Asteroid Family Identification

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2 authors:

Philippe Bendjoya
University of Nice Sophia Antipolis

Vincenzo Zappala
National Institute of Astrophysics

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Ph. Bendjoya
University of Nice

V. Zappalà
Astronomical Observatory of Torino

Asteroid families have long been known to exist, although only recently has the availability of new reliable statistical techniques made it possible to identify a number of very "robust" groupings. These results have laid the foundation for modern physical studies of families, thought to be the direct result of energetic collisional events. A short summary of the current state of affairs in the field of family identification is given, including a list of the most reliable families currently known. Some likely future developments are also discussed.

1. INTRODUCTION

The term "asteroid families" is historically linked to the name of the Japanese researcher Kiyotsugu Hirayama, who was the first to use the concept of orbital proper elements to identify groupings of asteroids characterized by nearly identical orbits (*Hirayama,* 1918, 1928, 1933). In interpreting these results, Hirayama made the hypothesis that such a proximity could not be due to chance and proposed a common origin for the members of these groupings. His interpretation, still accepted today and strengthened by theoretical and observational results described elsewhere in this book (*Zappalà et al.,* 2002; *Cellino et al.,* 2002), was that members of a family are the fragments produced by the disruption of a common parent body resulting from a catastrophic collision. In his classic analysis, Hirayama was able to identify a set of five families still known today as the Hirayama families: Eos, Themis, Koronis, Flora, and Maria.

Several years passed before the problem of asteroid family identification was again taken into consideration by *Brouwer* (1951), who substantially confirmed the Hirayama families, although he divided Flora into four different groups. Moreover, 20 new additional groups were added to the family list. Since then, and before 1990, several authors have proposed their own catalogs of families (*Arnold,* 1969; *Lindblad and Southworth,* 1971; *Carusi and Massaro,* 1978; *Williams,* 1979; *Kozai,* 1979; *Williams,* 1989). No uniform agreement has been achieved among the results of different analyses, and the discrepancies were generally very large. Authors using slightly different databases (in terms of numbers of objects) of asteroidal proper elements found numbers of families ranging from 15 (*Carusi and Massaro,* 1978) to 117 (*Williams,* 1989). Different methods of classification were used by different authors. Most of them were based on a visual analysis of the data, only in some cases complemented by statistical tests to evaluate the level of significance of the different groupings. Some kind of subjectivity was certainly present. Moreover, in some cases, the results of previous investigations were used to calibrate new identification methods. According to the original papers published in the literature, *Brouwer* (1951) used a fairly subjective criterion to subdivide the Flora family delineated by Hirayama. *Arnold* (1969) assumed that the asteroids are dispersed in the proper-element space in a Poisson distribution. *Lindblad and Southworth* (1971) calibrated their method in such a way as to find good agreement with Brouwer's results. *Carusi and Massaro* (1978) adjusted their method in order to again find the classical Hirayama families. *Williams* (1979, 1989) developed a technique based on a visual inspection of the data, complemented by an *a posteriori* statistical test. *Kozai* (1979) explicitly recognized the significant degree of arbitrariness present in his family identification technique. Of course, the quoted discrepancies shed serious doubts regarding the reliability of the proposed families, apart from the classical ones already found by Hirayama. The situation in 1989 is effectively summarized in *Valsecchi et al.* (1989).

The disagreement on the number of existing families and their associated memberships had not completely prevented any kind of physical analysis of the proposed groupings (*Zappalà et al.,* 1984), but there was widespread agreement that only the classical, very prominent Hirayama families could be taken seriously into consideration for detailed physical studies, whereas many of the other groupings proposed by different authors were much more questionable. *Chapman et al.* (1989) pointed out also a number of cosmochemical inconsistencies among several of these smaller families, including objects belonging to taxonomic classes hardly compatible in terms of likely thermal histories.

2. SITUATION AFTER 1990

Beginning in 1990, the situation in the field of family identification began to improve significantly. The main reason has been the development of more objective identification procedures. In addition, the availability of increasingly larger datasets of accurate proper elements has allowed the researchers to analyze increasingly larger samples of the whole asteroid population, giving them better possibilities to reliably identify the presence of dense, compact clusters.

