Optimization and Economics for Mobile Crowdsensing

(CS8695: Research in Computer Science)

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Outline

- Introduction
- 2 Background on Mobile Crowdsensing
- 3 Distributed Task Selection in MCS
- 4 Delay-Sensitive MCS

Research Vision: Putting Theory into Practice



- Improve the understanding of technological phenomena.
- Design new solutions for emerging applications.

Research Vision: Putting Theory into Practice

- Emerging applications:
 - Mobile crowdsensing
 - Mobile data offloading
 - Blockchain system
 - Unmanned aerial vehicles
 - Vehicular ad hoc networks
 - **.**..

Research methodologies:

- ► Game theory
- Markov decision process
- Optimization theory
- Algorithms
- Microeconomic theory







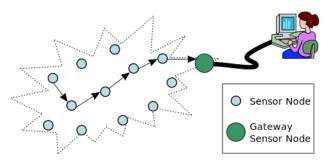




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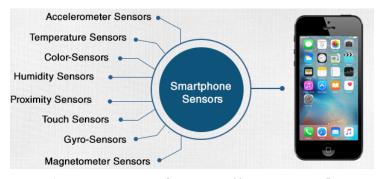
Traditional Sensing Architecture



Sensor networks (source: Wikipedia)

- Traditional sensing technologies leverage distributed sensors to acquire environmental information.
- Key issues: Limited spatial coverage, poor scalability, no mobility

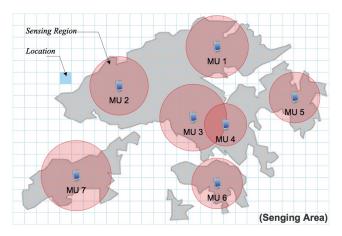
We Carry a Lot of Sensors



Sensors in smartphones (source: http://myphonefactor.in/)

Modern smartphones have a huge number of sensors.

Emergence of Mobile Crowdsensing (MCS)



Mobile Crowdsensing

• MCS: Individuals use their mobile devices to collectively sense and share information related to a certain phenomenon of interest.

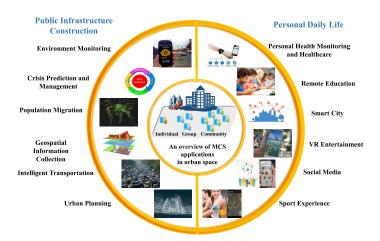
MCS: Benefits and Concerns

- Benefits
 - ► Good scalability
 - Good mobility

MCS: Benefits and Concerns

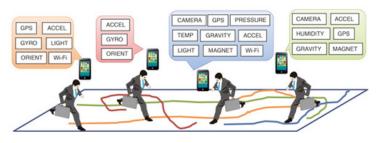
- Benefits
 - Good scalability
 - Good mobility
- Concerns
 - Excessive resource consumptions
 - Uneven and biased coverages
 - Large variations of sensing quality
 - Security and privacy
 - Incentives

Classification of MCS: Application Types



MCS applications: community vs. personal sensing (Source: Shu *et al.*, "When Mobile Crowd Sensing Meets Traditional Industry," *IEEE Access*, 2017)

Classification of MCS: User Efforts

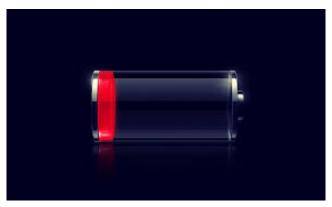


(Source: https://www.ntt-review.jp)

- Participatory sensing: Requires the active involvement of individuals to contribute sensor data.
- Opportunistic sensing: More autonomous with minimum user involvement.

Research Issues in Mobile Crowdsensing

Resource Consumption (This seminar)



(Source: Internet)

- MCS may consume significant amount of resources.
- How to incentivize users to contribute despite of the costs?

Private Information



(Source: Internet)

- Users will have private information regarding the capability of their devices and quality of their data.
- How to incentivize users to truthfully reveal private information?

User Effort



(Source: Internet)

- The quality of MCS data is related to user effort that may be difficult to verify.
- How to incentivize users to exert high level of efforts?

