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A classification of 6479 asteroids into families by means of the wavelet clustering method

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Abstract. — After the first use of the wavelet transform method for asteroid family identification (Bendjoya et al. 1991), an improved three-dimensional wavelet cluster analysis has been developed and tested on artificial families in Bendjoya et al. (1992). The success of these tests has provided us a reliable method to detect statistically significant groupings in the proper element space. This method has now been applied to a large set of proper elements of real asteroids, computed by the refined algorithm of Milani & Knežević, version 5.7 (Milani & Knežević 1992). The family search is performed among 6479 asteroids: 4168 numbered asteroids plus 2311 multi-opposition objects. The robustness of the detected significant groupings has been tested against several parameters: two levels of statistical significance versus the occurrence of chance groupings; the use of two different metrics in the proper element space; and the presence of "noise" caused by small variations in time of the proper elements. The derived robustness parameters have led to classify the set of asteroids in groupings named dynamical families and tribes, which correspond to different ranges of the robustness parameters. Twenty six dynamical families and 10 tribes have been identified.

Key words: methods: statistical — minor planets

1. Introduction

Asteroid family searches have been carried out many times, and extensive references on this subject can be found in Valsecchi et al. 1989. All of these works used sets of so-called proper elements, namely the quasi-invariants of asteroid motions, as the starting data for the classification of asteroids into families. However, as pointed out by Valsecchi et al., large discrepancies between the published classifications have been present, concerning not only the number of families but also their composition. Zappalà & Cellino (1992) argued that in general these discrepancies were rather due to the clustering methods used for the family search, than to the different ways to compute the proper elements.

A renewal of interest in this subject has occurred when two different methods of cluster analysis, applied to the same proper element set (Zappalà et al. 1990, and Bendjoya et al. 1991), have yielded results in substantial agreement. Zappalà et al. identified asteroid families by means of the well-known hierarchical clustering method (HCM) adapted by them to the specific needs of the problem on the other hand, Bendjoya et al. used for the first time the mathematical formalism of the wavelet transform to carry out a new kind of cluster analysis in the proper element space, aimed at identifying statistically significant families.

While this wavelet analysis method (W.A.M) gave a good agreement in the resulting family classification when compared to HCM, Bendjoya (1992) pointed out that it was not accurate and powerful enough to detect weakly contrasted families. The reason for this limitation is clear. The wavelet analysis was performed on the 3 planar projections of the proper element space, and the detected overdensities of points in these planes were used to reconstruct the three-dimensional families by taking their intersections. In this way, the projection procedure led to smooth out the fine structures of the distribution in the 3-dimensional space, thus making their detection hard or even impossible. The need for a new method, suitable for direct application to the $3D$ proper element space, was obvious. Hence, a new version of W.A.M. has been developed in Bendjoya et al. (1993), in order to perform the search of asteroid families directly in the $3D$ data space. This new W.A.M has been tested by applying it on artificial asteroid families, generated by a numerical asteroid break-up algorithm consistent with the results of laboratory impact experiments (Farinella et al. 1991). Several tests were devised, in order to point out the reliability and the limitations of W.A.M. Knowing a priori the members of the simulated families, the ability of W.A.M to rediscover them has been tested against different backgrounds, different sizes and different shapes of the families in the proper element space. Its ability to separate families which were partially overlapping or lying close to each other has also been taken into consideration.

Moreover, since the dynamical and statistical definition of asteroid families obtained by means of both HCM and W.A.M, is based on the choice of a metric (having the dimension of a velocity), the successful outcome of our tests is a convincing proof that our chosen metric (the "standard" metric of Zappalà et al.) does not introduce strong distortions of the proper elements space, that may hinder the identification of real families. A similar argument can be made for the mathematical algorithm converting osculating into proper elements: we showed that families generated in the osculating element space were successfully recovered both in this space and in the proper element space. It is to be noticed that the families have been searched, in the osculating element space, just after the simulated impact without considering any post collisional evolution of the members. Moreover, the simulations allowed us to estimate in a number of cases the abundance of interlopers (i.e., the objects detected as belonging to a family but not genetically related to it), and these results will be used here to assess the likelihood of interlopers in the families which has been detected. Finally, a comparison between the W.A.M and the HCM procedures was carried out, showing that both methods, while still presenting some (different) problems, are suitable for obtaining a reliable family identification, with a high degree of significance versus the occurrence of chance groupings.

It is worthwhile to stress that automated objective methods (such as W.A.M and HCM) have great advantages with respect to the more subjective procedures adopted in the past, since they do not require any "a priori" knowledge on the set of data, nor entail a "leaning process" based on the detection of the few, well known families. Furthermore, automated methods of investigation provide the possibility of testing the robustness of the detected grouping against several parameters. It has been sometimes claimed that human eyes are also a performing "clustering tool", and indeed they have been used for asteroid family search (Williams 1979, 1992); in this respect, one can notice that the wavelet transform algorithm has been used with success to model the mechanism of human eye analysis, which is based on a multi-scale analysis of the contrasts (Mallat 1987). It is thus possible to compare W.A.M with an objective eye analysis; a cross comparison of the results of such analyses of random distributions can lead to quantify the level of significance of the detected overdensities of points.

The present family search has been carried out using a sample of 6479 main-belt asteroids, including 4168 numbered objects and 2311 unnumbered multi-opposition ones. This sample of data is extracted from the proper elements set derived by means of the latest theory of Milani & Knežević (1992), corresponding to the 5.7 version of their algorithm. This new version is based on a new secular perturbation theory including the effects of all the outer planets (and partly the inner planets as well); new resonance monitoring schemes have also been introduced, leading to more accurate and more reliable proper elements. The proper element set, provided by Milani & Knežević includes 4535 objects among the 4722 numbered asteroids (at December 10, 1991), excluding the Trojans, the Hildas and the Earth-approaching bodies. Moreover, among the 12691 unnumbered asteroids observed so far, this data set includes 2459 objects observed at least during two oppositions. It has been excluded the single-opposition orbits, for which the quality of the osculating elements is too poor to derive from them accurate proper elements.

The file of proper elements derived by Milani and Knežević associates a quality code to each asteroid, which gives an indication of the non-convergence of the iterative procedure in the proper element computation and can be translated into an estimate of the accuracy of the proper elements. A quality code with value 20 corresponds to an oscillation of 0.01 between the values of the proper eccentricity and/or inclination provided by the last two iterations. Therefore, in agreement with the indications of Milani and Knežević, the asteroids with a quality code greater than 20 have been discarded from our sample.

In addition, owing to the expansion of the Hamiltonian in series of the eccentricities and inclinations on which Milani and Knežević's theory is based, one must consider as reliable only the proper elements of asteroids with an eccentricity and a sine of the inclination less than about 0.3, even if the associated quality code is smaller than 20. Although we have kept in mind this limitation, we have nevertheless performed the family search keeping in our sample the bodies having proper eccentricities and sine of the proper inclinations up to 0.45. This has been done in order to check whether the theory used to derive proper elements might artificially create significant overdensities at such inclinations and eccentricities. Among the 185 objects with a sine of the proper inclination, or a proper eccentricity greater than 0.3, only one grouping of 7 asteroids has been detected. This grouping is in the Phocaea region. This grouping has been known for a long time (see e.g. Williams 1971, 1979) as a group of asteroids isolated in this region by the presence of several resonances. Indeed as pointed out by Williams (1971), this depopulated region of the main belt is surrounded by the secular resonances ν_{16}, ν_5 and ν_6 and by the mean motion resonance 7:2. Valsecchi et al. (1989) proposed then to consider some dynamical process "pushing" asteroids in specific zones of the phase space. Moreover the high inclinations of these objects, and the presence of so many resonances, make the proper elements computed by Milani and Knežević very doubtful even "rubbish" (Milani private com.). These are the reasons why we do not consider this 7 member grouping as relevant and we will not mention it in the further discussion. Except the Phocaea grouping no other grouping has been detected at such high inclinations, which seems to confirm that Milani and Knežević's theory do not generate significant artificial groupings.

We have applied different alternative criteria (a stricter one and more relaxed ones) for the definition of what a "family" is, and we have estimated the number of expected interlopers in all the families. The robustness of the detected significant groupings of asteroids has been tested in three different ways: (i) using two different metrics to define distances in the proper element space; (ii) choosing two different thresholds for the statistical significance of the groupings; (iii) taking into account the variable accuracy of the proper elements, through the generation of "noisy" data. These different tests have allowed us to classify the detected significant groupings in three categories: "dynamical families", "tribes" and "marginally significant clusters" (the latter have been discarded a posteriori. The whole analysis will be described in more details in Sect. 2.

The results will be presented in Sect. 3, where we will also comment on the good agreement between most of our dynamical families and those resulting from the last classification of Zappalà et al. (1992). Finally, we shall mention some possible origin mechanisms for some dynamical families.

The comparison with Williams' classification (1991), using a very different set of data (about 1800 numbered objects plus 900 PLS asteroids that are not considered here) does not seem relevant. A deeper comparison, case by case, will be made in a forthcoming paper between the Zappalà et al. (1992) classification, Williams' one and the actual one.

2. The analysis

We recall that the wavelet clustering method consists in computing a wavelet coefficient at each node of a network superimposed to the set of the studied data. Let $C(a_0,\ b_0)$ be the onto wavelet coefficient computer at the node located in b_0 and for a given scale a_0 :

$$C(a_0,\ b_0) = K(a_0) \sum_{j=1}^{N} \psi_{a_0,\ b_0}(x_j)$$

where N is the number of points in the 3 dimensional proper element space, $K(a_0)$ is a normalization factor, x_j is the location of the j^{th} point of the data set, and $\psi_{a_0,\ b_0}$ is the analyzing wavelet dilated by a_0 and translated at the location b_0. In what concern our analysis, the so-called mexican hat wavelet has been chosen again:

$$\psi_{a,b} = \left(n - \frac{\|x-b\|^2}{a^2}\right) \exp\left(-\frac{\|x-b\|^2}{2a^2}\right)$$

where n is the dimension of the data space and where $\|x-b\|$ represents the distance between x and b expressed by means of a metric suited to the considered problem. Because of the property of the analyzing wavelet, which is a zero mean function, $C(a_0,\ b_0)$ is an indicator of the presence of overdensities with a characteristic size about a_0, in the vicinity of the considered node b_0. For different values of the scale parameter a (namely: 110 m/s, 160 m/s, 230 m/s, 320 m/s, 450 m/s and 640 m/s), a wavelet coefficient map is computed, in which the high positive values of the coefficients point out the presence of overdensities above the background at the considered scale.

A subset of these coefficients is selected when considered as significant versus chance. The degree of significance is obtained by computing a wavelet coefficient threshold from a wavelet analysis performed on a pseudo random distribution. This pseudo random distribution is built in following independently the marginal distributions in proper semi-major axis, proper eccentricity and proper inclination of the data set under study. The subsets of significant wavelet coefficients connected on the grid form the "skeletons", from which its possible to reconstruct the significant underlying structure of points. The analysis is made for a set of scales leading to a local embedded multi-scale analysis. One disposes then, of sets of significant embedded grouping of points at different scales in the proper element space. For some methods of investigation, the simultaneous presence of big and small families in an area under study, can influence and even spoil the detection of the smaller ones. The fact that the wavelet clustering method is a local and multi-scale analysis method avoids this bias, since the members of the big families and of the small ones are not detected over the same range of scales.

For the next step of the analysis, one needs a criterion to cut the hierarchy of embedded significant structures of asteroids exhibited by the wavelet analysis. Following the philosophy described in Bendjoya et al. (1991), a maximal scale ($a_{\max}$) is defined for a given zone and corresponds to the scale for which the different structures merge together. Like in Bendjoya et al. 1991, $a_{\max}$ is defined as the scale for which the number of supplementary asteroids (between one scale and the successive one) for the whole zone, is minimum. The determination of $a_{\max}$ allows to separate the different skeletons and hence the different significant groupings. Then, taking profit of the fine tuning developed during the simulated family study, we have considered that an asteroid belongs to a "family", if the latter is associated to a skeleton for each scale from the smallest one to $a_{\max}$. The most restrictive criterion, called hereafter crit. 1, imposes that the detected asteroid must be found a least at the three first scales in order to be listed as a family member. Once again from the work about family simulations, it appeared that a more relaxed criterion could be used. This second criterion, crit. 2, takes into account the asteroids associated to the same skeletons than with crit. 1 but which have been detected only from the second scale (160 m/s). This appears to be a reasonable approximation, since when we perform a "zoom" at a scale of the order of 110 m/sec it happens that the W.A.M coefficients start to be significantly influenced by the local distribution of single objects. The real and the quasirandom distributions look locally very similar, and some overdensities at this scale may loose their statistical significance. In the present analysis we have to take into account the possible post-collisional evolution of the families leading to an expected more diluted halo around a dense core. Taking into account this possibility, we have also considered the asteroids detected with still more relaxed criteria: crit. 3 and crit. 4. These latter two criteria are defined in the same way as crit. 2 but the smallest scales of detection are respectively 230 m/s and 320 m/s. More relaxed criteria are not considered since they would correspond to scales bigger than $a_{\max}$. Obviously we keep in mind that, the more relaxed is the criterion used, the greater is the expected number of interlopers. That is why the different robustness tests that we have applied, and that we are going to describe, have been done on the families defined with the most restrictive criterion: crit. 1. The members associated to these families and added by the other criteria will be given and will be used in a forthcoming paper about a detailed comparison on families defined by different methods.

The family search has been performed by means of the two metrics already used in a previous paper (Bendjoya et al. 1991) and called the "standard metric" and the "alternative metric".

$$d = na'\sqrt{k_1\left(\frac{\delta a'}{a'}\right)^2 + k_2(\delta e')^2 + k_3(\delta \sin i')^2} \quad (1)$$

where δ lies for the difference between the asteroid coordinates and corresponding coordinates of the node of computation, n and a' are respectively the mean motion and the proper semi major axis of this node. For the standard metric $k_1 = 5/4$, $k_2 = 2$ and $k_3 = 2$. The alternative metric is obtained with $k_1 = 1/2$, $k_2 = 3/4$ and $k_3 = 4$ which appears as a sufficiently different but satisfactory metric in order to test the robustness of the family versus the choice of the metric. These two metrics have been introduced by Zappalà et al. (1990) and are now largely used.