Proper elements are crucially important for family identification procedures, due to the fact that the orbital elements of the asteroids are subject to variations over different timescales, as a consequence of planetary perturbations. Assuming that a family is formed after the disruption of parent body, one can expect that the osculating (instantaneous) orbital elements of the fragments are very similar just after the family's formation, but then start to rapidly diverge as an effect of perturbations. If there were not any way to identify some conserved feature in the orbital evolution of the objects, one could expect not to be able to infer from the osculating elements of different objects at a given epoch strong clues about a possible common origin from a collisional event. Fortunately, the orbital evolution of the objects, at least in the case of "regular" orbits not affected by resonant conditions, is not totally unpredictable in practice over reasonably long timescales. In particular, some degree of orbital characterization describing the long-term behavior can be discerned by the proper elements. These are quasi-constant parameters representing a kind of average of the osculating elements over fairly long timescales, in which short-period and long-period fluctuations have been removed. A more exhaustive and rigorous treatment of asteroid proper elements can be found in *Knežević et al.* (2002). For family identification purposes, the important fact is that the osculating elements of family members vary as a function of time, but the proper elements do not; therefore, it is possible to analyze the orbital similarities of the objects by examining their proper elements, rather than their osculating elements.

The techniques of proper-element computations have improved over time, since the epoch of the first pioneering studies of Hirayama. In particular, the refinement of the dynamical theories and computational techniques has steadily produced increasingly larger datasets of asteroid proper elements.

On the other hand, the choice of the proper-element database was probably not the main source of the discrepancies in the family lists obtained by different authors before 1990, as mentioned in the previous section. In particular, *Zappalà et al.* (1992) show that by applying the new Hierarchical Clustering Method (HCM) (see below) to the dataset of proper elements used by *Williams* (1989), they can identify 26 families having more than 5 members. By applying the same HCM method to a different proper-element database containing a comparable (though not equal) number of objects, computed by *Milani and Knežević* (1990) using a different dynamical theory, the number of resulting families turns out to be 20. The number of families found in both experiments is 16. This is a good agreement, whereas the number of resulting families found by *Williams* (1989) by applying his own identification method to his proper-element database (the same just mentioned above) is 117. These results, summarized in Table 1, show that the choice of the proper elements was not the critical issue in explaining the observed discrepancies in family searches. Most of the problems were thus due to the adopted identification methods. In this respect, the situation changed quickly starting in 1990.

TABLE 1. Comparison between the numbers of families identified by applying the HCM method (see text) to different proper element databases, according to *Zappalà et al.* (1992).

	N ≥ 5	N ≥ 10
Williams	26	15
Milani-Knežević	20	14
Intersection	16	14

The first two lines indicate the numbers of families having numbers of objects (N) larger than 5 or larger than 10 found by using the Williams and the Milani-Knežević proper elements respectively. The third line indicates the number of common families found in both cases, being formed mostly or exclusively by objects present in both databases. The numbers shown indicate a good agreement, mainly when considering larger (more reliable) families.

2.1. Improved Identification Methods

To quantify the similarity of two orbits, a metric in the space of the proper elements (phase space) a', e', and i' (proper semimajor axis, eccentricity, and inclination respectively) must be defined. *Zappalà et al.* (1990) develops different choices for a metric in the phase space. All of them give quantifications of the "distance" between two orbits, in terms of quantities having all the dimensions of a velocity. The reason is that the well-known Gaussian equations define a relation between the change in orbital elements produced by a sudden velocity change and the components of this change. Therefore, differences in a', e', and i' can be measured in terms of velocity changes needed to produce these differences. This approach is very suitable for family-determination purposes, since the physical origin of a family is thought to be the breakup of its parent body. The different fragments would be expected to have small ejection velocities compared with the orbital velocity of their parent (this being one necessary condition for applying the Gaussian formulas). Hence, computing the mutual distances of the objects in the proper-element space allows for the detection of groups of asteroids that may be genetically related. Moreover, the knowledge of the velocity distribution within a family gives important constraints for any further kinematic modeling of these events (see *Zappalà et al.,* 2002).

A crucial issue when dealing with clustering identification in the proper-element space is the ability to evaluate the statistical significance of the detected groupings against chance fluctuations. This can be done by analyzing random distributions of simulated objects generated according to the constraint of reproducing separately the overall a, e, and i distributions of the real asteroids under consideration (quasirandom populations). The techniques for generating such quasirandom distributions were first developed by *Zappalà et al.* (1990). Using these techniques, no underlying model of statistical distribution of the objects (Gaussian or Poissonian) is needed, and the overall properties of the local distributions of the asteroids in different regions of the main belt are reproduced. By performing the clustering analysis on both the original dataset and several realizations of the quasirandom populations, it becomes possible to evaluate and quantify the probability that any given cluster is due to chance fluctuations. In particular, detected groupings having negligible probabilities of being due to chance are the obvious candidates to be considered as real, collisionally produced families.

