RYERSON UNIVERSITY

UNDERGRADUATE THESIS

Financial Performance of Renewable Energy Firms

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Intro

1.1 Abstract

This paper studies the impact of climate change on energy producing assets and explores alternative methods to mitigate the adverse effects of climate change to support the transition to lower carbon emitting economies. We investigate risk-adjusted returns of two distinct portfolios, one comprised of alternative energy stocks and the other comprised of traditional energy stocks. The study applies modern portfolio theory to construct CVaR efficient portfolios (where CVaR - Conditional Value at Risk - is the objective function to minimize) using historical stock prices. Backtesting of the portfolios is done to study their performance with real market data. The sample period used to backtest the efficient portfolios is from January 2013 to July 2018, where the portfolios will be rebalanced and their returns evaluated on a monthly basis. The resulting Sharpe Ratio for the traditional portfolio is three times higher than its counterpart. These findings suggests that investors, regardless of their exact preferences, will be more likely to invest in the "black" traditional portfolio over the "green" alternative energy portfolio. The long run implication of this is that little investment will be directed towards the alternative energy sector, and this lack of investment is yet another barrier that renewable technologies face. Therefore the implementation of carbon taxes, carbon mitigating policies, and/or creating subsidies for low carbon-footprint firms

may be needed to encourage investment into the renewable energies sector.

1.2 Introduction

According to US SIF (2010): the forum for sustainable and responsible development, socially responsible investments (SRIs) now account for over 10% of all assets under professional management in the United States (US SIF, 2010). Socially responsible investments, otherwise known as sustainable or ethical investing, is a label applied to any investment strategy which considers both returns and social/environmental good brought about when making investment decisions. Additionally, socially responsible investment assets have grown by more than one-third since 2005, while the broader universe of investable assets has remained relatively stagnant. These numbers suggest that there is a trend in the professional asset management business toward investing into firms that are concerned about environmental, social, and governance (ESG) issues.

The demand for green investments, in particular, has grown rapidly as society places more emphasis on environmental issues. Green investing is a relatively new subset of SRI, so it is not surprising that there is no formal definition of what consitutes a "green" firm and is largely a subjective assessment on the part of the investor. For the purpose of this paper, green investments will be defined as investments into firms that actively minimize resource usage, produce eco-friendly products or generate energy from renewable sources. Essentially, environmental factors are taken into consideration when deciding how to allocate funds. Despite their growth in popularity, the future of investment into green firms will likely depend on their ability to generate competitive returns.

Boulatoff and Boyer (2009) found that green firms underperformed comparable to Nasdaq firms by 9 percent over a five-year period ending in October 2008. Additionally, the green firms experience more volatility. However, it is possible that these firms are in the early stages of development, and are expected to incur substantial research and development costs. This paper will examine whether this trend of underperformance continues into the post recession era.

The purpose of this paper is to determine whether a portfolio of renewable energy companies can perform as well as a portfolio of their non-renewable counterparts in a post-recession era. By constructing portfolios and performing out-of-sample testing from January 4th, 2013 to July 1st, 2018, this paper examines risk-adjusted returns of both portfolios by measuring their respective Sharpe ratios.

1.3 Markowitz Portfolio Choice Theory Overview

In 1952, Harry Markowitz published his paper "Portfolio Selection" which introduced ideas that would later become the foundation of modern portfolio theory. The ideas were expanded upon and adopted heavily in the finance industry, and were deemed important enough to award a Nobel prize to Harry Markowitz, Merton Miller, and William Sharpe in 1990. Markowitz assumed that the rational investor will decide on a portfolio of investments at time t based on gains and losses that will be made at $t + \Delta t$, and will hold it for time horizon Δt. At time $t + \Delta t$, the investor will reconsider the situation and decide anew.

Markowitz argued that investors should make decisions based on the trade-off between risk and expected return. Expected return is defined as the expected price change plus any additional income over time horizon Δt such as dividend payments, divided by the price of the security at time t. He also suggested that risk should be measured by the variance of returns by finding the average squared deviation around the expected return.

After defining these terms, Markowitz reasoned that the rational investor would choose the portfolio with the minimal variance for a given expected return among all the possible set of portfolios. This set of all possible portfolio combinations that can be constructed is called the *feasible set*, seen as the grey area marked by point IV on figure 1.1 below. The set of all mean-variance efficient portfolios are the portfolios along the dark line between points II and III.

