Final Project

QF633 – C++ for Financial Engineering

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Code output:

ID	Trade*	PV	Delta	Vega
1	Swap USD-SOFR	139,769.3919	1,442.2532	0.0000
2	Swap USD-SOFR	-1,326,556.8802	-6,212.3893	0.0000
3	Swap SGD-SORA	-257,018.6551	1,504.6434	0.0000
4	Swap SGD-SORA	-69,988.1434	-6,750.6491	0.0000
5	Bond USD-GOV	475,676.4210	-392.4480	0.0000
6	Bond SGD-GOV	-1,562,036.8930	672.2137	0.0000
7	Bond SGD-MAS-BILL	99,784.6398	-5.0170	0.0000
8	TreeProduct APPL	-5,945.9712	-2.0464	-148.7167
9	TreeProduct SP500	48,765.8185	-60.9166	4,653.3414
10	TreeProduct STI	15,363.7581	-19.5587	1,537.6986
11	TreeProduct STI	-17,905.9034	25.8977	-2,569.4668
12	TreeProduct APPL	-5,945.9712	-2.0464	-148.7167
13	TreeProduct SP500	61,477.9702	-36.0960	4,337.6943
14	TreeProduct STI	19,218.6161	-12.2302	1,478.6226
15	TreeProduct STI	-21,602.4557	20.1409	-2,722.9097

**Note: Black Scholes was not used to compute the corresponding trade's PV as shown in the output table.*

General Observation

1. **Swaps:**
   - All swaps show a Vega of 0.00. This is expected as their value is not directly sensitive to changes in volatility of the underlying interest rates as compared to options. On the other hand, swaps are primary sensitivity to interest rate movements (as reflected in Delta).
   - Swaps are sensitive to Delta. A positive Delta (e.g., Swap USD-SOFR Trade 1: 1,442.25) suggests the swap's PV will increase if the underlying interest rate increases, while a negative Delta (e.g., Swap USD-SOFR Trade 2: -6,212.39) suggests that the PV will decrease with an increase in the underlying rate.
   - The PV signs for swaps are consistent with the chosen direction: "Pay" swaps typically have positive Delta because the present value benefits from increasing rates, while "Receive" swaps generally have negative Delta as their PV decreases when rates rise.
2. **Bonds:**
   - Similar to swaps, bonds exhibit a Vega of 0.00, as their PV value is not directly impacted by volatility in the same manner as options is. Bonds are also primarily sensitive to interest rate changes.
   - All bond Deltas are negative. This is typical for fixed-income securities; as an increase in interest rates would result in a fall in the present value of future bond cash flows, leading to a fall in the bond's price.
   - The negative Delta in bonds reflects their sensitivity to interest rate increases, which reduces the discounted present value of their fixed coupon payments and principal.
3. **Tree Products (APPL, SP500, STI):**
   - Only the Tree Products have non-zero Vega values. Notably, Vega measures sensitivity to volatility.
     - Positive Vega (e.g., TreeProduct SP500: Trades 9 and 13; TreeProduct STI: Trades 10 and 14) indicates that the value of these positions will increase if the implied volatility of the underlying asset increases.
     - Negative Vega (e.g., TreeProduct APPL: Trade 8; TreeProduct STI: Trades 11 and 15) suggests that the value of these positions will decrease if implied volatility rises.
     - This is consistent with option theory: long option positions typically have positive Vega, meaning they gain value when volatility increases, while short option positions generally have negative Vega, meaning they lose value when volatility increases.

Overall Portfolio Characteristics:
The portfolio is diversified across different asset classes (fixed income, derivatives) and currencies (USD, SGD). The presence of significant Vega values from the "Tree Products" suggests that the portfolio has exposure to market volatility. The combination of positive and negative PVs and Deltas across the various securities indicates a complex set of positions designed to manage risk and generate returns based on different market views (interest rate movements, volatility changes, and underlying asset price movements).

a. Comparing tree model price of European vs Black, and explain the difference

ID	Underlying	Option	Direction	Model	PV	Delta	Vega
8	APPL	Call	Short	European	-5,945.97	-2.05	-148.72
8	APPL	Call	Short	Black-Scholes	-5,945.97	-2.05	-148.72
9	SP500	Put	Long	European	48,765.81	-60.92	4,653.34
9	SP500	Put	Long	Black-Scholes	48,765.81	-60.92	4,653.34
10	STI	Put	Long	European	15,363.76	-19.56	1,537.70
10	STI	Put	Long	Black-Scholes	15,363.76	-19.56	1,537.70
11	STI	Put	Short	European	-17,905.90	25.90	-2,569.47
11	STI	Put	Short	Black-Scholes	-17,905.90	25.90	-2,569.47

All PVs, Delta and Vega are identical between the Tree model and Black-Scholes model which suggests that the number of steps, 50 chosen for the European tree model, was sufficient to approximate to the Black-Scholes accurately. This is because European options have no early exercise feature with no dividend assumed. This assumption is in alignment with the Black-scholes model. The convergence of the tree model to Black-Scholes is expected as the number of time steps increases, confirming that the binomial tree model is consistent and robust for European option pricing.

b. Comparing the American and European trade pv and explain the difference.

Generally, as seen, American options tend to have consistently higher PVs (Trades 13, 14, 15) than those that are European Options (Trades 9, 10, 11), especially for puts. The results are likely attributed to American puts being able to benefit from early exercise, especially in a falling market or with high dividends. As such, the value of early exercise is captured by the tree model, but not in European-style pricing. Early exercise is typically more valuable for puts than for calls, as puts can be exercised to lock in value if the underlying asset price falls significantly.

Additionally, it is noted that notably, the call options on AAPL (Trades 8 & 12) have identical PVs, indicating that either no dividends is expected AAPL, or that the market conditions don’t justify early exercise (e.g., deep out-of-the-money call).