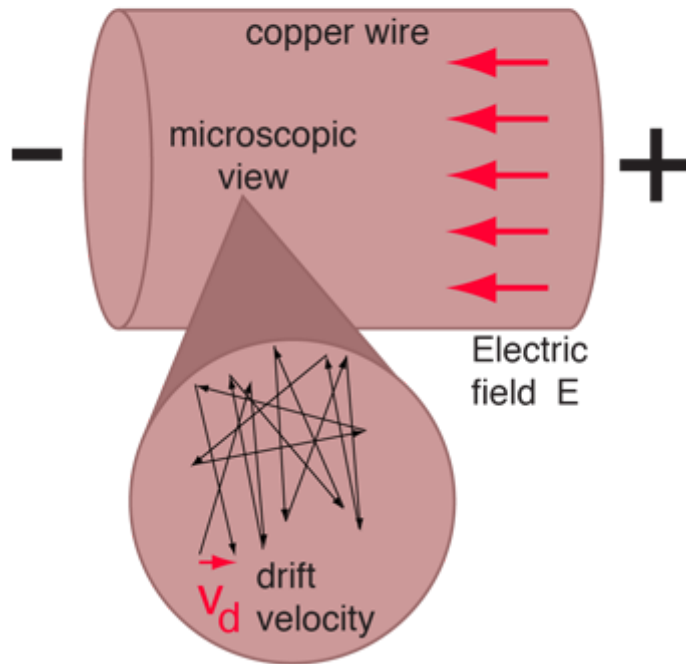


Microscopic View of Ohm's Law

The electron moves at the Fermi speed, and has only a tiny drift velocity superimposed by the applied electric field.



When [electric current](#) in a material is proportional to the [voltage](#) across it, the material is said to be "ohmic", or to obey [Ohm's law](#). A microscopic view suggests that this proportionality comes from the fact that an applied electric field superimposes a small drift velocity on the free electrons in a metal. For ordinary currents, this drift velocity is on the order of millimeters per second in contrast to the speeds of the electrons themselves which are on the order of a million meters per second. Even the electron speeds are themselves small compared to the speed of transmission of an electrical signal down a wire, which is on the order of the speed of light, 300 million meters per second.

The current density (electric current per unit area, $J=I/A$) can be expressed in terms of the [free electron density](#), as

$$J = nev_d \quad \begin{array}{l} n = \text{free electron density} \\ v_d = \text{drift velocity} \end{array}$$

The number of atoms per unit volume (and the number free electrons for atoms like copper that have one free electron per atom) is

$$n' = \frac{\begin{array}{c} \text{Avogadro's number} \\ (N_A \text{ atoms / mole}) \end{array} \begin{array}{c} \text{Density} \\ (\rho \text{ kg / m}^3) \end{array}}{\begin{array}{c} \text{Atomic mass} \\ A(\text{kg / mole}) \end{array}}$$

From the standard form of Ohm's law and [resistance](#) in terms of [resistivity](#):

[Index](#)

[Kittel](#)
Ch 10

[Kip](#)
Sec 7.4

$$I = \frac{V}{R} \quad ; \quad R = \frac{\rho L}{A}$$

$$J = \frac{V}{RA} = \frac{V}{\frac{\rho L}{A} A} = \frac{EL}{\rho L} = \frac{E}{\rho} = \sigma E$$

The next step is to relate the drift velocity to the electron speed, which can be approximated by the [Fermi speed](#):

$$v_F = \sqrt{\frac{2E_F}{m}} \quad \boxed{\text{Table}}$$

The drift speed can be expressed in terms of the accelerating electric field E, the electron mass, and the characteristic time between collisions.

$$v_d = \frac{eE}{m} \tau = \frac{eE}{m} \frac{d}{v_F}$$

The conductivity of the material can be expressed in terms of the Fermi speed and the mean free path of an electron in the metal.

$$\sigma = \frac{ne^2 d}{mv_F} \quad \boxed{\text{Table}}$$

[Numerical example for copper.](#) [Table of resistivities](#)

[Table of free electron densities](#)

[HyperPhysics***** Electricity and Magnetism](#)

R Nave

[Go Back](#)