The analysis has been performed with the standard metric with a threshold equal to 5/1000, then with a threshold equal to 1/1000. This means that the significant wavelet coefficients, and hence the associated groupings of asteroids, have less than 5/1000 (resp 1/1000) risks to be due to chance. This double analysis allows to test the robustness of the detected grouping versus the level of confidence associated to its detection. Then, the influence of the metric has been tested by performing a new analysis with the "alternative metric" and a statistical level of confidence of 5/1000. Finally the robustness of the significant groupings has been tested versus the accuracy of the proper elements of version 5.7. In this aim we have analyzed a new set of data derived from the original one, in taking into account the instability with respect to time of these proper elements. A deviation predicted by the model (Milani private communication) has been added randomly and independently to each proper element. The rms of these deviations are given by:

$$\begin{aligned}\sigma(e)^2 &= \frac{(0.021)^2}{(g-g_6)^4} + \frac{(0.0147)^2}{(g+g_5-2g_6)^2} + \frac{(1.8\ 10^{-5})^2}{(a-3.28)^4} \\ &+ \quad (0.5)^2 \sin^8(I) + (0.0005)^2 \\ \sigma(\sin(I))^2 &= \frac{(0.038)^2}{(g-g_6)^4} + (0.25)^2 \sin^8(I) - 1 + (0.0005)^2 \\ \underset{(in\ AU)}{\sigma(a)^2} &= \frac{(1\ 10^{-5})^2}{(a-3.28)^4} + (0.0005)^2\end{aligned}$$

This semi empirical formula takes into account: (i) the non linear effects of the $g - g_6$ secular resonance, (ii) the non linear forcing term $g + g_5 - 2g_6$, (iii) the truncation of powers of the inclination larger than 4 in the perturbing function (iv) the effects of the nearby mean motion resonances (mainly the 2:1 resonance), for more details see Milani & Knežević 1992. As pointed out by Milani and Knežević this estimation of the "noise" on the proper elements cannot be proven to held over time spans much longer than that of the used numerical integrations. Nevertheless the analysis of such a "noisy" data set, allows to estimate the influence of the latter effects on the robustness of a significant grouping.

The robustness of each significant grouping detected by means of the standard metric with a level of significance of 5/1000 and with crit. 1 are tested against (i) the noise of the proper elements (ii) the higher statistical level of confidence 1/1000 and (iii) the alternative metric. For each kind of comparison a robustness parameter R_x is computed. Let N_1 be the number of members of the significant grouping identified by the preliminary analysis, N_1' the number of members of the same grouping detected by a secondary analysis and N_c the number of asteroids in common:

$$R_x = 1 - \frac{1}{2}\left(\frac{N_1 - N_c}{N_1} + \frac{N_1' - N_c}{N_1'}\right)$$

Where x stands for s1, n or m2 (see later). A robustness parameter such defined gives more information than a simple percentage of common objects. Indeed, as an example, let us consider two cases leading to the same percentage of common objects: the first case being obtained with ($N_1 = 100$, $N_c = 80$, $N_1' = 100$) and the second one with ($N_1 = 100$, $N_c = 80$, $N_1' = 150$). Although we find 80% of common objects in both cases, the first robustness parameter will be 0.9 and the second one will be 0.666 which characterizes better the robustness of the considered grouping.

Instead of splitting the main belt in 7 zones as it has been done in the previous paper (Bendjoya et al. 1991), this time, only three regions are considered. They are defined from the mean motion resonances with Jupiter: a first one between the 4:1 and 3:1 mean motion resonances ($2.065 < a' < 2.501$), a second one between 3:1 and 5:2 Kirkwood gaps ($2.501 < a' < 2.825$), and the last one, between 5:2 and 2:1 gaps ($2.825 < a' < 3.278$). The merging of the seven previous zones in only three new ones, is made in order to avoid the "ghost family" effect (Milani et al. 1991) which occurs when the members of a very populated family represent a non negligible part of the studied data, leading to the non detection of the less populated families. On the other hand, the aim is to avoid the possibility to "loose" some members of families crossed by a mean motion resonance (like Eos). Figure 1a. and 1b. shows the whole set of analyzed data, projected in the (a', e') and (a', $\sin i'$) planes respectively with the limits of the three zones. The inner zone contains 2543 asteroids, the medium one 1789 and the outer 2147 asteroids.

Finally a rough estimation of the number of interlopers has been performed. An interloper has been defined as being an asteroid associated to a significant grouping by the analysis, but which is not a physically created fragment. One more time we used the results of the simulated family analysis, to estimate the number of interlopers for each groupings detected with the standard metric, applying crit. 1 and with a degree of significance of 5/1000. We have assumed that the number of interlopers depends on the volume of the considered grouping in the proper element space (V_g), and on the number of objects not belonging to the grouping, and considered as the local background (N_{bg}), in a fixed portion of volume (V_{bg}) centered on the barycentre of the grouping. The volume of the grouping V_g is estimated as follows: being given a significant grouping, the coordinates of the extreme members are determined leading to the knowledge of the expanse of this grouping, $\Delta a'$, $\Delta e'$, and $\Delta \sin i'$. V_g is the volume of the ellipsoid of semi axes $\Delta a'/2$, $\Delta e'/2$ and $\Delta \sin i'/2$. The volume V_{bg} in which we consider the local background is a cube of edge 0.2. Applying this procedure to the different cases of simulated families of Bendjoya et al. 1992, the percentage interlopers has been plotted as a function of $V_g \frac{N_{bg}}{V_{bg}}$. The line fitting these experimental values is shown in Fig. 2. This fit is done with a factor of correlation of value 0.947. The number of interlopers for each significant grouping of the main belt is then estimated from the fitted line of slope $\simeq 1.810^{-2}$. Quite obviously we do not extrapolate this "law", and estimate the rough number of interlopers expected when using the W.A.M, only for the values of $V_g \frac{N_{bg}}{V_{bg}}$ within the range of the ones computed for the simulations.

The results of the whole analysis are summarized in Tables 1a, 2a and 3a. The reader can find in these tables: (i) the name of the significant grouping with the standard metric at the level 5/1000 with crit. 1 (this name is the one of the least numbered asteroid of the grouping); (ii) the parameter of robustness versus noise Rn; (iii) the parameter of robustness versus the statistical level of significance Rs1; (iv) the same versus the metric Rm2; (v) the averaged robustness parameter $R (R = (R\text{s1} + R\text{m2} + R\text{n})/3)$; (vi) the number of asteroid that can be added to the grouping if crit.2 is applied; (vii) the same with crit. 3; (viii) the same with crit. 4; (ix) the estimated number of interlopers: Ninter. The averaged robustness parameters give global information on the reliability of the detected significant groupings. We have chosen to classify the detected groupings in three classes based on the robustness parameters. The "dynamical families" are the groupings which are still detected at the higher level of significance versus chance, in general these grouping are also robust versus the metric and the noise. The groupings that are not detected at the higher statistical level ($R_{s1} = 0$), but which are robust enough versus the two other tests ($R_n \neq 0$ and $R_{m2} \neq 0$) are called the "tribes". Finally the groupings which resist to no or only one test are called the "marginally significant clusters" and are discarded a posteriori. Indeed, they typically correspond to what an eye-analysis can define as a significant grouping without having the possibility to change any parameters of this analysis. We see here, the great advantage of an automated analysis, which allows to make cross comparisons in order to weight the quality of the detected groupings. This nomenclature is a little bit different than the one proposed by Farinella et al. (1991), but still compatible. The main discrepancy, between these two nomenclatures, comes from the fact that the one proposed here, results from several tests of robustness, while Farinella et al.'s one, lying on the HCM classification, is inspired by the sensitivity of the composition of the groupings to the determination of the level of selection. According to the variation of the amount of members as a function of the level chosen to define the groupings, they proposed to classify the groupings in "clans", "clusters" and "clumps". Nevertheless, the same kind of information can be obtained here, by looking at the number of members added successively by the more and more relaxed criteria. Of course the term of "asteroid family" is reserved, whatever the method of investigation used, to the groupings which present some physical evidences of a collisional origin, such as: cosmochemical compatibility of the members and/or mass or velocity distributions in good accordance with experiments of fragmentations.

Tables 1 (b to l), 2 (b to q) and 3 (b to l) give the composition of the dynamical families and of the tribes. The members added by the different more relaxed criteria are also given. The name of the least numbered asteroid is given with its coordinates (a', e', sin i') and the number of members. The multi-opposition asteroids have a number greater than 7000 and the reader can find a correspondence table between these numbers and their present nomenclature in appendix.

Finally Table 4 gives for each dynamical family and tribe the number of multi-opposition members and the corresponding families previously identified (when available) by Bendjoya et al. (1991).

3. The results

Twenty six dynamical families have been identified plus 10 tribes. This represents a total of 1950 asteroids (with crit. 1), that is to say $\simeq$ 30% of the whole set of data, 1754 asteroids belong to dynamical families and 196 to tribes. Over these 1950 asteroids, 648 are multi-opposition objects. The six dynamical families: Hertha, Flora, Eunomia, Eos, Themis, Koronis contain $\simeq$ 61% of the total number of "family" population. One can remark that even for the dynamical families the global robustness coefficients for the inner zone are in general lower than for the two other regions. This is essentially due to the metric robustness parameter which is lower. Here appears the sensitivity of the W.A.M to the choice of the metric. Indeed the sphere of activity of the analyzing wavelet is modified by the change of the metric, and hence the neighbourhood of the grouping can influence differently its detection according to the choice of the metric. This effect is less present in the HCM method used by Zappalà et al. since individual distances between asteroids are considered and not global distance as in the W.A.M.

3.1. *The inner zone*

This zone contains 7 dynamical families and 3 tribes. The maximum scale retained to separate the groupings, $a_{\max}$ is 320 m/s. Figures 3 show the dynamical families and the tribes identified in this zone with the most restrictive criterion, crit. 1. respectively in the (a', e') plane (Fig. 3a) and in the (a', sin i') plane (Fig. 3b), and Fig. 3c and Fig. 3d display the whole set members obtained by all the criteria, in the same planes respectively.

The results about the different tests of robustness are found in Table 1a and the memberships of the dynamical families are found in the Tables 1b to 1k.

Hertha dynamical family, with 145 members (82 multi-opposition objects) groups together the 32: Amalasuntha, 34: Polona and 35: 1969UN families found in the previous analysis of Bendjoya et al. (1991) (to which we will refer hereafter by "B.p.a" (Bendjoya et al. previous analysis). One can notice that already in B.p.a it had been mentioned that 34: Polona and 35: 1969UN families could merge in a larger grouping including the asteroids Nysa and Hertha. One more time the 44 Nysa asteroid can be added to the Hertha dynamical family when crit.2 is applied. Although this paper is not dedicated to a systematic study of each proposed family but rather to their presentation, we can point out here the need to make deeper physical or chemical investigations before concluding to the break-up origin of a family. Indeed the mixing of different taxonomic types in this dynamical family was already pointed out by Bell (1989). Quite obviously the kind of analysis performed in this work is unable to distinguish such discrepancies. This dynamical family is defined at 320 m/s and seems to be quite homogeneous from a dynamical point of view since only one skeleton is exhibited at each scale. Nevertheless its dependency on the applied criterion is strong and the number of expected interlopers is not negligible.

We can remark that two members of the family 33: Leonce found in B.p.a can be associated to Hertha if crit. 4 is used. It is to be noticed that this family had already given a rather low noise robustness parameter. This leads us to believe that the improvements of the proper elements brought by the version 5.7 of Milani and Knežević have corrected sufficiently the coordinates of the members of 33: Leonce to induce the disappearance of this family.

The Flora dynamical family is the biggest one found by the present analysis, with 368 members, 102 of which are multi-opposition objects. Flora groups together the 21: Lucretia, 22: Berolina, 23: Auravictrix, and 24: Iduberga families identified in B.p.a. The asteroids 281 Lucretia and 700 Auravictrix are detected with crit. 1, 422 Berolina with crit. 2 and 963 Iduberga with crit. 4. One more time, it had already be mentioned in B.p.a, the possibility to define a much more spread family than 22: Berolina which would have been composed by 403 members. The present $3D$ wavelet analysis, allows to point out that at the smallest studied scale (110 m/s) it exists 9 different skeletons of significant wavelet coefficients (see Bendjoya et al. 1992), which merge in a unique one for the larger scales. This is the track of the presence of several denser groupings within Flora which is in good accordance with the now commonly suggested multi-collisional-event origin of Flora. The presence, in this dynamical family, of the recently visited 951 Gaspra by the Galileo flyby, is in good accordance with the remarks of Williams (1992). He proposed to consider Gaspra to be a piece of the asteroid 8 Flora. His conclusions lie on the similar S type of 951 Gaspra and 8 Flora (Zellner et al. 1985), and also to the proximity of these two asteroids. This proximity appears also in the present analysis since these two asteroids belong to the same denser sub-grouping (formed by 39 members) of the identified Flora dynamical family. The number of interlopers cannot be extrapolated for Flora, because of its estimated volume which is of about three times the biggest volume of the simulated families. The high density of the background around Flora can be the explanation of the big amount of members added by the relaxed criteria.

The Vesta dynamical family is composed by 79 members with 43 multi-opposition objects. This dynamical family, also defined at 320 m/s, has been particularly pointed out by Zappalà et al. (1992) because of the abundance of the "exotic" V-type members (basaltic). The present composition of Vesta mixes objects such as 63 Ausonia with 4 Vesta which seems incompatible with the cratering event origin proposed for the Vesta family. One more time the need of deeper investigations is pointed out on this example. This dynamical family groups together the 31a: Vesta and 31b: Tinchen of B.p.a. Vesta is the second case, and the last one, for which the number of interlopers cannot be estimated. This time, the great number of asteroids in the background is responsible of this impossibility of extrapolating the curve giving the percentage of interlopers as function of $V_{\rm g}\frac{N_{\rm bg}}{V_{\rm bg}}$. Vesta presents a weaker sensitivity to the criteria than the two previous dynamical families, nevertheless 33 members can be added by the more relaxed criteria.

Erigone is a small dynamical family with 12 members. This dynamical family has not been identified by B.p.a. The low value of the metric robustness parameter is due to the fact that the analysis made with the second metric allows the merging of this dynamical family with Eurynome.

Within the Eurynome dynamical family, also defined at 320 m/s, three different skeletons are pointed out by the analysis, at the smallest studied scales, which merge in a unique one at the other scales. This seems to indicate that, this rather spread family can be due to a multi-collisional event or successive break-ups. The percentage of expected interlopers is of order of $\simeq 28\%$.

Metis makes a good "score" for the tests versus the statistical degree of significance and versus the noise, but is not found again with the second metric. This dynamical family, found at 230 m/s, is dense enough in comparison with the local background which allows to detect it at a higher level of confidence versus chance. Moreover Metis is not spoiled by the possible noise of the proper elements, but when the analysis is performed with the second metric its members cannot be rediscovered by applying crit. 1. This is a typical example of the sensitivity of the W.A.M to the metric as already mentioned. This sensitivity is also certainly due to the small number of members of this dynamical family located in a dense background.

