

S1 and S2 Heart Sound Segmentation Using Variational Mode Decomposition

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Abstract—Heart sound segmentation is the process of identifying the various events like S1, S2, S3, S4, and murmurs present in the phonocardiogram (PCG) signal. In this paper, a heart sound segmentation algorithm for segmenting S1 and S2 heart sound which corresponds to the systolic and diastolic activity of the heart is presented. The proposed method is based on variational mode decomposition (VMD) algorithm. The recorded PCG is decomposed into various modes using VMD. A novel criterion is proposed to select an appropriate mode which is based on center frequency and relative energy. Then the Shannon entropy envelope of the selected mode is computed. S1 and S2 heart sound is segmented by applying dynamic threshold on the Shannon entropy envelope. The segmentation algorithm is tested on both available standard database (Physionet, Pascal, Michigan, and eGeneralMedical) and recorded signal. The simulation results demonstrate that the proposed method achieved the sensitivity, positive predictivity, and the accuracy of above 95% with detection error rate of below 6%.

Keywords: Phonocardiogram (PCG), Variational mode decomposition (VMD), Single channel PCG segmentation.

I. INTRODUCTION

PCG is one of the most widely used physiological signals for cardiac monitoring due to its low cost and ease of acquiring it from the patients. PCG consists the various events like S1 sound, S2 sound, S3 sound, S4 sound and murmurs. The process of identifying various events in the PCG signal is termed as heart sound segmentation. It is one of the important step in automatic cardiac monitoring using PCG. Among the various events, S1 and S2 events are considered important as they correspond the systolic and diastolic activities of the heart [1]. PCG segmentation algorithms in literature can be broadly classified into two types: single channel segmentation and multichannel segmentation. In single channel approaches segmentation is done using only the recorded PCG signal, whereas in multichannel approaches segmentation is done using PCG along with other physiological signals like ECG, PPG and carotid pulse (CP). In [2], a multichannel approach is proposed where segmentation is done using PCG, ECG and CP. In [3], a multichannel approach is proposed where segmentation is done using PCG and carotid pulse. These multichannel approaches offer promising results but requires simultaneous recording of various physiological signals which may not be possible in ambulatory conditions. Therefore the focus of this work is on single channel PCG segmentation.

Initially PCG segmentation algorithms were based on computing simple features like absolute value, energy, Shannon entropy, Shannon entropy and autocorrelation on the recorded

PCG [4]–[9]. However, the performance of these methods deteriorated in the presence of noise and other artifacts. In order to overcome this difficulty, transform domain techniques were considered. In transform domain techniques the PCG signal is first transformed into various time-frequency components or modes using techniques like discrete wavelet transform (DWT), empirical wavelet transform (EWT), empirical mode decomposition (EMD), ensemble empirical mode decomposition (EEMD) and variational mode decomposition (VMD) [10]–[16]. Then various features are extracted from the transformed signal for segmentation. These transform domain techniques offer better performance because noise can be easily suppressed in the transformed domain. In [10], DWT has been used for segmentation and offers better performance, however, choosing the mother wavelet is a challenge. PCG segmentation based on EMD was proposed in [14], but, it suffers from mode mixing problem. In order to overcome this mode mixing problem, PCG segmentation based on EEMD was proposed in [15]. EEMD based heart sound segmentation outperforms the other existing approaches. However, EEMD is recursive and sensitive to noise. PCG segmentation using VMD was recently proposed in [16]. In [16], the PCG signal is first decomposed into various modes using VMD. Then the best mode is selected based on the following selection criteria: The error between the each of the modes and the actual signal is computed. The mode which has the least absolute error is selected. Then Shannon energy envelope is computed from which the S1 and S2 sounds are identified. The disadvantage of [16], is that the selection criterion proposed for selecting the suitable mode is not robust to noise. That is, it may fail for PCG signals corrupted with noise. Also Shannon energy used in [16], may not enhance the low amplitude informative events present in PCG which leads to increase in absolute timing error of start and end time instants of S1 and S2.

