correct the prior estimate based on the event triggering mechanism. So we consider the two cases of $\gamma_k = 1$ and $\gamma_k = 0$.

a) $\gamma_k = 1:\ L_k = L_{k-1} \bigcup \{\varepsilon_k = \varepsilon\}$

Note that $\varepsilon_k = \mathrm{H}x_k + v_k - \mathrm{H}\hat{x}_k^- = \mathrm{H}\tilde{x}_k^- + v_k$ is Gaussian conditioned on L_{k-1} with zero-mean, it is easily get the jointly Gaussian distribution of ε_k and x_k conditioned on L_{k-1} as following

$$p_{x_k,\varepsilon_k}(x, \varepsilon \mid L_{k-1}) \sim N\left(\begin{bmatrix} \hat{x}_k^- \\ 0 \end{bmatrix}, \begin{bmatrix} \mathrm{P}_k^- & \mathrm{P}_k^-\mathrm{H}^{\mathrm{T}} \\ \mathrm{HP}_k^- & \mathrm{HP}_k^-\mathrm{H}^{\mathrm{T}} + Q_v \end{bmatrix}\right) \tag{13}$$

According to (13) and lemma 2, we obtain

$$\mathrm{E}[x_k \mid L_{k-1}, \varepsilon_k = \varepsilon] = \hat{x}_k^- + \mathrm{P}_k^-\mathrm{H}^{\mathrm{T}}[\mathrm{HP}_k^-\mathrm{H}^{\mathrm{T}} + Q_v]^{-1}\varepsilon \tag{14}$$

$$\mathrm{E}[(x_k - \mathrm{E}[x_k \mid L_{k-1}, \varepsilon_k = \varepsilon])(*)^{\mathrm{T}} \mid L_{k-1}, \varepsilon_k = \varepsilon]$$
$$= \mathrm{P}_k^- - \mathrm{P}_k^-\mathrm{H}^{\mathrm{T}}[\mathrm{HP}_k^-\mathrm{H}^{\mathrm{T}} + Q_v]^{-1}\mathrm{HP}_k^- \tag{15}$$

In light of (14) and (15), let $K_k = \mathrm{P}_k^-\mathrm{H}^{\mathrm{T}}[\mathrm{HP}_k^-\mathrm{H}^{\mathrm{T}} + Q_v]^{-1}$, we have

$$\hat{x}_k = x_k^- + K_k\varepsilon_k,\ \mathrm{P}_k = \mathrm{P}_k^- - K_k\mathrm{HP}_k^- \tag{16}$$

b) $\gamma_k = 0:\ |\bar{\varepsilon}_k| \le \delta, \breve{L}_k = L_{k-1} \bigcup \{\gamma_k = 0\}$

From (13) we know $\mathrm{P}_k^\varepsilon = \mathrm{E}[\varepsilon_k^2 \mid L_{k-1}] = \mathrm{HP}_k^-\mathrm{H}^{\mathrm{T}} + Q_v$, and define $\bar{\varepsilon}_k = \varepsilon_k\varphi_k$, $\varphi_k = (\mathrm{P}_k^\varepsilon)^{-1/2}$ we have

$$\mathrm{E}[\bar{\varepsilon}_k \mid L_{k-1}] = \mathrm{E}[\varepsilon_k\varphi_k \mid L_{k-1}] = \varphi_k\mathrm{E}[\varepsilon_k \mid L_{k-1}] = 0,$$

$$\mathrm{E}[\bar{\varepsilon}_k^2 \mid L_{k-1}] = \mathrm{E}[(\varepsilon_k\varphi_k)^2 \mid L_{k-1}] = \varphi_k^2\mathrm{P}_k^\varepsilon = 1$$

So $\bar{\varepsilon}_k$ obeys the standard Gaussian distribution conditioned on L_{k-1}, i.e. $\bar{\varepsilon}_k \sim N(0,1)$, then, in light of lemma 1

$$\mathrm{E}[\bar{\varepsilon}_k^2 \mid \breve{L}_k] = \mathrm{E}[\bar{\varepsilon}_k^2 \mid L_{k-1}, |\bar{\varepsilon}_k| \le \delta] = 1 - \lambda(\delta) \tag{17}$$