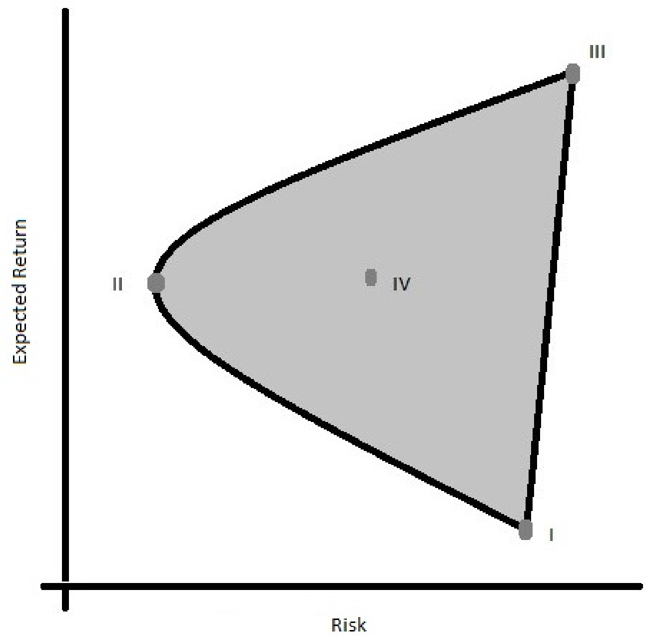


FIGURE 1.1: Feasible and Markowitz Efficient Portfolios

The set of mean-variance efficient portfolios is better known as the *efficient frontier*, and is where Markowitz reasoned that the rational investor will choose his or her portfolio from. Point II in figure 1.1 is often called the *global minimum variance portfolio* (GMV), as it is the portfolio along the efficient frontier with the smallest variance.

The process of finding this efficient frontier can vary depending on the tastes of the investor. In general, the process inputs of expected return, variance, and correlations are estimated from data and constraints are placed onto the portfolio. Constraints can be simple and straightforward such as no short selling allowed, or can be complicated as limiting values to rounded "lots". Then an optimization software is used to solve a series of optimization problems in order to generate an efficient frontier. The implementation of this optimization is not at all obvious, however the theory is relatively straightforward.

1.3.1 The Classical Framework for Mean-Variance Optimization

Suppose that an investor has to choose a portfolio of N risky assets. The investor's choice can be represented with weight vector $w = (w_1, w_2, ..., w_N)^T$, where each weight w_i represents the percentage of the portfolio invested in asset i and satisfies the equation

$$\sum_{i=1}^{N} w_i = 1$$

For now, short selling is permitted, meaning w_i can be negative. Suppose that asset returns $R = (R_1, R_2, ..., R_N)^T$ have expected returns $\mu = (\mu_1, \mu_2, ..., \mu_N)^T$ and an $N \times N$ covariance matrix given by

$$\Sigma = \begin{pmatrix} \sigma_{11} & \cdots & \sigma_{1N} \\ \vdots & & \vdots \\ \sigma_{N1} & \cdots & \sigma_{NN} \end{pmatrix}$$

where σ_{ij} denotes the covariance between assets i and asset j. Under these assumptions, the return of the portfolio is a random variable $R_p = \boldsymbol{w^T R}$

with expected return and variance given by

$$\mu_p = \boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{\mu}$$

$$\sigma_p^2 = \boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{\Sigma}\boldsymbol{w}$$

If variance is defined to be $\boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{\Sigma}\boldsymbol{w}$, then by following Markowitz the investor seeks the solution to

$$\underset{w}{\text{minimize}} \quad \boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{\Sigma}\boldsymbol{w} \tag{1.1}$$

subject to constraints

$$\mu_0 = \boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{\mu}$$

$$\boldsymbol{w}^{\boldsymbol{T}}\mathbf{1} = 1$$

where $\mathbf{1}^{\boldsymbol{T}} = [1, 1, ..., 1]$ and μ_0 is the target expected return chosen by the investor. The optimization problem (1.1) will be referred to as the *risk minimization formulation* of the classical mean-variance optimization problem. The constraints in this problem are minimal and therefore finding a solution does not require the use of a complex optimization program.

1.3.2 Investors can only invest in risky assets

Suppose the investor has to choose a portfolio consisting of n risky assets. The investor can find the set of portfolio weights that minimize the variance of a portfolio for each feasible expected return. Short selling is permitted for now, which means weights can be negative. If $w = (w_1, ..., w_n)^T$ is the vector of portfolio weights, where w_i is the weight in the *ith* asset, then once again expected return of the portfolio is defined by $\mu_p = w^T\mu$ and variance is given by $\sigma_p^2 = w^T\Sigma w$. The quantities μ and σ_p^2 cannot be observed directly and must be estimated using historical data. Then once they are found, the rational investor would seek to minimize risk for a given expected return.

Fabozzi and Sergio provide an analytic solution to the problem[1]:

$$w = g + h\mu_0$$

where g and h are the two vectors

$$g = \frac{1}{ac - b^2} \cdot \Sigma^{-1}[c\mathbf{1} - b\mu]$$

$$h = \frac{1}{ac - b^2} \cdot \Sigma^{-1}[a\mu - b\mathbf{1}]$$

and

$$a = \mathbf{1}^T\Sigma^{-1}\mathbf{1}$$

$$b = \mathbf{1}^T\Sigma^{-1}\mu$$

$$c = \mu^T\Sigma^{-1}\mu$$

Using the analytic solution greatly reduces the computational power needed to find the optimal set of weights. With the only risky assets constraint in place, the efficient frontier takes on the shape of a parabola in a expected return/standard deviation coordinate system.