Contribution of Paper

In this paper a robust S1 and S2 heart sound segmentation method based on VMD is presented. The robustness is due to the novel selection criterion that is proposed to select the appropriate mode. The proposed selection criterion is based on relative energy and center frequency of decomposed modes. For segmentation, Shannon entropy envelope is computed on the selected mode instead of Shannon energy envelope used in [16]. The Shannon entropy enhances the low amplitude informative events (like S1 and S2) and overcomes the dis-

advantage mentioned earlier. The proposed method is tested on both real time signals and signals taken from standard database (Physionet, Pascal, Michigan and eGeneralMedical). For real time acquisition of PCG, a hardware is designed using microphone (ABM-713-RC) and Arduino-Uno board is used for digitizing the continuous PCG.

This paper is organized as follows: In section II we presented the proposed algorithm of S1 and S2 heart sound segmentation using VMD. In section III we presented the simulation results of the proposed method. In section IV we concluded the contribution of the proposed method and future work.

II. PROPOSED METHODOLOGY

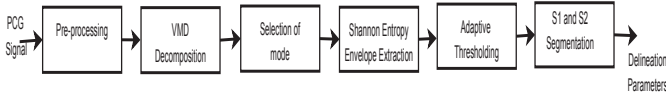


Fig. 1: Illustrates the block diagram of the proposed method.

In this section the proposed VMD based S1 and S2 heart sound segmentation method is presented. The block diagram of the proposed system is shown in Fig 1. From the Fig. 1, it can be seen that the proposed method is broadly a two-step process. In the first step, the PCG signal is decomposed into various modes using VMD and an appropriate mode is selected based on the proposed selection criterion. In the second step, Shannon entropy envelope is computed over the selected mode. The envelope signal is further processed to obtain the delineation parameters (start and end time instants of S1 and S2). These two steps are described in the following subsection.

A. VMD decomposition with proposed mode selection criteria

The recorded PCG signal is subjected to removal of mean and amplitude normalization in preprocessing unit. An example of preprocessed signal is shown in Fig. 2 (a). The preprocessed signal is then applied to VMD algorithm for decomposition into three modes. A brief description about the functionality of VMD algorithm is presented below and detailed information can be found in [17]:

Variational mode decomposition: Variational mode decomposition (VMD) algorithm decomposes the input real valued signal into user-defined number of principal modes. These modes are extracted concurrently and each mode is bandlimited about a center frequency. The ensemble of modes will reconstruct (either exactly or in least squares sense) the input real valued or composite signal.

The input PCG signal $p(t)$ is decomposed into m number of sub-signals or modes, d_m . For each mode d_m , the associated analytic signal is computed by using Hilbert transform to obtain the single sided frequency spectrum. Then, each mode is shifted to baseband by exponential tuning to the respective estimated center frequency. After that the bandwidth is estimated through the squared L^2 norm of the gradient

(i.e, H^1 Gaussian smoothness). Then, as represented in [17], the resulting constrained variational optimization problem is expressed as:

$$\min_{\{d_m\}, \{\omega_m\}} \left\{ \sum_{m=1}^M \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * d_m(t) \right] e^{-j\omega_m t} \right\|_2^2 \right\} \quad \text{s.t.} \quad \sum_{m=1}^M d_m(t) = p(t) \quad (1)$$

where $p(t)$, $d_m(t)$, and ω_m represents the PCG signal, m_{th} decomposed mode and its corresponding center frequency respectively.