Euterpe, also defined at 230 m/s, is composed by two skeletons at the scale 160 m/s containing a group of 4 close asteroids: 4579, 7810, 8025 and 9328. These 4 asteroids are rediscovered at the higher statistical level while the others are not. This dynamical family should rather be considered as a tribe since only the sub-grouping of the four above mentioned asteroids are statistical level "resistant".

Ella is a little tribe of 15 members which resists pretty well to the tests against the noise and against the metric, but is not sufficiently contrasted to be detected at the higher level of significance.

Lameia and Victoria have respectively 19 and 34 members and are both not sufficiently contrasted to be detected at the higher level significance which has made their classification in tribes. These latter 3 tribes are defined at 230 m/s and are insensitive to the different criteria.

Finally a good agreement is found between: Hertha, Flora and Vesta, and Erigone and the corresponding families found by Zappalà et al. on the same set of data with an improved hierarchical clustering method (private com.) A forthcoming paper, as already mentioned, will propose a detailed comparison.

3.2. *The medium zone*

This zone provides a large number of very robust dynamical families, since we find 14 dynamical families, and only 2 tribes. All the families found in B.p.a. in zone 4 are rediscovered. The dynamical families of this zone are also characterized by the fact that except for Eunomia a very few number of interlopers is expected for each one. The largest scale considered in this zone is $a_{\max} = 230$ m/s, hence all the dynamical families and tribes are defined at this scale, and only crit. 2 and crit. 3 can be considered. Except Eunomia, once again, and Valda, the groupings in this zone are less sensitive to the different applied criteria than the ones of the inner zone.

Figures 4 show the dynamical families and the tribes identified in this zone respectively, by means of crit. 1, in the (a', e') plane (Fig. 4a) and in the $(a', \sin i')$ plane (Fig. 4b) while Fig. 4c and Fig. 4d display the whole set members obtained by all the criteria, in the same planes respectively. The results about the different test of robustness are found in Table 2a, the memberships of the dynamical families are found in the Tables 2b to 2q.

Antonia is formed by two "adjacent" subgroups: one of 14 members (272 Antonia) and another of 7 members (363 Padua). This later is entirely rediscovered at the higher statistical level. Antonia has been identified in B.p.a as 43: Lydia.

Maria and Dora are two well contrasted dynamical families with almost the same global robustness parameter. They have been identified in B.p.a as 44: Maria and 45: Dora respectively.

Adeona identified as 41b: Adeona in B.p.a can still be seen as a "satellite" of Eunomia, just like Nina since they are 3 very near dynamical families that merge together at scale 320 m/s.

Bower is a tiny dynamical family of 5 members which are all rediscovered at the higher statistical level. This shows that the present automated procedure is able to detect very small but robust families, which have not to be discarded a priori. Such a small dynamical family can be enlarged to 10 members if we consider the more permissive criterion crit. 3., and would be a good candidate for physical parameter study in order to check if such a small and robust grouping is detected because of its collisional origin or if it is due to chance.

Geffion, identified as 42: Leto in B.p.a, is a 37 member dynamical family. Even at the smallest scale only one skeleton is exhibited, which implies a homogeneousness from a dynamical point of view.

Eunomia is the biggest family of the zone with its 156 members (71 multi-opposition objects). As previously mentioned, a not negligible amount of members can be added by applying the two different more relaxed criteria, and the number of expected interlopers represents $\simeq 25\%$ of the members defined by means of crit. 1. The present Eunomia is the 41a: Eunomia family identified in B.p.a. At the contrary of the Flora dynamical family the wavelet analysis does not point out for Eunomia the presence of denser subclusters, since even for the smallest studied scale only one skeleton is associated to this latter. This is the track of a relative homogeneity within Eunomia from a dynamical point of view. If we bring together this homogeneity of Eunomia with its heterogeneousness of taxonomic types, we can be lead to suggest that Eunomia could be due to a succession of catastrophic collisions occurred at different times. About this heterogeneousness it is to be noticed that in B.p.a the presence of 85 Io and 141 Lumen, both of C-type was pointed out as possible interlopers. These two asteroids do not belong anymore to the dynamical family Eunomia, and only 85 Io is one of the possible added members by crit. 2.

Chloris is a dynamical family of 11 members that has not been identified by Williams but is discovered by Zappalà et al. (1992) with the name Idelsonia.

The 17 members of Lilaea (5 multi-opposition objects) have not been found in B.p.a. This dynamical family has been identified by Zappalà et al. in their last identification (1992) with the name Merxia.

All the members of Misa defined by Zappalà et al. 1992 are rediscovered here. As for Dora and Adeona, this dynamical family is entirely rediscovered at the higher statistical level.

Agnia is composed by two sub-clusters associated to two different skeletons at 160 m/s: one with 15 members (2514 Taiyuan) and the other one with 20 members (847 Agnia). These two sub-clusters are contrasted enough to be rediscovered at a statistical level of 1/1000. These two sub-groups are associated to the same skeleton at 230 m/s. Moreover at this later scale, the tribe Concordia is also associated to the same skeleton. Concordia is also composed by two sub-clusters, also distinguishable at 160 m/s, : a sub-cluster of 9 members (58 Concordia) and the other one with 7 members (128 Nemesis). This time these two sub-clusters do not "survive" at the higher statistical level. It seems that they are four groupings, close enough to be detected at the scale 230 m/s by the same skeleton, but two of these four groupings are dense enough to be classified as a dynamical family composed by two "tangent" smaller dynamical families. The two other sub-groupings form a tribe of two "tangent" smaller tribes. As it is impossible to distinguish the membership of Agnia or of Concordia for the asteroids added by crit. 3, we only consider those added by crit. 2.

The 47 members of Valda are found in seven skeletons the scale 160 m/s merging in a unique one at the following scale of 230 m/s. Either a dense local background allows to make some bridges between not genetically related small groupings, or this can be the track of multi-collisional history. Nevertheless the number of members added by the more relaxed criteria and the non negligible number of expected interlopers seems to indicate the first hypothesis as the right one. Then we can notice that the two more robust groupings are on one hand a 21 member grouping (262 Valda) and on the other hand a 11 member one (1644 Rafita).

Finally Pandora is a 21 members tribe with only 2 multi-opposition objects. This rather spread tribe is very sensitive to the "noise" in the proper element computation, just as Mellena dynamical family.

Dora, Maria, Antonia, Adeona, Lilaea, Geffion, Eunomia, Chloris, Misa, Agnia and Valda are also found by Zappalà et al. in their classification of 1992 and their memberships are in good accordance with the present identification.

3.3. *The outer zone*

Five dynamical families have been identified in this zone with a high global robustness parameter, and 5 tribes must be added to the classification made in this zone. This zone contains the three big and well known Hirayama's families: Eos, Koronis and Themis. As in the inner, zone $a_{\max}$ is equal to 320 m/s. The number of expected interlopers is low for all the groupings of this zone, while the number of added members by means of the different criteria is large. Figures 5 show the dynamical families and the tribes identified in this zone, by means of crit. 1 respectively in the (a', e') plane (Fig. 5a) and in the $(a', \sin i')$ plane (Fig. 5b), Fig. 5c and Fig. 5d show the whole set members obtained by all the criteria, in the same planes respectively. The results about the different tests of robustness are found in Table 3a, the memberships of the dynamical families are found in the Tables 3b to 3k.

Veritas, already found as 72: Veritas in B.p.a is the most robust family of all since its global robustness parameter is 0.987.

Eos is a dynamical family of 219 members (only 28 multi-opposition). The 61: Eos family of B.p.a was composed by 172 numbered objects of the present Eos. This dynamical family is particular in the sense that it is crossed by the $g + s - g_6 - s_6$ resonance and also by the $g + s - g_5 + s_7$ resonance (Milani & Knežević 1992). This particularity will be considered in the following section with the cases of Eunomia, Agnia and Padua.

Koronis is a compact big dynamical family with 161 members (37 multi-opposition) which 24 asteroids can be added to, with crit. 2. This dynamical family was 51: Koronis in B.p.a. The other candidate of the Galileo flyby is 243 Ida which, as pointed out by Williams (1991) is a S-type asteroid as other members of Koronis.

Themis is the dynamical family the most sensitive to the choice of criteria since 99, 54 and 58 asteroids can respectively be added, by means of crit. 2, crit. 3 and crit. 4, to its 177 members defined with crit. 1. This particular sensitivity has been already pointed out in B.p.a. The discrepancies between the B.p.A determination and the one of Zappalà et al. (1990), has been explained by the probable "core-halo" structure of Themis. The previous wavelet analysis was unable to extract from the background the halo around the core which was the only part recognizable. The present 3D analysis is able to point out such a peculiar structure through the big amount of asteroids added by the different criteria and through the fact that the number of expected interlopers is low.

The dynamical family Hygiea is "tangent" to the tribe Isergina since these two groupings associated to two different skeletons at 160 m/s are detected by a unique skeleton at the successive scales. Their different behaviors against the statistical level robustness test allow to classify them into two the different classes: dynamical family and tribe respectively. Since the distinction between these two groupings is impossible from the scale 230 m/s we only consider the members added to both of them, by means of crit. 2.

Aralia and Brasilia are two tribes of respectively 13 and 23 members.

Meliboea was not discovered in B.p.a and the authors had proposed that this family, found at this time by Zappalà et al. (1990), was missed because of the projection effects that smoothed the contrast of this tribe. As it has been suggested in B.p.a, the 3D method is able to discover this 19 member family. Its zero statistical level robustness parameter indicates that this tribe is effectively weakly contrasted as expected.

Finally Benda is a tribe of 10 members with 4 multi-opposition objects.

734 Benda is a 78.6 km object and 861 Aida is a 70 km one.

One more time a good agreement with the last classification of Zappalà et al. (1992) is found for Veritas, Eos, Koronis Themis, Hygiea and Meliboea. We can already mention that, for Eos, Koronis and Themis the agreement is better if we consider the classification made by means of crit. 2.

3.4. *The resonant cases*

Eunomia is crossed by the $g_6 - g_5 + s - s_6$ resonance, Padua and Agnia, by the $g - g_6 + s - s_6$ resonance and Eos by the $g + s - g_6 - s_6$ resonance and also by the $g + s - g_5 + s_7$. For the close-to-resonance asteroids, the way that the proper elements have been computed is not correct. Therefore the quality code of the proper elements of such asteroids can be increased artificially, so much that they may have not been take into account in the set of data. In order to detected the possible "orphans" of these families, we have performed a new analysis, including the asteroids for which the quality code of the proper elements is greater than 20, and applying crit. 1.

Padua and Agnia are unchanged. For Eunomia 9 different members: 2822, 3041, 3080, 3569, 7000, 7917, 8401, 8587, and 9111 are within Eunomia.

For Eos the following asteroids are added to this dynamical family.

320 766 1075 1174 1552 1723 1753 2115 2453 2909
3140 3310 3328 3736 3876 4059 4115 4170 4600 7064
8441 8460 9085 9228 9623

4. Conclusion

The 3D wavelet clustering method has made its proofs of reliability on several "artificial" families which allows to be confident in the results proposed by the present work. The twenty six dynamical families and the 10 tribes presented here, have passed several tests against the level of significance versus chance, against the "noise" in the proper elements, and against two sufficiently different metrics.

A good agreement is obtained between most of the dynamical families and the classification of Zappalà et al. (1992), and most of the tribes have in common several elements with the classification of Williams (1979).

Moreover the present method allows to suggest or to predict different scenario of the formation of the proposed "families" (total disruption, multi-breaking events, structures in "core-halo").

The limitation of such an analysis has been pointed out, and the reader must keep in mind that no conclusion about the certainty of a break-up event can be made without deeper physical and chemical investigations such as albedos, colors spectra and taxonomic classifications.

Nevertheless the dynamical families and the tribes defined in this paper can be considered as a significant set of candidates for such investigations.

These different physical data will be analyzed (when available) in a forthcoming paper for each of the families presently proposed and a systematic comparison family by family will be done with the classifications of other family searchers.

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References

Bell J. 1989, Icarus 78, 426

Bendjoya Ph., Slezak E., Froeschlé Cl. 1991, A&A 251, 312

Bendjoya Ph., Cellino A., Froeschlé Cl., Zappalà V. 1993, A&A 272, 651-670

Bendjoya Ph. 1992, Celest. Mech., in press

Farinella P., Davis D.R., Cellino A., Zappalà V. 1991, "From cluster s to families: a new nomenclature" Proc. Asteroid Comets and Meteors 91, in press

Mallat S. 1987, "A compact multiresolution representation: The Wavelet Model. Proc. of the IEEE Computer Society Worshop on Computer Vision, IEEE Comput. Soc. Press, Washington, pp. 2-7

Milani A. and Knežević Z. 1992, Icarus, in press

Milani A., Farinella P., Knežević Z. 1992, "On the search for asteroid families" Comptes rendus de l'Ecole de Goutelas (Froeschlé Cl. and Benest D., eds.) ed. frontières

Valsecchi G.B., Carusi A., Knežević Z., Kresak L., Williams J.G. 1989, "Identification of asteroid dynamical families" in Asteroids II (Binzel R.P., Gehrels T. and Matthews M.S., eds) Univ. of Arizona Press, Tucson, pp. 368-385

Williams J.G. 1971, "Proper elements, families, and belt boundaries" in Physical Studies of Minor Planets (T. Gehrels, ed.) pp. 177-181 NASA SP-267, U.S. Government Printing Office, Washington D.C.