Further, we define $f_\delta := \Pr(-\delta \le \bar{\varepsilon}_k \le \delta \mid L_{k-1})$, using the conditional probability density function

$$p_{\bar{\varepsilon}_k}(\bar{\varepsilon} \mid \breve{L}_k) = \begin{cases} \dfrac{p_{\bar{\varepsilon}_k}(\bar{\varepsilon} \mid L_{k-1})}{f_\delta} & if\ |\bar{\varepsilon}_k| \le \delta \\ 0 & otherwise \end{cases} \tag{18}$$

$$\hat{x}_k = \mathrm{E}[x_k \mid \breve{L}_k] = \int_{-\delta}^{\delta} \mathrm{E}[x_k \mid L_{k-1}, \varepsilon_k = \varphi_k^{-1}\bar{\varepsilon}]p_{\bar{\varepsilon}_k}(\bar{\varepsilon} \mid L_{k-1})d\bar{\varepsilon} \tag{19}$$

From (14), (19) can be rewrite as following

$$\hat{x}_k = \frac{1}{f_\delta}\int_{-\delta}^{\delta}(\hat{x}_k^- + L_k\varphi_k^{-1}\bar{\varepsilon})p_{\bar{\varepsilon}_k}(\bar{\varepsilon} \mid L_{k-1})d\bar{\varepsilon}$$
$$= \hat{x}_k^- + \frac{K_k}{\varphi_k f_\delta}\int_{-\delta}^{\delta}\bar{\varepsilon}p_{\bar{\varepsilon}_k}(\bar{\varepsilon} \mid L_{k-1})d\bar{\varepsilon} = \hat{x}_k^- \tag{20}$$

where, $\int_{-\delta}^{\delta}\bar{\varepsilon}p_{\bar{\varepsilon}_k}(\bar{\varepsilon} \mid L_{k-1})d\bar{\varepsilon} = \mathrm{E}[\bar{\varepsilon}_k \mid L_{k-1}] = 0$.

From (4) and (20) we can obtain

$$\mathrm{P}_k = \mathrm{E}[\tilde{x}_k\tilde{x}_k^{\mathrm{T}} \mid \breve{L}_k] = \mathrm{E}[\tilde{x}_k^-(\tilde{x}_k^-)^{\mathrm{T}} \mid \breve{L}_k]$$
$$= \mathrm{E}[(\tilde{x}_k^- - K_k\varepsilon_k + K_k\varepsilon_k)(*)^{\mathrm{T}} \mid \breve{L}_k]$$
$$= \mathrm{E}[(\tilde{x}_k^- - K_k\varepsilon_k)(*)^{\mathrm{T}} \mid \breve{L}_k] + \mathrm{E}[K_k\varepsilon_k^2K_k^{\mathrm{T}} \mid \breve{L}_k] +$$
$$\mathrm{E}[(\tilde{x}_k^- - K_k\varepsilon_k)\varepsilon_k^{\mathrm{T}}K_k^{\mathrm{T}} \mid \breve{L}_k] + \mathrm{E}[K_k\varepsilon_k(\tilde{x}_k^- - K_k\varepsilon_k)^{\mathrm{T}} \mid \breve{L}_k] \tag{21}$$

According to lemma2,

$$\mathrm{E}\{(x_k - \mathrm{E}[x_k \mid L_{k-1}, \varepsilon_k = \varepsilon])(*)^{\mathrm{T}} \mid L_{k-1}, \varepsilon_k = \varepsilon\} = \mathrm{P}_k^- - K_k\mathrm{HP}_k^- \tag{22}$$

Notice that (14) we can easily obtain

$$x_k - \mathrm{E}[x_k \mid L_{k-1}, \varepsilon_k = \varepsilon] = x_k - \hat{x}_k^- - K_k\varepsilon = \tilde{x}_k^- - K_k\varepsilon \tag{23}$$

