Bayesian Optimisation of SLIP Model Parameters

Kaur Aare Saar, Dr Fumiya Iida (kas82@cam.ac.uk, fi224@cam.ac.uk)

Biologically Inspired Robotics Lab, Department of Engineering, University of Cambridge

Short Description

The Spring Loaded Inverted Pendulum (SLIP) gait model can be characterised with various parameters, including spring stiffness, mass of the robot, touchdown angle and leg length. Tuning the parameters can be time consuming and Bayesian Optimisation provides an efficient way for finding the optimal gait parameters.

List of scripts and functions

`BayesianOptimisationDemo1D.m` – Script for demonstration of Bayesian Optimisation in 1 dimension on a simple known function. The optimisation process is displayed in a figure.

`BO.m` – Main script to run Bayesian Optimisation of the SLIP model parameters. The parameters to optimise and the corresponding search ranges are defined in the script. The optimisation process and results are stored in *.mat files.

`SLIP_model.m` – Function that describes the dynamics of SLIP model. The arguments of the function are the parameters of the model. The function returns the motion data and the performance of the model.

`getNextSample.m` – Function that returns the next sampling point to maximise improvement of specified amount. The function argument is the Gaussian Processes Regression through existing sampling points.

`results.m` – The main function for displaying the optimisation results of any existing optimisation process. This includes a figure with colour map for parameters and a corresponding fitness function. A set of parameters and disturbance in initial conditions can be chosen to show animation and phase plot of the corresponding model.

`animation.m` – Function to animate the motion of SLIP model with specified parameters

`phaseplot.m` – Function to display phase plot of the SLIP model (ground height vs. velocity vector direction of the robot)

Bayesian Optimisation

Bayesian Optimisation is an optimisation method which minimises the number of sampling points required to find the maximum of the objective function. Gaussian process regression with the best-fitting Kernel parameters is found and fit through sampled points as described in [1]. For each point in the argument space the expected mean and standard error from the expected mean are found. The expected mean and error data is then used to obtain acquisition function which is then used to find the point where to sample next. [2]

There are different acquisition functions that can be used in the optimisation. One of the most common ones is the *improvement based acquisition function* with an *exploration-exploitation parameter*. This maximises the probability that the next sample is greater than the sum of the maximum sample that has already been found and a trade-off parameter. Normally the trade-off parameter is chosen to have greater values initially to support exploration (sampling the points in argument space with higher uncertainties) and to have smaller values in the later phases of optimisation to support exploitation (sampling the points in argument space close to already found maximum value). [2]

Figure 1 shows how the sampled data of a function can be modelled as Gaussian process and the calculated threshold. The plot below shows the probability that the value of the function is above the specified threshold. The next sampling point is chosen at the point where the probability is the highest. See more details of Bayesian optimisation in *Exercise 0*.

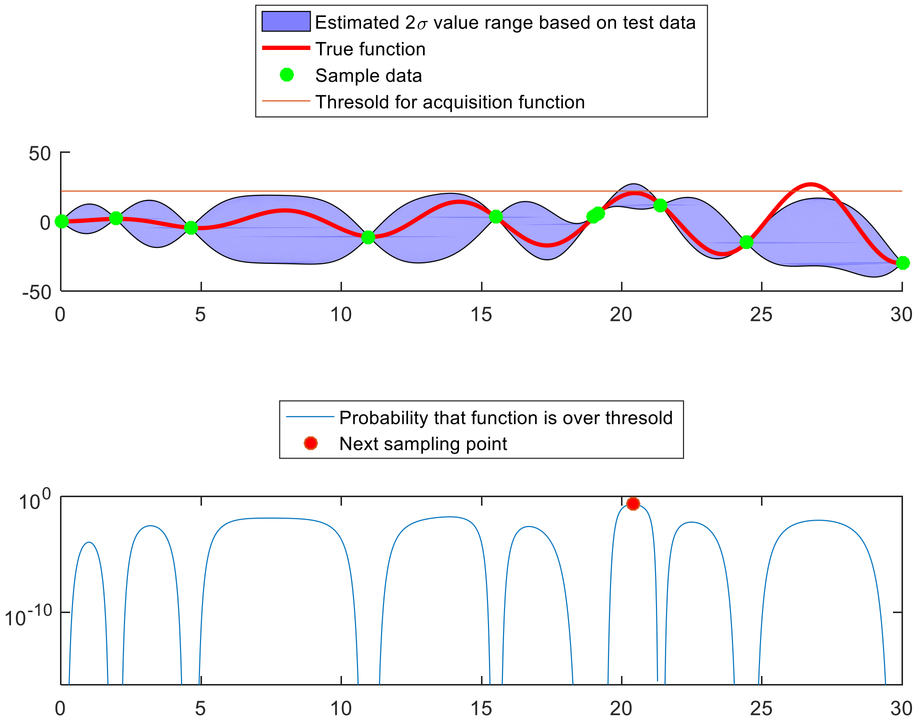

Figure 1. Bayesian Optimisation with the objective function to be maximised above and the corresponding acquisition function below