Microscopic View of Copper Wire

As an example of the [microscopic view of Ohm's law](#), the parameters for copper will be examined. With one free electron per atom in its metallic state, the electron density of copper can be calculated from its bulk density and its atomic mass.

$$n = \frac{(6.02 \times 10^{23} \text{ atoms / mole})(8.92 \times 10^3 \text{ kg / m}^3)}{63.5 \times 10^{-3} \text{ kg / mole}} = 8.46 \times 10^{28} / \text{m}^3$$

The [Fermi energy](#) for copper is about 7 eV, so the Fermi speed is

[Index](#)

[Kittel](#)
Ch 10

[Kip](#)
Sec 7.4

$$v_F = c \sqrt{\frac{2E_F}{mc^2}} = 3 \times 10^8 \text{ m/s} \sqrt{\frac{2 \times 7 \text{ eV}}{511000 \text{ eV}}} = 1.57 \times 10^6 \text{ m/s}$$

The measured conductivity of copper at 20°C is

$$\sigma = 5.9 \times 10^7 \text{ } \Omega\text{m}$$

The mean free path of an electron in copper under these conditions can be calculated from

$$d = \frac{\sigma m v_F}{n e^2} = \frac{(5.9 \times 10^7 \text{ } \Omega\text{m})(9.11 \times 10^{-31} \text{ kg})(1.57 \times 10^6 \text{ m/s})}{(8.46 \times 10^{28} \text{ } / \text{m}^3)(1.6 \times 10^{-19})^2} = 3.9 \times 10^{-8} \text{ m}$$

The drift speed depends upon the electric field applied. For example, a copper wire of diameter 1mm and length 1 meter which has one volt applied to it yields the following results.

$$R = \frac{L}{\sigma A} = \frac{1 \text{ m}}{(5.9 \times 10^7 \text{ } \Omega\text{m})(\pi \cdot 0.0005^2 \text{ m}^2)} = 0.0216 \Omega$$

For 1 volt applied this gives a current of 46.3 Amperes and a current density

$$J = 5.9 \times 10^7 \text{ A/m}^2$$

This corresponds to a drift speed of only millimeters per second, in contrast to the high Fermi speed of the electrons.

$$v_d = \frac{J}{ne} = \frac{5.9 \times 10^7 \text{ A/m}^2}{(8.46 \times 10^{28} \text{ } / \text{m}^3)(1.6 \times 10^{-19} \text{ C})} = 0.0043 \text{ m/s}$$

Caution! Do not try this at home! Dr. Beihai Ma of Argonne National Laboratory wrote to point out that the current density of 5900 A/cm² in this example is over ten times the current density of 500 A/cm² that copper can normally withstand at 40°F. So doing this in the laboratory might be too exciting. Thanks for the sanity check Dr. Ma.

(If you scale down the voltage applied so that the current is just 3 Amperes, current density 382 A/cm², so that the copper wire will remain intact, the calculated drift velocity is just 0.00028 m/s. This would be more typical for working conditions in this wire.)

What about quantum effects and Fermi-Dirac statistics?

The treatment of the microscopic Ohm's Law and drift velocity above is basically a classical treatment. But we know that the electrons in a metal obey [Fermi-Dirac statistics](#), and that at low temperatures, all the available electron energy levels are filled up to the [Fermi Level](#). We also know that this Fermi level (about 7 eV in copper) is very high compared to [room temperature thermal energy](#). Then how do we justify using the whole free electron population above in the calculation of drift velocity when for thermal interactions, only those electrons within about kT of the Fermi Level are accessible to the interaction?

One place where this situation is discussed is in the classic Solid State Physics book by Charles Kittel. He points out that because of the full population of conduction electrons up to the Fermi level, these electrons in the copper wire have a very high velocity, on the order of 1.6×10^6 cm/sec for copper. An externally applied electric field in a copper wire will exert a continuous force on the electrons and would continue to accelerate them if that acceleration were not randomized by the multiple collisions and interaction with the lattice. The observation of Ohm's Law experimentally shows us that an equilibrium current is achieved, and that the presumption that all the conduction electrons are participating allows us to project an effective drift velocity for these electrons under the influence of the applied electric field. It is noted that this is an entirely classical presumption.