Williams J.G. 1979, "Proper elements and family memberships of the asteroids" in Asteroids (Gehrel T., ed.) Univ. of Arizona Press, Tucson, pp. 1040-1063

Williams J.G. 1991, "Gaspra and Ida in families" submitted to Asteroids, Comets and Meteors 1991 (Harris A. and Bowell E., eds)

Williams J.G. 1992, Icarus 96, pp. 251-280

Zappalà V., Cellino A., Farinella P., Knežević Z. 1990, AJ. 100, 6, 2030

Zappalà V., Cellino A., Farinella P., Milani A. 1992, submitted to AJ

Zellner B., Tholen D.J., Tedesco E.F. 1985, Icarus 61, 355

Table 1a. Results for the inner zone: (i) the name of the significant grouping with the standard metric at the level 5/1000 with crit. 1 (this name is the one of the least numbered asteroid of the grouping); (ii) the parameter of robustness versus noise R_n; (iii) the parameter of robustness versus the statistical level of significance R_{s1}; (iv) the same versus the metric R_{m2}; (v) the averaged robustness parameter $R(R = (R_{s1} + R_{m2} + R_n)/3)$; (vi) the number of asteroid that can be added to the grouping if crit. 2 is applied; (vii) the same with crit. 3; (viii) the same with crit. 4; (ix) the estimated number of interpolers: Ninter

Name	Nmemb	Rn	Rs1	Rm2	R	N2	N3	N4	Ninter
135 **Hertha**	145	0.923	0.956	0.850	0.909	41	39	25	40
8 **Flora**	368	0.861	0.933	0.816	0.870	52	55	27	*
4 **Vesta**	79	0.888	0.876	0.833	0.866	17	8	8	*
163 **Erigone**	12	0.917	0.958	0.583	0.819	1	2	1	0
79 **Eurynome**	43	0.842	0.771	0.637	0.750	3	0	0	12
9 **Metis**	11	0.958	0.955	0.	0.638	0	0	0	0
27 **Euterpe**	17	0.947	0.318	0.535	0.600	1	1	0	1
435 *Ella*	15	0.969	0.	0.816	0.595	0	0	0	1
248 *Lameia*	19	1.	0.	0.769	0.590	0	0	0	1
12 *Victoria*	34	0.662	0.	0.532	0.398	1	0	0	9

Nmemb: the number of members
Rn: the noise robustness parameter
Rs1: the statistical level robustness parameter
Rm2: the metric robustness parameter
R: the averaged robustness parameter
N2: number of members added by means of crit. 2
N3: the same by means of crit. 3
N4 the same by means of crit. 4
Ninter: the number of expected interlopers

* see the text.

Table 1b to 1k. Lists of members of the dynamical families and tribes of the inner zone

Table 1b.

135 Hertha (*2.42845 .1746 .0464*) 145 members
135 650 878 1493 1650 1768 1896 1932 2026 2081 2139 2210 2278 2446 2462 2509 2561 2607 2664 2680 2710 2751 2775 2809 2818 2990 2998 3005 3114 3185 3384 3408 3463 3485 3486 3530 3583 3612 3857 3881 3891 3963 3997 3999 4026 4042 4173 4227 4251 4273 4425 4429 4458 4465 4476 4480 4515 4549 4589 4637 4646 4659 4697 7006 7065 7068 7079 7089 7104 7200 7231 7241 7248 7281 7304 7340 7387 7407 7415 7437 7516 7519 7578 7669 7713 7723 7725 7747 7748 7838 7863 7882 7891 7892 7911 7923 8030 8207 8242 8260 8274 8327 8335 8371 8428 8449 8459 8506 8558 8647 8651 8683 8702 8753 8782 8831 8836 8849 8863 8871 8885 9046 9048 9081 9151 9229 9264 9274 9275 9323 9356 9370 9373 9403 9437 9455 9498 9508 9549 9596 9634 9642 9649 9656 9676

members added by crit. 2
44 750 1511 1740 2313 2391 2527 2565 2662 2923 3000 3006 3048 3064 3172 3228 3566 3716 3818 3839 3983 7013 7107 7145 7411 7684 7729 7789 7848 7865 7902 8296 8527 8880 8883 8892 9002 9317 9338 9423 9594

members added by crit. 3
142 2170 2441 2728 2966 2973 2984 2994 3069 3091 3130 3293 3458 3467 3529 3541 3984 4027 4090 4494 7640 7682 7808 7823 8148 8208 8941 8971 9165 9213 9298 9333 9378 9379 9547 9559 9579 9593 9637

members added by crit. 4
877 1358 1378 1586 1924 1964 1972 2007 2066 2276 2279 3192 3227 4214 4265 4282 7086 7318 7477 7633 7867 8309 9329 9408 9644

Table 1b. continued

8 Flora (*2.20144 .1443 .0939*) 368 members
8 43 281 291 298 315 367 496 525 553 685 700 711 728 736
763 770 800 802 836 841 851 871 883 905 913 929 937 939 951
967 1026 1037 1047 1052 1055 1056 1060 1123 1133 1141 1185 1188 1225 1270
1274 1324 1335 1365 1370 1376 1387 1405 1418 1422 1446 1449 1451 1494 1514
1518 1523 1530 1549 1562 1590 1601 1619 1622 1631 1636 1652 1663 1675 1682
1696 1707 1733 1736 1738 1752 1763 1785 1790 1798 1806 1807 1810 1814 1820
1823 1829 1831 1842 1850 1855 1857 1967 1991 2019 2030 2031 2036 2071 2080
2088 2093 2094 2112 2119 2129 2130 2156 2162 2243 2280 2283 2287 2350 2383
2399 2410 2435 2445 2454 2460 2472 2473 2478 2510 2512 2538 2545 2575 2581
2583 2635 2647 2656 2665 2746 2764 2768 2784 2789 2812 2815 2828 2831 2841
2845 2847 2858 2873 2874 2880 2896 2897 2899 2902 2916 2942 2955 2989 2999
3002 3009 3023 3029 3031 3033 3058 3059 3067 3072 3100 3105 3108 3116 3138
3144 3165 3166 3180 3181 3201 3257 3260 3272 3282 3306 3340 3356 3359 3373
3390 3410 3414 3421 3448 3459 3478 3481 3500 3589 3605 3607 3640 3677 3680
3681 3690 3722 3749 3763 3764 3771 3807 3817 3825 3855 3875 3928 3943 3991
4012 4033 4062 4070 4080 4113 4146 4148 4150 4171 4185 4218 4263 4269 4278
4287 4296 4299 4322 4323 4362 4396 4422 4472 4488 4527 4536 4563 4605 4614
4631 4640 4650 4654 4669 4670 4691 4692 4703 4710 4720 7009 7021 7036 7042
7049 7052 7100 7109 7144 7162 7166 7191 7215 7240 7263 7276 7290 7323 7326
7334 7344 7352 7361 7371 7398 7440 7442 7453 7465 7481 7527 7554 7621 7622
7636 7642 7659 7670 7676 7730 7744 7759 7790 7893 7916 7979 7997 8019 8052
8095 8099 8152 8178 8203 8212 8300 8347 8392 8397 8402 8420 8424 8425 8479
8554 8569 8622 8650 8701 8769 8816 8841 8856 8873 8894 8912 8917 8995 9039
9082 9150 9155 9201 9219 9227 9231 9237 9283 9325 9354 9366 9375 9384 9387
9410 9454 9473 9475 9550 9568 9672 9681

members added by crit. 2
352 422 540 825 831 915 1016 1130 1249 1307 1338 1396 1412 1455 1472
1577 2017 2402 2438 2500 2609 2678 2682 2750 2887 2890 2975 3073 3244 3253
3301 3413 3533 3669 3698 3840 3919 4145 4204 4338 4443 4606 7057 7206 7644
8182 8183 8222 8277 8553 9011 9135

members added by crit. 3
823 864 1117 1219 1399 1527 1602 1699 1713 1997 2171 2285 2535 2564 2691
2839 2971 3412 3493 3520 3769 3822 3956 3986 4025 4048 4066 4373 4459 4478
4555 7232 7262 7300 7338 7366 7367 7386 7435 7532 7610 7905 7922 7924 7984
7998 8002 8103 8160 8278 8295 8544 8550 9247 9590

Table 1c.

members added by crit. 4
963 1088 1089 1344 1634 1899 2070 2362 2409 3083 3121 3133 3590 3831 3948
4445 4475 4594 7058 7291 7432 7990 8240 8263 8874 9013 9167

Table 1d.

4 Vesta (*2.36154 .0994 .1106*) 79 members
4 63 1906 1933 1959 1979 2024 2029 2086 2346 2419 2508 2511 2547 2590
2640 2974 3155 3268 3376 3477 3494 3498 3703 3720 3936 3944 3968 4005 4038
4147 4311 4444 4510 4518 4546 7078 7163 7184 7320 7322 7359 7444 7535 7769
7773 7779 7803 7805 7870 7951 8062 8083 8130 8153 8176 8211 8492 8499 8534
8540 8608 8620 8626 8649 8672 8706 8777 8866 8869 8893 9019 9073 9267 9380
9411 9427 9572 9612

members added by crit. 2
2011 2045 2799 3657 3697 3890 4684 7443 7556 7559 7658 7740 8047 8100 8198
8375 8814

members added by crit. 3
3007 4022 4524 7472 7626 7787 8339 8852

members added by crit. 4
1989 2468 2716 3263 3536 8498 9022 9161

Table 1e.

163 Erigone (*2.36718 .2099 .0826*) 12 members
163 933 2776 4229 4279 7167 7257 7363 7677 7817 8494 8557

members added by crit. 2
1448

members added by crit. 3
2348 3052

members added by crit. 4
4675

Table 1f.

79 Eurynome (*2.44425 .1775 .0870*) 43 members
79 477 917 1066 1077 1267 1454 1689 1928 2200 2572 2763 2821 2939 3038 3053 3081 3084 3381 3403 3433 3462 3574 3699 3942 3958 4437 4635 7119 7158 7350 7404 7521 7982 7988 8395 8637 8743 8950 9012 9044 9365 9627

members added by crit. 2
1217 1296 3018

Table 1g.

9 Metis (*2.38642 .1269 .0817*) 11 members
9 113 2532 2777 2853 7628 8228 9057 9425 9521 9661

Table 1h.

27 Euterpe (*2.34700 .1865 .0125*) 17 members
27 2838 3014 3289 3466 3473 3973 4364 4432 4579 4643 7165 7810 8025 8536 9156 9328

members added by crit. 2
4468

members added by crit. 3
8824

Table 1i.

435 Ella (*2.44950 .1219 .0287*) 15 members
435 1966 2190 2739 3123 4238 7321 7496 7727 7772 7943 8623 8958 8993 9004

Table 1j.

Lameia (*2.47093 .0731 .0852*) 19 members
248 752 1152 1375 1770 1923 2861 3056 3320 3547 3849 3869 4199 4434 7656 8889 9420 9578 9591

Table 1k.

12 Victoria (*2.33424 .1742 .1617*) 34 members
12 84 220 284 437 753 783 1507 1538 1709 1718 2424 2641 2733 3523 3592 3627 3795 3879 4272 4319 4711 4718 7573 7594 7970 8344 8867 9062 9118 9148 9230 9478 9610

members added by crit. 2
8539

Table 2a. The same as Table 1a but for the medium zone

Name	Nmemb	Rn	Rs1	Rm2	R	N2	N3	Ninter
272 Antonia	21	1.	0.952	0.976	0.976	5	3	0
170 Maria	52	0.951	0.981	0.972	0.968	4	3	2
668 Dora	44	0.966	0.966	0.966	0.966	2	5	0
145Adeona	26	0.948	1.	0.933	0.960	3	0	0
1639 Bower	5	1.	1.	0.857	0.952	1	4	0
1272 Geffion	37	0.959	0.959	0.910	0.943	3	1	0
15 Eunomia	156	0.931	0.961	0.921	0.938	21	12	40
779 Nina	8	0.944	0.938	0.900	0.927	2	1	0
410 Chloris	11	0.958	0.955	0.843	0.922	2	3	0
213 Lilaea	17	1.	0.853	0.824	0.892	0	0	0
569 Misa	13	0.729	1.	0.933	0.887	3	2	0
847 Agnia	35	0.874	0.868	0.884	0.875	1		4
262 Valda	47	0.743	0.734	0.801	0.759	13	6	13
869 Mellena	9	0.556	0.778	0.361	0.519	2	5	0
55 *Pandora*	21	0.536	0.	0.884	0.473	5	2	3
58 *Concordia*	16	0.840	0.	0.313	0.384	4		

Table 2b to 2q. The same as Tables 1b to 1k but for the medium zone

Table 2b.

272 Antonia (*2.77779 .0497 .0692*) 21 members
272 308 363 1420 1726 2306 2560 2930 2996 4124 4516 7339 7602 7632 7799 7925 8179 8391 8685 9195 9485

members added by crit.2
1372 2732 3020 8452 8518

members added by crit.2
203 673 1517

Table 2c.

170 Maria (*2.55376 .1039 .2595*) 52 members
170 472 575 616 660 695 714 727 787 875 879 897 1158 1160 1215 1677 1996 2151 2221 2429 2638 2865 2903 2962 3055 3066 3158 3159 3167 3594 3637 3786 3970 4099 4104 4122 4167 4673 7133 7153 7238 7528 7828 8043 8419 8682 8712 8842 9241 9309 9504 9685

members added by crit. 2
3332 4569 7324 9051

members added by crit. 3
652 3537 8090

Table 2d.

668 Dora (*2.79662 .1986 .1363*) 44 members
668 1414 1734 1795 1836 1970 2598 2807 2940 3534 3563 3611 3630 3775 3829 4135 4220 4343 4354 4356 4584 7329 7384 7523 7553 7612 7756 7866 8074 8137 8323 8582 8589 8596 8767 8854 8933 9140 9347 9353 9419 9512 9576 9602

members added by crit. 2
8503 9095

members added by crit. 3
351 826 2992 4534 8298

Table 2e.

145 Adeona (*2.67278 .1688 .2030*) 26 members
145 166 997 1238 1384 1783 1936 1994 2333 3096 3205 3238 3407 3445 3725 4157 4522 4582 4629 7002 7716 8199 8254 8806 9068 9417

members added by crit. 2
4523 9302 9390

Table 2f.

1639 Bower (*2.57334 .1408 .1586*) 5 members
1639 3815 4561 7711 8870

members added by crit. 2
4595

members added by crit. 3
342 544 1226 3528

Table 2g.

1272 Geffion (*2.78377 .1297 .1566*) 37 members
1272 1433 1751 1839 2053 2157 2373 2386 2493 2521 2559 2595 2631 2801 2875 2905 2911 2977 3724 3788 3860 3910 3964 4096 4182 4702 7286 7294 7546 7664 8268 8864 9042 9266 9614 9619 9640

members added by crit. 2
3853 4020 4417

members added by crit. 3
255

Table 2h.