SLIP model

The spring Loaded Inverted Pendulum (SLIP) gait model is one of the simplest bipedal gait models. It consists of a point mass with mass m connected to two weightless springs with stiffness k and uncompressed length l_0. The point mass is constrained to move in the sagittal plane. The motion of model can be described with three phases, each characterised with a different system of differential equations: (a) double stance phase – both legs are in contact with ground, (b) single stance phase – one leg is in contact with ground, the other leg is in swing, and (c) flight phase – neither of the legs is in contact with ground and the point mass in free motion. In this model, the impacts are considered to be completely elastic and springs have no damping. Therefore, the model conserves energy throughout the motion.

Hooke's law and Newton's 3rd law yield to set of differential equations for each phase with first support point at $(0\,,0)$ (if applicable), second support point at $(x_0\,,0)$ (if applicable) and the point mass at $(x\,,y)$.

Flight phase:

$$\ddot{x} = 0$$

$$\ddot{y} = -g$$

Single stance phase:

$$\ddot{x} = x\frac{k}{m}\left(\frac{l_0}{\sqrt{x^2+y^2}} - 1\right)$$

$$\ddot{y} = y\frac{k}{m}\left(\frac{l_0}{\sqrt{x^2+y^2}} - 1\right) - g$$

Double stance phase

$$\ddot{x} = x\frac{k}{m}\left(\frac{l_0}{\sqrt{x^2+y^2}} - 1\right) + (x - x_0)\frac{k}{m}\left(\frac{l_0}{\sqrt{(x-x_0)^2+y^2}} - 1\right)$$

$$\ddot{y} = y\frac{k}{m}\left(\frac{l_0}{\sqrt{x^2+y^2}} - 1\right) + y\frac{k}{m}\left(\frac{l_0}{\sqrt{(x-x_0)^2+y^2}} - 1\right) - g$$

The transitions between phases are either landings or take-offs. Landing occurs when the height of the mass gets smaller than the height determined by touchdown angle and leg length. Take-off occurs when the distance between support and the mass gets larger than the uncompressed spring length.

Two simplest gait patterns that can be identified are the running and the walking gait, first described in [3]. In walking gait single stance phase alternates with double stance phase. In running gait single stance phase alters with flight phase. The values of touchdown angle, spring stiffness and the total internal energy that provide stable gait patterns were first described in [4]. In addition, there are different skipping gait patterns where walking and running alternate with each other. For humans, running and walking are most prevalent gait patterns but in certain environments skipping could be preferred as described in [5].

The provided MATLAB code permits changing between all three phases: double stance, single stance and flight. Therefore, the gait with optimised parameters could be either walking, running or skipping depending on the initial conditions provided by user. Different stable limit cycles for running, walking and skipping were recorded and these can be found in `limitcycles` folder along with the respective phase plots and animations.

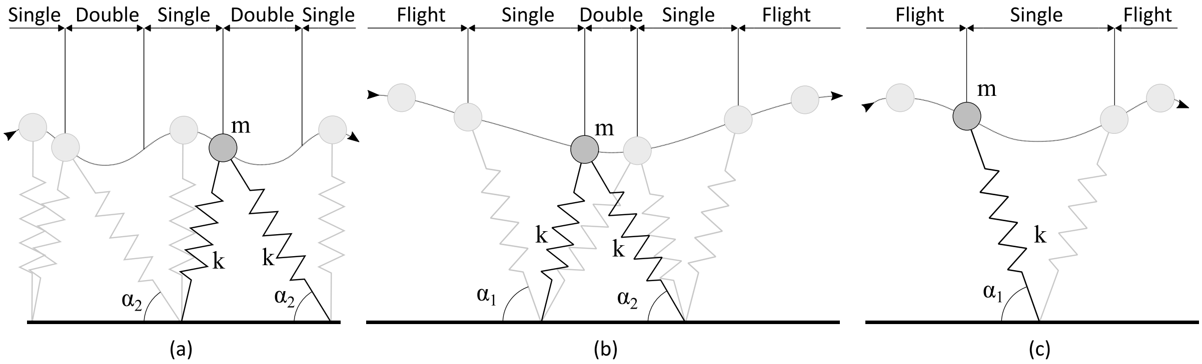

Figure 2. SLIP model gait patterns with transitions between phases. (a) walking, (b) skipping, (c) running.