Substituting (23) into (22), we have

$$\mathrm{E}[(\tilde{x}_k^- - K_k\varepsilon_k)(\tilde{x}_k^- - K_k\varepsilon_k)^{\mathrm{T}} \mid L_{k-1}, \varepsilon_k = \varepsilon] = \mathrm{P}_k^- - K_k\mathrm{HP}_k^-$$

Such that

$$\mathrm{E}[(\tilde{x}_k^- - K_k\varepsilon_k)(*)^{\mathrm{T}} \mid \breve{L}_k]$$
$$= \frac{1}{f_\delta}\int_{-\delta}^{\delta} \mathrm{E}[(\tilde{x}_k^- - K_k\varepsilon_k)(*)^{\mathrm{T}} \mid L_{k-1}, \varepsilon_k = \varphi_k^{-1}\bar{\varepsilon}]p_{\bar{\varepsilon}_k}(\bar{\varepsilon} \mid L_{k-1})d\bar{\varepsilon}$$
$$= \frac{1}{f_\delta}(\mathrm{P}_k^- - K_k\mathrm{HP}_k^-)\int_{-\delta}^{\delta} p_{\bar{\varepsilon}_k}(\bar{\varepsilon} \mid L_{k-1})d\bar{\varepsilon} = \mathrm{P}_k^- - K_k\mathrm{HP}_k^- \tag{24}$$

According to the definition of f_δ, we know that

$$\int_{-\delta}^{\delta} p_{\bar{\varepsilon}_k}(\bar{\varepsilon} \mid L_{k-1})d\bar{\varepsilon} = f_\delta.$$

$$\mathrm{E}[(\tilde{x}_k^- - K_k\varepsilon_k)\varepsilon_k^{\mathrm{T}}K_k^{\mathrm{T}} \mid \breve{L}_k] = \mathrm{E}[\tilde{x}_k^-\varepsilon_k^{\mathrm{T}}K_k^{\mathrm{T}} \mid \breve{L}_k] - \mathrm{E}[K_k\varepsilon_k^2K_k^{\mathrm{T}} \mid \breve{L}_k] \tag{25}$$

where

$$\mathrm{E}[\tilde{x}_k^- \varepsilon_k^{\mathrm{T}} K_k^{\mathrm{T}} \mid \breve{L}_k]$$

$$= \frac{1}{f_\delta} \int_{-\delta}^{\delta} \mathrm{E}[\tilde{x}_k^- \mid L_{k-1}, \varepsilon_k = \varphi_k^{-1}\bar{\varepsilon}]\bar{\varepsilon}\varphi_k^{-1} K_k^{\mathrm{T}} p_{\bar{\varepsilon}_k}(\bar{\varepsilon} \mid L_{k-1}) d\bar{\varepsilon}$$

$$= \frac{1}{f_\delta} \int_{-\delta}^{\delta} \mathrm{E}[x_k - \hat{x}_k^- \mid L_{k-1}, \varepsilon_k = \varphi_k^{-1}\bar{\varepsilon}]\bar{\varepsilon}\varphi_k^{-1} K_k^{\mathrm{T}} p_{\bar{\varepsilon}_k}(\bar{\varepsilon} \mid L_{k-1}) d\bar{\varepsilon}$$

$$= \frac{1}{f_\delta} \int_{-\delta}^{\delta} (\mathrm{E}[x_k \mid L_{k-1}, \varepsilon_k = \varphi_k^{-1}\bar{\varepsilon}] - \hat{x}_k^-)\bar{\varepsilon}\varphi_k^{-1} K_k^{\mathrm{T}} p_{\bar{\varepsilon}_k}(\bar{\varepsilon} \mid L_{k-1}) d\bar{\varepsilon}$$

In view of (14) and (18), above equation can be further organized into the following form

$$\mathrm{E}[(\tilde{x}_k^- - K_k\varepsilon_k)\varepsilon_k^{\mathrm{T}} K_k^{\mathrm{T}} \mid \breve{L}_k] = K_k \mathrm{E}[(\varphi_k^{-1}\bar{\varepsilon}_k)^2 \mid \breve{L}_k] K_k^{\mathrm{T}}$$
$$= K_k \mathrm{E}[\varepsilon_k^2 \mid \breve{L}_k] K_k^{\mathrm{T}} \tag{26}$$