15 Eunomia (*2.64368 .1462 .2263*) 156 members
15 473 630 812 1050 1193 1275 1329 1333 1425 1431 1495 1499 1503 1531 1775 1886 1926 2005 2079 2181 2302 2337 2381 2384 2490 2537 2649 2660 2672 2685 2743 2796 2810 2869 2915 2993 3017 3182 3242 3252 3286 3296 3305 3387 3488 3492 3518 3539 3662 3707 3729 3758 3767 3805 3892 3894 3909 3934 3961 3965 3974 3977 4085 4133 4149 4164 4254 4288 4358 4384 4399 4467 4502 4526 4567 4580 4601 4602 4611 4628 4636 4644 4695 4704 7014 7015 7028 7035 7053 7069 7080 7103 7152 7194 7218 7265 7279 7280 7301 7303 7315 7391 7410 7419 7518 7533 7569 7585 7614 7638 7955 7968 8011 8036 8055 8072 8151 8159 8270 8307 8333 8432 8474 8475 8481 8485 8613 8632 8641 8662 8692 8791 8901 8903 8929 8931 8936 8980 9034 9072 9097 9189 9197 9232 9271 9343 9389 9428 9528 9535 9601 9606 9667 9675 9687

members added by crit. 2
85 1106 1346 1458 1554 2304 2463 2822 2970 2988 3041 3487 3569 3714 3779 4191 4275 7255 8177 9193 9617

members added by crit. 3
1284 2842 3503 3711 3870 7024 7114 7506 7802 8628 8966 9098

Table 2i.

779 Nina (*2.66554 .1792 .2679*)8 members
779 1326 1441 1505 1719 3702 8167 8224

members added by crit. 2
2771 3197

members added by crit. 3
8354

Table 2j.

410 Chloris (*2.72671 .2491 .1604*) 11 members
410 521 1403 1534 1613 4585 8229 8959 9183 9235 9261

members added by crit. 2
4200 8631

members added by crit. 3
109 187 9278

Table 2k.

213 Lilaea (*2.75290 .1437 .0996*) 17 members
213 808 1327 1662 1791 2042 2427 3439 3472 4325 4479 4662 7277 7833 8367 8433 8568

Table 2l.

569 Misa (*2.65761 .1799 .0395*) 13 members
569 2289 3027 3035 3889 4047 7862 7894 8465 8515 8713 8754 9030

members added by crit. 2
8003 9499 9671

members added by crit. 3
4108 7701

Table 2m.

847 Agnia * (*2.78281 .0680 .0624*) 35 members
847 1228 2085 2354 2401 2514 3349 3430 3491 3701 3718 3734 3844 3987 4051 4237 4261 4426 4619 7150 7237 7314 7319 7406 7692 7712 7849 7854 8313 8705 9154 9490 9522 9551 9586

members added by crit. 2
7357

Table 2n.

* **847 Agnia** (*2.78281 .0680 .0624*) 20 members
847 1228 2085 2354 2401 3349 3430 3491 3701 3734 3844 3987 4051 4261 4426 4619 7150 7237 7314 7319

* **2514 Taiyuan** (*2.65090 .0641 .0468*) 15 members
2514 3718 4237 7406 7692 7712 7849 7854 8313 8705 9154 9490 9522 9551 9586

* see text

262 Valda * (*2.55359 .1967 .122*) 47 members
262 518 1018 1205 1325 1391 1501 1587 1644 1658 1888 1931 2118 2167 2174 2766 2834 2980 2997 3206 3444 3600 3678 3862 4114 4154 4528 4639 7033 7224 7418 7424 7928 7957 8037 8517 8629 8656 8731 8834 8846 8939 9035 9078 9358 9361 9626

members added by crit. 2
678 765 1756 2447 3474 3748 4612 7688 8530 8726 8886 8942 9024

members added by crit. 3
2497 4129 7473 8132 9050 9416

***262 Valda** (*2.55359 .1967 .122*) 21 members
262 518 1018 1205 1325 1391 1501 1888 1931 2118 2167 2174 2766 2834 3206 3444 3678 3862 4154 7424 7928

* **1644 Rafita** (*2.54759 .1778 .1373*) 11 members
1644 1658 2980 2997 3600 4639 7033 7224 7418 8037 8656

* see text

Table 2o.

869 Mellena (*2.69200 .2231 .1335*) 9 members
869 1294 1479 1525 2109 3184 4274 4540 7980

members added by crit. 2
1277 1555

members added by crit. 3
3295 7011 7154 7630 7996

Table 2p.

55 Pandora (*2.75973 .1093 .1223*) 21 members
55 123 200 359 378 424 614 741 872 1716 1813 2141 2251 2567 2730 3256 3508 3897 3924 7128 8606

members added by crit. 2
2715 3294 3389 7515 8143

members added by crit. 3
1176 7603

Table 2q.

58 Concordia * (*2.69998 .0808 .0818*) 16 members
58 128 380 1039 1135 2138 3827 3886 4630 7421 7436 7830 7835 8029 8439 9330

members added by crit. 3
3065 7625 8405 8964

* **58 Concordia** (*2.69998 .0808 .0818*) 9 members
58 380 1039 1135 2138 4630 7421 7830 9330

* **128 Nemesis** (*2.75012 .0909 .0902*) 7 members
128 3827 3886 7436 7835 8029 8439

* see text

Table 3a. The same as Table 1a but for the outer zone

Name	Nmemb	Rn	Rs1	Rm2	R	N2	N3	N4	Ninter
490 **Veritas**	12	0.962	1.	1.	0.987	1	0	0	0
221 **Eos**	219	0.970	0.998	0.936	0.968	38	14	16	5
158 **Koronis**	161	0.969	1.	0.923	0.964	25	12	13	0
24 **Themis**	177	0.969	0.994	0.856	0.940	99	54	58	16
10 **Hygiea**	29	0.931	0.821	0.809	0.854	8			4
293 *Brasilia*	13	0.923	0.	0.964	0.629	0	0	0	0
137 *Meliboea*	19	0.947	0.	0.902	0.616	0	1	3	0
973 *Aralia*	23	0.935	0.	0.845	0.593	1	0	1	1
1271 *Isergina*	19	0.974	0.	0.806	0.593	4			1
734 *Benda*	10	1.	0.	0.600	0.533	1	0	0	0

Table 3b to 3l. The same as Tables 1b to 1k but for the outer zone

Table 3b.

490 Veritas (*3.17494 .0651 .1576*) 12 members
490 844 1086 2147 2428 2934 3090 3542 7229 7619 8285 9255

members added by crit. 2
1351

Table 3d.

221 Eos (*3.01244 .0879 .1770*) 219 members
221 339 450 513 520 529 562 573 579 590 608 633 639 651 653 661 669 742 775 798 807 833 876 890 1087 1105 1112 1129 1186 1199 1207 1210 1220 1286 1287 1291 1297 1339 1353 1364 1388 1410 1413 1416 1434 1464 1485 1533 1557 1604 1605 1641 1649 1654 1711 1732 1737 1758 1767 1780 1786 1787 1801 1812 1826 1844 1852 1861 1882 1887 1903 1957 2027 2052 2091 2111 2124 2136 2179 2180 2191 2206 2216 2263 2309 2315 2345 2358 2387 2400 2413 2416 2425 2443 2459 2471 2476 2522 2523 2531 2578 2600 2618 2622 2632 2646 2677 2686 2690 2706 2711 2740 2752 2767 2787 2804 2836 2889 2907 2928 2957 2982 3003 3028 3088 3126 3168 3190 3194 3214 3232 3237 3250 3312 3318 3329 3357 3366 3425 3469 3505 3506 3560 3570 3582 3620 3638 3713 3750 3772 3774 3820 3887 3896 3914 3950 3955 3992 4041 4052 4058 4074 4077 4100 4102 4118 4163 4207 4210 4271 4395 4427 4453 4473 4493 4498 4505 4529 4532 4537 4542 4557 4559 4574 4575 4593 4598 4668 4714 4717 7047 7072 7161 7188 7362 7412 7441 7537 7607 8000 8165 8431 8462 8472 8488 8508 8804 8968 9017 9064 9127 9172 9254 9339 9341 9510 9595 9604

members added by crit. 2
1095 1234 1265 1532 1834 1845 1910 2020 2475 2562 2661 3830 4078 4223 4290 4346 4357 4485 4568 4576 7094 7187 7655 7764 7778 7785 7933 7956 7961 8218 8227 8244 8301 8638 9169 9228 9292 9345

members added by crit. 3
677 1033 1148 1799 2530 4410 4513 4681 7592 7820 8283 8627 8853 8916

members added by crit. 4
179 1588 1755 1971 1992 3468 4044 4093 4381 7801 8595 8781 9008 9093 9609 9670

Table 3e.

158 **Koronis** (*2.86884 .0455 .0375*) 161 members
158 167 208 243 263 277 321 452 462 534 658 720 761 832 975
993 1079 1100 1223 1245 1289 1336 1350 1363 1389 1423 1442 1482 1570 1618
1635 1741 1742 1745 1762 1802 1824 1848 1878 1894 1913 1955 2051 2092 2117
2123 2144 2155 2188 2209 2224 2225 2226 2230 2300 2338 2377 2470 2506 2555
2574 2589 2620 2626 2713 2726 2729 2785 2811 2814 2833 2837 2901 2924 2931
2958 2963 2969 2985 3016 3019 3032 3117 3191 3195 3207 3226 3303 3334 3337
3377 3380 3386 3409 3436 3457 3515 3516 3545 3623 3726 3765 3778 3780 3781
3791 3856 3975 4076 4123 4195 4206 4259 4260 4345 4351 4365 4447 4506 4507
4623 4651 4664 4696 7019 7157 7177 7293 7332 7370 7392 7456 7492 7503 7541
7542 7736 7742 7757 7777 7909 7926 7949 8005 8024 8032 8034 8336 8411 8513
8520 8549 8890 8967 9003 9158 9321 9439 9474 9558 9585

members added by crit. 2
311 1497 1774 1835 2160 2700 2742 3261 3804 3941 4084 4286 4316 4389 4656
7010 7369 7700 7868 8266 8368 8858 9461 9462 9502

members added by crit. 3
811 1029 1725 2319 2498 3143 4627 7483 7737 7809 8314 9206

members added by crit. 4
1443 1912 2541 2591 3374 4241 4441 7770 9031 9033 9516 9546

Table 3f.

24 **Themis** (*3.13459 .1514 .0189*) 177 members
24 62 90 171 222 379 383 461 468 492 515 526 621 656 710
767 846 936 938 954 991 1003 1027 1074 1082 1259 1302 1445 1487 1539
1576 1615 1623 1669 1686 1687 1691 1778 1782 1788 1805 1851 1939 1953 2009
2016 2039 2046 2058 2153 2163 2165 2182 2203 2217 2222 2228 2248 2256 2264
2270 2293 2297 2310 2325 2361 2372 2404 2418 2439 2450 2461 2489 2492 2499
2519 2524 2525 2528 2534 2551 2587 2627 2718 2749 2769 2781 2800 2803 2884
2894 2919 3049 3128 3174 3208 3245 3276 3292 3399 3441 3499 3502 3591 3615
3666 3705 3790 3799 3832 3878 3884 3930 3980 3981 4009 4013 4061 4073 4079
4139 4174 4176 4301 4344 4361 4366 4470 4571 4616 4633 4657 4683 4685 7102
7201 7258 7259 7336 7375 7385 7460 7476 7687 7908 7944 7976 7985 8057 8084
8138 8147 8359 8374 8384 8451 8500 8514 8547 8621 8764 8953 8970 8992 9060
9088 9163 9311 9316 9376 9440 9446 9483 9519 9523 9580 9650

members added by crit. 2
316 431 637 885 996 1073 1247 1340 1383 1581 1674 1684 1698 1764 1895
1898 2010 2114 2220 2240 2250 2336 2405 2517 2533 2549 2592 2723 2757 2863
2882 2918 2978 3008 3061 3148 3154 3179 3186 3297 3358 3418 3495 3507 3543
3599 3621 3797 3814 3852 3898 3899 3946 3962 4192 4193 4198 4211 4306 4334
4406 4474 4545 4592 4642 7084 7175 7181 7221 7249 7337 7399 7405 7514 7855
8038 8040 8046 8119 8126 8156 8169 8190 8269 8434 8509 8545 8604 8724 8729
9027 9049 9394 9481 9489 9501 9529 9571 9645

members added by crit. 3
104 268 561 848 946 981 1253 1633 1986 2003 2164 2197 2238 2342 2505
2563 2657 2673 2688 2708 2722 2848 3213 3264 3597 3766 3785 4098 4126 4153
4234 4385 4393 4412 4463 4677 7155 7170 7313 7353 7382 7409 7463 7466 7811
7938 8020 8338 8353 8378 8442 9282 9436 9518

members added by crit. 4
223 555 1229 1462 1624 1956 2142 2237 2659 2721 2877 2981 2986 3054 3071
3164 3241 3274 3423 3449 3504 3601 3859 3916 3922 3933 4151 4187 4242 4337
4622 4658 7016 7101 7185 7217 7374 7383 7395 7497 7743 7842 7860 8042 8053
8086 8230 8249 8310 8511 8512 8551 8810 8837 8960 9005 9513 9561

Table 3g.

10 Hygiea (*3.14228 .1342 .0884*) 29 members
10 333 1107 1109 2436 2615 3319 3626 4687 7073 7117 7234 7316 7428 7431 7605 7618 7681 7714 7719 7839 7977 7994 8134 8304 8308 8373 8994 9314

members added by crit. 2
100 538 1209 1611 4530 7599 7874 9318

Table 3h.

1271 Isergina (*3.13526 .1023 .0995*) 19 members
1271 1298 1599 2747 3078 3157 3215 3947 4120 4449 7273 7910 8464 8577 8881 9138 9186 9476 9573

members added by crit. 2
1961 3754 7761 8646

Table 3i.

293 Brasilia (*2.86187 .1283 .2586*) 13 members
293 392 697 804 1521 2326 2378 3044 3971 3985 4162 4461 4471

Table 3j.

137 Meliboea (*3.11859 .1851 .2510*) 19 members
137 768 791 1369 1452 1498 1794 2040 2152 2374 2829 3650 4004 8026 8078 8122 8357 9184 9426

members added by crit. 3
3106

members added by crit. 4
199 786 7564

Table 3k.

973 Aralia (*3.22516 .0801 .2791*) 23 members
973 1070 1306 1330 1371 1567 1832 2145 2211 2414 3419 3471 3871 3902 3967 4436 7572 7646 7971 8409 8914 8976 9094

members added by crit. 2
9102

members added by crit. 4
1520

Table 3l.