1.3.3 Investors can also invest in the risk-free asset

What if investors could also invest in the risk-free asset? To accommodate this option into the mathematics, the optimization problem is no longer subject to the constraint $w^T\mathbf{1} = 1$ and instead we use $1 - w^T\mathbf{1}$ which is the proportion of wealth invested into the risk-free asset. If the risk-free asset has a return R_f, expected return of the portfolio is now defined

$$\mu_p = (1 - w^T\mathbf{1})R_f + w^T\mu$$

[1]This problem is solved by method of Lagrange multipliers. See chapter 7 in Sergio M. Focardi and Frank J. Fabozzi, *The Mathematics of Financial Modeling and Investment Management* (Hoboken, NJ: John Wiley Sons, 2004)

where $(1 - w^T\mathbf{1})R_f$ and $w^T\mu$ are the returns of the risk-free asset and risky assets respectively. The original optimization problem was

$$\underset{w}{\text{minimize}} \quad \boldsymbol{w^T \Sigma w} \tag{1.2}$$

subject to constraints

$$\mu_0 = \boldsymbol{w^T \mu}$$

$$\boldsymbol{w^T 1} = 1$$

The new optimization problem becomes

$$\underset{w}{\text{minimize}} \quad \frac{1}{2}\mathbf{w^T}\Sigma\mathbf{w} + \lambda[\bar{\mu}_p - (R_f + \mathbf{w^T}(\mu - R_f\mathbf{1}))] \tag{1.3}$$

where $\bar{\mu}_p$ is the required return on the portfolio and λ is the Lagrange multiplier removing the need for explicit constraints on the problem. Optimal weights can be computed with the explicit formula

$$w^* = \lambda\Sigma^{-1}(\mu - R_f\mathbf{1})$$

where

$$\lambda = \frac{\bar{\mu}_p - R_f}{\alpha_2 - 2\alpha_1 R_f + \alpha_3 R_f^2} \tag{1.4}$$

with the scalars defined as $\alpha_1 = \mu^T\Sigma^{-1}\mathbf{1}$, $\alpha_2 = \mu^T\Sigma^{-1}\mu$, and $\alpha_3 = \mathbf{1}^T\Sigma^{-1}\mathbf{1}$. (1.2) shows that the weights of the risky assets in any minimum variance portfolio are proportional to the vector $\Sigma^{-1}(\mu - R_f\mathbf{1})$ with a proportionality constant λ. Then, with a risk-free asset, all minimum variance portfolios are a combination of risky and risk-free assets, this portfolio is called the the tangency portfolio, otherwise known as the market portfolio.

