$$\dot{x} = h + rx - x^2$$

a) Plot the bifurcation diagram for i) $h<0$, ii) $h=0$, and iii) $h>0$.

First find fixed points: $f(x) = h + rx - x^2 = 0$

$$x^2 - rx - h = 0$$

$$x^* = \frac{r \pm \sqrt{r^2 - 4(1)(-h)}}{2(1)} = \frac{r \pm \sqrt{r^2 + 4h}}{2}.$$

Since bifurcation is in terms of r, write

$$x^*(r) = \frac{r}{2} \pm \sqrt{\left(\frac{r}{2}\right)^2 + h}.$$

The bifurcation occurs when $\left(\frac{r_{crit}}{2}\right)^2 + h = 0$, therefore

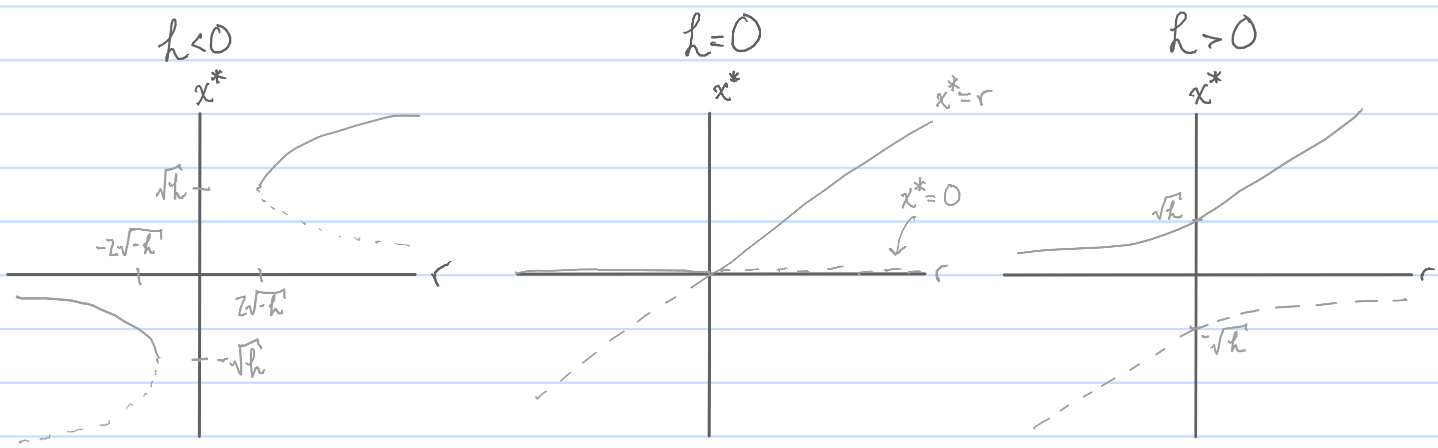
$$r_{crit} = \pm 2\sqrt{-h} \qquad \text{(doesn't exist for } h>0\text{)}$$

Stability is found using linear stability analysis:

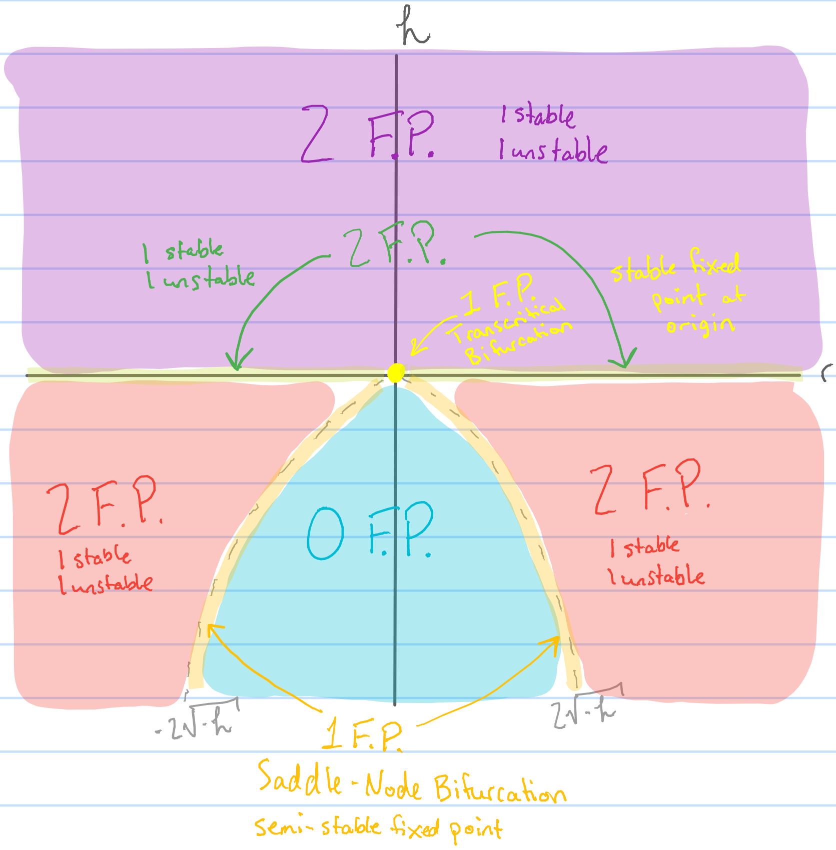
$$f'(x^*) = -2x + r,$$

which implies that x^* is stable when $-2x > r$. We can evaluate this expression for test points in each case. For $h<0$, let $h=-1$ and $r=\pm 4$. For $h>0$, let $h=1$ and $r=\pm 4$.

The bifurcation diagrams are



b)


h
2 F.P.
1 stable
1 unstable
1 stable
1 unstable
2 F.P.
1 F.P.
Transcritical Bifurcation
stable fixed point at origin
r
2 F.P.
1 stable
1 unstable
0 F.P.
2 F.P.
1 stable
1 unstable
$-2\sqrt{-h}$
$2\sqrt{-h}$
1 F.P.
Saddle-Node Bifurcation
semi-stable fixed point