734 Benda (*3.15180 .0781 .1057*) 10 members
734 861 2219 2520 3342 3649 7450 7850 8895 8949

members added by crit. 2
2184

Table 4. For each dynamical families and tribes the number of multi-opposition members and the corresponding families previously identified (when available) by Bendjoya et al. (1991)

Name	Nmemb	Nmo	family version 4.2
Hertha	145	82	32 + 34 + 35
Flora	368	102	21 + 22 + 23 +24
Vesta	79	43	31a + 31b
Erigone	12	7	
Eurynome	43	15	
Metis	11	6	
Euterpe	17	6	
Ella	15	9	
Lameia	19	5	
Victoria	34	11	
Antonia	21	10	43
Maria	52	14	44
Dora	44	23	45
Adeona	26	7	41b
Bower	5	2	
Geffion	37	11	42
Eunomia	156	71	41a
Nina	8	2	
Chloris	11	5	
Lilaea	17	5	
Misa	13	7	
Agnia	35	16	46
Valda	47	19	
Mellenaa	9	1	
Pandora	21	2	
Concordia	16	7	
Veritas	12	4	72
Eos	219	28	61
Koronis	161	37	51
Themis	177	43	71
Hygiea	29	20	
Brasilia	13	0	
Meliboea	19	6	
Aralia	23	7	
Isergina	19	9	
Benda	10	4	

Nmemb: number of members
Nmo: number of multi-opposition objects
family version 4.2: The corresponding family (when available) from Bendjoya et al. 1991 analysis

Appendix

7002	1925BA	7181	1975BP1	7337	1978RL1	7477	1979TA	7669	1981EJ1	7838	1981EA3
7006	1929TD1	7184	1975QC	7338	1978RX1	7481	1979UQ	7670	1981EN1	7839	1981EG3
7009	1931FC	7185	1975RP	7339	1978RM2	7483	1979UD1	7676	1981EZ1	7842	1981EA4
7010	1931TS1	7187	1975SS	7340	1978RE3	7492	1980BB	7677	1981EC1	7848	1981ER4
7011	1931TC2	7188	1975SA1	7344	1978RC9	7496	1980EB	7681	1981ER1	7849	1981EY4
7013	1931UB	7191	1975TE	7350	1978SB3	7497	1980FB	7682	1981EV1	7850	1981EA4
7014	1931UD	7194	1975TQ3	7352	1978SH3	7503	1980FO1	7684	1981EY1	7854	1981EV4
7015	1931VS	7200	1975UE	7353	1978SN4	7506	1980FV2	7687	1981EQ1	7855	1981EX4
7016	1932CY	7201	1975UF	7357	1978SP5	7514	1980GO	7688	1981ER1	7860	1981EQ4
7019	1933UM1	7206	1975VS5	7359	1978SU5	7515	1980KM	7692	1981ED1	7862	1981ES4
7021	1935SC	7215	1976EB	7361	1978SD7	7516	1980LU	7700	1981EY1	7863	1981ET4
7024	1936QE1	7217	1976GH2	7362	1978SN7	7518	1980LE1	7701	1981EH2	7865	1981EA4
7028	1939UB	7218	1976GN2	7363	1978SS7	7519	1980NB	7711	1981EF2	7866	1981ED4
7033	1942CG	7221	1976GU3	7366	1978SB8	7521	1980PW	7712	1981EL2	7867	1981EQ4
7035	1943DL	7224	1976GL8	7367	1978TB2	7523	1980PV1	7713	1981EO2	7868	1981ER4
7036	1943EN	7229	1976QL2	7369	1978TR2	7527	1980RJ	7714	1981ER2	7870	1981EX4
7042	1949QL	7231	1976SJ	7370	1978TT2	7528	1980RU	7716	1981EX2	7874	1981EW4
7047	1950HJ	7232	1976SG2	7371	1978TW2	7532	1980RO2	7719	1981EE2	7882	1981FP
7049	1951WH	7234	1976SW3	7374	1978TO8	7533	1980SD	7723	1981ET2	7891	1981JE2
7052	1953TS2	7237	1976SV1	7375	1978TV8	7535	1980SJ	7725	1981EB2	7892	1981JS2
7053	1953UD	7238	1976US1	7382	1978VT4	7537	1980TH	7727	1981EH2	7893	1981JB3
7057	1955SF	7240	1976UP2	7383	1978VD5	7541	1980TH3	7729	1981EK2	7894	1981JE3
7058	1961BC	7241	1976UG1	7384	1978VE5	7542	1980TX3	7730	1981ET2	7902	1981QE1
7065	1965SO	7248	1976YY	7385	1978VG5	7546	1980TC5	7736	1981EH2	7905	1981QH2
7068	1966CL	7249	1976YP1	7386	1978VL5	7553	1980VX1	7737	1981EL2	7908	1981QE3
7069	1966CM	7255	1977DR1	7387	1978VS5	7554	1980XX	7740	1981ET2	7909	1981QP3
7072	1967GM1	7257	1977DL3	7391	1978VW6	7556	1980YC	7742	1981EW2	7910	1981QT3
7073	1967JP	7258	1977DQ3	7392	1978VD7	7559	1981DM	7743	1981EX2	7911	1981RF
7078	1968OA1	7259	1977DN4	7395	1978VJ8	7564	1981DB1	7744	1981EC2	7916	1981RR3
7079	1968QE	7262	1977EL	7398	1978VT9	7569	1981DC2	7747	1981EJ2	7922	1981SY1
7080	1969GD	7263	1977EO	7399	1978VG1	7572	1981DG3	7748	1981EK2	7923	1981SE2
7084	1969TQ1	7265	1977EF1	7404	1978VL1	7573	1981EN	7756	1981EN2	7924	1981SU2
7086	1969TT1	7273	1977FN1	7405	1978VP1	7578	1981EE1	7757	1981EO2	7925	1981SD4
7089	1969TN4	7276	1977NN	7406	1978VY1	7585	1981EK4	7759	1981ET2	7926	1981SA5
7094	1970PS	7277	1977PE1	7407	1978VE1	7592	1981EL5	7761	1981EY2	7928	1981SC7
7100	1971RA	7277	1977PE1	7409	1978XQ	7594	1981ER5	7764	1981ED2	7933	1981TJ4
7101	1971SS1	7277	1977PE1	7410	1979EL	7599	1981EK7	7769	1981ER2	7938	1981UT7
7102	1971SN2	7279	1977QY	7411	1979FD2	7602	1981EV7	7770	1981ET2	7943	1981US1
7103	1971SX3	7280	1977QF1	7412	1979FQ2	7603	1981EZ7	7772	1981EZ2	7944	1981VK
7104	1971TF	7281	1977QK1	7415	1979HE3	7605	1981EM8	7773	1981EA2	7949	1981WA1
7107	1971UM	7286	1977RG	7418	1979KD	7607	1981ES8	7777	1981EP2	7951	1981WF9
7109	1971UQ	7290	1977RF2	7419	1979KG	7610	1981EV8	7778	1981EV2	7955	1982BS
7114	1972AU	7291	1977RD3	7421	1979KQ	7612	1981EY8	7779	1981EX2	7956	1982BW
7117	1972HR	7293	1977RW6	7424	1979ML	7614	1981EB9	7785	1981EU2	7957	1982BE1
7119	1972KL	7294	1977RY6	7428	1979MU2	7618	1981EQ9	7787	1981EF3	7961	1982BQ4
7128	1973AW3	7300	1977TC1	7431	1979MK3	7619	1981ES9	7789	1981EX3	7968	1982EF
7133	1973RF	7301	1977TD1	7432	1979MP3	7621	1981EV9	7790	1981EY3	7970	1982FC
7144	1973SO3	7303	1977TQ6	7435	1979MJ5	7622	1981EW9	7799	1981EO3	7971	1982FJ
7145	1973SR3	7304	1977UD	7436	1979ML5	7625	1981EL1	7801	1981EZ3	7976	1982FP3
7150	1973SC6	7313	1978NY7	7437	1979MM5	7626	1981EP1	7802	1981EB3	7977	1982FX3
7152	1973SR6	7314	1978ON	7440	1979MB6	7628	1981ES1	7803	1981EQ3	7979	1982JE1
7153	1973TP	7315	1978OP	7441	1979MH6	7630	1981EX1	7805	1981EU3	7980	1982JR1
7154	1973UC	7316	1978PJ2	7442	1979MR6	7632	1981EZ1	7808	1981ED3	7982	1982OF
7155	1973UB5	7318	1978PX2	7443	1979MS6	7633	1981EB1	7809	1981EH3	7984	1982PC
7157	1973UJ5	7319	1978PY2	7444	1979MX6	7636	1981EE1	7810	1981EK3	7985	1982PR
7158	1974FJ	7320	1978PD3	7450	1979OQ5	7638	1981EH1	7811	1981EL3	7988	1982RK1
7161	1974ME	7321	1978PO3	7453	1979OK1	7640	1981EO1	7817	1981EJ3	7990	1982RO1
7162	1974MG	7322	1978PW3	7456	1979QP	7642	1981EA1	7820	1981ER3	7994	1982SE1
7163	1974OE	7323	1978PX3	7460	1979QZ1	7644	1981EF1	7823	1981EB3	7996	1982SJ1
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7167	1974SF	7329	1978RR	7466	1979QX9	7656	1981EC1	7833	1981EM3	8000	1982SG4
7170	1974SP1	7332	1978RZ	7472	1979SU2	7658	1981EE1	7833	1981EM3	8002	1982SV5
7175	1974SB5	7334	1978RH1	7473	1979SL7	7659	1981EF1	7833	1981EM3	8003	1982SX5
7177	1974VS	7336	1978RK1	7476	1979SU1	7664	1981EX1	7835	1981EW3	8005	1982ST6

Appendix. continued

8011	1982TG1	8211	1985CZ1	8409	1986RB1	8569	1987VB	8842	1989CH1	9012	1989WL
8019	1982UP	8212	1985CA2	8411	1986RF1	8577	1987WJ1	8846	1989CM1	9013	1989WR
8020	1982UV1	8213	1985DX2	8419	1986TG1	8582	1987YH	8849	1989CY1	9017	1989WE1
8024	1982UP2	8222	1985GO	8420	1986TK1	8589	1988AL	8852	1989CS2	9019	1989WJ1
8025	1982UQ3	8224	1985GU1	8424	1986TJ2	8595	1988AE5	8853	1989CX2	9022	1989WV1
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8029	1982UX5	8228	1985JK	8428	1986TL4	8604	1988BZ1	8856	1989CD4	9027	1989WU2
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8032	1982UQ6	8230	1985JY	8432	1986TT1	8608	1988BX3	8863	1989EM	9031	1989XA
8034	1982UT6	8240	1985PD2	8433	1986TB1	8613	1988BH5	8864	1989EV	9033	1989XC
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8042	1982UR1	8260	1985RS1	8442	1986VT	8626	1988CX3	8871	1989EL6	9044	1989YN
8043	1982UX1	8263	1985RU2	8446	1986WG	8627	1988CD4	8873	1989FA	9046	1989YR
8046	1982VZ	8266	1985RC4	8449	1986WO1	8628	1988CK4	8874	1989FH	9048	1989YH1
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8103	1983RP2	8309	1985VF1	8499	1987QG2	8685	1988PM2	8916	1989NG1	9095	1990FM1
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8122	1983TR2	8313	1985XR	8503	1987QF7	8701	1988RP1	8929	1989RH	9098	1990FT1
8126	1983VN7	8314	1985XS	8506	1987QW7	8702	1988RR2	8931	1989RD1	9102	1990HR
8130	1983WM	8323	1986CG	8508	1987QY1	8705	1988RU3	8933	1989RB2	9118	1990MN
8132	1983XE	8327	1986CS1	8509	1987RG	8706	1988RG4	8936	1989SA	9127	1990OO
8134	1983XG	8333	1986EZ	8511	1987RY	8712	1988RT6	8939	1989SE	9135	1990OJ2
8137	1983XH1	8335	1986EZ1	8512	1987RZ	8713	1988RU6	8941	1989SJ	9138	1990OH4
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8143	1984DE	8338	1986EE5	8514	1987RC1	8726	1988RK1	8949	1989SA3	9148	1990QC1
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8159	1984FS	8359	1986PW4	8534	1987SN3	8767	1988VB	8966	1989TR1	9161	1990QL3
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8165	1984JA2	8367	1986QY	8539	1987SD4	8777	1988VO1	8968	1989TG1	9165	1990QY3
8167	1984OA	8367	1986QY	8540	1987SE4	8781	1988VB3	8970	1989UD	9167	1990QM4
8169	1984QJ	8368	1986QB1	8544	1987SE7	8782	1988VD3	8971	1989UM	9169	1990QV4
8176	1984SA1	8371	1986QT1	8545	1987SS9	8791	1988VD5	8976	1989UY	9172	1990RB
8177	1984SC1	8373	1986QJ2	8547	1987SN1	8804	1988XW1	8980	1989UT2	9183	1990SH1
8178	1984SF1	8374	1986QP2	8549	1987SM1	8806	1989AG	8992	1989UE7	9184	1990SZ1
8179	1984SG1	8375	1986QQ2	8550	1987SR1	8810	1989AQ	8993	1989UF7	9186	1990SF2
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8183	1984SR2	8384	1986QY4	8553	1987SM1	8816	1989AX1	8995	1989UE8	9193	1990SN4
8190	1984SY5	8391	1986RP1	8554	1987SQ1	8824	1989AW6	9002	1989VQ	9195	1990SU1
8198	1984UX1	8392	1986RS1	8557	1987SX1	8831	1989BY	9003	1989VR	9197	1990TF
8199	1984UX2	8395	1986RS2	8558	1987UG	8834	1989BN1	9004	1989VV	9201	1990TS
8203	1984WM1	8397	1986RU2	8568	1987UF5	8836	1989CA	9005	1989VX	9206	1990TK1
8207	1985CG	8402	1986RB5	8568	1987UF5	8837	1989CM	9008	1989WB	9213	1990TF4
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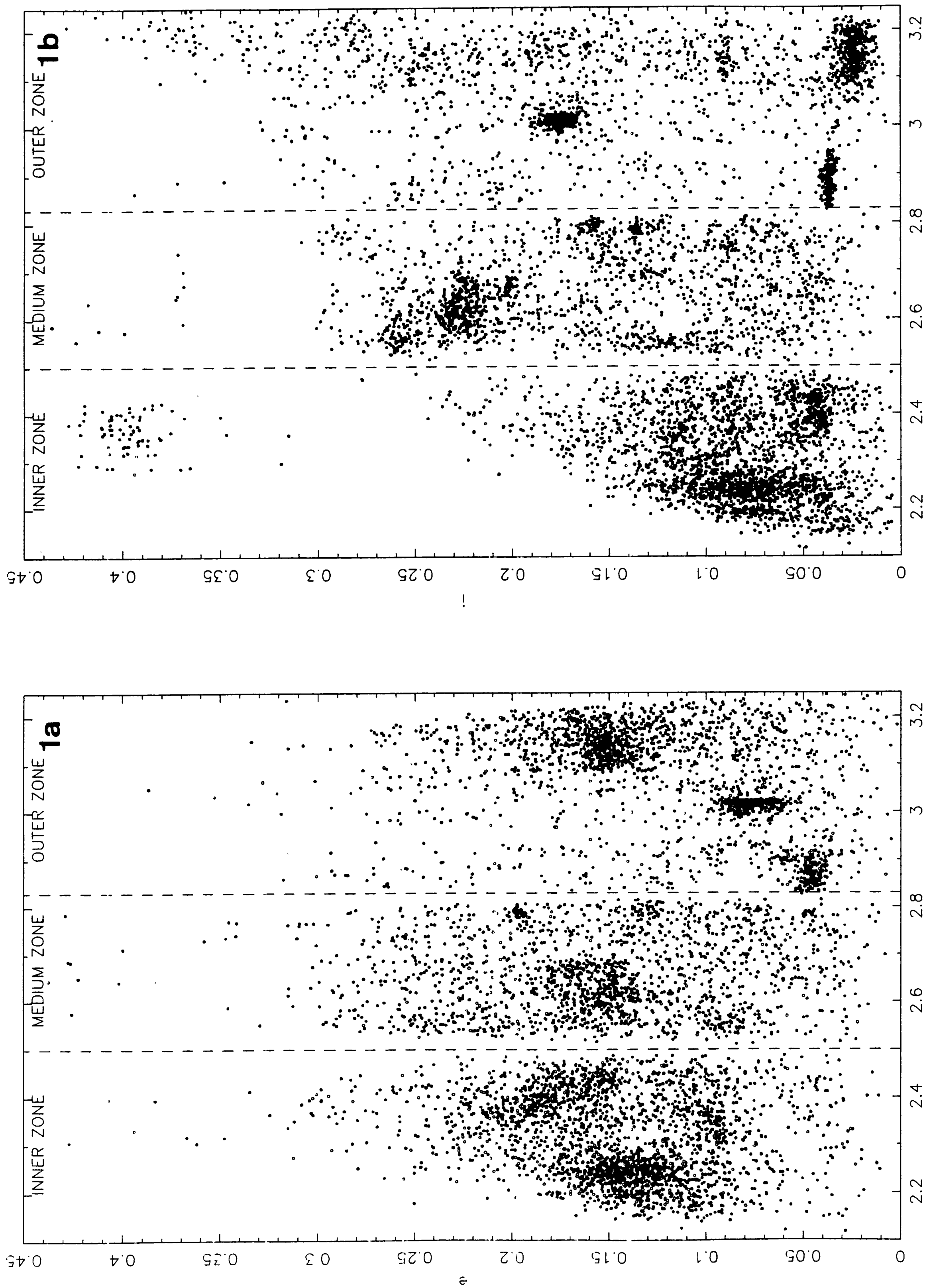