Long-term Participation



(Source: Internet)

- A MCS campaign might require long-time efforts from users.
- How to incentivize users to stay in the system long enough?

User Cooperation



(Source: Internet)

- Some MCS applications require coordination among user to avoid waste of resources and achieve large coverage.
- How to incentivize users to choose the best tasks considering the systems' needs and their own capabilities?

Data Diversity



(Source: Internet)

- Diverse data improve the accuracy and effectiveness of MCS applications.
- How to incentive users with diverse backgrounds to accomplish the same MCS task?

Bounded Rationality



(Source: Internet)

- Users are not fully rational in terms of risk and cognitive reasoning capabilities.
- How to consider the realistic bounded rational behavior of users?

Delay Consideration (This seminar)



(Source: Internet)

- Some tasks have stringent delay constraints
- How to tradeoff resource consumption (in sensing and reporting data)
 vs. delay requirements?

Security and Privacy



(Source: Internet)

- Reported data contains personal information such as location.
- How to protect the security and privacy of personal data?

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Commercial MCS Platforms

- Example: Gigwalk and Field Agent:
 - Audit task: Fact finding and data gathering.
 - * E.g., Checking on-shelf availability and prices of style gel.
 - ▶ Research task: Collection of users' opinions and insights.
 - ★ E.g., Surveys and shop-alongs.





©Gigwalk and Field Agent

Distributed Task Selection

 Collection of location-dependent and time-sensitive information related to stores and products of customers.



© Appirio Hub and Field Agent

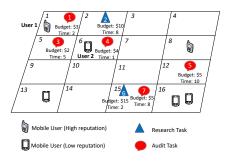
Research Problem

How should we allocate tasks to mobile users?

Task Allocation in Mobile Crowdsensing

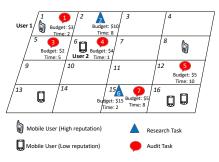
- Related works on task allocation in MCS:
 - ► Fair and energy-efficient task allocation [Zhao INFOCOM'14].
 - Social surplus maximization for location-dependent task scheduling [He TVT'17].
 - Multi-task assignment with spatiotemporal correlation [Wang TMC'19]
- Main ideas:
 - ▶ Centralized task allocation → Distributed task selection.
 - ▶ No time constraint → Time-sensitive tasks.
- First study on these aspects motivated by commercial applications.

System Model: Tasks



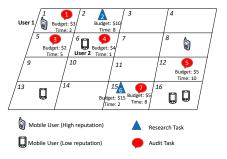
- Location-dependent and time-sensitive task $k \in \mathcal{K}$:
 - ▶ Location $I[k] \in \mathcal{L}$.
 - ▶ Execution time $t[k] \in \mathcal{T}$.
 - ▶ Budget $\rho[k] \ge 0$. Shared by users performing the same task.
 - ★ E.g. I[5] = 12, t[5] = 10, and $\rho[5] = 5 .
 - Difficulty level: E.g., Audit and research.

System Model: User i



- Initial location $l_i^{\text{init}} \in \mathcal{L}$. E.g., $l_2^{\text{init}} = 6$.
- Destination θ_i with prior distribution $p(\theta_i) \geq 0$.
- Movement time $\Delta_i[I,I'] \geq 1$.
- Switching cost $c_i[k, k'] = m_i[I[k], I[k']] + e_i[k']$ if $k \neq k'$.
 - ▶ Movement cost $m_i[I, I']$
 - Execution cost: $e_i[k]$
- Reputation \Longrightarrow Eligible tasks $\mathcal{K}_i \subseteq \mathcal{K}$.
 - ▶ E.g. $\mathcal{K}_1 = \{1, 2, 3, 4, 5, 6, 7\}$ and $\mathcal{K}_2 = \{1, 3, 4, 5, 7\}$.

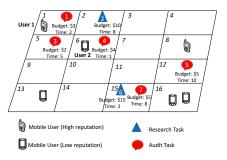
System Model



Question

Given the rewards, locations, and execution times of tasks, how should each user moves and selects tasks to maximize his utility?

