Deep-Sea Research, 1968, Vol. 15, pp. 497 to 503. Pergamon Press. Printed in Great Britain.

INSTRUMENTS AND METHODS

Calibration curves for thermistors

JOHN S. STEINHART* and STANLEY R. HART*

(*Received* 15 *April* 1968)

Abstract—A function for use in interpolating thermistor calibrations has been found which is suitable for use with a wide variety of thermistors for ranges of a few degrees to a few hundred degrees. It is

$$T^{-1} = A + B \log R + C (\log R)^3$$

where T is the Kelvin temperature, R the resistance, and A, B and C constants. This function possesses a number of desirable properties and fits thermistor data considerably better than many such functions now in use.

INTRODUCTION

THERMISTORS, in recent years, have become by far the most common devices for the precision measurement of temperature in limnology and oceanography and their use is widespread in many other fields as well. Because they are inexpensive, while still retaining high resolution of temperature, they are commonly used in the measurement of heat flow, small temperature differences in currents, and in the study of long-term temperature variation in deep water as well as the microstructure of the temperature gradient. These applications and others frequently require precision of 0·001°C and multiple sensors which are to be intercompared. Calibration procedures to maintain laboratory precision sufficient to produce field measurements to 0·001°C are well known and demanding, but the fitting of curves to calibration data remains a difficult problem. It is to this problem we address ourselves.

The customary representation for temperature dependence of resistivity, ρ, in a semiconductor is

$$\rho^{-1} = F(T) \exp(-\Delta E/2kT) \tag{1}$$

where ΔE is the energy gap between the valence band energies and the conduction band energies, T is the Kelvin temperature, and k is Boltzman's constant. $F(T)$ is itself a function of temperature; thus even in simple compounds the theory does not provide a suitable, explicit law to which calibration data may be fitted. ROBERTSON *et al.* (1966) point out that the spinel structure of thermistors used in precision temperature measurement does not have simple, sharply defined energy bands and the ΔE term is consequently in doubt, even if the composition is accurately known.

Several attempts have been made to find empirical " laws " relating temperature to resistance for thermistors (BECKER, GREEN and PEARSON, 1946 ; BOSSON, GUTMANN and SIMMONS, 1950; ROBERTSON *et al.*, 1966). We find it more fruitful to treat the problem simply as one of curve fitting and searched for a relationship which would fit the data as well as possible but was still simple to use. To this end we set up criteria for choice of a relationship which would be most desirable for interpolation of temperature measurements.

These criteria, with reasons for their choice, are as follows :

(1) A single smoothly varying relationship for the entire temperature range. A common practice at many institutions consists of fitting and smoothing a series of functions (or the same function several times) for the total range of temperature involved. This technique can produce very close fits but may disguise systematic deviations in some subregion of the temperature span. Ideally these deviations should be revealed as consistent residuals.

*Department of Terrestrial Magnetism, Carnegie Institution of Washington, Washington, D.C., U.S.A.

(2) Absence of the " plus-minus-plus " effect. This is another way of saying that a proposed fitting function must have the right shape. For example, in a very small temperature span the simple exponential relationship $\log R = A + BT^{-1}$ (A, B are constants) will produce satisfactorily small residuals but the residuals will have a consistent bias of sign (plus-minus-plus) and for experimental data this may be the only clue that the curve is simply the wrong shape for the data even though the residuals are small.

(3) The Kelvin temperature an explicit function of the resistance. This criteria is desirable because the fitted function will be used to calculate the temperature for a measured resistance. In a computer this is not a serious problem, although it would be a slight advantage in ease of calculation and in time used, but should it be necessary to calculate a few temperatures on a desk calculator or slide rule, algebraic reduction of implicit formulae can be very time consuming.

(4) A stable, relatively simple mathematical form. Simplicity of form is obviously desirable on aesthetic grounds, but is also required so that it may be easily determined that the fitted calibration curve does not contain unwanted maxima, minima, points of inflection or any singularities in the region of interest. Further, a chosen function ought not to change much if the number of points used in fitting is changed. Of the many kinds of functions used in curve fitting, some have the unfortunate property of great sensitivity to addition or removal of a point in a particular region. It is this property we wish to avoid.

(5) The chosen function must admit of linear fitting procedures. This property is desirable so that least squares procedures may be used and goodness of fit compared, even if the statistics of fit are not formally valid as a maximum likelihood estimate.