Fig. 1. The whole data set in the (a', e') plane (1.a) and in the plane $(a', \sin i')$ (1.b)

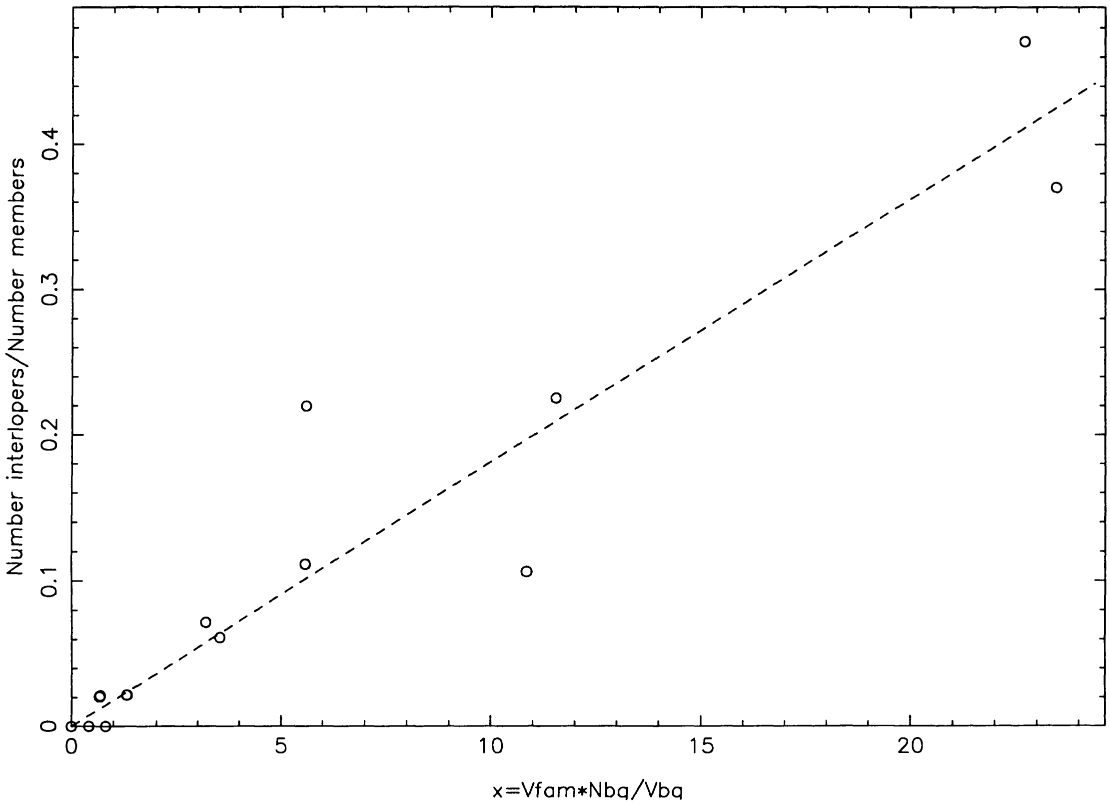

Fig. 2. The percentage of interlopers determined from the simulated families (Bendjoya et al. 1992) as a function of $V_g N_{bg}/V_{bg}$.

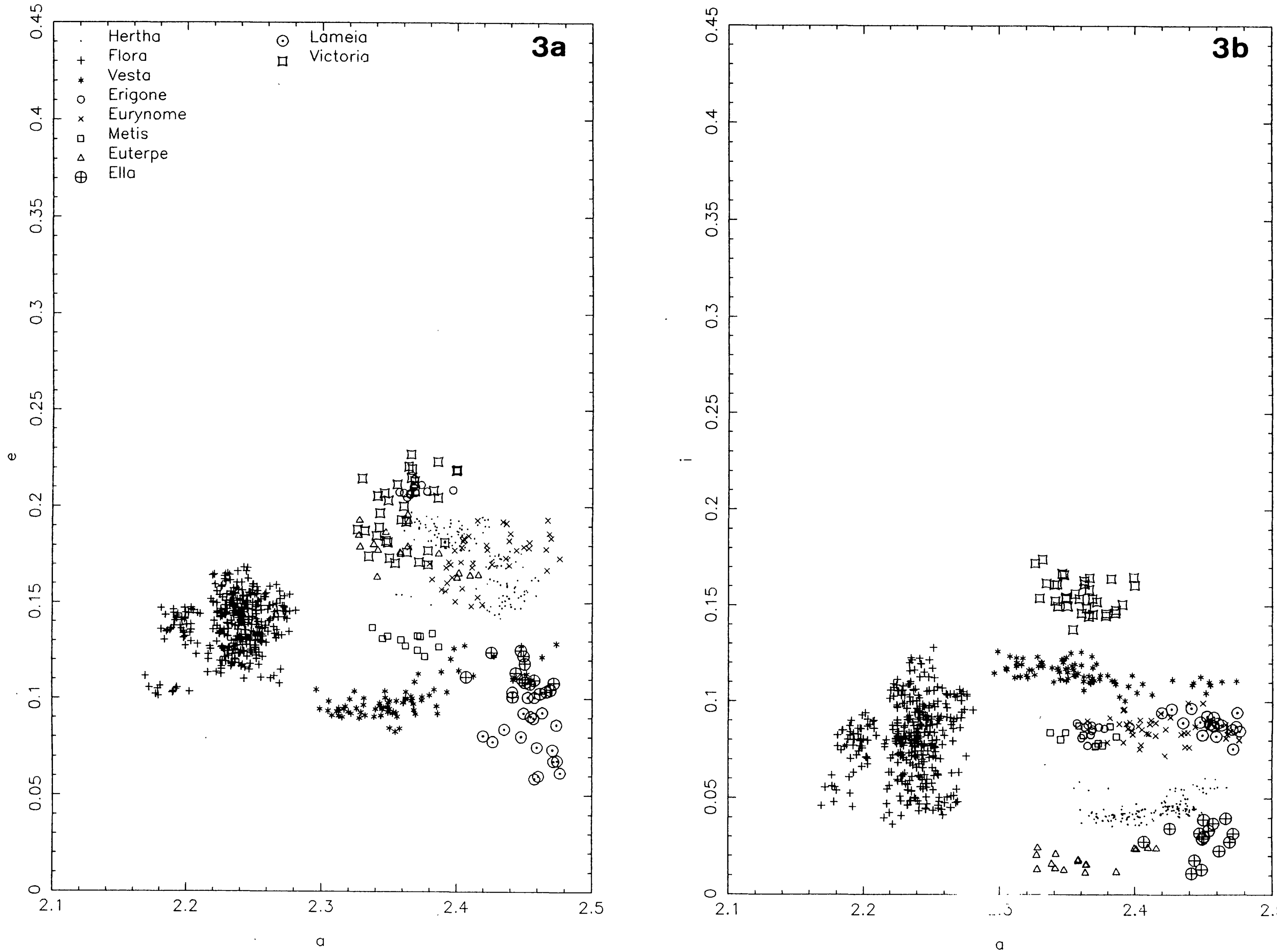

Fig. 3. The dynamical families and the tribes of the inner zone determined with crit. 1 in the (a', e') plane (3.a) and in the plane $(a', \sin i')$ (3.b). The same but by means of all the more relaxed criteria (3.c) and (3.d)