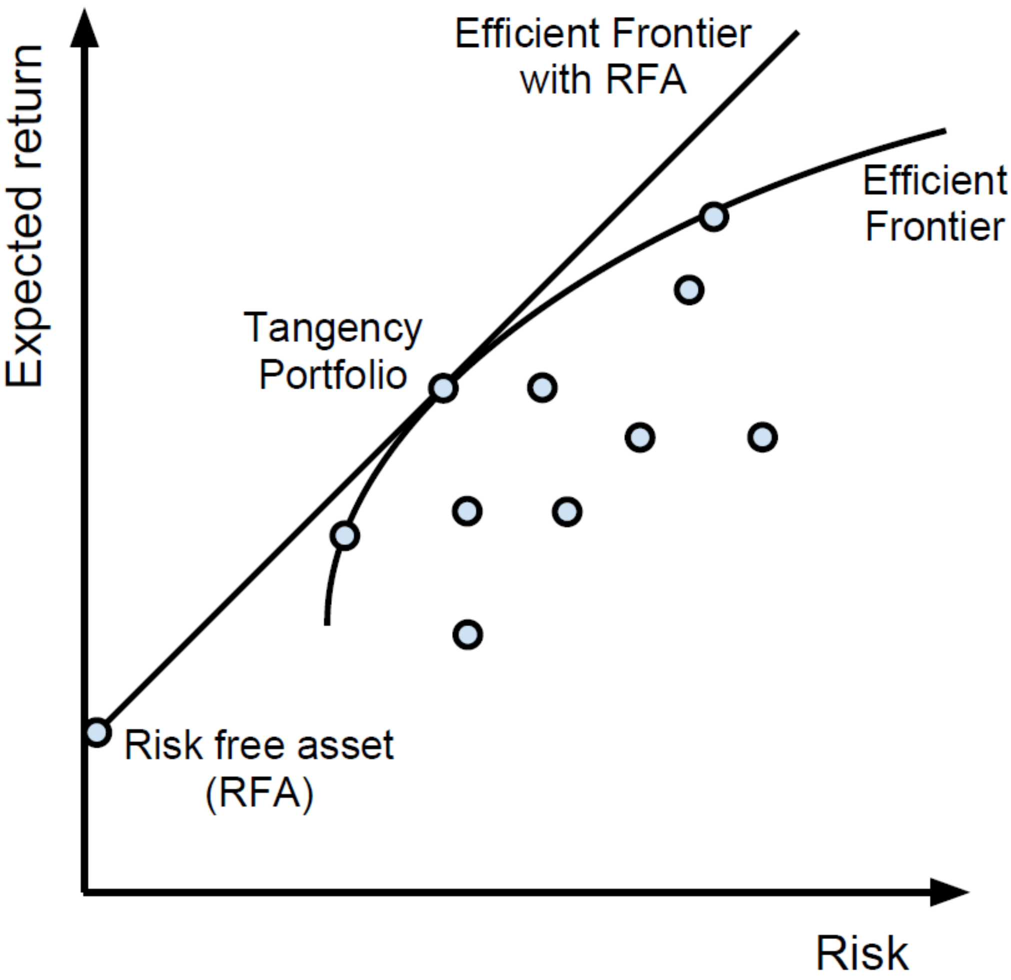


FIGURE 1.2: Efficient frontier with and without risk-free asset. Source: http://blog.andersen.im/2012/06/modern-portfolio-theory/

The weights of the market portfolio are given by

$$w_R^M = \frac{1}{\mathbf{1}^T \Sigma (\mu - R_f \mathbf{1})} \cdot \Sigma^{-1}(\mu - R_f \mathbf{1})$$

or by solving the *maximal Sharpe Ratio optimization problem*. The Sharpe ratio is a calculation that measures the actual return of an investment beyond the risk free rate adjusted for its risk. It is a measure that looks at both returns and risk in tandem, and was developed by William Sharpe in 1966. The Sharpe ratio is given by $Sharpe = \frac{R_p - R_f}{\sigma_p}$ then the optimization problem becomes

$$\begin{aligned} \underset{w}{\text{minimize}} \quad & \frac{w^T\mu - R_f}{\sqrt{w^T\Sigma w}} \\ \text{subject to} \quad & w^T\mathbf{1} = 1 \end{aligned} \tag{1.5}$$

Once the market portfolio is determined along the efficient frontier, portfolios to the left represents combinations of risky and risk-free assets where portfolios to the right are so-called leveraged portfolios, that is portfolios that borrow money buy risky assets. Practically speaking, a manager needs to make a decision on how much wealth to allocate between the risky and risk-free assets, and then construct the risky portfolio by allocating the weights upon the set risky assets available for investment.

1.3.4 Short Sales Constraint

Previously there were no restrictions imposed on the portfolio weights other than having them sum to one. However in practice, there are many situations where short selling an asset is not allowed either because its policy or because the particular asset class is difficult to short. When investors cannot short assets, we require portfolio weights $w_i \geq 0$, $i = 1, 2, \ldots, n$. There is no analytic, closed form solution to find the optimal portfolio and one must resort to numerical optimization techniques. With no risk-free assets available for investment, the problem is formulated as

$$\underset{w_i \geq 0, i=1,\ldots,n}{\text{minimize}} \quad \frac{1}{2}w^T\Sigma w + \lambda_1(\bar{\mu}_p - w^T\mu) + \lambda_2(1 - w^T\mathbf{1}) \tag{1.6}$$

and if risk-free assets are available for investment, then the problem is

$$\begin{aligned} \underset{w_i \geq 0, i=1,\ldots,n}{\text{minimize}} \quad & \frac{1}{2}w^T\Sigma w + \lambda[\bar{\mu}_p - (R_f + w^T(\mu - R_f\mathbf{1}))] \\ \text{subject to} \quad & 1 - w^T\mathbf{1} \geq 0 \end{aligned} \tag{1.7}$$

As expected, the efficient frontier with the short sales constraint is below the frontier without the constraint.

1.4 Conditional Value-At-Risk (CVaR) Optimization

The Mean-Variance method uses variance as a measure of risk, whereas the CVaR method will use Conditional Value-At-Risk as its measure. To introduce CVaR, first we must define Value-At-Risk. Value-At-Risk is one of the most widely-known measure risk measure first developed by JP Morgan and made available through the RiskMetrics^{TM} software (Longerstaey, 1996). VaR allows us to measure percentiles of loss distributions and helps firms predict maximum expected loss at a specified confidence level (e.g 95%) over some time horizon (1 or 10 days for example). Mathematically, VaR is defined as

$$VaR_{1-\epsilon}(R_p) = min\{R|P(-R_p \geq R) \leq \epsilon\}$$

where P is the probability function and $\epsilon \in [0, 1]$. This statement emphasizes the fact that the $(1-\epsilon)$-VaR is the value of R such that the probability that the possible portfolio loss $(-R_p)$ exceeds R is less than ϵ. It is common to assume that portfolio values are lognormally distributed, however this assumption typically does not hold true especially when the portfolio has short positions. More importantly, VaR does not take into account the *magnitude* of the losses beyond the VaR value.