In no sense are we proposing in this way to find a " law " of thermistor behavior. The search is for a satisfactory linearization of the problem satisfying as many of the above criteria as possible. If such a function can be found it can be tested for suitability as an empirical law by examining how well it is able to extrapolate values outside the calibration range. Failure to predict accurately in extrapolation would show that a function was a poor " law " no matter how well it worked for interpolation.

DATA

As data for study of the fitting functions, we used calibration data on about twenty thermistors over a 7 deg C range (0°–7°C), two thermistors over a 35 deg C range (0°–35°C), and published data of Bosson, Gutmann and Simmons (1950) for two thermistors over ranges of 170 deg C and 200 deg C (hereafter referred to as BGS data). These results were rechecked for our own thermistors with two sets of later calibration data, and tried on some calibration data kindly supplied to us by other workers (Simmons, private communication : Jessop, private communication). All results were in general agreement : thus for purposes of illustration only three sets of representative data will be discussed (Table 1).

Table 1. *Pertinent facts for the thermistors.*

	T3	S4	BGS
Maker	*Fenwall*	*Fenwall*	*Western Electric*
Identification	G244H	K824A	4756-12 No. 1 material
Type	Glass bead	Glass bead	Unknown
Calibration range	0°–7°C	0°–35°C	– 68°–134°C

Our calibration data were obtained from a laminar flow constant temperature bath which will maintain ± 0·002°C for 10–20 min and ± 0·0004°C for 2 min or more. Calibration was against a quartz oscillator thermometer with a resolution of 0·0001 deg C, and an estimated precision of ± 0·0003 deg C. For T3 the quartz oscillator and the thermistor were inserted in the same copper block within the constant temperature bath. The time constants of the thermistor assembly and the quartz thermometer are both about 1 sec. For thermistor S4 two different quartz oscillators were employed during the calibration and the copper block was not used. The estimated precision for this calibration set is ± 0·002 deg C.

Absolute accuracy is much poorer than these figures but is not important for these tests. Non-linearity of frequency vs. temperature could be significant and at these levels of precision is difficult to assess. However, we used three different oscillators during the tests and found no significant difference in the residuals which could be interpreted as deviations from linearity.

PROCEDURE

With equation (1) as a guide we examined about 100 different relationships between resistance and temperature with from two to five fitted constants. In each case a multiple regression program was used to test the relationship against several sets of calibration data. We soon found that criteria (3) could be satisfied and attention was concentrated on functions which were explicit functions of temperature. Of four or five reasonably good fits, the one consistently the best is

$$T^{-1} = A + B \log R + C (\log R)^3 \tag{2}$$

where T^{-1} is the inverse Kelvin temperature, R is the resistance, and A, B, and C constants to be fitted. For most experimental data the resistance (or the temperature) may not enter an expression more than twice without encountering serious computational difficulties. This results from the determinant of the coefficient matrix in the regression analysis becoming numerically a very small number, and shows that, even though the terms (of a power series for example) may be formally independent, the presence of real scatter in the data makes the terms appear dependent for fitting purposes. For data of very high quality and wide temperature span three or occasionally four entries of resistance may be used and we determined from these cases that the addition of the $(\log R)^2$ term in equation (2) degraded the fit.

RESULTS

The fit of equation (2) to thermistors T3 and S4, listed in Table 1, is shown in Table 2. The numerical coefficients are given in the Appendix. Clearly the fit is good and the mean residuals are approximately the same size as the experimental precision.

Table 2. *Results of fitting equation* (2).

Thermistor T3				*Thermistor* S4			
R_{obs} (VA^{-1})	T_{obs} (°C)	T_{est} (°C)	Obs-est (°C residual)	R_{obs} (VA^{-1})	T_{obs} (°C)	T_{est} (°C)	Obs-est (°C residual)
6304·8	− 0·0046	− 0·0005	− 0·00002	5088·45	− 0·0070	− 0·0037	− 0·00327
5716·8	2·1856	2·1857	0·00006	5087·9	0·0000	− 0·0011	0·00117
5394·0	3·5004	3·4999	0·00047	4581·9	2·4845	2·4812	0·00329
5325·5	3·7905	3·7904	0·00017	4397·4	3·4645	3·4657	− 0·00121
5277·4	3·9970	3·7904	0·00014	4357·1	3·6864	3·6870	− 0·00067
5215·1	4·2675	4·2675	0·00000	4247·5	4·3032	4·3011	0·00202
5168·6	4·4715	4·4719	− 0·00038	3813·1	6·9282	6·9278	0·00035
5046·7	5·0174	5·0179	− 0·00045	3504·3	9·0130	9·0138	− 0·00080
4663·8	6·8358	6·8357	0·00005	3236·5	11·0004	11·0019	− 0·00151
4616·0	7·0747	7·0746	0·00009	2880·7	13·9579	13·9582	− 0·00032
				2569·1	16·9169	16·9163	− 0·00053
				2293·3	19·9040	19·9034	0·00058
				1999·7	23·5792	23·5785	0·00066
				1787·6	26·6461	26·6466	− 0·00054
				1573·7	30·2011	30·2019	− 0·00083
				1454·5	32·4361	32·4364	− 0·00033
				1334·6	34·9111	34·9102	0·00087
Mean residual 0·00018 without regard to sign				Mean residual 0·00111 without regard to sign			