System Model



Question

Given the rewards, locations, and execution times of tasks, how should each user moves and selects tasks to maximize his utility?

- Coordination of multiple rational users in task selection.
- Game theory would be a useful methodology for analysis.

Game Theory: A Brief Introduction

- Game theory is the study of mathematical models of strategic interaction among rational decision-makers.
- Three key components:
 - ► A set of players
 - An action/strategy space for each player
 - ► A utility function for each player
- Example: Prisoner's Dilemma

	Not Confess	Confess
Not Confess	(-1, -1)	(-5,0)
Confess	(0, -5)	(-3, -3)

Strategies

Best response strategy

$$a^*(b) = \arg\max_{a \in \mathcal{A}} U_1(a, b)$$

- ▶ Best response might not be unique
- Dominant strategy
 - ▶ A player's strategy that always leads to a payoff **no worse** than his any other strategies, **independent** of his opponents' strategies

	Not Confess	Confess
Not Confess	(-1, -1)	(-5,0)
Confess	(0, -5)	(-3, -3)

Nash Equilibrium

• Formally, $(a^* \in \mathcal{A}, b^* \in \mathcal{B})$ is a Nash equilibrium (NE) if

$$U_1(a^*,b^*) \geq U_1(a,b^*)$$
, for any $a \in \mathcal{A}$

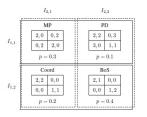
and

$$U_2(a^*,b^*) \geq U_2(a^*,b), \text{ for any } b \in \mathcal{B}$$

 No player has the incentive to unilaterally deviate from his/her strategy to obtain a higher payoff.

Bayesian Game: More advanced formulation

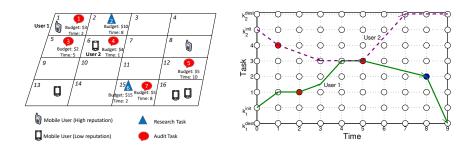
- Assumption so far: All players know what game is being played.
- In practice: Players may have uncertainty about other players.
 - ▶ E.g., Destination θ_i with prior distribution $p(\theta_i) \ge 0$.



© Yoav Shoham

- In game theory, a Bayesian game is a game in which players have incomplete information about the other players.
 - ▶ Model the uncertainty of player i by a type space Θ_i
 - ▶ Model the common prior distribution $p(\theta_1, ..., \theta_I)$ of types

Task-Time Route as the Action



• Task-time route of a user is a sequence of task-time points

$$\left(\left(k_{i}^{\mathsf{init}}, t_{i}^{\mathsf{init}}\right), \left(k_{i}^{1}, t_{i}^{1}\right), \left(k_{i}^{2}, t_{i}^{2}\right), ..., \left(k_{i}^{n}, t_{i}^{n}\right), \left(k_{i}^{\mathsf{dest}}, t_{i}^{\mathsf{dest}}\right)\right).$$

Bayesian Game

Task Selection Game (TSG)

- Players: Mobile users.
- Actions: Task-time routes $\mathbf{r} = (r_1, \dots, r_l)$.
- Utilities: The vector $\mathbf{U} = (U_i, \forall i \in \mathcal{I}).$

Bayesian Game

Task Selection Game (TSG)

- Players: Mobile users.
- Actions: Task-time routes $\mathbf{r} = (r_1, \dots, r_l)$.
- Utilities: The vector $\mathbf{U} = (U_i, \forall i \in \mathcal{I})$.
- Types: The set $\Theta = \Theta_1 \times ... \times \Theta_I$ possible destinations.
- Prior information: The common prior $p(\theta_1, \dots, \theta_I)$ of types.

$$U_i(\mathbf{r}, \theta) = \underbrace{\sum_{a=1}^{n} \frac{\rho[k_i^a]}{\omega[(k_i^a, t_i^a), \mathbf{r}, \theta]}}_{\text{Total reward}} - \underbrace{\sum_{a=1}^{n-1} c_i[k_i^a, k_i^{a+1}]}_{\text{Total switching cost}}.$$

How to analyze a game?

