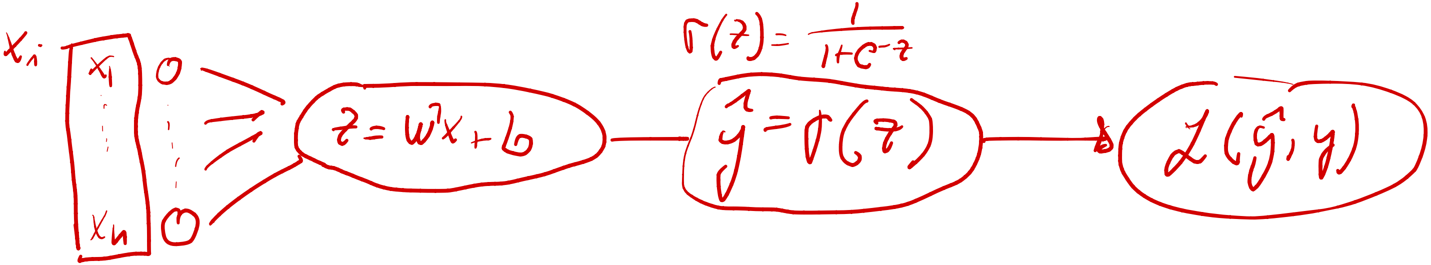

$$J = -\frac{1}{m} \sum_{i=1}^{m} \left[y_i \log(\hat{y}_i) + (1-y_i) \log(1-\hat{y}_i) \right]$$

$$\frac{d\mathcal{L}}{d\hat{y}} = -d\left[y \log(\hat{y}) + (1-y) \log(1-\hat{y}) \right] = -\frac{y}{\hat{y}} + \frac{1-y}{1-\hat{y}}$$

$$= \frac{-y(1-\hat{y}) + \hat{y}(1-y)}{\hat{y}(1-\hat{y})} = \frac{-y + \cancel{y\hat{y}} + \hat{y} - \cancel{\hat{y}y}}{\sim}$$

$$\boxed{\frac{\hat{y} - y}{\hat{y}(1-\hat{y})}}$$

$$\frac{\partial L}{\partial z} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial z} = \left(\frac{\hat{y} - y}{\cancel{\hat{y}(1-\hat{y})}}\right) \cdot \left(\cancel{\hat{y}(1-\hat{y})}\right) = \boxed{\hat{y} - y}$$

$$\frac{\partial \hat{y}}{\partial z} = \partial\left[\frac{1}{(1+e^{-z})}\right] = \partial\left[(1+e^{-z})^{-1}\right] = -(1+e^{-z})^{-2}(-e^{-z})$$

$$\boxed{\frac{e^{-z}}{(1+e^{-z})^{2}}} = \left(\frac{1}{1+e^{-z}}\right)\left(1 - \frac{1}{1+e^{-z}}\right) = \boxed{\hat{y}(1-\hat{y})}$$

$$\frac{\partial \mathcal{L}}{\partial w} = \underbrace{\frac{\partial \mathcal{L}}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial z}} \cdot \frac{\partial z}{\partial w} = \boxed{(\hat{y} - y) \cdot X}$$

$$\frac{\partial z}{\partial w} = \frac{\partial}{\partial w}\left[w^{T} x + b \right] = X$$

$$\frac{\partial z}{\partial b} = 1$$

$$\frac{\partial \mathcal{L}}{\partial b} = (\hat{y} - y)(1) = \boxed{\hat{y} - y}$$

Algoritmo (Reg. Log)

for t = 1 to max_iter :

$$z = w.T @ X + b$$

$$\hat{y} = \sigma(z)$$

$$dz = \hat{y} - y$$

$$dw = \frac{1}{m} X @ dz.T$$

$$db = \frac{1}{m} np.sum(dz)$$

$$w = w - lr * dw$$

$$b = b - lr * db$$

$$\sigma = \frac{1}{1 + e^{-z}}$$

$$\sigma = (1 + e^{-z})^{-1}$$