Table 3 shows the results of fitting equation (2) to the Bosson, Gutmann and Simmons data. To show the degree of improvement, the results from equation (2) are compared with the " law " proposed by Bosson, Gutmann and Simmons and presently in use by ROBERTSON *et al.* (1966).

Table 3. Comparison of fits for two equations.

Observed temperature (°K)	*Observed resistance* (VA^{-1})	BGS *equation* $\log R = A + B/(T + \theta)$ % discrepancy $= \frac{100\,(R_{est} - R_{obs})}{R_{obs}}$ (%)	*Equation* (2) $T^{-1} = A + B \log R + C (\log R)^3$ % discrepancy $= \frac{100\, T_{obs} - T_{est})}{T_{obs}}$ (%)
204·71	10,558,700	+ 0·61	− 0·011
209·17	7,368,300	+ 1·06	+ 0·036
216·95	4,130,100	− 0·18	− 0·015
224·16	2,465,400	− 0·36	− 0·011
232·54	1,398,800	− 0·51	− 0·010
240·05	864,970	− 0·56	− 0·009
247·74	540,510	− 0·28	+ 0·010
257·18	315,040	− 0·44	− 0·005
266·48	190,430	− 0·27	+ 0·001
276·34	115,210	− 0·05	+ 0·007
286·27	71,585	+ 0·12	+ 0·011
296·35	45,383	+ 0·37	+ 0·020
308·54	27,163	+ 0·31	+ 0·006
320·24	17,104	+ 0·44	+ 0·010
331·57	11,289	+ 0·04	− 0·027
344·07	7297·4	+ 0·19	− 0·014
357·16	4761·1	+ 0·25	− 0·004
369·36	3282·5	+ 0·10	− 0·008
382·29	2263·8	0·00	− 0·002
394·74	1615·7	− 0·09	+ 0·010
407·17	1177·6	− 0·37	+ 0·007

Standard relative error, BGS equation : $\left[\frac{\Sigma (R_{est} - R_{obs})^2/R^2}{21}\right]^{\frac{1}{2}} = 0{\cdot}00395$

Standard relative error, equation (2) : $\left[\frac{\Sigma (T_{est} - T_{obs})^2/T_{obs}^2}{21}\right]^{\frac{1}{2}} = 0{\cdot}00016$

Because of the different form of the two equations quantitative comparisons are not exact but the improvement in fit of equation (2) over the BGS equation is a factor of 10 or more. The mean residual for the equation (2) fit is 0·03°K and is about the same as the experimental precision quoted by Bosson *et al.* for their data; (temperatures $\pm$ 0·02 deg K; resistance $\pm$ 0·02%). From all three thermistors we thus conclude that it is not possible to find an equation that fits more closely unless more precise data are available. We did find in our search several relations that fitted almost as well, but none was consistently good that also satisfied criterion 3.

Thus far we have clearly satisfied criteria 1, 3 and 5. Inspection of Tables 2 and 3 indicates that there is no marked plus-minus-plus effect in the residuals. There is a sign consistency for the BGS residuals, which can be completely changed by refitting after removing the points at 209°K and 331°K. In any case the effect is extremely small.

INTERPOLATION AND EXTRAPOLATION

To satisfy the remaining one of our criteria we examined the behavior of the fitted curves as successive deletions of experimental points were carried out in an arbitrary manner, but preserving, as far as possible, the total range of calibration. The results of this reduction in numbers of observed points are shown in Fig. 1. The clear result is that equation (2) fits the observed points well within the limits of experimental precision even though very few points are used to make the fit. The single set of three points for the BGS data that exceeds the experimental precision includes the poor point at 209·17°K, which in many fitted equations appears to be in error by about twice the experimental precision. Further examination of Fig. 1 suggests that the best experimental strategy is to extend the calibration range outside the range in which measurements are to be made—and that only a few points are needed to validate the fitting procedure.