- Game properties:
 - ► Existence of NE?
 - ► Uniqueness of NE?
 - ► Convergence to NE?



[FIG1] A (nonexhaustive) methodology for proving the existence of a pure Nash equilibrium in strategic games. The reader has to refer to the acronyms and references used in the section "Existence."

How to analyze a game?

- Game properties:
 - Existence of NE?
 - Uniqueness of NE?
 - ► Convergence to NE?
- Common checks for existence of a pure NE:
 - (QG) Quasi-concave utility functions
 - ▶ (PG) Potential game
 - ► (SMG) Supermodular game



[FIG1] A (nonexhaustive) methodology for proving the existence of a pure Nash equilibrium in strategic games. The reader has to refer to the acronyms and references used in the section "Existence."

Samson Lasaulce, Mérouane Debbah, and Eitan Altman, "Methodologies for analyzing equilibria in wireless games", IEEE Signal Processing Magazine, vol. 26, no. 5, Sept. 2009.

Bayesian potential game

• Bayesian game Ω is a Bayesian potential game if there exists an exact potential function $\Phi(\mathbf{r}, \theta)$ such that

$$U_{i}(\mathbf{r}_{i},\mathbf{r}_{-i},\boldsymbol{\theta}) - U_{i}(\mathbf{r}'_{i},\mathbf{r}_{-i},\boldsymbol{\theta}) = \Phi(\mathbf{r}_{i},\mathbf{r}_{-i},\boldsymbol{\theta}) - \Phi(\mathbf{r}'_{i},\mathbf{r}_{-i},\boldsymbol{\theta}),$$

$$\forall \mathbf{r}_{i},\mathbf{r}'_{i} \in \mathcal{R}_{i}(\boldsymbol{\theta}_{i}), \forall \boldsymbol{\theta} \in \Theta, \forall i \in \mathcal{I}.$$
(1)

• Change in utility function = Change in potential function

Convergence

Question 1

Will the strategy profile of the users converge using best response updates?

Convergence

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Convergence

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Will the strategy profile of the users converge using best response updates?

Theorem 1

Every task selection game possesses the finite improvement property.

- FIP: Best response updates always converge to a pure Bayesian NE within a finite number of steps, independent of the initial strategy profile or players' updating order.
- Idea: Show that TSG is a Bayesian potential game.

Convergence Speed

Question 2

What is the speed of convergence?

Convergence Speed

Question 2

What is the speed of convergence?

Theorem 2

A best response update can be computed in $\mathcal{O}(LK^3T^3)$ time.

- Best response updates can be found in polynomial time.
- Idea: Show that the problem of finding a best response update can be reformulated as a shortest path problem.

Centralized Task Allocation: Benchmark

- Service provider centrally allocates tasks to users.
- CTA benchmark: Globally optimal solution of social surplus maximization problem.

Theorem 3

The problem of finding the social surplus maximization solution of the TSG is NP-hard.

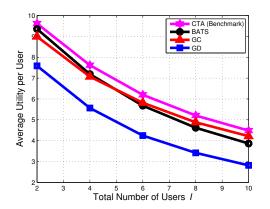
 Idea: Show that finding the social surplus maximization solution in a special case of a TSG can be transformed into a 3-dimensional matching decision problem, which is NP-complete.

Performance Evaluations

- Compare four schemes:
 - ▶ BATS: Our proposed scheme based on task selection game.
 - ► CTA benchmark: Global optimal centralized solution.
 - ▶ GC (Greedy centralized): Greedy algorithm on social surplus max.
 - ▶ GD (Greedy distributed): Each user chooses the earliest task without any coordination.
- Setting:
 - ▶ 1 km × 1 km region.
 - ▶ Movement cost: $m_i[I, I'] = c_i^{\text{move}} \text{dist}(I, I')$.
 - ▶ Movement time: $\Delta_i[I, I'] \propto \text{dist}(I, I')$

Performance Evaluations: Average Utility

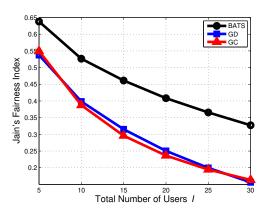
Setting: K = 5 tasks and movement cost $c^{\text{move}} = 1$.