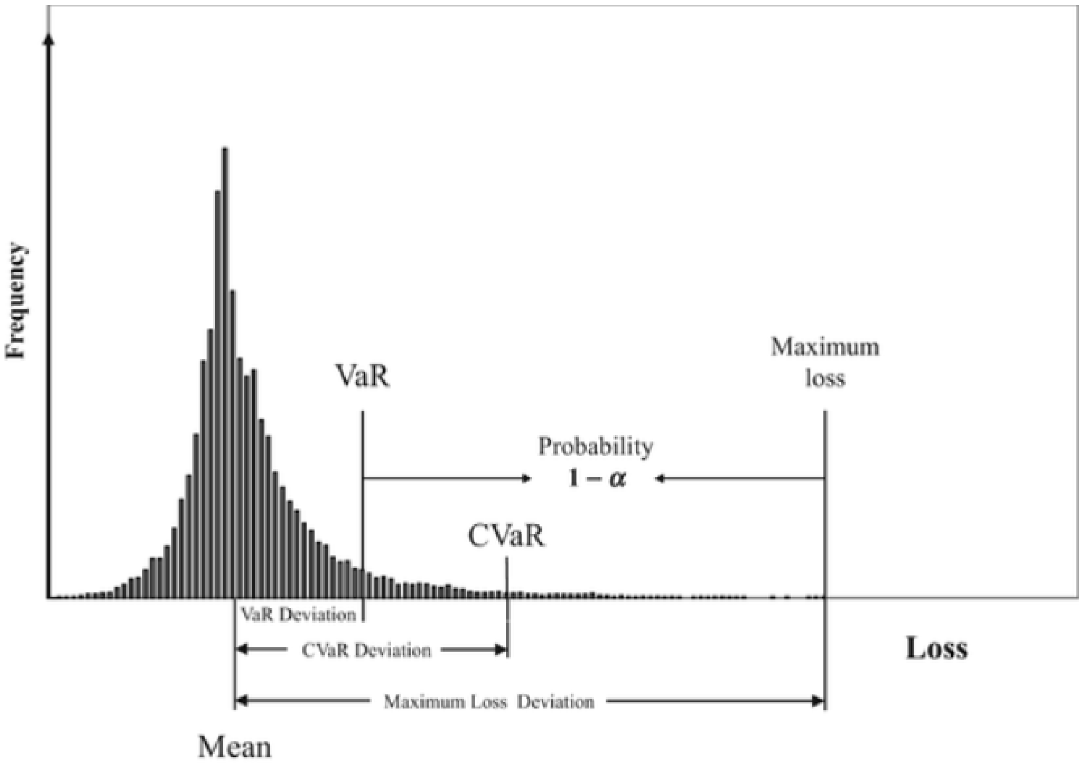


FIGURE 1.3: Graphical Illustration of CVaR, note that it measures tail risk beyond the VaR cutoff. Source: Serraino et al. (2008)

In effect, the tails of the distribution are ignored when investors use VaR to determine which portfolio they prefer. This shortfall in VaR led to the development of Conditional Value-at-Risk. The new risk measure - CVaR - developed by Artzner and Heath, is a coherent[2] risk measure (Artzner et al, 1999) defined:

$$CVaR_{1-\epsilon}(R_p) = \mathbb{E}\left[-R_p | -R_p \geq VaR_{1-\epsilon}(R_p)\right]$$

[2]A risk measure p is called a *coherent* if it satisfies the following properties:

1. *Monotonicity.* If $X \geq 0$, then $p(X) \leq 0$
2. *Subadditivity.* $p(X+Y) \leq p(X) + p(Y)$
3. *Positive homogeneity.* For any real number c, $p(cX) = cp(X)$
4. *Translational invariance.* For any real number c, $P(X+c) \leq p(X) - c$

where X and Y are random variables. In words, these properties can be interpreted as: (1) If returns are only positive, then risk of loss is non-positive; (2) the risk of a portfolio of two assets should be less than or equal to the sum of the risk of the individual assets; (3) if the portfolio is increased by some factor c, the risk becomes increased by the same factor; and (4) cash or another risk-free asset does not contribute to portfolio risk. For more information on coherent measures of risk, see *Coherent Measures of Risk* by Artzner et al (1999).

CVaR measures the expected amount of loss while in the tail of the distribution of possible portfolio losses beyond the portfolio VaR. In financial math literature, CVaR is commonly referred to as the expected shortfall, expected tail-loss, or tail VaR. The most common values for $(1-\epsilon)$ are 90%, 95%, and 99%. Graphically, the efficient frontier of a CVaR portfolio with $(1-\epsilon) = 90\%$ will be above the frontier with $(1-\epsilon) = 99\%$. When asset returns are normally distributed, the mean-variance and mean-CVaR approach yield similar results. Otherwise, the results will be different because of how the risk is measured. Mean-variance defines risk to be the variance which incorporates both the right and left tail, the potential gain and the loss, whereas CVaR only considers the tail of the distribution where the losses occur. Let $R_p = w^T R$ where $R = (R_1, \ldots, R_n)^T$ where R_i is the return on asset i for $i = 1, \ldots, n$. The portfolio optimization problem becomes

$$
\begin{aligned}
\underset{w}{\text{minimize}} \quad & \{CVaR_{1-\epsilon}(w^T R)\} \\
\text{subject to} \quad & w_i \geq 0, i = 1, \ldots, n, \\
& w^T \mu \geq \bar{\mu}_p, \\
& w^T \mathbf{1} = 1
\end{aligned}
\tag{1.8}
$$

But since no analytic solution exists, one must resort to optimization techniques which are often difficult to apply in practice. Krokhmal et al. (2002) suggested a scenario-based optimization based on the approximation

$$
\mathbb{E}\left[-R_p | R_p \leq -VaR_{1-\epsilon}(R_p)\right] \approx \frac{1}{q\epsilon} \sum_{i=1}^{q} w^T R^i
$$

where $R^i = (R^i_1, \ldots, R^i_n)^T$ is the ith scenario for asset returns and q is the quantile of the return distribution given the $(1-\epsilon)$ confidence level. For example if $\epsilon = 0.1$, and we have 100 scenarios then $q = 10$ and the summation is over the lowest 10 portfolio returns.