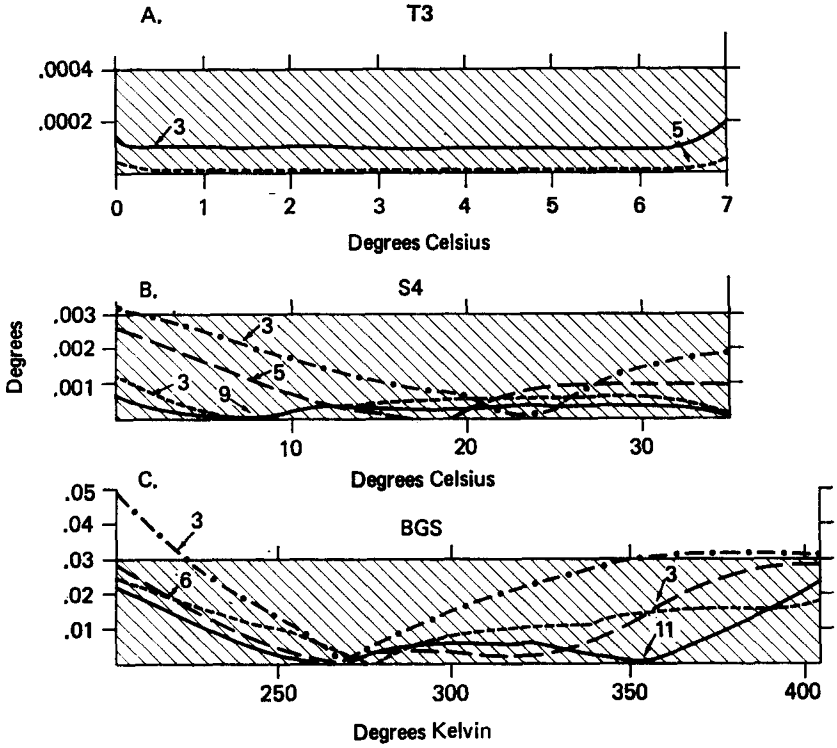

Fig. 1. Interpolation behavior of equation (2). The ordinate is the difference in temperature between predicted temperatures using a small number of points in fitting equation (2) and the predicted values using all experimental points. The numbers show the number of points used in each fit and the hatched areas show the limits of experimental precision. Differences are without regard to sign.

For extrapolation the situation is quite different. Figure 2 shows the departures to be expected when a fitted curve is used to predict temperatures outside the range over which the calibration was done. These curves were obtained by taking small segments of the calibration data and fitting equation (2). The fitted equation was used to predict values outside the range from which it was derived and the predictions compared to observations. The results are plotted in terms of multiples of the experimental precision so that the three sample thermistors may be compared. For S4 and BGS two different sub-ranges were treated in this way and both results are shown. In general the extrapolations are reasonable for 15 to 30 per cent beyond the calibration range. Beyond that point they deteriorate rapidly. As suggested earlier, this also shows that equation (2) is not very useful as a " law," even though it is an excellent fitting function.