- Average utility decreases with the level of contention.
- GC and BATS achieve similar average user utility as CTA benchmark.

Performance Evaluations: Fairness

Setting: K = 10 tasks and movement cost $c^{\text{move}} = 0.1$.



- BATS: Highest fairness (: each user has an equal chance to update his strategy profile).
- GC: Lowest fairness (: some users may not be allocated with tasks).

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Reporting in Delay-sensitive MCS

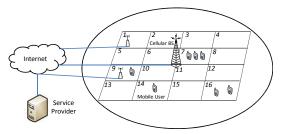
- After sensing, users still need to report data to service provider.
 - ▶ Offline mode (e.g., in Gigwalk): Sense at a location without Internet connectivity and upload later by a deadline.
- User should choose between cellular and Wi-Fi by considering his mobility, network availability, and deadline.





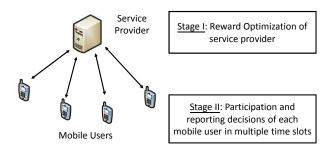
Network Setting

- Locations $\mathcal{L} = \{1, \dots, 16\}$, where cellular is always available.
- Free Wi-Fi availability is location-dependent:
 - $\mathcal{L}^{(1)}=\{1,9\}$ and $\mathcal{L}^{(0)}=\mathcal{L}\backslash\mathcal{L}^{(1)}$ in the figure.



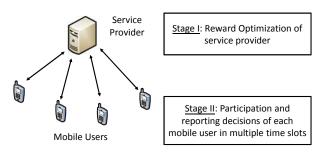
- Time slots $\mathcal{T} = \{1, \dots, T\}$.
- Users $\mathcal{I} = \{1, \dots, I\}$ are at locations of interest when t = 1.

Stage I



- Stage I: Service provider chooses reward to maximize its profit.
- Delay-Sensitive MCS: Value of data degrades with time.
- Reward $r = \theta^{t-1}R$ for t < T and r = 0 otherwise.
 - ► *R* > 0: Initial reward.
 - ▶ $0 < \theta \le 1$: Discount factor.
 - ► T: Deadline.

Stage II



- Each user $i \in \mathcal{I}$ is associated with:
 - ▶ Sensing cost σ_i : Energy cost for performing the sensing task.
 - ► Cellular transmission cost *c_i*: Payment of cellular data.
 - ▶ Mobility pattern: User moves around locations \mathcal{L} in \mathcal{T} time slots.

Stage II: Tradeoff of Delay-Sensitive MCS

- Aim: To achieve a good tradeoff for a user between
 - ► Lower upload cost: Wait for Wi-Fi and upload data with zero cost.
 - ▶ Higher reward: Upload early with cellular for a higher reward.



Stage II: Tradeoff of Delay-Sensitive MCS

- Aim: To achieve a good tradeoff for a user between
 - ► Lower upload cost: Wait for Wi-Fi and upload data with zero cost.
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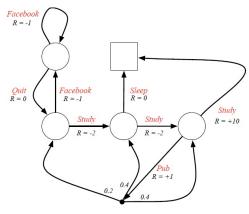
Stage II's Question

Given the reward scheme, should user *i* participate in sensing? If so, should it report by cellular, by Wi-Fi, or not to report?

Markov decision process (MDP)

- MDP: Computational tools for sequential decision problems by considering both the short-term and long-term consequences of a chosen action.
- Five key ingredients in a sequential decision problem:
 - ▶ Decision epoch $t \in \mathcal{T}$: Time points at which decisions can be made.
 - ★ Can be either finite or infinite, and either discrete or continuous.
 - State s ∈ S: It summarizes the past information that is relevant for future optimization.
 - ▶ Action $a \in A$: Actions that the decision maker can take.
 - State transition probability $p_t(s'|s, a)$: It represents that the probability that the system will be in state s' at time t+1 if action a is taken in state s at time t.
 - ▶ Reward $r_t(s, a)$: Immediate reward gained by choosing action a in state s at time $t \in \mathcal{T}$.