As the Eq. (1) is a constrained optimization problem, the classic quadratic penalty term and Lagrangian multipliers, λ are used to render the problem unconstrained [16]. The augmented Lagrangian \mathcal{L} is as follows:

$$\mathcal{L}(\{d_m\}, \{\omega_m\}, \lambda) := \alpha \sum_m \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * d_m(t) \right] e^{-j\omega_m t} \right\|_2^2 + \left\| p(t) - \sum_k d_m(t) \right\|_2^2 + \left\langle \lambda(t), p(t) - \sum_k d_m(t) \right\rangle \quad (2)$$

where α represents the data fidelity controlling parameter. As explained in [17], the minimization problem of Eq. (1) is found as the saddle point of augmented Lagrangian \mathcal{L} (i.e, Eq. (2)) in a sequence of iterative sub-optimizations called alternate direction method of multipliers (ADMM). Then the minimization of Eq. (2) with respect to d_m results in mode update equation as follows:

$$\hat{d}_m^{n+1}(\omega) = \frac{\hat{p}(\omega) - \sum_{i \neq m} \hat{d}_i(\omega) + \frac{\hat{\lambda}(\omega)}{2}}{1 + 2\alpha(\omega - \omega_m)^2} \quad (3)$$

where $\hat{d}_m^{n+1}(\omega)$, $\hat{p}(\omega)$, $\hat{\lambda}(\omega)$, and $\hat{d}_i(\omega)$ are the Fourier transforms of $d_m^{n+1}(t)$, $p(t)$, $\lambda(t)$, and $d_i(t)$ respectively.

Similarly as explained in [17], the minimization of Eq. (2) with respect to ω_m results in center frequency update equation as follows:

$$\omega_m^{n+1} = \frac{\int_0^\infty \omega |\hat{d}_m(\omega)|^2 d\omega}{\int_0^\infty |\hat{d}_m(\omega)|^2 d\omega} \quad (4)$$

By using the aforementioned algorithm presented in [17], the recorded PCG is decomposed into three modes. The values of parameters used for decomposition using VMD is presented in Table I. An example of decomposition of modes and their corresponding frequency spectrum are shown in Fig. 2. From the frequency spectrum of Fig. 2, it can be observed that, lower modes consists the more information about low frequency components present in the signal. In higher modes (see ‘mode2’ and ‘mode3’ frequency spectrum in Fig. 2), the high frequency components are distracting the low frequency information of heart sound signal. So the selection of suitable

mode for heart sound segmentation is the challenging task to be performed.

From Fig. 2, it can be observed that the lower modes consists the low center frequency value, whereas higher modes consists high center frequency value. Another observation from Fig. 2 is, the energy of the higher modes which consists high frequency components is less than the energy of the lower modes which consists the lower frequency components. From this observation, a novel criterion of selection of suitable mode is proposed. The criterion is, the mode which consists maximum relative energy and center frequency of the range between 10-70 Hz (usually S1 and S2 are within the range of 10-50 Hz [18]) is selected for further processing to segment the heart sound signal. From Fig. 2, it can be observed that the first mode is selected which satisfies the proposed criterion. The proposed selection of mode is tested on noisy PCG and abnormal PCG which are shown in Fig. 4, and Fig. 6.

B. Shannon entropy based heart sound segmentation

In this subsection, Shannon entropy based S1 and S2 heart sound segmentation is presented. Here for the notation convenience, the selected first mode is denoted as discrete signal $p_1[n]$. The signal $p_1[n]$ is subjected to a non-linear amplitude transformation to emphasize the informative amplitude levels present in the $p_1[n]$. In this work, Shannon entropy is considered for non-linear amplitude transformation. The rationale behind choosing Shannon entropy is, it has ability to enhance the informative low amplitude segments (S1 and S2 sound) of the heart sound signal. However, this feature will also enhance the low amplitude noise segments present in the heart sound signal. Therefore $p_1[n]$ is subjected to a fixed threshold to suppress the noise. Let $p_{th}[n]$ be the signal after thresholding. Then Shannon entropy is computed as,