The parameters that are varied during optimisation are the spring stiffness, touchdown angle while walking and touchdown angle when running. Leg length and the mass of the robot are considered to be constant due to dimensional analysis.

During optimisation processes the performance of the model with specified parameters is assessed. The best characterisation of the performance was concluded to be the distance covered before losing stability, however different fitness functions could be used (e.g. number of steps or time before losing stability). The chosen fitness function is evaluated for the parameter set under evaluation and with pre-defined initial conditions subject to disturbances.

Analysing the results

Three pre-run optimisation processes with different initial conditions are provided with the code in `data/BO_Data_xxx.mat` files. These results can be reviewed using `results(DatasetNumber)` function. Use `DatasetNumber=1` for walking gait model, `DatasetNumber=2` for skipping gait model and `DatasetNumber=3` for running gait model.

The figure created when using the results function displays three plots. The first plot shows the progress of parameter values with optimisation iteration. As exploitation starts the values of parameters are not exposed to large changes in the values. The second plot shows the fitness function evolution with optimisation iteration. The third plot allows user to choose a parameter set and disturbance in initial conditions for animation and displaying phase plot to visualise the performance.

Exercises

Some sample exercises are provided to familiarise the user with the code. For all exercises open the main function directory in MATLAB. The code was created using MATLAB 2016a, including Statistics Toolbox and Neural Network Toolbox. The minimum requirement for using Gaussian process regression fitting function `fitrgp` present in the code is MATLAB 2015b.

Exercise 0 – Simple Bayesian Optimisation in 1D

Run `BayesianOptimisationDemo1D.m` script. Wait until a figure with two plots appears. Click on the figure to proceed to next iteration of Bayesian iteration. See how the optimisation successfully finds the function maximum. To stop the optimisation, close the figure window or press Ctrl-C in MATLAB command line.

Try to change the objective function, noise level, kernel type and exploration/exploitation coefficient (all defined in the script initialisation) to see how it affects the performance of optimisation.

Exercise 1 – Results of Bayesian Optimisation SLIP model

Run `results(1)` from MATLAB command window to load an existing Bayesian Optimisation of walking gait model. Wait until figure with three plots appears. Click on the bottom plot to choose parameters for optimisation and wait for animation to start in a new figure. x-axis corresponds to respective parameter set. The locomotion is more likely to fail if a parameter set with low performance (see 2nd plot) is chosen or if the magnitude of the applied disturbance is higher.

Run `results(2)` and `results(3)` to see results for skipping and running gait model respectively.

Exercise 2 – Running Bayesian Optimisation of SLIP model

Run script `BO` to iterate Bayesian Optimisation of SLIP model. During each iteration the parameters which give the highest improvement are chosen to be sampled next. The parameters and respective performance are displayed in the command window. The results are saved in the `data` directory as `BO_Data_xxx.mat` where *xxx* is the dataset number and can be reviewed later using the `results` function as shown in the previous exercise.

Try to alter the `InitialConditions` variable and run `BO` to see which initial conditions yield a stable gait and what the gait looks like. In general, $x = dy/dt = 0$ as initial conditions are defined at the gait apex.

Experiment with the `searchrange` matrix to change the search range for parameters to optimise. If the lower bound and the upper bound are equal the variable is constrained. Wider ranges may cause unnecessary exploration in the early stages of optimisation and thus make the process slower to converge, but with too narrow a range there is a risk that the optimum parameters are not in the range. In addition, there are also physical constraints: the spring constant has to be positive and touchdown angle α must satisfy $0° < \alpha < 90°$.

Try to alter the parameters in `BO` script section *Performance assessment and optimisation parameters* to see how it affects the optimisation process. E.g. change the amount of disturbance added to initial conditions. If there is less disturbance added, the number of tests required could be less as the outcome of the tests is likely to be more consistent.

Summary

Bayesian Optimisation provides an efficient way for tuning parameters of the SLIP model. The provided MATLAB code could tune the gait model parameters for different initial conditions and hence produce different stable motions, including both walking and running gaits.

Acknowledgements

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References

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