Automated techniques managed by the computer present several advantages with respect to techniques based mostly on visual and subjective inspection of the data: (1) Because they are implemented in the form of computer algorithms, they are much less prone to subjectivity and more easily reproducible. (2) Alternative choices of the metric can be easily made and immediately used to produce alternative results. In this way, it is possible and easy to test the sensitivity of the obtained classifications upon the choice of the metric. (3) Tests of the robustness of the results as a function of the nominal uncertainties of the data can be easily performed. (4) An overall quantitative parameterization of the overall robustness and statistical significance of the identified groupings can be "objectively" defined.

The following section recalls the most important identification techniques developed by different authors starting in 1990. Most of them have been found to produce results in very good agreement, producing an important database of reliable families to be investigated from the point of view of physical characterization by means of different techniques (described in other chapters in this book).

2.2. Hierarchical Clustering Method (HCM)

This method is based on the classical tree construction for hierarchical classification purposes. *Zappalà et al.* (1990) modifies, adapts, and optimizes this method for the identification of significant groupings in the three-dimensional proper-element space. The tree is obtained by agglomerating, at each step of the procedure, the two nearest objects of the considered sample into a single object and reiterating the process until only one object survives. At each step, the distance $d(i \cup j, k)$ between an agglomerated $i \cup j$ object and the generic object k is defined to be equal to the minimum between d(i, k) and d(j, k). The results are given in the form of stalactite diagrams like that shown in Fig. 1. By comparing the stalactites obtained from the real dataset and those obtained from quasirandom simulated populations, it is then possible to cut the hierarchy to derive clusters that are statistically significant. The only crucial choice that must be taken by the human operator is the final assessment of the critical distance level, below which all the existing groupings are assumed to be real families. As mentioned above, this level corresponds to the deepest level reached by a set of simulations of quasirandom populations of fictitious objects. In this context, the choice to be made is not the critical distance level itself, but the minimum number of objects required to exist at the deepest levels of the quasirandom populations. In the successive implementations of this technique (*Zappalà et al.,* 1990, 1995; *Zappalà and Cellino,* 1994), the choice of the critical distance levels has generally been made according to restrictive criteria, leading in some cases to likely underestimation of the family memberships (*Migliorini et al.,* 1995).

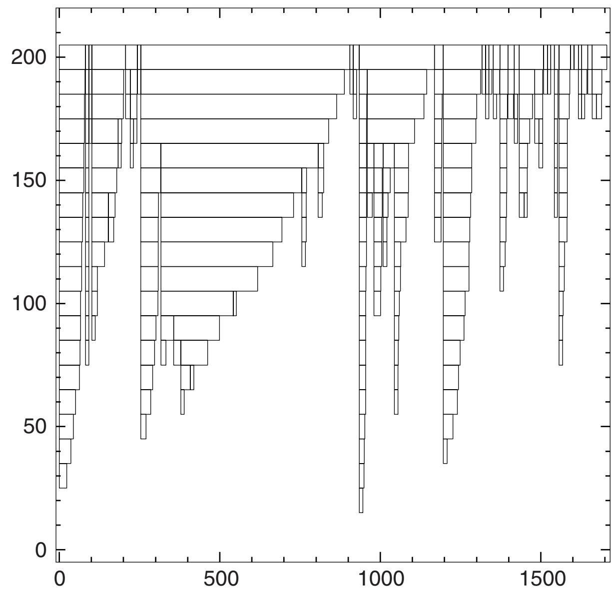

Fig. 1. Example of a typical stalactite diagram, referring to the asteroids belonging to the intermediate region of the main belt ("zone 4" according to *Zappalà et al.,* 1995). The different stalactite "branches" correspond to different identified families.

2.3. Wavelet Analysis Method (WAM)

This method is fully described *by Bendjoya et al.* (1991, 1993). It is a density-evaluation method based on the use of a particular function called a "wavelet" having a characteristic size defined according to the adopted metric. The use of the wavelet transform allows to detect local overdensities of points belonging to a given N-dimensional space (in this case the a', e', and i' space) at different scales. By superimposing a grid in the phase space, it is possible to compute for each node of the grid a wavelet coefficient. This coefficient is the sum of the contribution of each data point weighted by the zero mean wavelet function. The higher the value of this coefficient, the denser a grouping in the vicinity of the node. The closer to zero the coefficient, the more uniform the local distribution. By using the same technique on pseudorandom distributions, one can derive a quantified criterion to retain significant coefficients characterizing a given set of asteroid proper elements. These significant coefficients allow one to reconstruct the underlying clusters and to assign a level of detection significance against chance.