$$S_{et}[n] = -|p_{th}[n]| \log(|p_{th}[n]|), \quad (5)$$

The Shannon entropy envelope $S_{et}[n]$ is smoothed using a zero phase forward and reverse filtering. For the zero phase forward and reverse filtering a rectangular window of length 50 ms with overlap of 1 ms is used. The selected first mode PCG and smoothed Shannon entropy envelop (SSEE) are shown in Fig. 3. (a) and Fig. 3. (b) respectively. A gated signal is obtained by applying an adaptive thresholding (mean value of SSEE) on the smoothed Shannon entropy envelope and it is shown in Fig. 3. (c). In Fig. 3. (c), the gated signal is shown in red color. For enhancing the informative events, $p_1[n]$ is multiplied with the obtained gating signal. The gating signal is subjected to derivative filter (first order forward difference (FOFD)) to emphasize large slope between the successive points of the gating signal. The resultant signal will consists of impulses with alternative positive and negative impulses. The time instants of impulses are projected onto the $p_1[n]$ and shown in Fig. 3. (d) in red colored circles. These time instants are termed as delineation parameters of heart sound signal, which represents the start and end time instants of S1 and S2 heart sound shown in Fig. 3. (d).

TABLE I: Parameter values used for VMD

Name of the parameter	Value
Data fidelity balancing parameter (α)	10000
Number of modes (M)	3
Time step (τ)	0
DC	0
Initial values of center frequencies of modes	0
Tolerance of convergence	10^{-6}

III. RESULTS AND DISCUSSION

In this section the simulation results of the proposed S1 and S2 heart sound segmentation is presented. The proposed method is tested in three different experiments. In experiment 1, the proposed VMD based heart sound segmentation is tested on the real-time normal PCG signal. In experiment 2, the proposed method is tested on noisy PCG. In experiment 3, the proposed method is tested on abnormal PCG. Real-time database is recorded from the self designed heart sound acquisition system. Physionet, Pascal, Michigan, and eGeneralMedical heart sound databases are also used for testing the proposed heart sound segmentation method.

A. Experiment 1:

In this experiment, the segmentation of recorded normal PCG signal is presented. The simulation results of the proposed method on recorded normal PCG signal is shown in Fig. 2. and Fig. 3. As explained earlier, the normal PCG signal is decomposed using VMD into three modes as shown in Fig. 2 (a)-(d). The proposed novel criterion based on relative energy and center frequency of modes is performed and first mode is selected as a dominant mode. The selected dominant mode is shown in Fig. 2 (e). Then the Shannon entropy envelopogram (SEE) is computed on the dominant mode and by using zero phase forward and reverse digital filtering SEE is smoothed. The dominant mode and smoothed SEE are shown in Fig. 3 (a) and Fig. 3 (b) respectively. An adaptive thresholding is applied on smoothed SEE to obtain gating signal and it is shown in Fig. 3 (c) (gating signal is shown in red color). In the final step, the first order forward difference is performed on obtained gating signal which results the start and end points of S1 and S2 heart sound events present in the normal PCG signal. The dominant mode along with the detected start and end time instants (red colored circles) of S1 and S2 heart sound is shown in Fig. 3 (d).

B. Experiment 2:

In this experiment, the recorded normal PCG signal is undergone for noise stress test. An additive white Gaussian noise (SNR=0dB) is added to the recorded normal PCG signal. The simulation results of the proposed method on noisy heart sound signal is shown in Fig. 4 and Fig. 5. As explained in experiment 1, the noisy PCG is decomposed into three modes by using VMD shown in Fig. 4 (a)-(d). From the frequency spectrum of modes shown in Fig. 4 (g)-(i), it can be observed