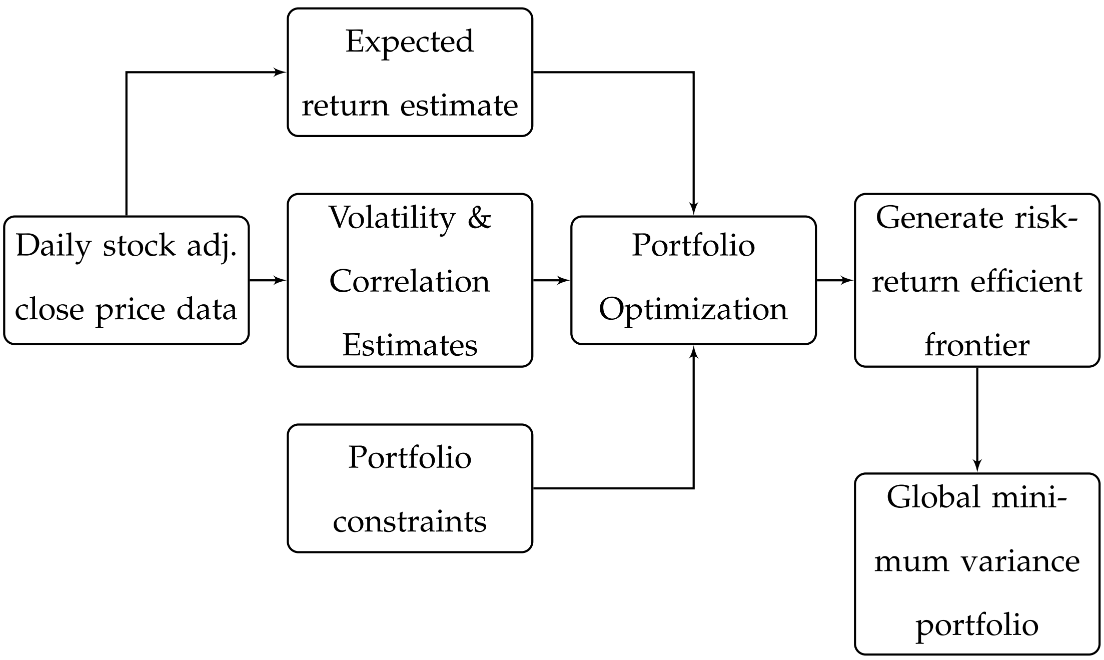
1.5 Data and Results

Two portfolios were constructed: a "black" portfolio of companies who generate energy using non-renewable sources (coal, petroleum, etc), and a "green" portfolio of companies who generate a majority of their energy through the use of renewable energy sources (geothermal, solar, biomass, etc). The exact composition of each portfolio and a short description of their assets are shown below:

Black Portfolio		
Firm	Ticker	Operations
Anadarko Petroleum Corp.	APC	Crude oil, petroleum
Devon Energy Corporation	DVN	Crude oil, petroleum
Chevron Corp.	CVX	Chemicals, petroleum
NextEra Energy	NEE	Crude oil, petroleum
Noble Energy Inc.	NBL	Crude oil, petroleum
Pioneer Natural Resources	PXD	Petroleum, natural gas
Valero Energy Corp.	VLO	Crude oil, petroleum
Exxon Mobil Corp.	XOM	Crude oil, petroleum

Green Portfolio		
Firm	Ticker	Operations
Ormat Technologies, Inc.	ORA	Geothermal, solar
Renewable Energy Group, Inc.	REGI	Renewables
Ballard Power Systems	BLDP	Fuel Cells
Enphase Energy	ENPH	Photovoltaics
First Solar Holding, LLC.	FSLR	Photovoltaics
Green Plains Renewable Energy, Inc.	GPRE	Ethanol
Ocean Powers Technology, Inc.	OPTT	Wave
SunPower Corporation	SPWR	Photovoltaics

The daily stock close price data was retrieved from Yahoo Finance and imported into excel documents. The portfolio optimization process is summarized by the following flow chart.