2.4. D Criterion

This method was first developed by *Lindblad and Southworth* (1971) and was later applied to increasingly larger datasets in its most recent applications by *Lindblad* (1992, 1994). It is based on the so-called D criterion, first developed to identify meteor streams (*Southwork and Hawkins,* 1963). The D criterion coupled with the neighbor-linking technique is basically another cluster-analysis method based on a different definition of the metric in the proper-element space. Also in this case, the analysis is carried out by means of a computer algorithm. The only human input is the choice of the cut-off distance used to reject spurious groupings.

3. PRESENT STATE OF THE ART

The HCM, WAM, and D-criterion techniques have been repeatedly applied in recent years to increasingly larger proper-element datasets.

The first applications of the methods were performed on a dataset of about 4000 proper elements (*Zappalà et al.,* 1990; *Bendjoya et al.,* 1991; *Linblad,* 1994). These data were taken from a version of the proper-element dataset computed by *Milani and Knežević* (1990). The results from the three techniques were found to be generally in excellent agreement, both in terms of number of identified families (about 15 having a high statistical reliability) and in terms of derived memberships.

The next step was a study devoted to test the performances of HCM and WAM in the case of a number of fictitious situations, in which artificial families were generated by using a suitable breakup model (*Bendjoya et al.,* 1993). The family members being known *a priori*, it was possible to test the effectiveness of HCM and WAM under different conditions including different compactness of the simulated families and different, more or less dense, background populations. The ability of the methods to separate overlapping and anisotropic families was also tested. The results confirmed the overall reliability of HCM and WAM, and their mutual consistency leading to an increased confidence in the results previously obtained in the case of real asteroids. The tests also suggested some possible improvements in the implementation of the two techniques, and this was taken into account in the next application of the methods to a new, larger dataset of asteroid proper elements, including about 6000 asteroids with a refined proper-element computation theory (*Milani and Knežević,* 1992). Again, the results were found to be in good agreement (*Zappalà and Cellino,* 1994; *Bendjoya,* 1993).

The most recent application of HCM and WAM was performed on a dataset of more than 12,000 proper elements (*Zappalà et al.,* 1995). Even if the difficulty of separating different groupings mutually overlapping was found to grow with the number of available data, this analysis was able to again give results in a very good mutual agreement. Most of the most-recent physical analyses of asteroid families have been based on the family lists published in the above paper.

Also in this latest implementation of the HCM and WAM techniques, robustness coefficients were derived for each grouping in order to assess the stability against modifications of several parameters, including choice of metric, noise in the proper elements, and statistical detection threshold. Table 2 gives a summary of the most-robust families found by both identification methods. These families can be considered the most important candidates for physical studies, and most of them have already been analyzed (see *Zappalà et al.,* 2002) and observed extensively, mainly by means of spectroscopic techniques (*Bus et al.,* 1996; *Cellino et al.,* 2002).

TABLE 2. List of the most reliable families found by at least one of the HCM and WAM methods (see text) in the *Zappalà et al.* (1995) classification.

Family Identification		Members	
HCM	WAM	HCM	WAM
8 Flora	43 Ariadne	604	575
44 Nysa	135 Hertha	381	374
4 Vesta	4 Vesta	231	242
163 Erigone	163 Erigone	45	49
1 Ceres	93 Minerva	89	88
170 Maria	170 Maria	77	83
668 Dora	668 Dora	77	79
145 Adeona	145 Adeona	63	67
808 Merxia	808 Merxia	26	29
569 Misa	569 Misa	25	27
410 Chloris	410 Chloris	21	27
1644 Rafita	1644 Rafita	21	23
1128 Astrid	1128 Astrid	10	11
24 Themis	24 Themis	550	517
221 Eos	221 Eos	477	482
158 Koronis	158 Koronis	325	299
137 Meliboea	137 Meliboea	13	16
845 Naema	845 Naema	6	7
20 Massalia	20 Massalia	49	45
15 Eunomia	15 Eunomia	439	303
110 Lydia	110 Lydia	26	50
128 Nemesis	58 Concordia	20	38
1639 Bower	342 Endymion	10	15
10 Hygiea	10 Hygiea	103	175
490 Veritas	92 Undina	22	36
293 Brasilia	293 Brasilia	10	18

The columns list the family identifications according to the two methods and the numbers of members found by each method. Only families having an intersection (numbers of objects in common) of 75% (upper block) or 50% (lower block) are listed. These families represent therefore the most prominent and reliable groupings currently identified in the main belt.