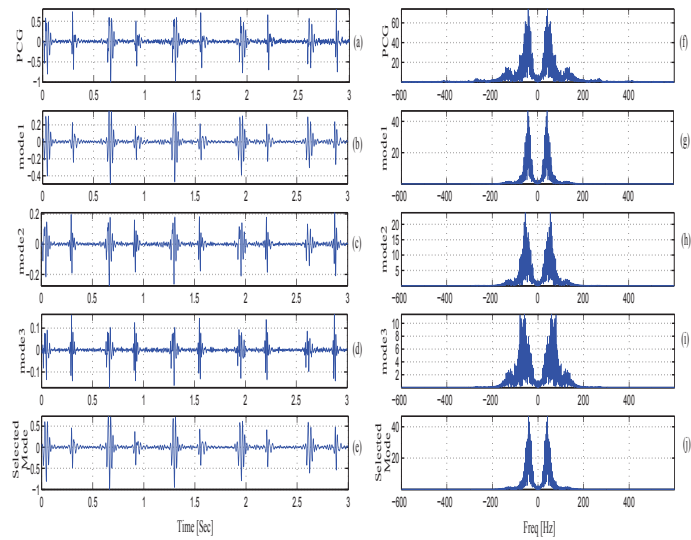


Fig. 2: Illustrates the VMD based decomposition and selection of mode of recorded normal PCG. (a) Recorded PCG signal. (b)-(d) VMD based decomposed modes of PCG. (e) Selected mode. (f)-(j) Frequency spectrum of (a)-(e).

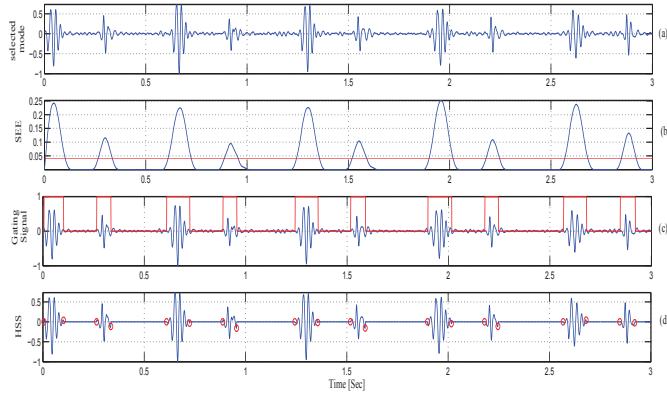


Fig. 3: Illustrates the Shannon entropy based S1 & S2 heart sound segmentation from selected mode in Fig. 1. (a) Selected mode of recorded normal PCG. (b) Smoothed Shannon entropy envelopogram of selected mode with mean as threshold (represented in red colored horizontal line). (c) Selected mode along with gating signal (red colored signal). (d) PCG with start and end points of S1 and S2 (pointed with red circles).

that the S1 and S2 sound components are accumulate with the lower modes, whereas the high frequency additive white Gaussian noise components are accumulate with the higher modes. Hence, this experiment highlighted the robustness of VMD to the high frequency noise. Then the dominant mode is selected based on the proposed center frequency and relative energy criterion and is shown in Fig. 4 (e). The selection of appropriate mode under this scenario may fails in the existing work mentioned in [16]. The dominant mode is further

processed to Shannon entropy based S1 and S2 heart sound detection method as explained in experiment 2. The simulation results of detected start and end time instants of S1 and S2 events from the dominant mode is shown in Fig. 5.

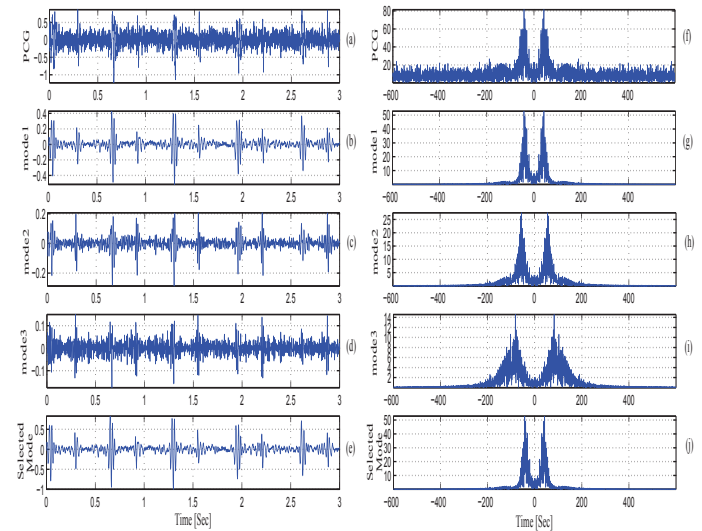


Fig. 4: Illustrates the VMD based decomposition and selection of mode of recorded PCG with AWGN (SNR=0 (dB)). (a) Recorded PCG with AWGN. (b)-(d) VMD based decomposed modes of (a). (e) Selected mode. (f)-(j) Frequency spectrum of (a)-(e).