Initially, the efficient frontier will be computed using the historical data from 1/04/2012 to 1/04/2013 which consists of 251 data points of stock adjusted close prices, then assume the investor is highly risk averse and chooses the global minimum variance portfolio. Due to the frequency of the stock data (daily), I compute daily returns by finding the logarithmic difference because logarithmic returns are symmetric (positive and negative percent changes of equal magnitude cancel each other out, unlike ordinary returns). For example, the return vector would be calculated by

$$r_t = log\left[\frac{S_t}{S_{t-1}}\right]$$

Where r_t would be the rate of return on day t and S_t is the stock price at time t. Since only risky assets are available for investment in both of the sets of available investments, the annualized expected return[3] of the black and green portfolio's assets are

$$\mu_b = \begin{array}{cccccccc} APC & DVN & CVX & NEE & NBL & PXD & VLO & XOM \end{array}$$
$$\left[\begin{array}{cccccccc} 0.0769 & 0.0495 & 0.0586 & 0.1124 & 0.0998 & 0.1636 & 0.1761 & 0.0426 \end{array}\right]$$

and

$$\mu_g = \begin{array}{cccccccc} ORA & REGI & BLDP & ENPH & FSCR & GPRE & OPTT & SPWR \end{array}$$
$$\left[\begin{array}{cccccccc} 0.1692 & 0.1614 & 0.1930 & -0.0652 & 0.0473 & 0.0991 & -0.0551 & 0.0192 \end{array}\right]$$

where μ_b, μ_g denote annualized expected returns of the black and green

[3] Annualized expected return found by using the following formula

$$E[r]_{yearly} = \left[(E[r]_{daily} + 1)^{252} - 1\right]$$

portfolio respectively with annualized[4] covariance matrices

$$\Sigma_b = \begin{pmatrix} 0.1580 & 0.1153 & 0.0687 & 0.0295 & 0.1152 & 0.1173 & 0.0859 & 0.0599 \\ 0.1153 & 0.1492 & 0.0675 & 0.0281 & 0.1133 & 0.1148 & 0.0816 & 0.0591 \\ 0.0687 & 0.0675 & 0.0645 & 0.0248 & 0.0683 & 0.0694 & 0.0573 & 0.0507 \\ 0.0295 & 0.0281 & 0.0248 & 0.0496 & 0.0303 & 0.0292 & 0.0302 & 0.0251 \\ 0.1152 & 0.1133 & 0.0683 & 0.0303 & 0.1450 & 0.1172 & 0.0841 & 0.0605 \\ 0.1173 & 0.1148 & 0.0694 & 0.0292 & 0.1172 & 0.1807 & 0.0896 & 0.0601 \\ 0.0859 & 0.0816 & 0.0573 & 0.0302 & 0.0841 & 0.0896 & 0.1581 & 0.0519 \\ 0.0599 & 0.0591 & 0.0507 & 0.0251 & 0.0605 & 0.0601 & 0.0519 & 0.0583 \end{pmatrix}$$

$$\Sigma_g = \begin{pmatrix} 0.0683 & 0.0031 & 0.0223 & -0.0037 & 0.0342 & 0.0265 & 0.0224 & 0.0412 \\ 0.0037 & 0.2498 & -0.0158 & -0.0012 & -0.0148 & 0.0003 & -0.0061 & -0.0146 \\ 0.0227 & -0.0158 & 0.5742 & 0.0000 & 0.0613 & 0.0489 & 0.0386 & 0.0938 \\ -0.0037 & -0.0012 & 0.0000 & 0.7124 & 0.00927 & -0.0110 & -0.0073 & -0.0025 \\ 0.0342 & -0.0148 & 0.0613 & 0.00927 & 0.2921 & 0.0613 & 0.0371 & 0.207 \\ 0.0265 & 0.0003 & 0.0489 & -0.0110 & 0.0613 & 0.2214 & 0.0226 & 0.0784 \\ 0.0224 & -0.0061 & 0.0386 & -0.0073 & 0.0371 & 0.0226 & 1.4365 & 0.0595 \\ 0.0412 & -0.0146 & 0.0938 & -0.0025 & 0.207 & 0.0784 & 0.0595 & 0.3910 \end{pmatrix}$$

[4]Annualized covariance can be found by multiplying daily covariance by 252, the average number of trading days in a year. Monthly covariance can be annualized by multiplying each element in the matrix by 12.

with correlation matrices

$$\rho_b = \begin{pmatrix} \mathbf{1.0000} & 0.7506 & 0.6807 & 0.3331 & 0.7611 & 0.6939 & 0.5435 & 0.6236 \\ 0.7506 & \mathbf{1.0000} & 0.6879 & 0.3268 & 0.7706 & 0.6990 & 0.5312 & 0.6334 \\ 0.6807 & 0.6879 & \mathbf{1.0000} & 0.4383 & 0.7065 & 0.6425 & 0.5674 & 0.8260 \\ 0.3331 & 0.3268 & 0.4383 & \mathbf{1.0000} & 0.3580 & 0.3090 & 0.3412 & 0.4672 \\ 0.7611 & 0.7706 & 0.7065 & 0.3580 & \mathbf{1.0000} & 0.7241 & 0.5552 & 0.6583 \\ 0.6939 & 0.6990 & 0.6425 & 0.3090 & 0.7241 & \mathbf{1.0000} & 0.5301 & 0.5858 \\ 0.5435 & 0.5312 & 0.5674 & 0.3412 & 0.5552 & 0.5301 & \mathbf{1.0000} & 0.5402 \\ 0.6236 & 0.6334 & 0.8260 & 0.4672 & 0.6583 & 0.5858 & 0.5402 & \mathbf{1.0000} \end{pmatrix}$$