As Kittel further examines electrical conductivity from the point of view of Fermi-Dirac statistics, he makes the following comment: "It is a somewhat surprising fact that the introduction of the Fermi-Dirac distribution in place of the classical Maxwell-Boltzmann distribution usually has little influence on the electrical conductivity, often only changing the kind of average used in the specification of the relaxation time. One might have expected at first sight to find a more drastic change because with the Fermi-Dirac distribution only those electrons near the Fermi surface can participate in collision processes. "

Kittel argues that the exclusion principle does not prevent the electric field intervention since it acts on each electron in the distribution to produce the same velocity change. There is always a vacant state ready to receive the electron which is changing its state under the action of the electric field, the vacancy being produced by the simultaneous change of the state of another electron. In contrast to a random thermal collision process, the single direction electric force produces electron excess states A and electron deficiency states B which allow the specific collisions required to restore equilibrium. The exclusion principle does prevent many collisions, but does allow those collisions needed to restore equilibrium. The bottom line is, except for the detailed nature of the relaxation time, the situation is essentially the same as in the classical situation, and leads to the same conductivity expression and average drift velocity.

[HyperPhysics](#)***** [Electricity and Magnetism](#)

R Nave

[Go](#)
[Back](#)

Free Electron Density in a Metal

The free electron density in a metal is a factor in determining its electrical conductivity. It is involved in the [Ohm's law behavior](#) of metals on a microscopic scale. Because electrons are fermions and obey the Pauli exclusion principle, then at 0 K temperature the electrons fill all

[Index](#)

[Kittel](#)
Ch 10

[Kip](#)
Sec 7.4

available energy levels up to the [Fermi level](#). Therefore the free electron density of a metal is related to the Fermi level and can be calculated from



A metal with Fermi energy $E_F = \text{[]} \text{ eV}$
will have free electron density $n = \text{[]} \times 10^{\text{[]}} / \text{m}^3$.

[Table of Fermi Energies](#)

Alternatively, if you can identify the number of electrons per atom that participate in conduction, then the free electron density can just be implied from the atomic mass and mass density of the material. The number of atoms per unit volume can be implied from the atomic mass and bulk density of the material:

$$n' = \frac{\overset{\text{Avogadro's number}}{(N_A \text{ atoms / mole})} \overset{\text{Density}}{(\rho \text{ kg / m}^3)}}{\underset{\text{Atomic mass}}{A(\text{kg / mole})}}$$

[Periodic Table of Elements](#)

A metal with bulk density $\rho = \text{[]} \text{ kg/m}^3$
and atomic mass $A = \text{[]} \times 10^{-3} \text{ kg/mole}$
will have a number of atoms per unit volume $n' = \text{[]} \times 10^{\text{[]}} / \text{m}^3$.

The number of atoms per unit volume multiplied by the number of free electrons per atom should agree with the free electron density above.

While these two approaches should be in agreement, it may be instructive to examine both for self-consistency.

Consider the element [zinc](#) with a tabulated Fermi energy of 9.47 eV. This leads to a free electron density of

$$n = \frac{8\sqrt{2}\pi(.511 \text{ MeV})^{3/2}}{(1240 \text{ eV nm})^3} \left[\frac{2}{3}(9.47 \text{ eV})^{3/2} \right] = 132 / \text{nm}^3 = 1.32 \times 10^{29} / \text{m}^3$$

From the Periodic Table, the density of zinc is 7140 kg/m³ and its atomic mass is 65.38 gm/mole. The number of atoms per unit volume is then

$$n_{\text{atoms}} = \frac{\rho N_A}{A} = \frac{(7140 \text{ kg / m}^3)(6.02 \times 10^{23} \text{ atoms / mole})}{65.38 \times 10^{-3} \text{ kg / mole}} = 6.57 \times 10^{28} \text{ atoms / m}^3$$

The number of free electrons per zinc atom to make these consistent is

$$\frac{n}{n'} = \frac{1.32 \times 10^{29}}{6.57 \times 10^{28}} = 2.01 \text{ free electrons per atom}$$

This number is what we would expect from the electron configuration of zinc, (Ar)3d¹⁰4s², so these two approaches to the free electron density in a metal are consistent.

[HyperPhysics](#)***** [Electricity and Magnetism](#)

R Nave

[Go](#)
[Back](#)