Substituting (26) into (25) we have

$$\mathrm{E}[(\tilde{x}_k^- - K_k\varepsilon_k)\varepsilon_k^{\mathrm{T}} K_k^{\mathrm{T}} \mid \breve{L}_k] = 0 \tag{27}$$

In the same way

$$\mathrm{E}[K_k\varepsilon_k(\tilde{x}_k^- - K_k\varepsilon_k)^{\mathrm{T}} \mid \breve{L}_k] = 0 \tag{28}$$

From (17) we have

$$\mathrm{E}[K_k\varepsilon_k^2 K_k^{\mathrm{T}} \mid \breve{L}_k] = \mathrm{E}[K_k(\varphi_k^{-1}\bar{\varepsilon}_k)^2 K_k^{\mathrm{T}} \mid \breve{L}_k] = (1-\lambda(\delta))\varphi_k^{-2} K_k K_k^{\mathrm{T}} \tag{29}$$

Substituting (24), (27)-(29) into (21) and noting that $\varphi_k = (\mathrm{P}_k^\varepsilon)^{-1/2}$, $\mathrm{P}_k^\varepsilon = \mathrm{H}\mathrm{P}_k^-\mathrm{H}^{\mathrm{T}} + Q_v$, (21) can be reorganized as following

$$\mathrm{P}_k = \mathrm{P}_k^- - \lambda(\delta) K_k \mathrm{H}\mathrm{P}_k^- \tag{30}$$

Next, let's combine above two cases, from (16), (20) and (30), (12) can be readily achieved. Proof is completed.

Remark 2: From Theorem1, we find that P_k is a function that ends up depending on δ. By tuning the value of δ, we can make a tradeoff between the sensor communication rate γ and the estimation quality by P_k. In fact, the selected δ value can be considered as the boundary that allows estimation error.

Theorem 2: Consider the situation of remote state estimation with the event triggered mechanism (6). Under the assumption1-3, the average communication rate γ from sensor to estimator in (7) is

$$\gamma = 2\Phi(\delta) \tag{31}$$

Proof: Since γ_k is a binary random variable that its value is 0 or 1, and has $\Pr(\gamma_k = 1 \mid L_{k-1}) = \Pr(|\bar{\varepsilon}_k| > \delta \mid L_{k-1})$ .We know $\Pr(\gamma_k = 0 \mid L_{k-1}) = \Pr(|\bar{\varepsilon}_k| \le \delta \mid L_{k-1}) = 1 - 2\Phi(\delta)$, such that

$$\Pr(\gamma_k = 1 \mid L_{k-1}) = 1 - \Pr(\gamma_k = 0 \mid L_{k-1}) = 2\Phi(\delta)$$

Proof is completed.

Remark 3: Due to $\delta > 0$, the tail probability function $\Phi(\delta)$ is decreasing function. It's easily to see that the larger the δ, the smaller the average communication rate. However, the communication rate is determined by the communication resources.

Corollary 1: Under the given a communication rate $\bar{\gamma}$, the optimal threshold in terms of MMSE is computed by

$$\delta^* = \Phi^{-1}(\frac{\bar{\gamma}}{2}) \tag{32}$$

IV. Simulation

Consider the following linear discrete-time stochastic system (1)-(2), where parameters of system are as follows

$$\mathrm{A} = \begin{bmatrix} 0.3 & -0.9 \\ 0 & 1 \end{bmatrix}, \mathrm{H} = [1 \quad 0]$$

In simulation, we let $Q_w = I_2$ (I_n denotes n by n identity matrix), $Q_v = 2$, and initial values $x_0 = [0 \quad 0]^{\mathrm{T}}$, $\mathrm{P}_{x_0} = I_2$. We take 100 sampling data to simulation. The Kalman filter with innovation-based triggering as $\delta = 0.5$ is shown in Fig.1. It is clear that proposed Kalman filter has a good state tracking performance.