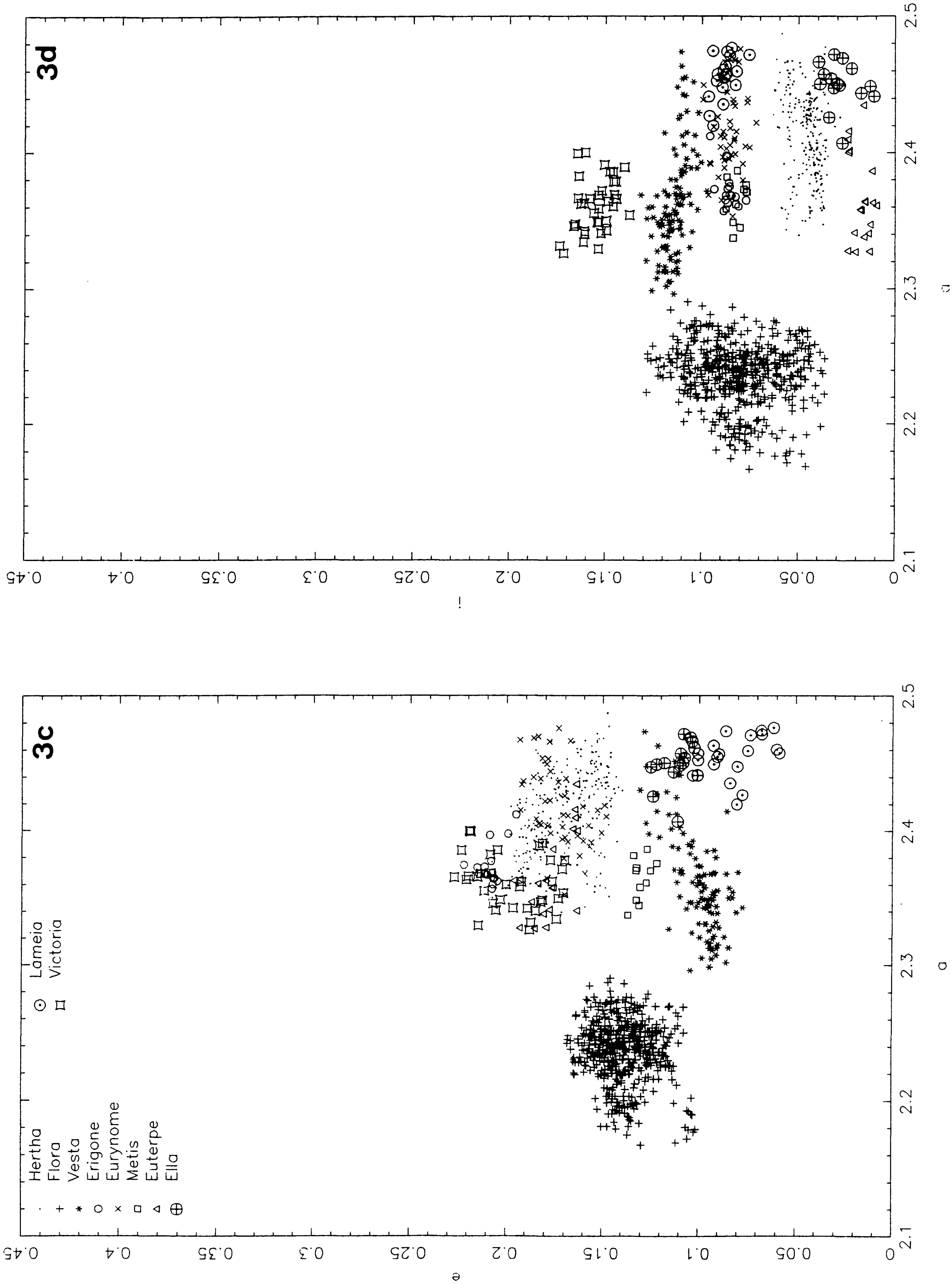

Fig. 3. continued

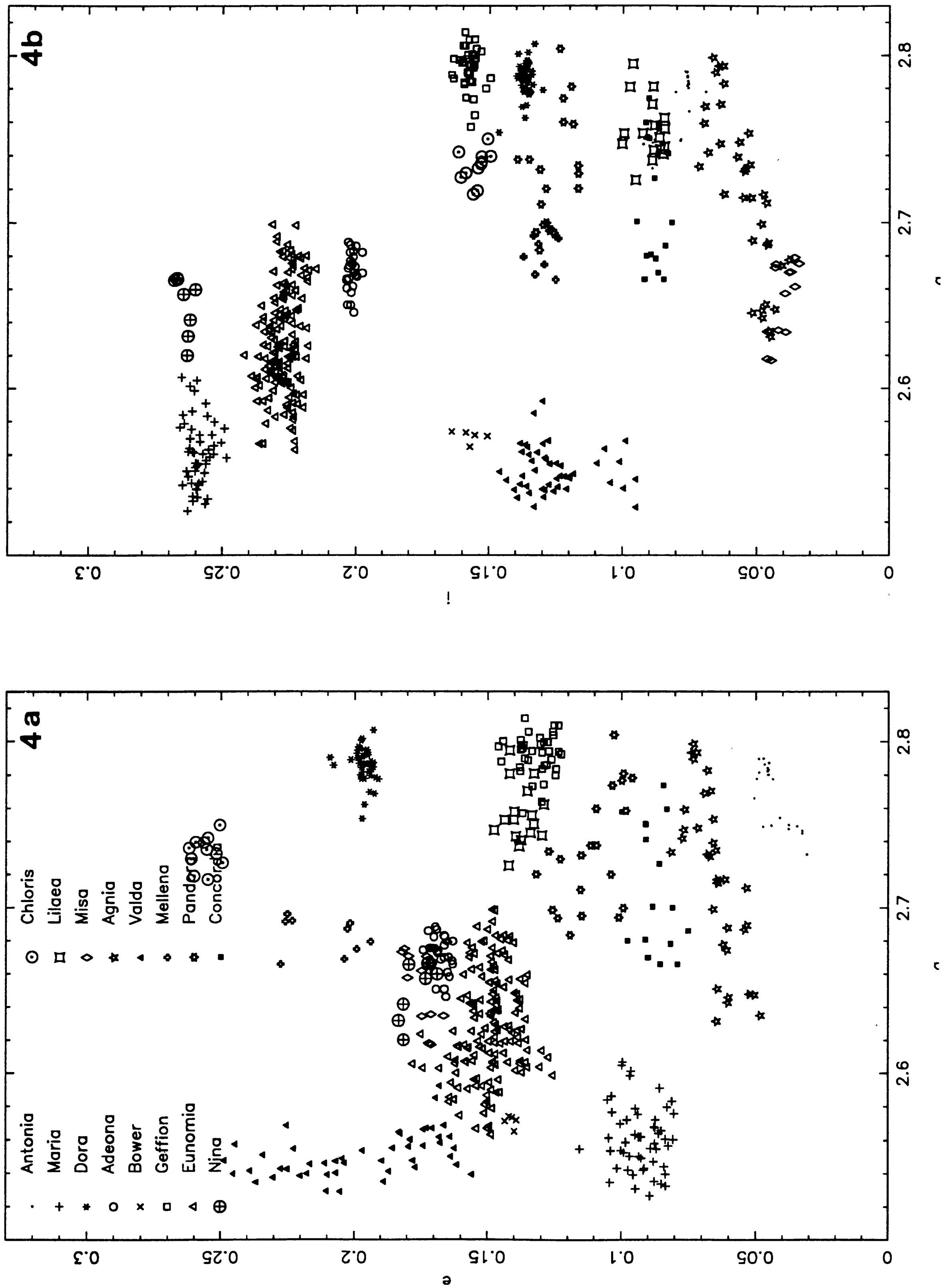

Fig. 4. The same as Fig. 3 but for the medium zone

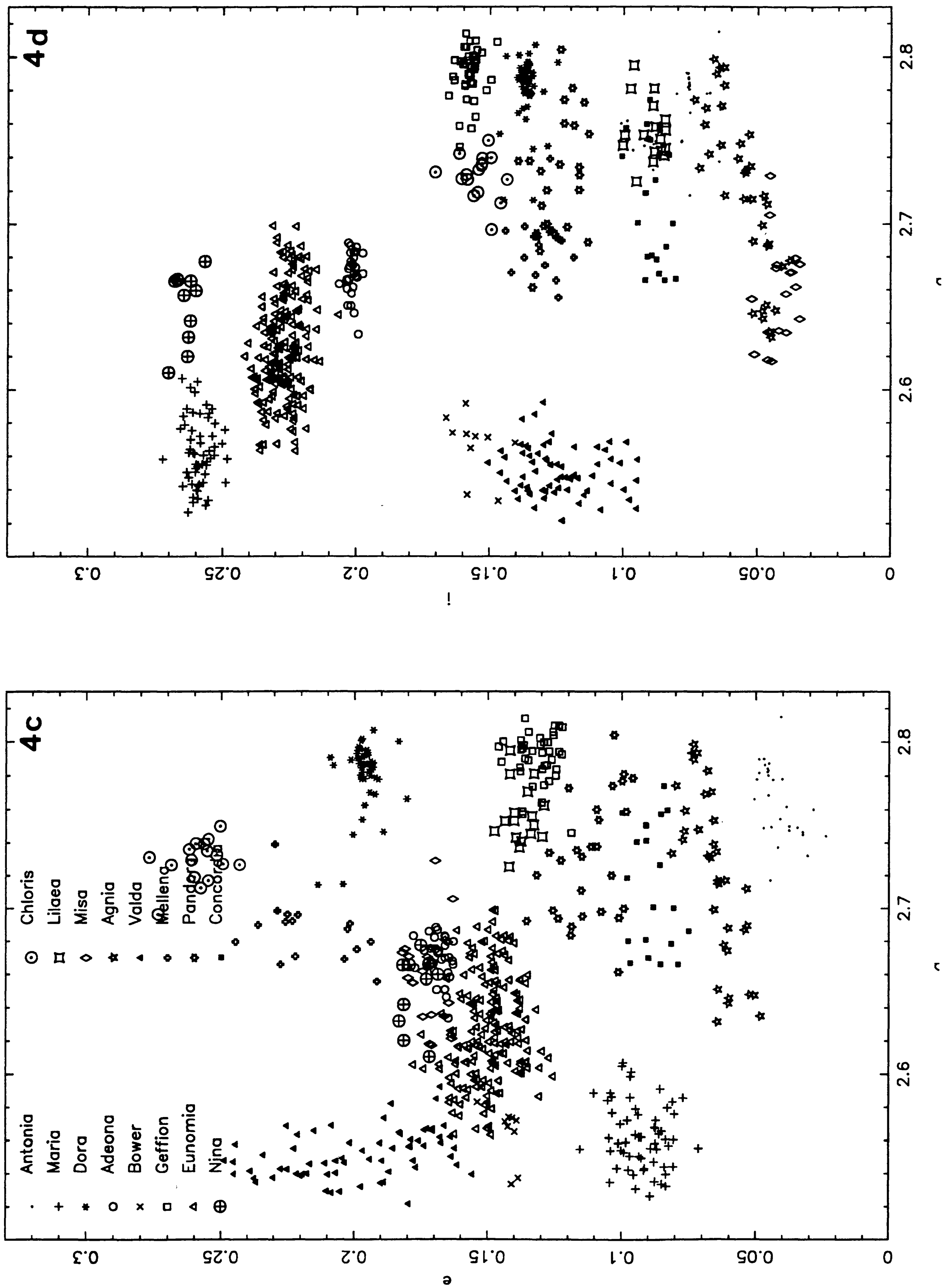

Fig. 4. continued

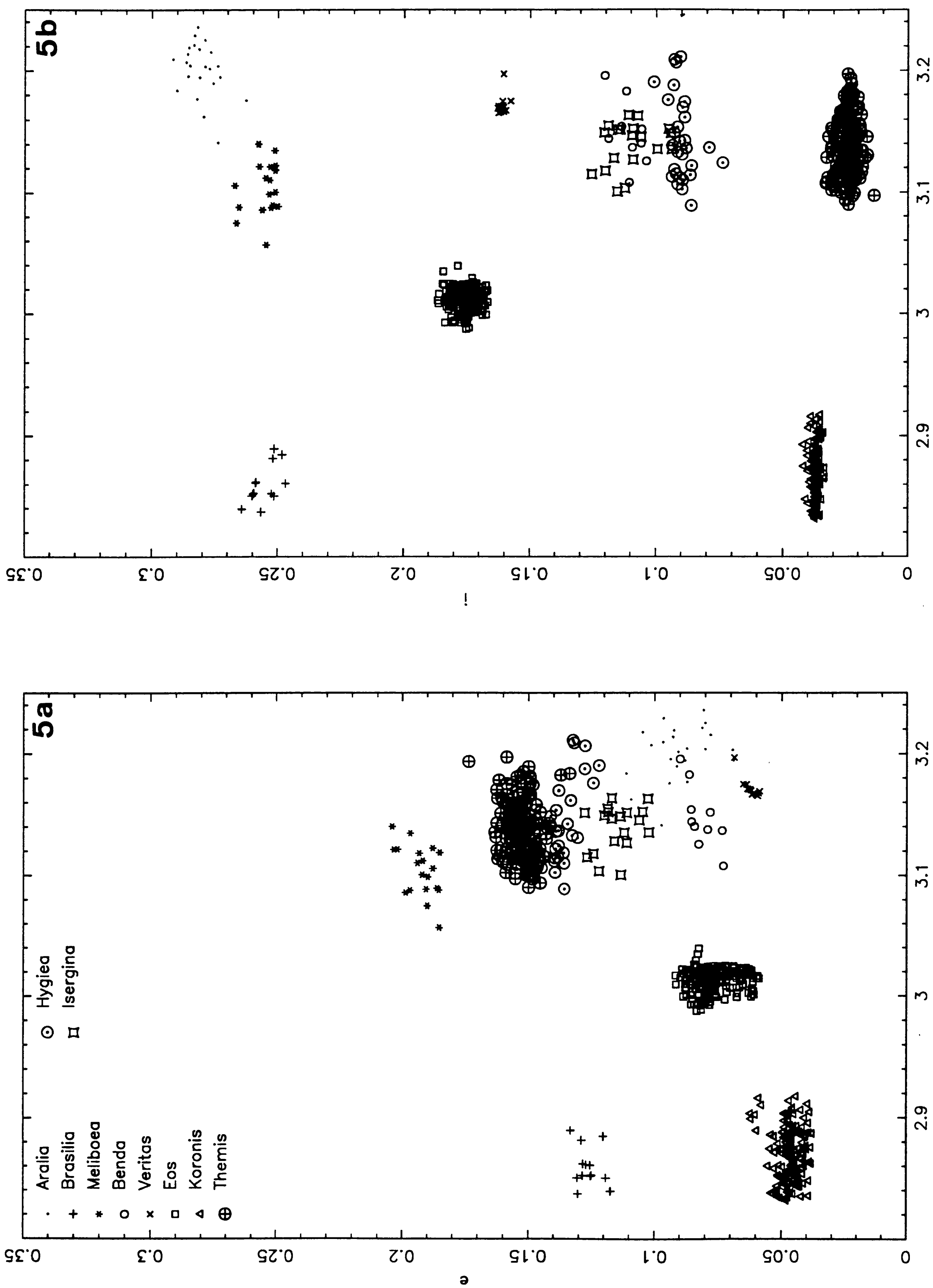

Fig. 5. The same as Fig. 3 but for the outer zone

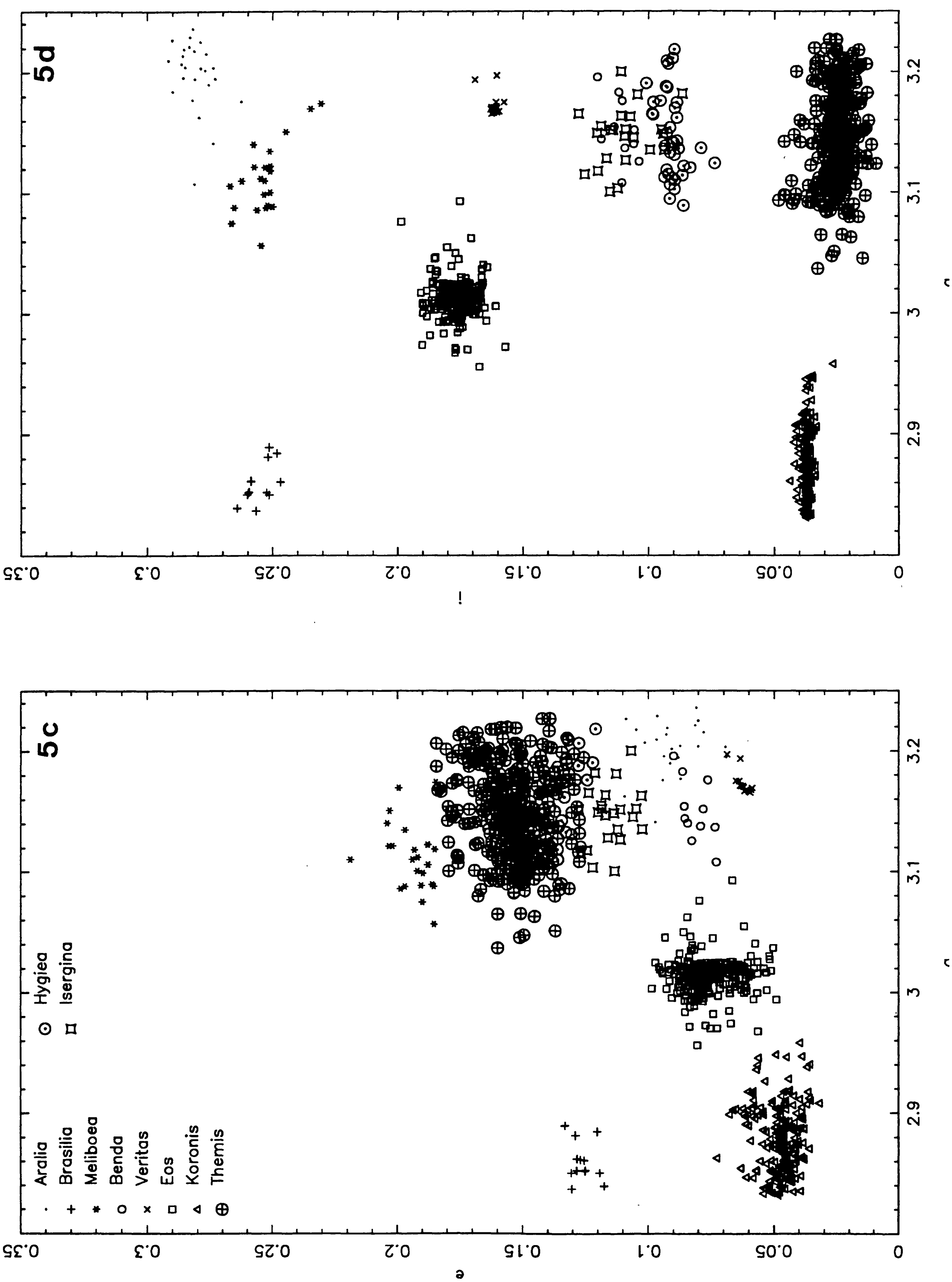

Fig. 5. continued