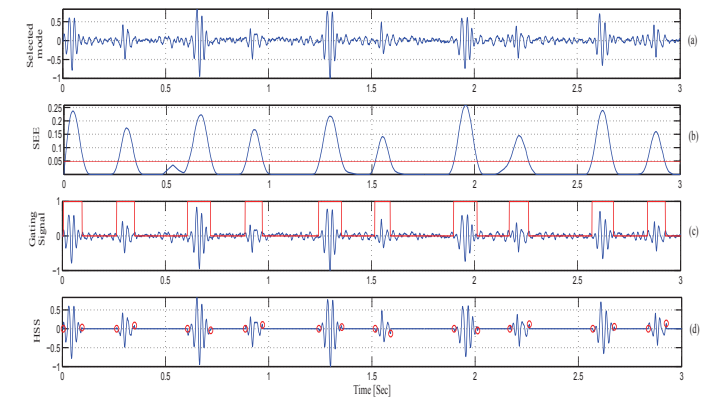


Fig. 5: Illustrates the Shannon entropy based S1 & S2 heart sound segmentation from selected mode in Fig. 3. (a) Selected mode of noisy PCG. (b) Smoothed Shannon entropy envelopogram of selected mode with mean as threshold (represented in red colored horizontal line). (c) Selected mode along with gating signal (red colored signal). (d) PCG with start and end points of S1 and S2 (pointed with red circles).

C. Experiment 3:

In this experiment, the segmentation of S1 and S2 heart sound in the abnormal PCG is discussed. The abnormal PCG is collected from the eGeneralMedical heart sound database and

it consists the systolic murmur. As discussed in experiment 1, the abnormal PCG is decomposed into three modes using VMD. The abnormal PCG and its decomposed modes are shown in Fig. 6 (a)-(d). From the frequency spectrum of decomposed modes shown in Fig. 6 (g)-(i), it can be observed that the high frequency murmurs are associated with the higher decomposed mode, whereas the S1 and S2 sounds associated with the lower mode. The dominant mode is selected by the proposed criterion and is shown in Fig. 6 (e). Then the Shannon entropy based heart sound segmentation explained in experiment 1 is performed on the obtained dominant mode and the results are shown in Fig. 7. The start and end time instants of abnormal PCG is shown in Fig. 7 (d).

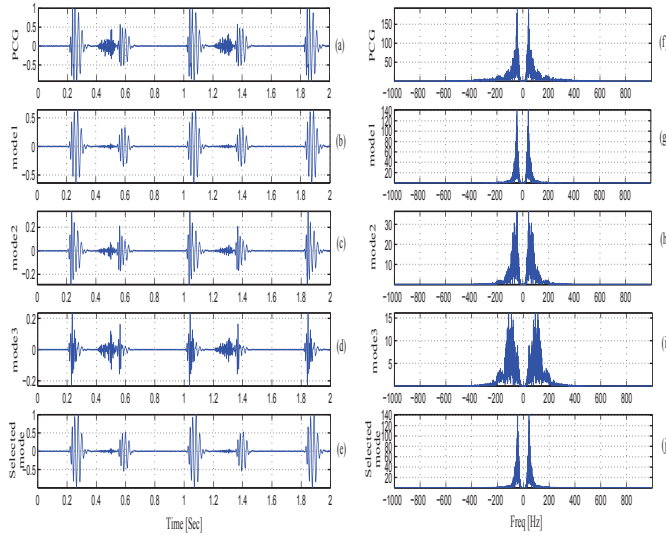


Fig. 6: Illustrates the VMD based decomposition and selection of mode of abnormal PCG collected from eGeneral (Subject No. 2). (a) Abnormal PCG . (b)-(d) VMD based decomposed modes of (a). (e) Selected mode. (f)-(j) Frequency spectrum of (a)-(e).