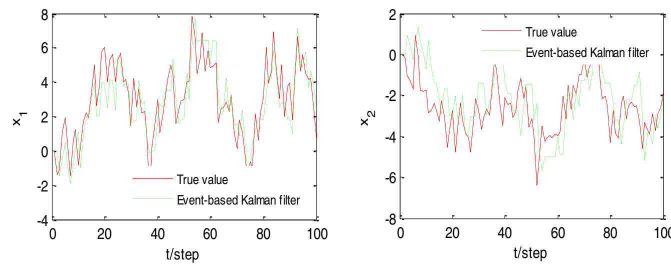

Figure 1. Kalman filter with innovation-based triggering: the first state component (left) and the second state component (right).

In terms of theorem 2, when $\delta = 0.5$, the theoretical value of average sensor-to-estimator communication rate $\gamma = 0.6$ can be calculated. We define the experimental value of the average communication rate as $\gamma_{EXP} = \frac{1}{k}\sum_{j=1}^{k}\gamma_j$, which asymptotically converges to $\gamma = 0.6$ as shown in Fig.2.

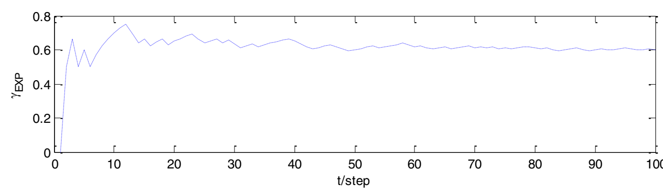

Figure 2. Average communication rate in the experiment.

We compare the estimation performance of standard Kalman filter and event-based filters with different thresholds by performing 1000 times Monte Carlo experiments, then the mean square errors (MSEs) can be calculated by (33) as shown in Fig. 3.

$$MSE_k = \frac{1}{M}\sum_{j=1}^{M}[(x_k - \hat{x}_k)^{\mathrm{T}}(x_k - \hat{x}_k)]$$
$$M = 1000,\ k = 1,\cdots,100 \tag{33}$$

From Fig. 3, we can clearly see that the estimated accuracy of the standard Kalman filtering is better than that of the Kalman filter with innovation-based triggering. Also seen in Fig. 3 is that estimation precision of event-based Kalman filter is reduced with the increase of threshold, this is because the average communication rate can be reduced as the threshold increased. However, compared with standard Kalman filter, the estimated accuracy of event-based Kalman filtering is slightly reduced, but it greatly reduces the average communication rate and saves the network resources. From the overall system level, reducing the communication rate may be more conducive to improving the performance of the system.

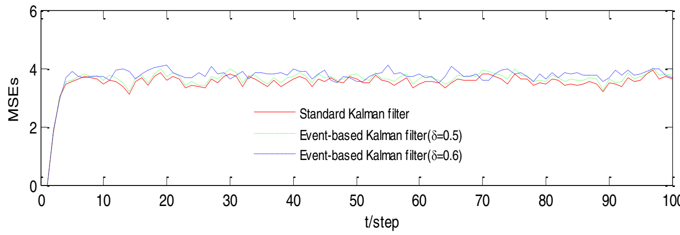

Figure 3. Comparison of MSEs of standard Kalman filter and event-based Kalman filter with different thresholds.

Next, we will compared the algorithms in our paper and [7] under the same average communication rate $\gamma = 0.6$. The comparison of MSEs simulated by 1000 Monte Carlo runs is drawn in Fig. 4. Because we exploit this extra information that is implied by the triggering condition when $\gamma_k = 0$ to help reduce the estimation error, however, the literature [7] does not. Thus, Fig. 4 shows clearly that our filter has better accuracy than the one in [7].

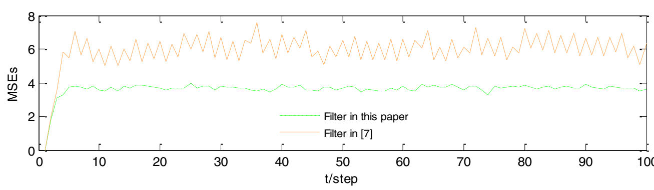

Figure 4. Comparison of MSEs of filters in this paper and [7].