Example



©David Silver

- Decision epoch $t \in \mathcal{T}$
- State $s \in S$
- Action $a \in A$
- State transition probability $p_t(s'|s, a)$
- Reward $r_t(s, a)$

Stage II's Solution: Markov Decision Process

- 1) Decision epochs: $t \in \mathcal{T} = \{1, \dots, T\}$.
- 2) State: s = (k, l)
 - k (Reporting status):
 k = 0 (Not yet reported) or k = 1 (Reported).
 - / (Location index).
- 3) Action: $a \in A_k \subseteq \{0,1\}$
 - a = 0 (idle) or a = 1 (report).

Markov Decision Process (cont'd)

• 4) State transition probability: $\mathbb{P}((k', l') | (k, l), a)$

$$\mathbb{P}((k',l') | (k,l), a) = q_i(l' | l) \chi(k' | k, a),$$

• $\chi(k' | k, a)$: Change in reporting status with action.

$$\chi(k' \mid k, 1) = \begin{cases} 1, & \text{if } k' = 1, \\ 0, & \text{otherwise,} \end{cases}$$

and

$$\chi(k' \mid k, 0) = \begin{cases} 1, & \text{if } k' = k, \\ 0, & \text{otherwise.} \end{cases}$$

• $q_i(l'|l)$: Obtain based on a user's mobility pattern.

Markov Decision Process (cont'd)

• 5) Surplus in each time slot $t \in \mathcal{T}$: Reward - Transmission cost

$$\phi_t(k, l, a) = \begin{cases} \theta^{t-1}R, & \text{if } a = 1, k = 0, \text{and } l \in \mathcal{L}^{(1)}, \\ \theta^{t-1}R - c_i, & \text{if } a = 1, k = 0, \text{and } l \in \mathcal{L}^{(0)}, \\ 0, & \text{if } a = 0. \end{cases}$$

General Case: Discounted Reward

(Optimization problem) To maximize a user's expected surplus:

$$\xi_i^* = \underset{\pi \in \Pi}{\operatorname{maximize}} \, E_{\mathbf{s}_1}^{\pi} \left[\sum_{t=1}^T \phi_t \big(\mathbf{s}_t^{\pi}, \delta_t \big(\mathbf{s}_t^{\pi} \big) \big) \right].$$

- ▶ Policy $\pi = (\delta_t(k, l), \forall k \in \mathcal{K}, l \in \mathcal{L}, t \in \mathcal{T})$: Reporting decision rules at all the states and time slots.
- ▶ Solved by finite-horizon MDP to obtain optimal policy π^* .

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$$\xi_i^* = \underset{\pi \in \Pi}{\operatorname{maximize}} \, E_{\mathbf{s}_1}^{\pi} \Bigg[\sum_{t=1}^T \phi_t \big(\mathbf{s}_t^{\pi}, \delta_t (\mathbf{s}_t^{\pi}) \big) \Bigg].$$

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- ▶ Solved by finite-horizon MDP to obtain optimal policy π^* .

Theorem 1 (Optimal Policy)

- (Participation Decision): Participate if $\xi_i^* \geq \sigma_i$.
- (Reporting Decision): If participate, report with optimal policy π^* .

Special Case: Non-Discounted Reward

- Reward scheme: Report by deadline T to obtain reward $r \geq 0$.
- p_i : Probability of not meeting Wi-Fi in T time slots.

Theorem 2 (Optimal Participation and Reporting Decisions)

- (a) For a small reward $0 \le r < c_i$:
 - Participation: Participate if $r \ge \frac{\sigma_i}{1-\rho_i}$.
 - Reporting: Wait for Wi-Fi until time T, when it will not report.