In addition to the families listed in Table 2, many less-robust groupings were also found and identified as "clumps," "tribes," or "marginal groupings" (see *Zappalà et al.,* 1995). Even in the case of these less-robust groupings, some good agreement could be found between the two techniques, but discrepancies also were found to become important. Of course, the real reliability of these groupings must wait for future, likely imminent, investigations. The interested reader will find the entire set of family members posted on the Web at http://www.obs-azur.fr/cerga_bdd.html.

The investigations mentioned above were based on datasets of proper elements in which high-inclination and high-eccentricity objects [having sin(i) and/or e > 0.3] were not included. The high-e, high-i region of the proper-element space has been investigated in detail by other working teams. In particular, the identification of a family associated with the asteroid Pallas was made by *Lemaître and Morbidelli* (1994) and later confirmed by *Bus* (1999, see below). Also, the possible existence of Hungaria (*Lemaître,* 1994) and Hansa (*Hergenrother et al.,* 1996) families has been proposed. The presence of these groupings has been proposed based on analyses of a set of proper elements computed by *Lemaître and Morbidelli* (1994).

Family searches have not been limited to the main-belt population, but have been performed also in the Trojan clouds by *Milani* (1993). The existence of families among Trojans raises interesting questions about a possible relation with the formation of short-period comets (*Marzari et al.,* 1995).

We should also mention an another important contribution in this field by *Migliorini et al.* (1995), who made a statistical analysis of published families in order to derive the likely number of random interlopers as a function of member sizes. The problem of interlopers is important since one of the ultimate goals of asteroid family studies is to reconstruct the original collisional events from which families were created. An interloper is an asteroid having nothing to do with the real family, but sharing by chance the same orbital proper elements. An estimate of the number of interlopers as a function of the diameter is fundamental in determining the correct barycenter and size distribution of the families. This investigation showed that in most cases the family identification criteria were not too liberal, leading to modest numbers of predictable interlopers. At the same time, it was also shown that in several cases (like Flora), the nominal membership numbers were likely too conservative, and the nominal member lists in these cases are expected to severely underestimate the real family membership.

4. FUTURE WORK

After the most recent application of the HCM and WAM techniques, the number of asteroids for which reliable proper elements have been computed has been steadily increasing. Currently (July 2001), the dataset of proper elements includes about 24,000 numbered asteroids and some thousands of unnumbered objects.

Therefore, it seems that it is time to propose a new family classification, to make it possible to confirm previous results and assess the real family status of several marginal groupings identified in previous searches. Some technical problem will certainly arise due to the large size of the available database. In particular, it is easy to expect that the mutual overlapping of different families will make it hard to separate the memberships of single objects in the overlapping regions. This problem has already been encountered by previous investigations, mainly in the inner belt, where the asteroid inventory is complete down to small sizes. A good example is given by the mutually overlapping Mildred and Polana families (formerly known as the "Nysa clan" following the nomenclature proposed by *Farinella et al.,* 1992), as shown by *Cellino et al.* (2001). This example suggests that in future family classifications, a major support to purely statistical data will come from observational data, mainly spectroscopy and spectrophotometry. In other words, we will be able to separate the memberships of families overlapping in the space of the orbital proper elements only when additional information on the reflectance properties of the objects is available. *Bus* (1999) has been able to show how a hybrid classification method, in which purely spectral information is added to proper-element data, can be developed. Such techniques will likely play an important role in the near future (see also *Cellino et al.,* 2002).

Family members have been found to account for an increasingly larger fraction of the considered asteroid samples in the analyses based on applications of HCM and WAM from 1990 to 1995. Apart from complications due to modifications of the identification algorithms (increasingly more conservative for increasing sizes of the proper-element databases), this generally means that the fraction of family members tends to increase with increasing faintness (decreasing size) of the objects. This is an important and still-controversial issue, covered in more details in *Zappalà et al.* (2002). It will be very interesting to check whether the above-mentioned trend is still apparent when trying to classify families using updated and larger databases. According to *Zappalà and Cellino* (1996), families' members might dominate the whole asteroid population at sizes <10 km. If this can be proved to be true, there are relevant implications for our overall understanding of the asteroid population.

Nearly 100 years after the first discoveries by Hirayama, asteroid families are still a very important branch of modern asteroid science. Being able to reliably identify them is the first necessary prerequisite for any further physical characterization study.

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