The performance metrics of the proposed S1 and S2 heart sound segmentation is presented in Table II. The proposed method is tested on real-time database acquired from the self designed hardware of PCG acquisition system. The proposed method also tested on standard databases like Physionet, Pascal, Michigan, and eGeneralMedical. A total number of 500 segments is tested by using the proposed algorithm. Here, the following benchmark performance metrics are used to evaluate the performance of the proposed algorithm: sensitivity (Se), positive predictivity (+P), accuracy (Acc.) and detection error rate (DER). The performance metrics are computed by,

$$Se.(%) = \frac{TP}{TP + FN} \times 100 \quad (6)$$

$$+P.(%) = \frac{TP}{TP + FP} \times 100 \quad (7)$$

$$Acc.(%) = \frac{TP}{TP + FP + FN} \times 100 \quad (8)$$

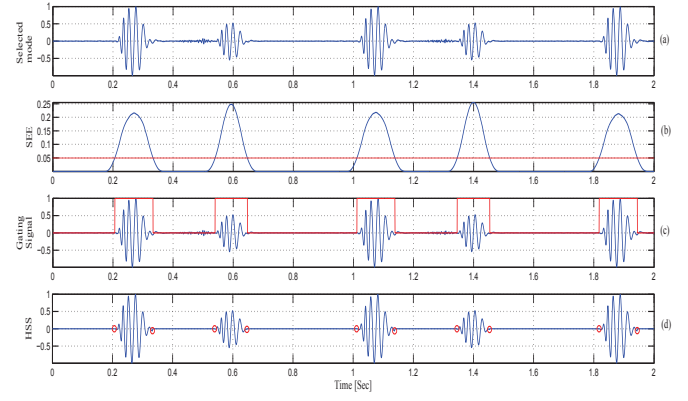


Fig. 7: Illustrates the Shannon entropy based S1 & S2 heart sound segmentation from selected mode in Fig. 3. (a) Selected mode of abnormal PCG. (b) Smoothed Shannon entropy envelopogram of selected mode with mean as threshold (represented in red colored horizontal line). (c) Selected mode along with gating signal (red colored signal). (d) PCG with start and end points of S1 and S2 (pointed with red circles).

$$DER.(%) = \frac{FP + FN}{TP} \times 100 \quad (9)$$

where, ‘TP’ indicates true positive, ‘FP’ indicates the false positive and ‘FN’ indicates the false negative. In the performance metrics table ‘sub.’ indicates the subject and ‘Seg.’ indicates the number of segments in the particular subject.

Here, ‘Se.’ indicates the sensitivity of the proposed algorithm, which means that the percentage of correctly detected PCG segments out of the total segments present in the original PCG signal. Similarly, ‘+P’ indicates the positive predictivity of the proposed algorithm, which means that the percentage of correctly detected segments out of the total number of segments identified by the proposed algorithm.

TABLE II: Performance metrics of proposed method.

Database	Seg.	TP	FP	FN	Se	+P	Acc.	DER
Physionet	100	98	4	1	98.90	96.07	95.14	5.1
Pascal	100	99	0	1	99	100	99	1
Michigan	100	100	0	0	100	100	100	0
eGeneral	100	100	0	0	100	100	100	0
Realtime	100	100	3	0	100	97.08	97.08	3

IV. CONCLUSION

In this work VMD based S1 and S2 heart sound segmentation is performed. A novel criterion for selecting appropriate mode for heart sound segmentation is proposed. In the part of the work, a hardware is developed for acquisition of normal PCG signal. The simulation results of the proposed method carried out on real-time normal PCG, PCG with noise and abnormal PCG collected from standard database, and it is validated with the performance metric table. The proposed algorithm achieved the sensitivity, positive predictivity and the accuracy of above 95% with considerably low detection error rate of below 6%. In future work VMD based murmur

identification will be contributed for heart sound segmentation frame work.

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