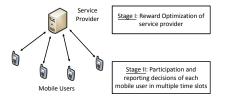
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Theorem 2 (Optimal Participation and Reporting Decisions)

- (a) For a small reward $0 \le r < c_i$:
 - Participation: Participate if $r \ge \frac{\sigma_i}{1-p_i}$.
 - Reporting: Wait for Wi-Fi until time T, when it will not report.
- (b) For a large reward $r \geq c_i$:
 - ▶ Participation: Participate if $r \ge \sigma_i + p_i c_i$.
 - Reporting: Wait for Wi-Fi until time T, when it will use cellular to report.

Stage I: Service Provider's Profit Maximization



- SP may have incomplete information about the users:
 - ▶ Sensing cost $\sigma \in S$ with distribution $f(\sigma)$.
 - ▶ Cellular transmission cost $c \in C$ with distribution f(c).
 - ▶ Wi-Fi unavailability $p \in [p^{\min}, p^{\max}]$ with distribution f(p).

Stage I's Question

Given the incomplete user information, how much reward should the SP offer to maximize its expected profit?

Profit Maximization

Probability of receiving n reports with reward r:

$$\mathbb{P}(n,r) = \binom{l}{n} \rho(r)^n (1 - \rho(r))^{l-n}.$$

- ho(r): Probability of reporting data given reward r.
- SP chooses a reward to maximize its expected profit:

$$\underset{r \geq 0}{\text{maximize}} \quad \phi(r) \triangleq \sum_{n=0}^{l} \Big(u(n) - r \times n \Big) \mathbb{P}(n, r).$$

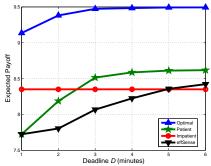
- \triangleright u(n): Utility function when n reports are received.
- ightharpoonup r imes n: Total payment to the users.
- Challenges:
 - $ightharpoonup \phi(r)$ may be discontinuous and non-concave.
 - ▶ Large search space of $r \ge 0$ numerically.

Stage II's Performance Evaluations

- Compare four schemes:
 - Optimal: Our proposed algorithm.
 - Patient:
 - * Always participate.
 - ★ Wait for Wi-Fi until the last time slot to report.
 - ★ If not available, report with cellular if $\theta^{T-1}R \ge c_i$.
 - Impatient:
 - ★ Always participate.
 - * Report at t=1 with Wi-Fi (if $l \in \mathcal{L}^{(1)}$) or cellular (if $l \in \mathcal{L}^{(0)}$).
 - 4 EffSense [Wang IEEE TSMCS'15]:
 - ★ Participate if $r \ge \sigma_i + (1 p^{\text{wifi}})^T c_i$.
 - * Wait for Wi-Fi until the last time slot to report.
- Setting:
 - Grid topology with L = 16 possible locations.
 - Probability that Wi-Fi is available $p^{\text{wifi}} = 0.4$.

Performance Evaluations: Deadline

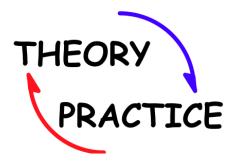
Setting: R = 25, $\theta = 0.9$, $\sigma_i = 10$, and $c_i = 11$.



- Optimal, patient, effSense: User payoff increases with deadline.
 - : Higher chance to meet Wi-Fi.
- Our proposed scheme achieves the maximal expected user payoff.
- Impatient: Independent of deadline.

Conclusions

- Distributed task selection in MCS (non-cooperative game theory).
- Delay-sensitive MCS (Markov decison process).



Thank you!

Bayesian Nash Equilibrium

- Strategy $s_i : \Theta_i \to \mathcal{R}_i$: Specifies the action for each possible type.
 - ▶ Strategy space S_i .
- Expected utility $EU_i(s) = \sum_{\theta \in \Theta} U_i(s(\theta), \theta) p(\theta)$.

Definition

The strategy profile s^* is a pure strategy Bayesian NE (BNE) if

$$m{s}_i^* = rg \max_{m{s}_i \in \mathcal{S}_i} EU_i(m{s}_i, m{s}_{-i}^*), \, orall \, i \in \mathcal{I}.$$