V. Conclusion

In this paper, an approximation MMSE Kalman filter with innovation-based triggering is designed for the linear discrete-time stochastic system under the assumption of the predicted distribution that is to be Gaussian. Although, compared with standard Kalman filter, the estimation accuracy of event-based Kalman filtering is slightly reduced, but it greatly reduces the average communication rate and saves the network resources. Judged from the overall system level, reducing the communication rate may be more conducive to improving the performance of the system. It's also worth mentioning that we consider the extra information that is implied by the triggering condition to help reduce the estimation error. Therefore, the accuracy of the designed filter is higher than that in the literature [7]. Furthermore, it is provided that the average communication rate under a given threshold and the optimal threshold value in the case of known communication rate. The algorithm of this paper can be generalized to vector measurement and multi-sensor systems, though the derivation should be more complicated.

Acknowledgment

This paper was jointly supported by National Science Foundation of China (No.61673117); Provincial Key Project (No. KJ2017A332, No. KJ2016A549), Fuyang Normal University Production and Research Cooperation Project (No.600201), National Undergraduate Innovative Program (No.201710371007), Fuyang Normal University Project (2016FSKJ16).

References

[1] H.B. Li, Z.Q Sun, F. C Sun, "Networked control systems: an overview of state-of-the-art and the prospect in future research," Control Theory & Applications, vol.27, No.2, pp. 238-243, 2010.

[2] Q. B. Ge, C. L. Wen, XU T. L. Xu, "Networked Kalman filtering with combined constraints of bandwidth and random delay," Information Sciences, vol. 55, No. 10, pp. 2343-2357, 2012.

[3] K. Y. You, L. H. Xie, "Survey of recent progress in networked control systems," Acta Automatica Sinica, vol. 39, No. 2, pp. 101-118, 2013.

[4] J.J. Yan, Y.Q. Xia, C.L. Wen et al., "Networked control system of quantitative control strategy analysis and design," Science press, Beijing, China, 2016.

[5] Y. Q. Long, Q. Ling, W. Zheng, "L2 stability analysis of networked control systems based on event-triggering," Information and control, vol. 45, No. 2, pp. 171-176, 2016.

[6] W. Peng, D. Y. Chen, W. H. Sun and G. Q. Zhang, "Distributed adaptive channel allocation in multi-radio wireless sensor networks," Journal of communication, vol. 11, No. 11, pp. 984-991, 2016.

[7] S. Trimpe, M. C. Campi, "On the choice of the event trigger in event-based estimation," In: International Conference on Event-based Control, Communication, and Signal Processing (EBCCSP), pp.1-8. IEEE, Krakow, Poland , 2015.

[8] S. Trimpe, R. D'Andrea, "Event-based state estimation with variance-based triggering," IEEE Transaction on Automatic Control, vol. 59, no. 12, pp. 3266-3281, Dec. 2014.

[9] S. Trimpe, "Event-based state estimation with switching static-gain observers," in Proc. of the 3rd IFAC Workshop on Distributed Estimation and Control in Networked Systems, pp. 91-96, Sep. 2012.

[10] M. Miskowicz, "Send-on-delta concept: an event-based data reporting strategy," Sensors, vol. 6, no. 1, pp. 49-63, 2006.

[11] J. Marck and J. Sijs, "Relevant sampling applied to event-based state estimation," in 4th International Conference on Sensor Technologies and Applications Processing, pp. 618–624. IEEE, Venice, Italy, Jul. 2010.

[12] J. F. Wu, Q. S. Jia, K. H. Johansson, and L. Shi, "Event-Based Sensor Data Scheduling: Trade-off between communication rate and estimation quality," IEEE Transactions on Automatic Control, vol. 58, no. 4, pp. 1041-1046, 2013.

[13] S. Simo, "Bayesian filtering and smoothing(1nd edn)," National Defense Industry Press, Beijing, 2015.