$$\rho_g = \begin{pmatrix} \mathbf{1.0000} & 0.0287 & 0.1145 & -0.0167 & 0.2414 & 0.2148 & 0.0714 & 0.2509 \\ 0.0287 & \mathbf{1.0000} & -0.0419 & -0.003 & -0.0550 & 0.0014 & -0.0103 & -0.0468 \\ 0.1145 & -0.0419 & \mathbf{1.0000} & 0.0000 & 0.1497 & 0.1371 & 0.0426 & 0.1980 \\ -0.0167 & -0.0030 & 0.0000 & \mathbf{1.0000} & 0.0203 & -0.0278 & -0.0072 & -0.0048 \\ 0.2414 & -0.0550 & 0.1497 & 0.0203 & \mathbf{1.0000} & 0.2410 & 0.0573 & 0.6129 \\ 0.2148 & 0.0014 & 0.1371 & -0.0278 & 0.2410 & \mathbf{1.0000} & 0.0401 & 0.2662 \\ 0.0714 & -0.0103 & 0.0426 & -0.0072 & 0.0573 & 0.0401 & \mathbf{1.0000} & 0.0794 \\ 0.2509 & -0.0468 & 0.1980 & -0.0048 & 0.6129 & 0.2662 & 0.0794 & \mathbf{1.0000} \end{pmatrix}$$

Once daily variance and expected return are estimated, they must be monthualized to best suit the monthly re-balance strategy. Since there are typically 22 trading days in any given month, the daily variance will be multiplied by 22 to scale it to match the holding period of 1 month until the portfolio is rebalanced. The monthly expected returns will be scaled by using the daily expected return by the following formula:

$$E[r]_{monthly} = \left[(E[r]_{daily} + 1)^{22} - 1 \right]$$

If, for example, our strategy involved a yearly re-balance, then the matrices would be multiplied by 252 to annualize them. There were also NaN rows within the returns array which are removed before any parameters are estimated. Both the Mean-Variance and CVaR optimization approach will be used to create the efficient frontiers. The out-of-sample test will run from 01/04/2013 to 06/01/2018 - and since the portfolio will be re-balanced and its value evaluated on a monthly bases - there will be 66 data points in total of monthly returns. With the set of monthly returns over the sampling period, the Sharpe ratio can be calculated directly. The assumed risk free rate used to calculate the Sharpe ratios will be $R_f = 0$. Since the monthly returns are used to calculate the Sharpe ratio, it makes sense to look at what the 1 month return is on a treasury bill. The 3 month Canadian treasury bill between January 2013 to October 2018 - the same period as the backtesting - on average offered a monthly return of 0.065%. Since it is not even a tenth of a percent, assuming $R_f = 0$ seems reasonable.

TABLE 1.1: Sharpe Results.

Portfolio	**Monthly Sharpe**	**Annualized Sharpe**
Black	0.435	1.506
Green	0.358	1.240

1.6 Conclusion

This paper studies the monthly returns of two separate CVaR optimal portfolios and compares their Sharpe Ratios after an investment period starting from January 2013 to July 2018. Observe that the annualized Sharpe ratio[5] is 21% higher in the case of the black portfolio. Given this information, investors will be more likely to invest in the black portfolio when

[5]Annualized Sharpe estimated by multiplying the monthly Sharpe ratio by $\sqrt{12}$, which is allowed when assuming that portfolio returns are a Wiener Process - in which volatility scales with the square-root of time (Lo, 2003).

comparing their risk-adjusted returns. Boulatoff and Boyer (2009) found that between 2003-2008, green firms underperformed and they still do post-recession which makes them less attractive as investments. Institutional investors are becoming more inclined to make socially responsible investments but do not wish to sacrifice returns in the process. To address this potential conflict, the implementation of a carbon tax in combination with carbon mitigation policy may provide a financial incentive to invest in firms with a relatively low carbon footprint.

In December 2016, the Government of Canada along with most provinces and territories have agreed to the Pan-Canadian Framework on Clean Growth and Climate Change. The key mission of the framework is to reduce greenhouse gas (GHG) emissions by putting a price on carbon pollution. In effect, firms that produce high amounts of GHG will pay more in taxes than a firm that produce lesser amounts of GHG, creating an incentive to reduce emissions. It is not yet concrete how the extra tax revenue will be distributed, but one possibility is to funnel the revenue into research and development of low carbon emitting technologies and support clean growth. None of the assets used in this paper are from Canadian stock exchanges, and therefore it is unlikely that the assets from the green portfolio used in the analysis benefited from the Pan-Canadian framework implemented in 2016.

A similar framework could be implemented across the U.S, as it is one of the few large industrialized nations that does not have a carbon tax. A simple solution could be to implement a federal carbon emissions tax. This would create a financial incentive to reduce carbon emissions while imposing a cost to firms that produce high levels of GHG.

1.7 References

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