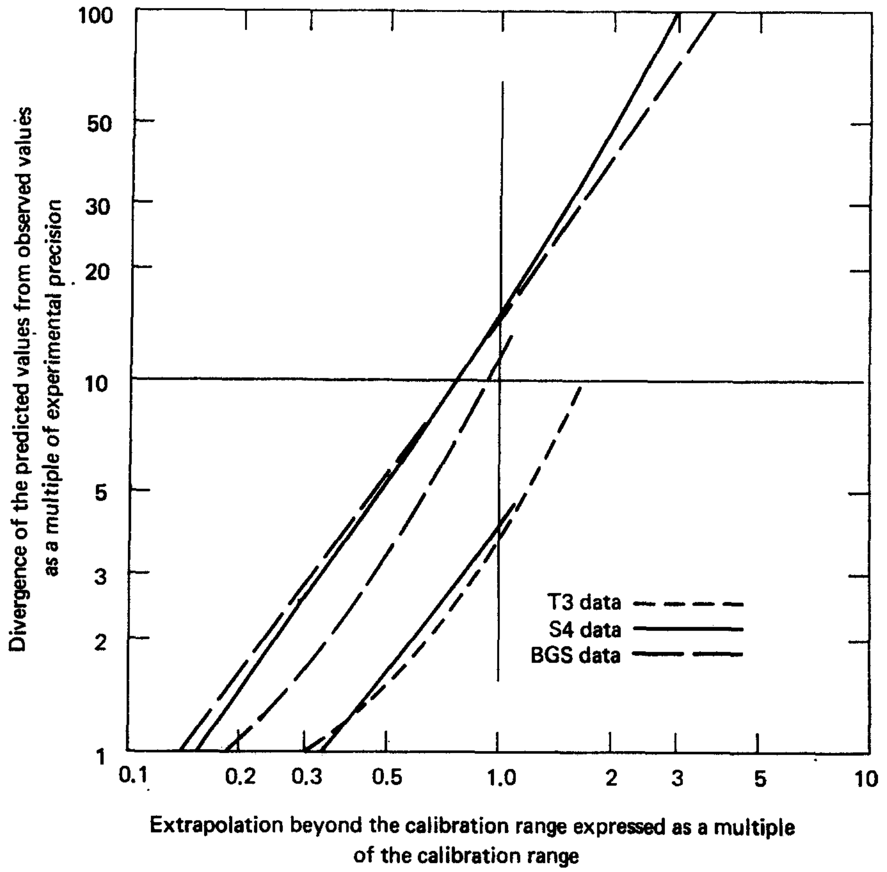

Fig. 2. Extrapolation behavior of equation (2). Normalized curves for all three thermistors for extrapolation beyond the range of calibration. Note that the curve first appears on this graph when it exceeds the experimental precision. See text for derivation.

There is some suggestion in the results that the temperature–resistance relationship for thermistors is best represented as a power series of odd powers of log R. Only the BGS data covered a sufficient range to test this hypothesis further. Table 4 shows the standard relative error for several multiple term fits. The results show that the even power terms do not improve the fit so well as the odd powers. It supports the suggestion that an exact relationship would be representable as a power series of odd powers.

It was pointed out to us by Dr. Richard P. von Herzen that odd power series expansions of several trigonometric functions of log R would have the general form of equation (2). We have tested sin,

Table 4. Comparative fits of several equations involving powers of log R to Bosson, Gutmann and Simmons data.

Equation	*Standard relative error*
$T^{-1} = A + B \log R$	0·006012
$T^{-1} = A + B \log R + C (\log R)^2$	0·000469
$T^{-1} = A + B \log R + C (\log R)^3$	0·000162
$T^{-1} = A + B \log R + C (\log R)^2 + D (\log R)^3$	0·000472
$T^{-1} = A + B \log R + C (\log R)^3 + D (\log R)^5$	0·000091

tan, $\sin^{-1}$, $\tan^{-1}$ and at least these simple examples disagree with the fitted coefficients in sign, or by several orders of magnitude or both.

CONCLUSION

An extensive examination of calibration functions has yielded a function suitable for calibration curves for precision thermistor temperature measurements. This function is recommended to workers making such precision measurements as its properties have been examined for a variety of data and a variety of thermistors.

Acknowledgements—We appreciate the opportunity to try this function on unpublished calibration data loaned to us by Dr. G. Simmons of Massachusetts Institute of Technology and Dr. A. Jessop of the Dominion Observatory of Canada. Copies of computer programs used in this paper may be obtained from the authors.

REFERENCES

Becker J. A., C. B. Green and G. L. Pearson (1946) Properties and uses of thermistors—thermally sensitive resistors. *Trans. Am. Inst. Elec. Engrs*, **65**, 711–725.

Bosson G., F. Gutmann and L. M. Simmons, (1950) A relationship between resistance and temperature of thermistors. *J. appl. Phys.*, **21**, 1267–1268.

Robertson E. C., R. Raspet, J. H. Swartz and M. E. Lillard (1966) Properties of thermistors used in geothermal investigations. *U.S. Geol. Surv. Bull.*, 1203–B, 34 pp.

APPENDIX

The table below gives the coefficients for equation (2) obtained by least squares procedures from a multiple regression program used by the authors.

	A all $\times 10^{-3}$	B all $\times 10^{-3}$	C all $\times 10^{-9}$
BGS Western Electric	0·792008	0·231076	84·261
T3	1·258294	0·263120	150·370
S4	1·168483	0·280480	158·816